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Exploitation, Skills, and Inequality

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Abstract

This paper uses a computational framework to analyse the equilibrium dynamics of exploitation and inequality in accumulation economies with heterogeneous labour. A novel index is presented which measures the intensity of exploitation at the individual level and the dynamics of the distribution of exploitation intensity is analysed. Various taxation schemes are analysed which may reduce exploitation or inequalities in income and wealth. It is shown that relatively small taxation rates may have significant cumulative effects on wealth and income inequalities. Further, taxation schemes that eliminate exploitation also reduce disparities in income and wealth but in the presence of heterogeneous skills, do not necessarily eliminate them. The inegalitarian effects of different abilities need to be tackled with a progressive education policy that compensates for unfavourable circumstances.

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1 Introduction

The common view—at least in economics, both in the mainstream but also for most heterodox scholars—is that the concept of exploitation cannot be defined coherently because of the logical flaws in the labour theory of value. Moreover, the notion of exploitation is considered to be metaphysical and obscure especially outside of economies with the simplest assumptions on preferences, technology and behaviour.

In particular, heterogeneous skills are usually deemed to pose insurmountable problems for the concept of exploitation. At the most general level, Marxian exploitation identifies a discrepancy between the labour 'given' by agents, in some relevant sense, and the labour 'received' by them, in some relevant sense. In simple economies with homogeneous labour, the agents' exploitation status is measured focusing on labour *time*. If individuals possess different skills, however, how should the amounts of labour given and received by them be measured? In units of labour time, or rather in terms of *effective*—or *skill-adjusted*—labour?

According to Roemer [17, 19], exploitation should be measured in units of effective labour but the concept of exploitation thus defined is not normatively meaningful, and the elimination of capitalist exploitation does not necessarily lead to a just society. In fact, an exploitation-free allocation requires income to be allocated in proportion to labour contributed and, in the presence of heterogeneous skills, this implies an unequal income distribution—a phenomenon that Roemer has dubbed 'socialist' exploitation. Actually, using a simple model of the U.S. economy, Roemer [19] has shown that, rather surprisingly, the elimination of exploitation would lead to *higher* income inequality than was actually experienced in the United States. This is an unpalatable conclusion for socialists and egalitarians, especially if skills are inherited and not acquired.

In this paper, we analyse the concept of exploitation in economies with heterogeneous agents and skills. Following Roemer ([19], p.16), we assume that "Agents are endowed with skill levels s, distributed according to a distribution function F on $\mathbb{R}_+$. If an agent with skill level s works for L time units, then she produces sL units of labour, measured in efficiency units". We thus consider economies with heterogeneous *skills*, but not heterogeneous *labour inputs*: agents perform the same tasks in production but some are more productive than others.[1]

To be sure, the use of heterogeneous labour inputs in production raises important issues in exploitation theory. It is unclear, for example, how labour performed by agents with different abilities in production can be made uniform.[2] This is the so-called 'problem of the reduction of complex labour to simple labour' and it raises

[1]Thus, for example, we cannot distinguish between agents with an unusual aptitude for mathematics and those with an unusual aptitude in basketball.

[2]We use here the term 'ability' to denote the different types of productive labour that agents can undertake (e.g. engineers, manual workers, IT experts, and so on), while we use 'skill' to denote the individuals' capacity to translate the same type of labour input into effective units of labour. We are grateful to an anonymous referee for suggesting this distinction.

some relevant conceptual and formal problems in exploitation theory. We do not address these issues in this paper.

We focus on economies with heterogeneous skills for theoretical reasons. First of all, this choice allows us to compare our results directly with Roemer [19], and more generally with several other of Roemer's models addressing exploitation theory (see, for example, Roemer [17], section 6.1; [17], Chapter 9 and section 10.3) and the related notion of 'proportional solution' (Moulin and Roemer [13]; Roemer and Silvestre [21]; Roemer [20]).

Second, our choice clearly defines the scope of our analysis thus enhancing conceptual clarity and precision. We thus reach possibly more limited but clearer and stronger conclusions, focusing on the normative and positive issues that skill inheritance, creation, and acquisition raise in exploitation theory. As noted earlier, skill heterogeneity is widely considered to pose major problems for the concept of exploitation and therefore a satisfactory solution to these problems is arguably a necessary first step in the construction of a general theory of exploitation.

Third, the focus of this paper is primarily normative, and we interpret the concept of exploitation as a measure of the injustices that may characterise advanced economies. From this perspective, the analysis of heterogeneous skills, rather than differential labour inputs, allows us to focus sharply on the core distinction between innate or inherited skills, and skills that are instead the product of training and education, consistent with the modern theory of equality of opportunity and with the Kantian approach recently developed by Roemer [20]. Nonetheless, preliminary work suggests that many of our key insights can be generalised to economies with heterogeneous labour inputs (see our companion paper, Yoshihara and Veneziani [28]).

In economies with heterogeneous agents and differential skills, we provide a notion of exploitation that is logically coherent, well-defined, and firmly anchored to empirical data. Indeed, we show that exploitation can be defined both at the aggregate and at the individual level by means of an *exploitation index* which measures an agent's effective labour per unit of income received. For each individual, this index can be measured based on available empirical data, and its distribution can be analysed with the standard tools of the theory of inequality measurement.

Further, contrary to Roemer [17, 18, 19], we show that the notion of exploitation is normatively relevant, and the analysis of the distribution of the exploitation index yields distinct insights on the injustices that characterise advanced economies and on the effects of redistributive policies. On the one hand, we argue that Roemer's [19] negative conclusions on the inequalities persisting in the socialist allocation critically depend on his specific modelling framework, including his assumptions on preferences, technology and—crucially—the distribution of skills. In his analysis, Roemer assumes that the US labour market is perfectly competitive and high salaries reflect high skills. This is both theoretically and empirically doubtful. On the other hand, as Roemer ([19], p.24) himself notes, even granting that "the socialist allocation, given the distribution of skills in the United States today, would bring with it a relatively high degree of income inequality, ... [one may object that] under socialism, that distribution of skills would change". Yet, he focuses on static, one period economies which cannot address the issue of evolution of the distribution of skills, income, wealth, and exploitation.

In this paper, we analyse a dynamic generalisation of Roemer's [17] accumulating economy with heterogeneous maximising agents. We assume that initial aggregate capital mimics the empirical wealth distribution for the U.S. and calibrate the distribution of skills in relation to wealth, such that the initial distribution of income is close to the empirical distribution of income for the U.S. Given the complexity of the model, we analyse the dynamics of the economy computationally, which allows us to derive definite conclusions on the distributive variables. The simulations confirm that indeed exploitation, income inequality and wealth inequality provide rather different normative insights, and socialists and egalitarians may face trade-offs when implementing various policies.

Nonetheless, with a more realistic distribution of skills, Roemer's [19] negative conclusions are significantly qualified. Whether exploitation disappears due to overaccumulation leading to the disappearance of profits, or by means of wealth taxation, income and wealth inequalities in the socialist allocation are nowhere close to the values in Roemer [19]. The *static* trade-offs are much less severe than suggested by Roemer [19]. Furthermore, if a fraction of the revenues from wealth taxation are devoted to education and the growth of skills, it can be shown that, *dynamically*, the trade-off becomes less severe over time and can be led to vanish in the long run. Socialists and egalitarians may not face a major conundrum after all.

Another contribution of the paper is methodological. Our analysis shows that computational methods can yield relevant insights in Marxian economics, and in social economics more generally. Computational techniques can be extremely useful as a device to generate thought experiments and to address some issues that cannot be easily tackled analytically. Given the complexity of our models, for example, they allow us to derive clear conclusions on our definition of exploitation, on the distribution of the exploitation index and on the dynamics of inequalities and exploitation. Pioneering work applying computational methods to Marxian theory includes Wright [31, 32, 33], Cogliano [3], and Cogliano and Jiang [4], though they focus on price and value theory and the circuit of capital rather than exploitation and class. More related to our work is a recent article by Cogliano et al. [5], which focuses on the mechanisms guaranteeing the persistence of exploitation in competitive economies with homogeneous labour. Our analysis here is more general as it includes heterogeneous skills and alternative taxation schemes.

2 The framework

In this section, and in the next, we set out the basic framework and the main definitions focusing on an economy with stationary population, technology, preferences, consumption norms, and labour endowments, and without taxes or an educational sector—the *basic economy*. This is for analytical clarity, as the basic economy provides a theoretical benchmark and starting point for our analysis. However, the framework, concepts, and definitions can be easily extended and the results derived continue to hold in more general economies.

Consider a dynamic extension of Roemer's [17] accumulating economy with a labour market and only one good produced and consumed.[3] In every period $t = 1, 2, \ldots$, there is a set $\mathcal{N} = \{1, ..., N\}$ of agents in the economy where ν denotes a generic agent. At the beginning of each t, every agent can produce by activating a Leontief production technique (A, L), where A is the amount of the produced input necessary to produce one unit of output and L is the amount of effective (or skill-adjusted) labour necessary to produce one unit of output. We assume that the economy can produce a surplus $(0 < A < 1)$ and labour is indispensable $(L > 0)$.

In every t, agents are characterised by their endowment of labour time $\zeta^\nu > 0$, a skill factor $s^\nu > 0$, and capital endowment $\omega^\nu_{t-1} \geqq 0$. Agents are endowed with the same amount of labour time which is normalised to one: $\zeta^\nu = 1$ for all $\nu \in \mathcal{N}$. The skill factor s^ν of any agent modifies their labour endowment so that the endowment of *effective labour* of any agent ν is $l^\nu \equiv s^\nu \zeta^\nu = s^\nu$. The distribution of agents' effective labour and wealth endowments at the beginning of t are given by $\Pi = (l^\nu)_{\nu\in\mathcal{N}}$ and $\Omega_{t-1} = \left(\omega^\nu_{t-1}\right)_{\nu\in\mathcal{N}}$, respectively. An agent $\nu \in \mathcal{N}$ endowed with $\left(l^\nu, \omega^\nu_{t-1}\right)$ can engage in three types of production activity: she can sell a quantity z^ν_t of her labour power; she can hire others to operate a technique (A, L) at the level y^ν_t; or she can work on her own to operate (A, L) at the level x^ν_t. Total effective labour performed by agent ν at t comprises both self-employed labour and labour sold on the market, and is denoted by $\Lambda^\nu_t \equiv Lx^\nu_t + z^\nu_t$.

Following Roemer [16, 17], we assume that production takes time and current choices are constrained by past events. To be precise, wages are paid ex post and $w_t \geqq 0$ denotes the nominal wage rate at the end of t, but every agent must be able to lay out in advance the operating costs for the activities she chooses to operate using her wealth W^ν_{t-1}. Letting $p_t \geqq 0$ denote the price of the produced commodity at the end of t and beginning of $t+1$, the market value of agent ν's endowment—her wealth—is $W^\nu_{t-1} \equiv p_{t-1}\omega^\nu_{t-1}$. The wealth that is not used for production activities can be invested to purchase goods to sell at the end of the period, δ^ν_t.

Our main behavioural assumption postulates that agents wish to maximise their wealth,[4] subject to consuming a strictly positive amount b of the consumption good per unit of effective labour performed, where b identifies a socially-determined basic consumption standard incorporating social norms, culture, and so on. The subsistence constraint introduces a positive lower bound on the real wage and it generalises

[3] Given our focus on the dynamics of exploitation, the one-good assumption yields no loss of generality. The model can be extended to include n commodities, albeit at the cost of a significant increase in technicalities and computational intensity.

[4] "Accumulate, accumulate! That is Moses and the prophets!" (Marx [11], p. 742).

Roemer's accumulation economies, which can be seen as a special case of our model when $b = 0$. Further, this assumption introduces a certain degree of heterogeneity in consumption, despite the fact that agents have identical preferences, because it implies that agents with higher skills end up consuming more, per unit of time, than less skilled ones. This seems reasonable because in a competitive economy, high-skilled agents receive a higher salary per unit of *time* which reflects their productive contribution.[5] It is important to note, however, that our assumptions on consumption are made mostly for technical convenience and with no significant loss of generality (see section 10.5 below).

Formally, in every t, given prices (p_{t-1}, p_t, w_t), every agent $\nu \in \mathcal{N}$ chooses $\xi_t^\nu \equiv (x_t^\nu; y_t^\nu; z_t^\nu; \delta_t^\nu)$ to maximise her wealth subject to purchasing b per unit of effective labour performed (1) and to the constraints set by her capital (2) and effective labour capacity (3). Formally, every ν solves the following programme MP_t^ν:

$$\max_{\xi_t^\nu \in \mathbb{R}_+^4} W_t^\nu = p_t \omega_t^\nu$$

subject to

$$\begin{aligned} p_t x_t^\nu + [p_t - w_t L]\, y_t^\nu + w_t z_t^\nu + p_t \delta_t^\nu &= p_t b \Lambda_t^\nu + p_t \omega_t^\nu && (1)\\ p_{t-1} A x_t^\nu + p_{t-1} A y_t^\nu + p_{t-1} \delta_t^\nu &= p_{t-1} \omega_{t-1}^\nu, && (2)\\ L x_t^\nu + z_t^\nu &\leqq l^\nu \equiv s^\nu. && (3) \end{aligned}$$

Let $\mathcal{A}^\nu(p_{t-1}, p_t, w_t)$ be the set of actions ξ_t^ν that solve MP_t^ν at prices (p_{t-1}, p_t, w_t). Let $(p, w) \equiv \{(p_t, w_t)\}_{t=1,\ldots}$ and let $(x^\nu; y^\nu; z^\nu; \delta^\nu) \equiv \xi^\nu = \{\xi_t^\nu\}_{t=1,\ldots}$. A *basic accumulation economy* is defined by agents $\mathcal{N}$, technology (A, L), effective labour endowments Π, and initial capital endowments Ω_0; and is denoted as $E(\mathcal{N}; (A, L); b; \Pi, \Omega_0)$, or, as a shorthand notation, E_0. We suppose that the economy can produce a surplus: $(1 - bL) > A$ or, equivalently, $1 - vb > 0$, where $v = L(1 - A)^{-1}$ denotes the embodied labour value.

Let $x_t \equiv \sum_{\nu \in \mathcal{N}} x_t^\nu$, and likewise for y_t, z_t, δ_t, ω_t, c_t, Λ_t, and l. Based on Roemer [17], the equilibrium notion can be defined.

Definition 1. *A reproducible solution (RS) for $E(\mathcal{N}; (A, L); b; \Pi, \Omega_0)$ is a vector (p, w) and associated actions $(\xi^\nu)_{\nu \in \mathcal{N}}$, such that at all t:*

(a) $\xi_t^\nu \in \mathcal{A}^\nu(p_{t-1}, p_t, w_t)$, for all $\nu \in \mathcal{N}$ (individual optimality);

(b) $A(x_t + y_t) + \delta_t \leqq \omega_{t-1}$ (capital market);

(c) $Ly_t = z_t$ (labour market);

(d) $(x_t + y_t) + \delta_t \geqq b\Lambda_t^\nu + \omega_t$ (goods market).

[5] Of course, workers with very low skill levels may end up consuming very little. This may be unrealistic but it is just an artefact of our normalisation of skill levels and labour endowments, and the model could be modified to assume $s^\nu \geq 1$ for all $\nu \in \mathcal{N}$.

At a RS, in every period: (a) all agents optimise; (b) aggregate capital is sufficient for production plans; (c) the labour market clears; (d) aggregate supply is sufficient for consumption and accumulation plans. E_0 can be interpreted either as a sequence of generations living for one period or as an infinitely-lived economy analysed in a sequence of temporary equilibria.

It may be argued that the concept of RS imposes stringent requirements on agents' rationality and expectation formation. For, agents trade in the goods and labour market at the beginning of each period based on expectations of prices that will form at the end of the period, and in equilibrium these expectations are exactly correct. Two points should be made here to motivate the focus on RS's, and our behavioural assumptions. First, formally, because we consider one-good accumulation economies with myopically optimising agents, it imposes much less stringent rationality and consistency requirements than standard macroeconomic models with agents maximising an intertemporal utility function. Indeed, as shown below, provided agents correctly expect a non-negative profit rate and a real wage above the minimum standard at the end of the period, their choices will be optimal even if their expectations turn out not to be perfectly accurate. Second, theoretically, our purpose is to analyse the dynamic equilibrium trajectories of Marxian economies and their exploitation structures as defined by Roemer [16, 17]. It is therefore appropriate, at least as a first step, to adopt a theoretical framework as close as possible to Roemer's, including—crucially—his Marxian equilibrium concept.

For any (p, w), the profit rate at t is $\pi_t = \frac{p_t - p_{t-1}A - w_t L}{p_{t-1}A}$. Given the structure of the economy, we shall focus on equilibria with strictly positive prices, so that the profit rate is well defined at all t.[6] By constraints (1) and (2), it immediately follows that at any RS, only (p_t, w_t) matter for individual choices at all t and so we can take the produced commodity as the numéraire, setting $p_t = 1$, all t.[7] Let the normalised price vector be denoted as $(\mathbf{1}, \widehat{w})$, where $\mathbf{1} = (1, 1, ...)$ and, at any t, $\widehat{w}_t$ is the real wage rate and $\pi_t = \frac{1-A-\widehat{w}_t L}{A}$. In what follows, with a slight abuse in notation, in the analysis of individual choices at t, we shall simply refer to the price vector $(1, \widehat{w}_t)$.

Given the previous observations and constraints (1)-(2), it follows that at any RS, for all $\nu \in \mathcal{N}$ and all t, the following equation must hold

$$\omega_t^\nu = [1 - A - \widehat{w}_t L] (x_t^\nu + y_t^\nu) + (\widehat{w}_t - b)(L x_t^\nu + z_t^\nu) + \omega_{t-1}^\nu. \tag{4}$$

Equation (4) has a number of implications.[8] First, it is immediate to prove from

[6] It immediately follows from MP_t^ν that if there is some t' such that $p_{t'} = 0$, then at any RS it must be $p_t = 0$ for all $t > t'$.

[7] Differences in beginning-of-period prices, p_{t-1}, and end-of-period prices, p_t, are inconsequential for agents' choices. At the beginning of t, given p_{t-1} and the expected (p_t, w_t), for every unit of wealth stored to be sold at the end of t one foregoes A^{-1} units of output produced at the end of t. Therefore one invests productively (rather than storing the good) provided $(p_t - w_t L) A^{-1} \geqq p_t$: beginning of period prices do not enter the decision.

[8] The proofs of all of the following claims and of Theorems 1 and 2 below are straightforward extensions of the proofs in Cogliano et al. [5] and are therefore omitted.

$(1-bL) > A$ that at any RS, if $\omega_{t-1} > 0$, then $\widehat{w}_t \geqq b$ and $\pi_t \geqq 0$, all t. Second, at any t, if the profit rate is strictly positive, then all wealth is used productively, and if the wage rate is above the minimum standard b, then the labour constraint (3) binds, for all agents at the solution to MP_t^ν. Formally: at any t, if $\pi_t > 0$, then $A(x_t^\nu + y_t^\nu) = \omega_{t-1}^\nu$, all $\nu \in \mathcal{N}$; and if $\widehat{w}_t > b$, then $Lx_t^\nu + z_t^\nu = l^\nu$, all $\nu \in \mathcal{N}$. Third, in equilibrium, at any t, the maximum wealth accumulated by any agent is

$$\omega_t^{*\nu} = (1+\pi_t)\,\omega_{t-1}^\nu + (\widehat{w}_t - b)\,l^\nu,$$

and therefore the growth rate of capital for each agent is

$$g_t^\nu = \pi_t + (\widehat{w}_t - b)\,\frac{l^\nu}{\omega_{t-1}^\nu},$$

while the aggregate growth rate of the economy is $g_t = \pi_t + (\widehat{w}_t - b)\,\frac{l}{\omega_{t-1}}$.

We conclude the analysis of the basic economy by characterising its equilibria.

Theorem 1. *Let $\left((\mathbf{1}, \widehat{w}), (\xi^\nu)_{\nu\in\mathcal{N}}\right)$ be a RS for E_0. At any t:*

(i) If $\pi_t > 0$ and $\widehat{w}_t > b$, then $l = LA^{-1}\omega_{t-1}$;

(ii) If $l > LA^{-1}\omega_{t-1} > 0$ then $\widehat{w}_t = b$;

(iii) If $l < LA^{-1}\omega_{t-1}$ then $\pi_t = 0$.

Theorem 1 defines the theoretical framework for the analysis of the dynamics of the economy. Although it only identifies necessary conditions for the existence of a RS, it does shed some light on how to construct the dynamic general equilibria. Consider part (ii). Suppose $l > LA^{-1}\omega_{t-1}$, some t. If $\widehat{w}_t = b$, then $\pi_t = \pi^{\max} \equiv \frac{1-A-bL}{A} > 0$ and labour performed does not produce any net income for accumulation, and for all $\nu \in \mathcal{N}$, any $(0; y_t^\nu; z_t^\nu; 0)$ with $Ay_t^\nu = \omega_{t-1}^\nu$ solves MP_t^ν. Therefore since $Ay_t = \omega_{t-1}$ and $l > LA^{-1}\omega_{t-1}$, we can choose a suitable profile $(z_t^\nu)_{\nu\in\mathcal{N}}$ such that $Ly_t = z_t$ and all conditions of Definition 1 are satisfied at t.

Consider part (iii). Suppose $l < LA^{-1}\omega_{t-1}$, some t. If $\pi_t = 0$, then $\widehat{w}_t = \frac{1}{v} > b$ and capital holders are indifferent between using their wealth productively and just carrying it for sale at the end of the period, and for all $\nu \in \mathcal{N}$, any $(0; y_t^\nu; z_t^\nu; \delta_t^\nu)$ with $z_t^\nu = l^\nu$ solves MP_t^ν. Therefore since $z_t = l$ and $l < LA^{-1}\omega_{t-1}$, we can choose a suitable profile $(y_t^\nu)_{\nu\in\mathcal{N}}$ such that $Ly_t = z_t$ and all conditions of Definition 1 are satisfied at t.

3 Exploitation

The concept of exploitation can now be introduced. In what follows, exploitation status is defined in every period t: this is a natural assumption if the model describes a series of one-period economies, otherwise it reflects a focus on *within period* exploitation.[9] Definition 2 identifies exploitation status in terms of the bundles of goods that an agent *can* purchase with her income. More precisely, following Veneziani and Yoshihara [29], at any RS (p, w) and for all $\nu \in \mathcal{N}$, let c_t^ν satisfy $p_t c_t^\nu = p_t \omega_t^\nu + p_t b \Lambda_t^\nu - p_t \omega_{t-1}^\nu$ for every t. Then, Definition 2 is an extension of Roemer [17] to economies with heterogeneous labour and $b > 0$.

Definition 2. *Agent ν is exploited at t if and only if $\Lambda_t^\nu > vc_t^\nu$; she is an exploiter if and only if $\Lambda_t^\nu < vc_t^\nu$; and she is neither exploited nor an exploiter if and only if $\Lambda_t^\nu = vc_t^\nu$.*

Definition 2 identifies exploitation status focusing on *effective* labour. According to Definition 2, the concept of exploitation measures discrepancies in the amount of labour that agents *contribute* to the economy and the amount that they receive, via their income.

To be sure, it may be argued that the rationale behind a focus on labour flows is unclear. In the accumulation economy, wealthy capitalists are symmetric to high-skilled workers: both potentially enjoy premiums by virtue of their superior endowments. The productive contribution of high-skill workers is similar to the productive contribution of high-wealth capitalists: the contributions of both hinge on endowment differentials that they did not previously work to create, and both must forego a current benefit (current consumption of physical goods for the capitalist, current consumption of leisure for the high-skill worker) to make this contribution.[10]

In this paper we do not aim to provide a thorough defence of the normative foundations of the concept of exploitation (for a discussion, see Veneziani and Yoshihara [30]). Nonetheless, a few points are worth making to justify our focus on Definition 2. First, although wealthy capitalists are symmetric to high-skilled workers in that they both contribute some productive factors, they are different in that one class's endowment is alienable while the other's is inalienable. In exploitation theory, this asymmetry carries significant normative weight in that skills and labour belong to agents and define their identity in ways that assets do not.

Second, Definition 2 incorporates an important normative intuition, which may be called the '*contribution view*': an efficient and exploitation-free allocation coincides with the *proportional solution*, a well-known fair allocation rule whereby every agent's income is proportional to her labour contribution to the economy (Roemer and Silvestre [21]). Proportionality is a widely held normative principle, whose philosophical foundations can be traced back to Aristotle, and it can be justified in terms of the Kantian categorical imperative (Roemer [20]). The contribution principle ('To each according to his contribution') is also one of the principles of justice analysed by Marx in the *Critique of the Gotha programme* [10]. Although Marx [10] considers

[9] For a discussion of *within period* and *whole life* exploitation, see Veneziani [25, 26].

[10] We are grateful to any anonymous referee for raising this issue.

it as bourgeois, advocating instead the need principle ("from each according to his ability, to each according to his need"), he does argue that the contribution principle is appropriate for a socialist society during the transition to a fully just society (for a discussion see Cohen [6]). As Roemer ([19], p.14) notes, "Socialism, as defined by Marx, was an economic system in which capitalist exploitation had been eliminated. This means that the distribution of society's output to its producers was in proportion to the value of labour they expended in its production".

Finally, as argued by Veneziani and Yoshihara [28], Definition 2 is the natural extension of all of the classic definitions of exploitation, and it is the approach adopted in much of the literature on exploitation in economies with heterogeneous skills.[11] An economy with no exploitation, "a socialist economy, is one in which output received by each individual is proportional to the efficiency units of labour that she expends in production" (Roemer [19], p.15).

Theorem 2 characterises the exploitation status of every agent, based on their wealth per unit of labour performed $\frac{\omega_{t-1}^{\nu}}{\Lambda_t^{\nu}}$:

Theorem 2. *Let* $\left((\mathbf{1}, \widehat{w}), (\xi^{\nu})_{\nu \in \mathcal{N}}\right)$ *be a RS for* E_0*. At any* t*, if* $\pi_t > 0$*:*
(i) agent ν *is an exploiter* $\Leftrightarrow \frac{\omega_{t-1}^{\nu}}{\Lambda_t^{\nu}} > \frac{1}{\pi_t} \frac{[1-\widehat{w}_t v]}{v}$*;*
(ii) agent ν *is exploited* $\Leftrightarrow \frac{\omega_{t-1}^{\nu}}{\Lambda_t^{\nu}} < \frac{1}{\pi_t} \frac{[1-\widehat{w}_t v]}{v}$*;*
(iii) agent ν *is neither exploited nor an exploiter* $\Leftrightarrow \frac{\omega_{t-1}^{\nu}}{\Lambda_t^{\nu}} = \frac{1}{\pi_t} \frac{[1-\widehat{w}_t v]}{v}$*.*

Theorem 2 generalises analogous results by Roemer [17] as it allows for unemployed labour. If $\Lambda_t^{\nu} = l^{\nu}$, all $\nu \in \mathcal{N}$, then by Theorem 2 exploitation status is determined by the ratio of capital and labour *endowments* as in Roemer [17]. If the economy is characterised by unemployed labour, however, $\Lambda_t^{\nu} < l^{\nu}$ for at least some $\nu \in \mathcal{N}$ and exploitation status is determined by the ratio of the capital endowment *and labour performed*, $\frac{\omega_{t-1}^{\nu}}{\Lambda_t^{\nu}}$.

Theorem 2 holds if $\pi_t > 0$. If $\pi_t = 0$ then $\widehat{w}_t = (1/v) > b$ and $\Lambda_t^{\nu} = vc_t^{\nu}$ for all $\nu \in \mathcal{N}$, and no exploitation exists in the economy according to Definition 2. This correspondence between profits and exploitation is a standard result in Marxian theory (for a discussion, see Veneziani and Yoshihara [27]).

Theorem 2 provides important normative insights on certain structural injustices that emerge in capitalist economies. Yet, an exclusive focus on the sets of exploiters and exploited agents yields a rather partial, coarse picture of the structure of exploitative relations: two economies with similar numbers of agents belonging to each set may still be very different. Based on Definition 2, it is possible to extend the normative reach of the concept of exploitation and provide a finer and more comprehensive picture of exploitative relations. For Definition 2 allows us to move beyond

[11] Indeed, based on the axiomatic analysis developed by Yoshihara [34] and Veneziani and Yoshihara [27, 29], it follows that Definition 2 is the only definition of exploitation that all of the classic approaches would support in the context of our one-good accumulation economy.

a purely aggregate analysis and explore the exploitation status of every agent. This immediately raises the issue of the measurement of the *intensity of exploitation*, both at the individual and at the aggregate level. It is certainly desirable to have a notion of exploitation that allows us to make statements such as "agent A is less exploited than agent B", or "Economy C is more exploitative than economy D", or "Economy E is becoming increasingly exploitative over time".

Based on Definition 2, we examine an index that measures exploitation intensity for each individual:

$$\varepsilon_t^\nu = \frac{\Lambda_t^\nu}{vc_t^\nu},$$

An analysis of exploitation status at the individual level raises a number of novel, interesting issues in exploitation theory both at the theoretical and at the empirical level, and it highlights some important formal and conceptual similarities between exploitation theory and the theory of inequality measurement. According to the exploitation intensity index ε_t^ν, an agent ν is exploited if and only if $\varepsilon_t^\nu > 1$, whereas she is an exploiter if and only if $\varepsilon_t^\nu < 1$. Yet, the index provides a much finer and more nuanced description of exploitative relations. For each individual, the exploitation index is a well defined magnitude based on available empirical data and the distribution of the exploitation indices can be analysed with the standard tools of the theory of inequality measurement. Just like for income inequalities, one can analyse differences in exploitation intensity across countries, or the evolution of exploitation intensity within a given country over a certain period of time by focusing on the distribution of ε_t^ν. What is the appropriate way of capturing the key characteristics of $(\varepsilon_t^\nu)_{\nu\in\mathcal{N}}$? In this paper, we focus on the Gini coefficient of $(\varepsilon_t^\nu)_{\nu\in\mathcal{N}}$, denoted as γ_t^ε, but alternative measures can be used. We return to this issue in the concluding section.

4 The benchmark simulation routine

This section presents the benchmark routine used for all simulations in the paper, unless otherwise stated.[12] All simulations run for $T = 50$ periods. In each period t, the subsistence level b_t serves as a lower limit for the wage $\widehat{w}_t$, with $\widehat{w}_t = b_t$ for any t in which the economy is capital constrained, $\widehat{w}_t = 1/v_t$ for any t in which the economy is labour constrained, and $b_t \leqq \widehat{w}_t \leqq 1/v_t$ for any t in which the economy is on the knife-edge.

Lemma 3 of Cogliano et al. [5] proves that if $(x_t^\nu; y_t^\nu; z_t^\nu; \delta_t^\nu)$ solves MP_t^ν, then there is another vector $(0; y_t'^\nu; z_t'^\nu; \delta_t^\nu)$ which solves MP_t^ν. In the simulations, this allows us to select one of the many solutions of MP_t^ν by setting $x_t^\nu = 0$ for all $\nu \in \mathcal{N}$. Specifically, at any t, we set $\xi_t^\nu = \left(0; A_t^{-1}\omega_{t-1}^\nu; \frac{L_t A_t^{-1}\omega_{t-1}}{l} l^\nu; 0\right)$, $\xi_t^\nu =$

[12] All simulations are done using *Mathematica* version 11 and the code is available from the authors upon request.

$\left(0; \frac{l}{L_t A_t^{-1}\omega_{t-1}} A_t^{-1}\omega_{t-1}^{\nu}; l^{\nu}; \left(1 - \frac{l}{L_t A_t^{-1}\omega_{t-1}}\right)\omega_{t-1}^{\nu}\right)$, or $\xi_t^{\nu} = \left(0; A_t^{-1}\omega_{t-1}^{\nu}; l^{\nu}; 0\right)$, for all ν, depending on whether the economy is capital constrained, labour constrained, or on the knife-edge. This specification of agents' optimal choices guarantees that Definition 1 is always satisfied across all simulations, and it has no implications for the analysis of exploitation, because the agents' exploitation status does *not* depend on the specific solution to MP_t^{ν} considered.

The simulations begin with data on $(\mathcal{N}; (A, L); b; \Pi, \Omega_0)$ and we set $N = 100$, $A = 0.5$, $L = 0.25$, and $b = 1.9$. The distribution of initial aggregate capital Ω_0 mimics the empirical wealth distribution for the U.S. (Allegretto [1]) and is derived as in Cogliano et al. [5]. At $t = 0$, ω_0 is distributed such that there are five groups of agents. The first group comprises 50% of the total population and agents in this group are assigned $\omega_0^{\nu} = 0$. The top 1% of agents are assigned 40% of ω_0, the next 4% are assigned 30% of ω_0, the next 15% are assigned 20%, and the remaining 10% of ω_0 is distributed to the remaining 30% of N.[13]

The skill factors s^{ν} are generated such that the initial distribution of income $\left((1+\pi_t)\omega_{t-1}^{\nu} + \widehat{w}_t\Lambda_t^{\nu}\right)_{\nu\in\mathcal{N}}$ is close to the empirical distribution of income for the U.S. Using the same sorting of agents as in the determination of Ω_0, at $t = 0$ an initial aggregate skill endowment $s = 750$ is distributed across agents so that agents in the first quintile of the wealth distribution are assigned 8.12% of s, the second quintile is assigned roughly 18.93% of s, the third quintile is assigned 28.56% of s, and the fourth quintile is assigned roughly 30.79% of s. These s^{ν} are increasing over the first 80% of agents. The next fifteen percent of agents are assigned roughly 13.42% of s, the next four percent of agents are assigned 0.15% of s, and the top one percent of agents are assigned whatever remains of s, which is inevitably the smallest share of all agents. The skill factors over the final 20% of agents are decreasing in magnitude. Within each group of agents there is a degree of randomness in the assigned skill factors so that agents have different s^{ν} and s^{ν} are increasing within each group.

This assignment of skill factors results in the highest skills existing at the top of the fourth quintile of agents, thus these agents have the highest labour income $\widehat{w}_t\Lambda_t^{\nu}$. After the fourth quintile, the agents' skill levels s^{ν} are decreasing in wealth as capital income $(1+\pi_t)\omega_{t-1}^{\nu}$ begins to make up a larger portion of agents' income, with the top one percent of agents deriving nearly all of their income from capital.

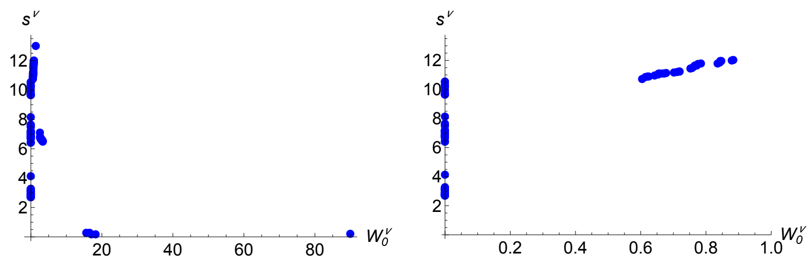
Figure 1 plots individual endowments: each point in the diagram denotes an individual's skill level and her wealth. The left panel displays the entire distribution of endowments. Given the highly skewed distribution of wealth, the right panel focuses only on the skill levels of agents with very low wealth.

Given this determination of $(s^{\nu})_{\nu\in\mathcal{N}}$, the initial distribution of income shares $\left(\frac{(1+\pi_1)\omega_0^{\nu} + \widehat{w}_1\Lambda_1^{\nu}}{\sum_{\nu}(1+\pi_1)\omega_0^{\nu} + \widehat{w}_1\Lambda_1^{\nu}}\right)_{\nu\in\mathcal{N}}$ is close to the empirical distribution in the U.S: in a typi-

[13] There can be some variation in the initial distribution of wealth across models due to the randomness built into the procedure. However, the differences are sufficiently small that simulation results are unaffected and comparable across models.

Figure 1: Skills versus wealth for typical simulation

(a) Complete distribution (b) Close-up



cal run, at $t = 1$, the bottom quintile earns around 3.5-4% of aggregate income, the second quintile earns 8.5-9%, the third quintile 14-15%, the fourth quintile 18-20%, and the fifth quintile 52-55%. The top 5% of agents receives roughly 37% of aggregate income and the top 1% receives 20-22%.[14] These figures are close to those reported by the U.S. Bureau of the Census [23] and to different measures of the income share of the top 1% reported by Mishel et al. [12] (Table 2AA) for recent years. The Gini coefficient of the initial distribution of income is slightly higher than, yet close to, that reported by the Census Bureau [24] and Guzman [9].

Given this initialisation procedure, the economy is initially capital constrained since $l > L_0 A_0^{-1}\omega_0$ and begins far from the knife-edge condition $l = L_0 A_0^{-1}\omega_0$, which allows for the examination of the evolution of exploitation.

5 The basic model

The simulation of the basic model is initialised as described in section 4. It runs by first checking whether the economy is capital constrained, labour constrained, or on the knife-edge and determines $\widehat{w}_t$ and π_t accordingly. Agents then solve MP_t^ν, their endowments update according to equation (4), and the simulation repeats as necessary.

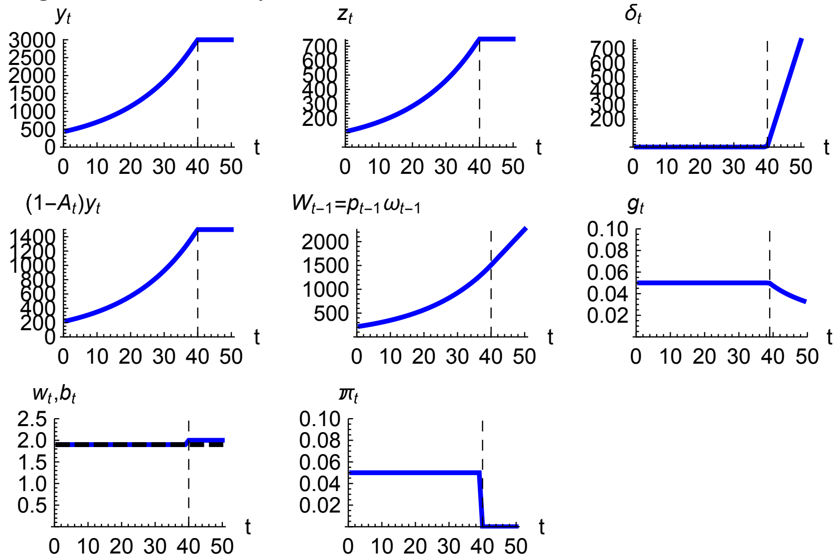
Figure 2 reports the summary results for the basic model. The simulation shows steady growth of activity levels (y_t, z_t) and net output $(1 - A)y_t$ until the economy becomes labour constrained—denoted by the vertical dashed line in the diagrams.[15] The growth rate of aggregate endowments and the profit rate are also steady as long as the simulation is capital constrained.

Figure 3(a) shows that the structure of exploitation is relatively stable as long as the economy is capital constrained, but as soon as labour becomes scarce exploitation disappears. Figure 3(b) displays the distribution of the exploitation intensity

[14]The small variation in the income distribution across these groups in different simulations is due to the small degree of randomness in the determination of individual endowments.

[15]Given our construction of the agents' optimal choices, $x_t = 0$ at all t and therefore the results for this variable are not shown.

Figure 2: Summary results - Basic model with skilled-labour



index ε_t^ν over t. The vertical axis in figure 3(b) shows the agents numbered 1 to 100 arranged by their initial wealth so that the 100th agent is the wealthiest—this ordering is abbreviated as $\nu(\Omega_0)$. Figure 3(b) shows a clear pattern of exploitation until the economy becomes labour constrained. Exploited agents experience $\varepsilon_t^\nu > 1$ consistently, while exploiters experience $\varepsilon_t^\nu < 1$ for all t, until the economy becomes labour constrained. The presence of heterogeneous skills does not significantly alter the structure of exploitative relations: the wealthiest agents exploit and the poorest ones are exploited. The Gini coefficient of exploitation intensity γ_t^ε—not shown—is steady at 0.05746 while the simulation is capital constrained, and zero thereafter.

Figure 4(a) shows that the Gini coefficient of wealth, denoted as γ_t^W, is stable until the economy is labour constrained, at which point wealth inequality begins to steadily decline as all agents accumulate. Figure 4(b) shows the distribution of wealth for select t.

Figure 5 shows the distribution of income shares $\frac{(1+\pi_t)\omega_{t-1}^\nu+\widehat{w}_t\Lambda_t^\nu}{\sum_\nu (1+\pi_t)\omega_{t-1}^\nu+\widehat{w}_t\Lambda_t^\nu}$ during the simulation. There is noticeable income inequality, which remains somewhat stable until the economy becomes labour constrained, at which point income inequality starts declining.

In summary, two general conclusions can be drawn from our results. First, skill heterogeneity poses no insurmountable conceptual problems for exploitation theory: the notion of exploitation remains theoretically robust, conceptually well defined, and grounded on empirically measurable magnitudes. Second, compared with the basic model in Cogliano et al. [5], the incorporation of skills provides a more complex picture from a normative perspective. For the simulations clearly show that exploitation, income inequality, and wealth inequality provide rather different normative insights. Before it becomes labour constrained, the economy displays significant, steady levels

Figure 3: Exploitation - Basic model with skilled-labour

(a) Exploitation status (b) ε_t^ν

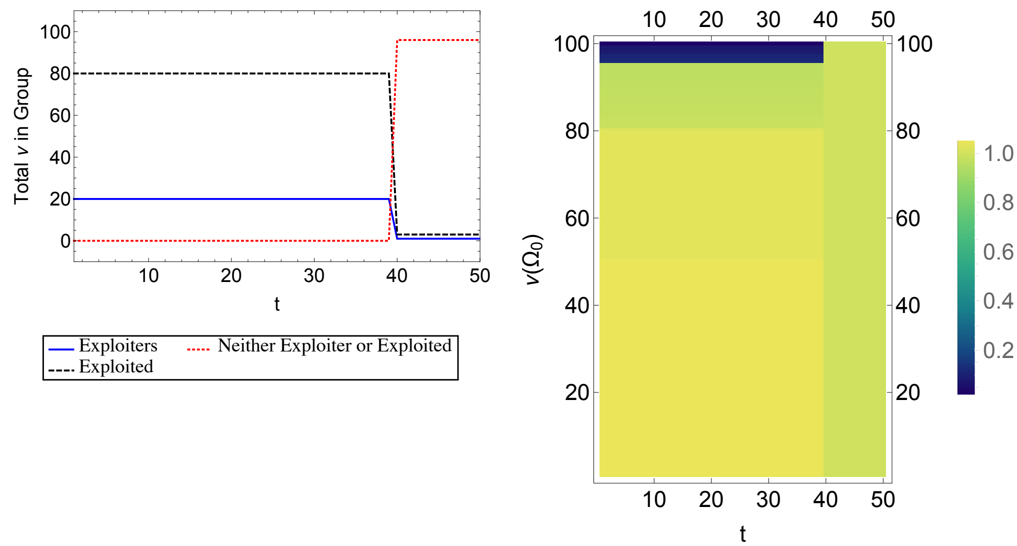
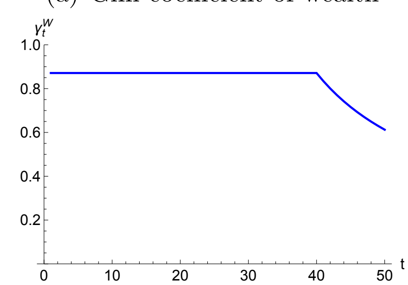
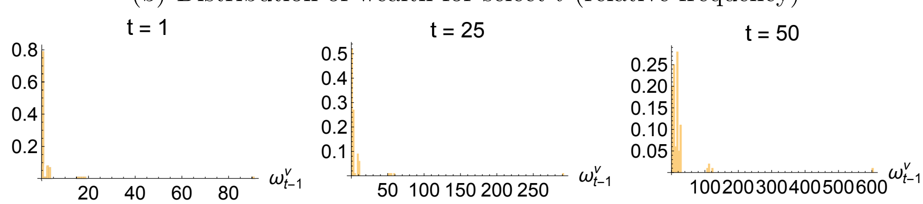


Figure 4: Distribution of wealth - Basic model with skilled-labour

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

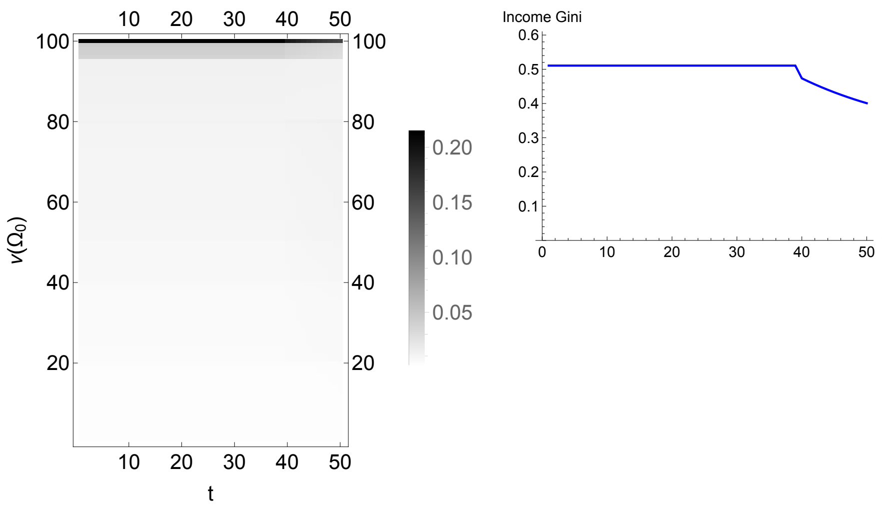


of exploitation, income inequality, and wealth inequality. Yet, while exploitation disappears as soon as the economy is labour constrained, income and wealth inequality decrease over time but do not go to zero. As Roemer [19] has argued, socialists and egalitarians may face some trade-offs when implementing various policies.

Of course, the disappearance of exploitation (and profits) does not mean that capitalist economies have some inherent mechanism that eventually ensures the establishment of a just distribution of income and labour. This result may be seen as an

Figure 5: Distribution of income - Basic model with skilled-labour

(a) Distribution of income shares over t (b) Gini coefficient of income



artefact of the linearity of the model which leads agents to overaccumulate, and our results suggest to explore the mechanisms that guarantee the persistence of (capital scarcity and) exploitation in capitalist economies.[16] Perhaps more importantly for our purposes, until the economy becomes labour constrained, it displays relatively high levels of exploitation and inequality. What kind of policies can be implemented to alleviate this? And are there any policies that can tackle at the same time exploitation and inequalities? We turn to these questions in the next sections, where we extend the model to include three different types of redistributive wealth taxes.

6 The economy with wealth taxes

In this section we analyse the effect on exploitation and inequality of a wealth taxation scheme similar to that proposed by Piketty [15]. We assume that wealth taxes are paid at the end of t once agents have solved MP_t^ν and determined their wealth $p_t\omega_t^\nu$ for the next time period $t+1$. The tax scheme works to redistribute wealth from relatively wealthy agents to agents with less wealth. Given that all agents consume at subsistence b according to how much labour they perform, any transfer that a relatively poor agent receives adds to their wealth. Because wealth taxes and transfers do not affect consumption or labour supply decisions, they do not affect the agents' optimal ξ_t^ν. Thus wealth taxes can be easily incorporated into the simulations without altering the optimisation programme, equilibrium conditions, or definition of exploitation.

There is an additional reason to focus on a scheme of wealth taxation that has no

[16] We return to this issue in the concluding section.

effect on the rate of accumulation. Exploitation can be eliminated either by pushing profits to zero or by redistributing wealth so as to make labour performed proportional to income. Our wealth taxation scheme allows us to separate the dynamics of exploitation arising from changes in profitability (and capital scarcity), and the dynamics of exploitation determined by changes in wealth distribution. The details of the tax structure are as follows.

Let the wealth tax that any agent ν pays at the end of any t be denoted by τ_t^ν, with the distribution of tax rates denoted by $(\tau_t^\nu)_{\nu\in\mathcal{N}}$. All $\nu \in \mathcal{N}$ are assigned a tax rate according to where they fall in the pre-tax distribution of wealth at the end of t, $(\omega_t^\nu)_{\nu\in\mathcal{N}}$. Agents with pre-tax wealth at or below the median face $\tau_t^\nu = 0$, agents with wealth between the median and the 75th percentile face $\tau_t^\nu = 0.005$, agents with wealth at or above the 75th percentile up to and including the 99th percentile face $\tau_t^\nu = 0.02$, and agents at the top one percent of the wealth distribution face $\tau_t^\nu = 0.05$.[17] At the end of every t, taxes are collected from agents for whom $\tau_t^\nu > 0$ and the wealth collected through taxes is evenly redistributed to the $\nu \in \mathcal{N}$ for whom $\tau_t^\nu = 0$.

The simulation is initialised as in section 4. The summary results concerning the behaviour of the aggregate and distributive variables are qualitatively the same as the basic model (see Figure 2) and not pictured here. This confirms that wealth taxes are macroeconomically neutral, which allows us to focus exclusively on the distributional effects of taxation.

Figure 6(a) reports the post-tax dynamics of exploitation. As in the basic model, there is a consistent structure of exploitation as long as the simulation remains capital constrained, but as the simulation evolves and taxes redistribute wealth, the number of exploited agents decreases as the number of exploiters rises. Figure 6(b) shows the post-tax distribution of ε_t^ν over the simulation. Two features of $(\varepsilon_t^\nu)_{\nu\in\mathcal{N}}$ are worth emphasising. First, as in the basic model, exploitation does not disappear until the simulation is labour constrained. Unlike in the basic model, in which $(\varepsilon_t^\nu)_{\nu\in\mathcal{N}}$ is constant over time, however, the distribution of exploitation intensity varies during the simulation as a result of wealth taxation.

Second, as expected, the wealthiest agents do not experience exploitation (despite their relatively low skills), but as the simulation progresses social relations become less exploitative and they come closer to the exploitation threshold as their wealth is taxed away. Perhaps more surprisingly, agents at the bottom of the wealth distribution do not experience exploitation after $t > 1$ either, due to the receipt of wealth transfers and their relatively low skills. The behaviour of ε_t^ν at the extremes of the distribution creates an interesting situation where agents with mid-range values of s^ν and little to no wealth experience exploitation most intensely. Some agents within this group possess enough wealth to pay taxes while others possess no wealth and receive transfers, however, this group effectively constitutes a "middle class" that, due to their skill levels, perform the most effective labour relative to their potential

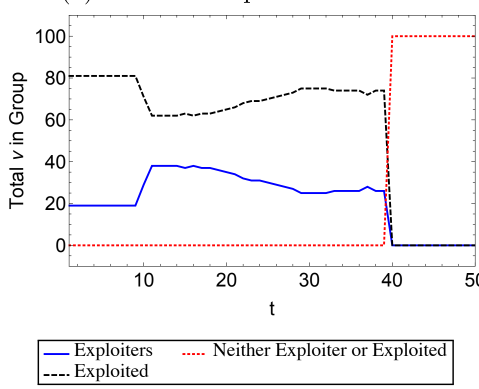
[17]The top tax rate on wealth is equal to the minimum suggested by Piketty ([15], p. 455).

means of consumption.

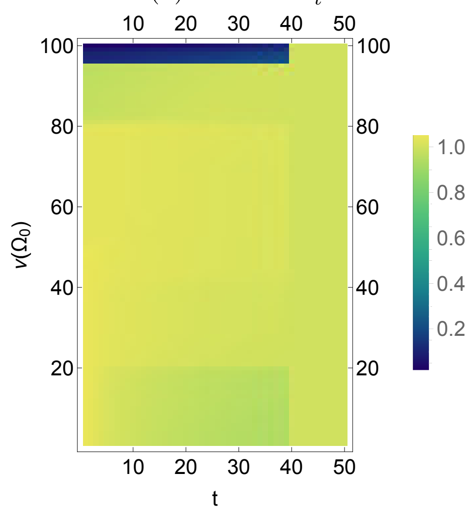
Figure 6(c) shows post-tax values of γ_t^{ε}, which decreases only slightly over the course of the simulation until the economy becomes labour constrained. The slight "saw-tooth" pattern in γ_t^{ε} is the result of agents shifting between different wealth tax rates as their endowments are redistributed, thereby altering vc_t^{ν}.

Figure 6: Exploitation - Model with wealth taxes

(a) Post-tax Exploitation status



(b) Post-tax ε_t^{ν}



(c) Gini coefficient of ε_t^{ν}

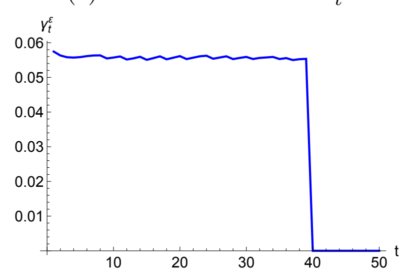
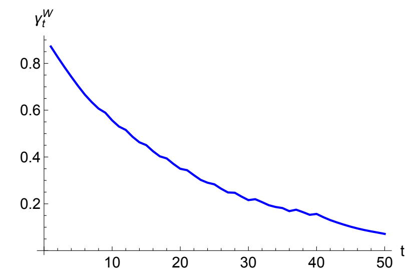


Figure 7(a) shows the Gini coefficient of wealth, γ_t^W; figure 7(b) shows the distribution of wealth for select t. While wealth taxation has a relatively small impact on exploitative relations, γ_t^W steadily, and rapidly, declines over t, clearly showing how effective even small tax rates can be in reducing wealth inequality.

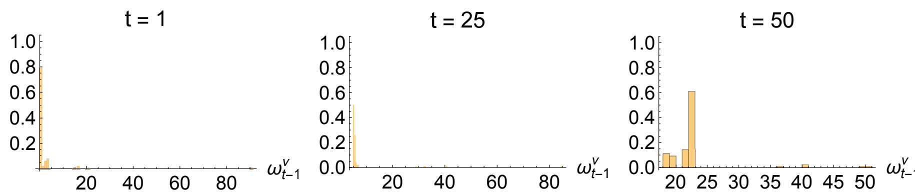
Figure 8 shows the dynamics of the post-tax distribution of income. Figure 8(a) shows the post-tax distribution of income shares for all t: as all agents start accumulating thanks to wealth redistribution, the distribution of income shares shrinks over t, and inequality decreases as non-labour income becomes more equal. Any residual income inequality is due to skill differentials, but is much smaller than at the beginning of the simulation. Once the simulation is labour constrained, capital income is zero for all agents, and income inequality depends only on $(s^{\nu})_{\nu \in \mathcal{N}}$. Figure 8(b) shows

Figure 7: Distribution of wealth - Model with wealth taxes

(a) Gini coefficient of wealth



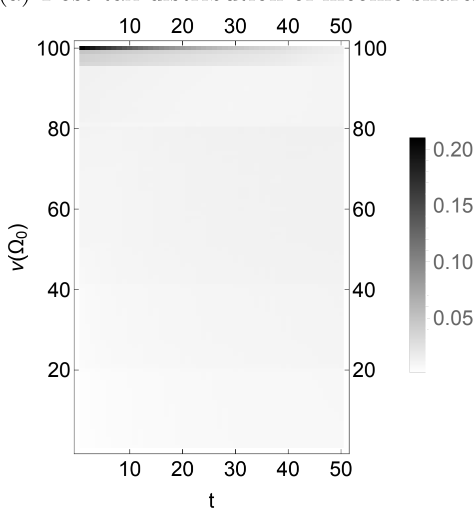
(b) Distribution of wealth for select t (relative frequency)



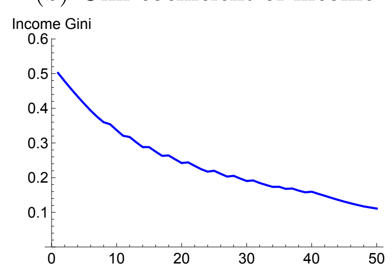
the dynamics of the post-tax Gini coefficient of income, which confirms the pattern just described.

Figure 8: Distribution of income - Model with wealth taxes

(a) Post-tax distribution of income shares



(b) Gini coefficient of income



In summary, the model shows the effectiveness of rather modest wealth taxes of the type suggested by Piketty [14, 15]. Given the very low taxation levels chosen, in every period, wealth taxation has negligible effects: in every period, the pre-tax and post-tax distributions of income, wealth and exploitation are extremely similar. Yet, wealth taxes have significant cumulative effects over time yielding major reductions in wealth and income inequality in a relatively short period of time.

Nonetheless, two features of the taxation scheme analysed should be noted. First, although Piketty-type taxes have significant effects on inequalities, they do not eliminate wealth inequality completely, except in the very long run. Second, a generic tax on wealth does not alter the fundamentally exploitative structure of a capitalist economy, and exploitation disappears only towards the end of the simulation when accumulation drives profits to zero. In the next two sections, we explore two alternative tax schemes to address these issues: a more robust wealth taxation scheme to eliminate wealth inequalities in a finite number of periods and a tax scheme specifically meant to elilminate exploitation.

7 The economy with wealth taxes to equalise wealth

In this section we extend the basic model to incorporate wealth taxes that are meant to quickly eliminate wealth inequality. At the end of any period t, after agents solve MP_t^ν, let $\omega_t'^\nu$ be the endowment of any $\nu \in \mathcal{N}$ according to equation (4). The taxation scheme is formalised in Rule 1:

Rule 1 (**Wealth Equality**). *If, at the end of t, $\pi_t > 0$ and the Gini coefficient of $(\omega_t')_{\nu\in\mathcal{N}}$ is positive, $\gamma_t^{\omega'} > 0$, then $\omega_t'^\nu$ is taxed at rate τ_t^ν according to where agent ν falls in the wealth distribution:*

$$\begin{aligned}
\tau_t^\nu &= \beta_t \left(1 - \frac{\mu_t\left[\omega_t'\right]}{\omega_t'^\nu}\right) && \textit{if and only if } \omega_t'^\nu > \mu_t\left[\omega_t'\right], \\
\tau_t^\nu &= 0 && \textit{if and only if } \omega_t'^\nu \leq \mu_t\left[\omega_t'\right],
\end{aligned}$$

where $\beta_t = \min\{0.05t, 1\}$ and $\mu_t\left[\omega_t'\right]$ denotes the mean of $(\omega_t')_{\nu\in\mathcal{N}}$.

At all t, let N_t^0 denote the number of agents with pre-tax wealth less than the average. For all $\nu \in \mathcal{N}$, agent ν's post-tax wealth at period $t+1$, ω_t^ν, is determined as follows:

$$\begin{aligned}
\omega_t^\nu &= (1-\tau_t^\nu)\omega_t'^\nu && \textit{if and only if } \omega_t'^\nu > \mu_t\left[\omega_t'\right], \\
\omega_t^\nu &= \omega_t'^\nu + \frac{\sum_{\nu\in\mathcal{N}} \tau_t^\nu \omega_t'^\nu}{N_t^0} && \textit{if and only if } \omega_t'^\nu < \mu_t\left[\omega_t'\right], \\
\omega_t^\nu &= \omega_t'^\nu && \textit{if and only if } \omega_t'^\nu = \mu_t\left[\omega_t'\right].
\end{aligned}$$

According to Rule 1, agents with optimal pre-tax wealth $\omega_t'^\nu$ above the average pay a tax rate such that their wealth for $t+1$ is brought closer to the average by a distance determined by β_t. Agents with wealth $\omega_t'^\nu$ below the average pay no taxes and receive an equal share of the total tax revenue, $\sum_{\nu\in\mathcal{N}} \tau_t^\nu \omega_t'^\nu$. Agents with $\omega_t'^\nu = \mu_t\left[\omega_t'\right]$ pay no taxes and receive no transfers. Rule 1 runs as long as both $\gamma_t^{\omega'}$ *and* the profit rate are positive, so that wealth is not taxed when the economy is labour constrained (i.e. for $t > 40$).[18]

The simulation is initialised as in section 4, and it occurs in the following sequence: (1) check whether the economy is capital constrained, labour constrained, or on the knife-edge and set $\widehat{w}_t$ and π_t accordingly; (2) solve MP_t^{ν}; (3) check that $\gamma_t^{\omega'} > 0$ and $\pi_t > 0$, and, if appropriate, use Rule 1 to redistribute wealth; (4) repeat as necessary.

The summary results are qualitatively the same as in the basic model (see Figure 2) and are therefore omitted. Figure 9 shows the after-tax dynamics of exploitation. As expected, wealth redistribution quickly reduces the number of exploited agents (figure 9(a)) and the distribution of ε_t^{ν} is more compressed than in the basic model (figure 9(b)), yet a robust middle class of skilled agents exists which remains exploited—albeit at a low level—even when wealth is equalised, until the economy becomes labour constrained and profits vanish. As figure 9(c) shows γ_t^{ε} declines and quickly reaches a stable level just above 0.04. Rule 1 reduces overall inequality in exploitation intensity, also shown in figure 9(d), but does not eliminate it entirely.

Figure 10 shows the Gini coefficient of wealth, γ_t^W, and the distribution of ω_{t-1}^{ν} for select t. As expected, γ_t^W sharply decreases over time and falls to zero in twenty time periods.[19]

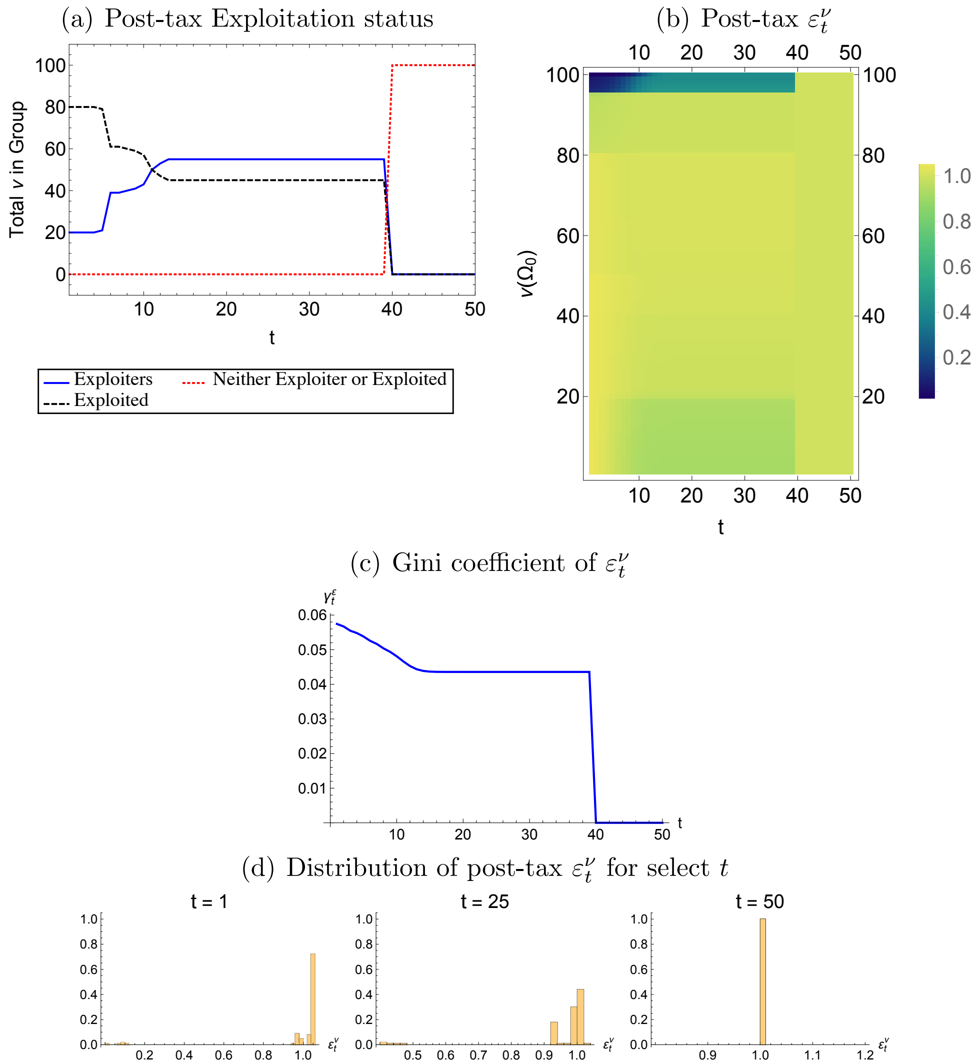
Figure 11 shows the post-tax income distribution over the simulation. By sharply reducing inequalities in capital income, Rule 1 has a strong egalitarian effect on the income distribution to a point where shares of aggregate post-tax income range from 0.00515 to 0.0154 when wealth equality is achieved. Income inequality is not negligible—some agents earn twice as much as others due to skill differentials—but it is a dramatic improvement over the laissez faire basic model.

As noted, Rule 1 does not eliminate exploitation. The redistribution of wealth affects agents with the lowest skill levels at the very bottom and top of the income distribution, while agents with the highest skills are largely unaffected. It is not clear what is necessarily desirable from a societal point of view. While some may find it desirable to achieve wealth equality, this equal right to returns from wealth creates "an unequal right for unequal labour" (Marx [10], p. 24) and simple-minded wealth egalitarianism may create inequalities along the lines of skill and ability.

[18] The type of tax rules used in this paper are designed to achieve a particular allocation of wealth over a pre-determined time period, as captured by β_t. It is possible for there to be some overshooting of the target allocation during the transition phase, prior to $\beta_t = 1$. Specifically, some agents who begin the simulation with wealth below the average and receive transfers via taxes may have wealth above the average post-transfer. This is due to the size of the wealth transfer being uniform across agents at any t when using Rule 1. However, any overshooting is small and transitory, and does not alter the results once the desired allocation is achieved.

[19] Note that γ_t^W increases after $t = 40$ in figure 10(a) because Rule 1 is run only while $\pi_t > 0$. This is done because our primary focus is on the dynamics of exploitation, and exploitation disappears once the economy becomes labour constrained and profits vanish (see also the last panel of figure 10(b) and the diagrams in figure 11).

Figure 9: Exploitation - Model with wealth taxes and wealth equality



(a) Post-tax Exploitation status

(b) Post-tax ε_t^{ν}

(c) Gini coefficient of ε_t^{ν}

(d) Distribution of post-tax ε_t^{ν} for select t

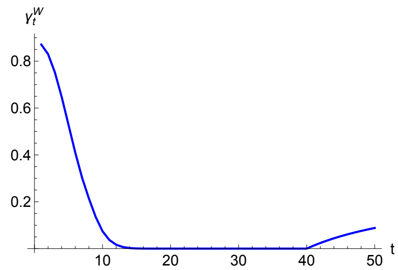
8 The economy with a socialist allocation

In this section, we extend the basic model to include wealth taxes aimed at achieving a socialist allocation. Roemer [19] defines a *socialist allocation* as one in which agents receive a share of total output proportional to their effective labour performed.[20] From Theorem 2, this means that at any period t such that $\pi_t > 0$, a socialist allocation

[20]Whereas in communism the allocation of economic goods will be independent of productive contributions. "From each according to his ability, to each according to his needs!" (Marx [10], p. 24).

Figure 10: Distribution of wealth - Model with wealth taxes and wealth equality

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

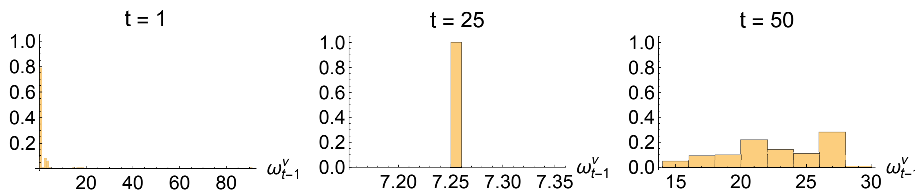
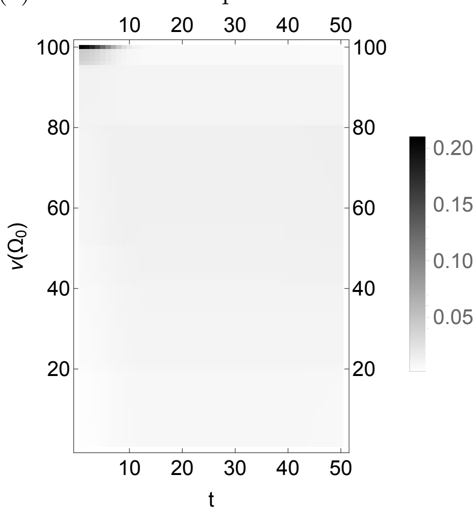
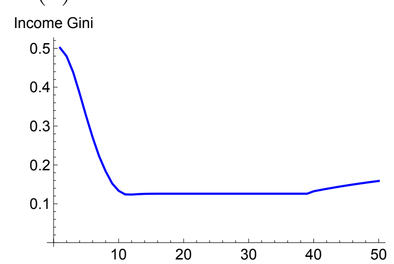


Figure 11: Distribution of income - Model with wealth taxes and wealth equality

(a) Distribution of post-tax income over t



(b) Gini coefficient of income



can be achieved only if, for any $\nu \in \mathcal{N}$,

$$\omega_{t-1}^{\nu} = \Lambda_t^{\nu} \frac{1}{\pi_t} \left(\frac{1 - \widehat{w}_t v}{v} \right). \tag{5}$$

In other words, a socialist taxation scheme must bring wealth to a level proportional to the effective labour performed by each agent. At first sight, equation (5) seems to suggest that, at the end of every period t (and the beginning of period $t+1$), the calculation of the relevant tax rates would require anticipating the equilibrium labour supply of all agents (Λ^{ν}_{t+1}) and the equilibrium distribution ($\widehat{w}_{t+1}$,π_{t+1}). As it turns out, this is unnecessary in our model and at the end of any t, a well-defined socialist taxation scheme can be defined based on past observed variables.

For all $\nu \in \mathcal{N}$, at any t, let Ψ^{ν}_t be defined as follows:

$$\Psi^{\nu}_t = \Lambda^{\nu}_t \frac{\omega_t}{Ly_t}.$$

This expression for Ψ^{ν}_t denotes the wealth of each agent *at the beginning of* $t+1$ that is consistent with a socialist allocation at $t+1$. To see this, note that in equation (5) $\Lambda^{\nu}_t \frac{1}{\pi_t}\left(\frac{1-\widehat{w}_t v}{v}\right) = \Lambda^{\nu}_t \frac{A}{L}$. Then, observe that in any period t such that $\pi_t > 0$, at a RS, $\Lambda^{\nu}_t \frac{\omega_t}{Ly_t} = \Lambda^{\nu}_t \frac{Ay_{t+1}}{Ly_t}$. Therefore $\Psi^{\nu}_t = \Lambda^{\nu}_{t+1}\frac{1}{\pi_{t+1}}\left(\frac{1-\widehat{w}_{t+1}v}{v}\right) = \Lambda^{\nu}_{t+1}\frac{A}{L}$ if and only if $\frac{\Lambda^{\nu}_t}{Ly_t} = \frac{\Lambda^{\nu}_{t+1}}{Ly_{t+1}}$, and the latter equality holds in equilibrium as long as $\pi_t > 0$, and the economy is capital constrained.

Recall that, at the end of any t, for any $\nu \in \mathcal{N}$, ω'^{ν}_t is ν's endowment according to equation (4). We consider the following tax scheme:

Rule 2 (**Socialist Allocation**). *Consider any period t such that $\pi_t > 0$. For any $\nu \in \mathcal{N}$, τ^{ν}_t is determined according to where agent ν falls in the distribution of ω'^{ν}_t:*

$$\begin{aligned} \tau^{\nu}_t &= \beta_t\left(1 - \frac{\Psi^{\nu}_t}{\omega'^{\nu}_t}\right) && \textit{if and only if} \quad \omega'^{\nu}_t > \Psi^{\nu}_t, \\ \tau^{\nu}_t &= 0 && \textit{if and only if} \quad \omega'^{\nu}_t \leq \Psi^{\nu}_t, \end{aligned}$$

where $\beta_t = \min\{0.05t, 1\}$.

Let $\mathcal{N}^0_t \subset \mathcal{N}$ be the subset of agents with $\omega'^{\nu}_t < \Psi^{\nu}_t$ at t. The wealth any agent has available at $t+1$ is:

$$\begin{aligned} \omega^{\nu}_t &= (1-\tau^{\nu}_t)\omega'^{\nu}_t && \textit{if and only if} \quad \omega'^{\nu}_t > \Psi^{\nu}_t, \\ \omega^{\nu}_t &= \omega'^{\nu}_t + \frac{l^{\nu}}{\sum_{\nu\in\mathcal{N}^0_t} l^{\nu}} \sum_{\nu\in\mathcal{N}} \tau^{\nu}_t \omega'^{\nu}_t && \textit{if and only if} \quad \omega'^{\nu}_t < \Psi^{\nu}_t, \\ \omega^{\nu}_t &= \omega'^{\nu}_t && \textit{if and only if} \quad \omega'^{\nu}_t = \Psi^{\nu}_t. \end{aligned}$$

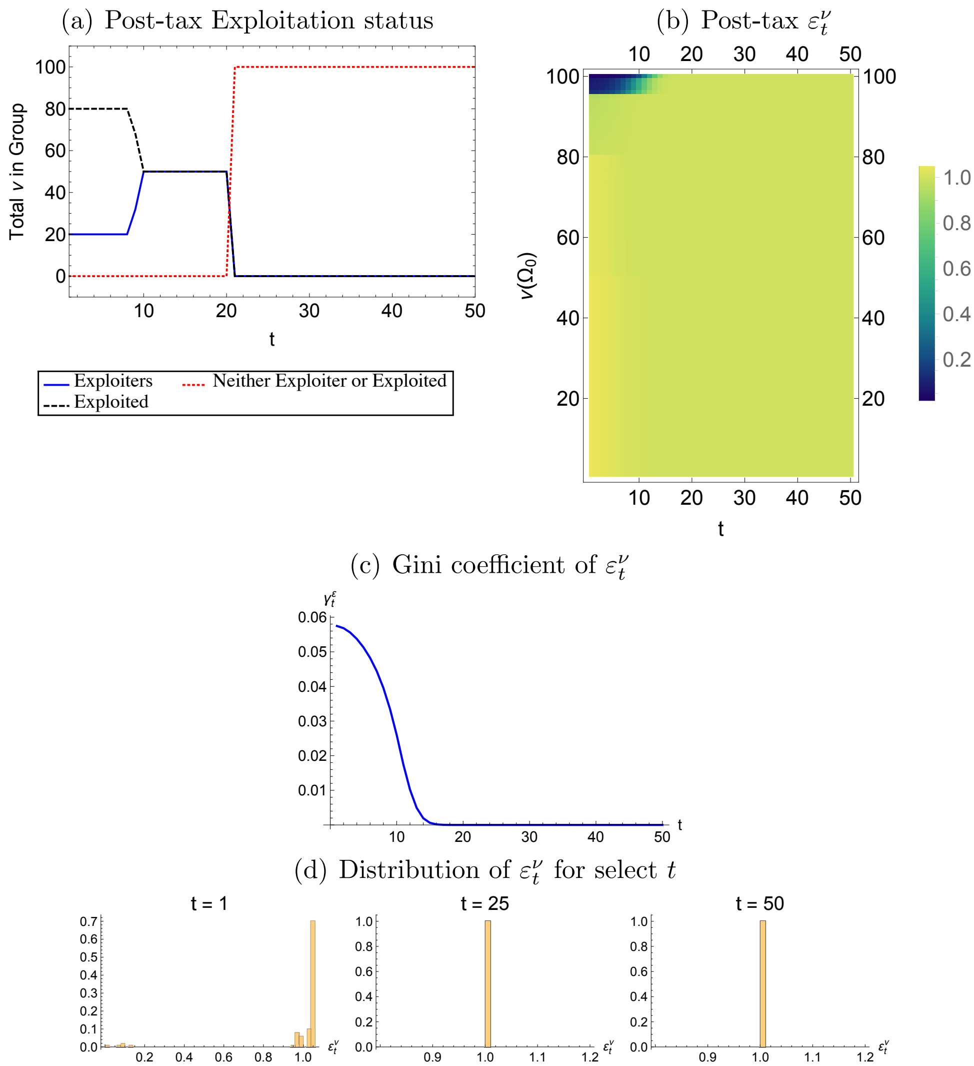
According to Rule 2, agents with wealth greater than the level consistent with a socialist allocation are taxed, while those whose wealth is below the level consistent with a socialist allocation receive a portion of total tax revenues, $\sum_{\nu\in\mathcal{N}} \tau^{\nu}_t \omega'^{\nu}_t$, consistent with their share of effective labour in the subset of relatively poor agents $\mathcal{N}^0_t$. Thus, wealth is redistributed until each agent holds wealth in proportion to their effective labour performed, as specified in equation (5).[21]

[21] As with Rule 1, Rule 2 also holds the possibility for overshooting the target allocation of wealth prior to $\beta_t = 1$. It is possible for wealth transfers to be large enough to overshoot Ψ^{ν}_t in the case of certain agents who might begin the simulation with small but positive amounts of wealth. Because the amount of wealth transferred to agents in $\mathcal{N}^0_t$ at any t is proportional to their share of effective labour in $\mathcal{N}^0_t$ it is possible that they receive a transfer large enough to push their wealth above Ψ^{ν}_t. However, any overshooting is small and quickly correct in subsequent time periods, and it is purely transitory since there can be no overshooting once $\beta_t = 1$.

The simulation is initialised as in section 4 and it occurs in the following sequence: (1) check whether the economy is capital constrained, labour constrained, or on the knife-edge and set $\widehat{w}_t$ and π_t accordingly; (2) solve MP_t^ν; (3) check whether $\omega_t'^\nu \neq \Psi_t^\nu$ for any $\nu \in \mathcal{N}$ and $\pi_t > 0$, and, if appropriate, apply Rule 2; (4) repeat as necessary.

Taxes are, again, macroeconomically neutral: the summary results of the simulations are qualitatively the same as in the basic model (see Figure 2) and are therefore omitted. Figure 12 shows the dynamics of post-tax exploitation. Figure 12(a) shows exploitation status. As expected, wealth redistribution quickly ends exploitation while moving all agents to the middle classes, as seen in figure 12(b), where $\varepsilon_t^\nu = 1$ for all ν by $t = 20$. This is confirmed by γ_t^ε in figure 12(c).

Figure 12: Exploitation - Model with wealth taxes and socialist allocation



(a) Post-tax Exploitation status

(b) Post-tax ε_t^ν

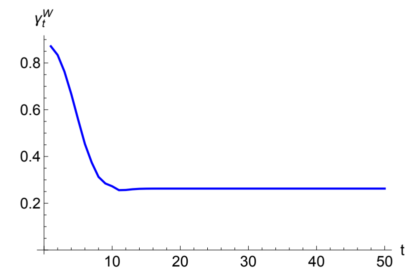
(c) Gini coefficient of ε_t^ν

(d) Distribution of ε_t^ν for select t

Figure 13 shows the Gini coefficient of wealth, γ_t^W, and the distribution of ω_{t-1}^ν for select t: γ_t^W reaches its minimum value of 0.2559 at $t = 11$, and while this is much lower than at the start of the simulation, it is not insignificant. Thus, the socialist allocation is not fully consistent with views calling for wealth equality. As Rule 1 shows, wealth equality leads to inequalities of other kinds.

Figure 13: Distribution of wealth - Model with wealth taxes and socialist allocation

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

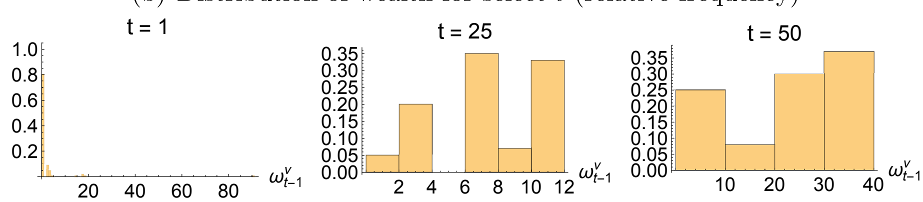
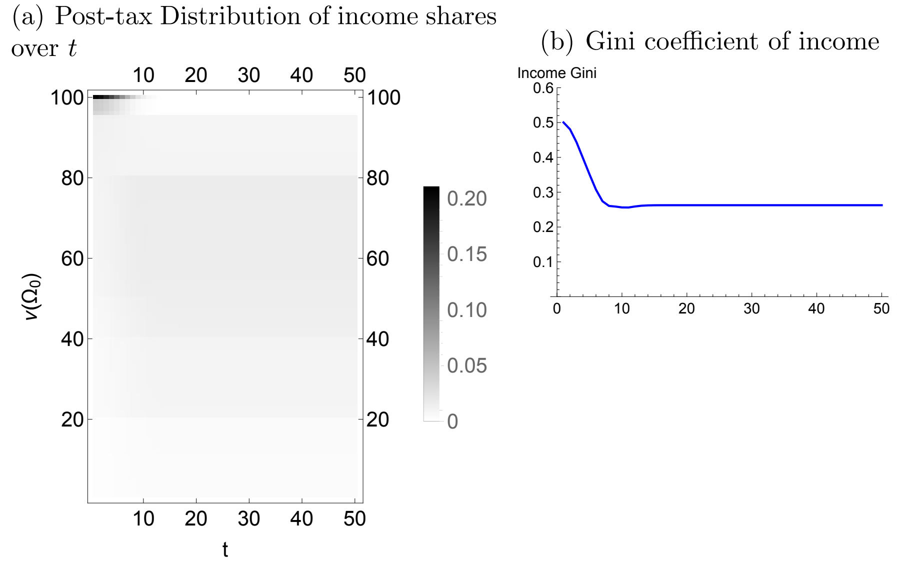


Figure 14 shows the distribution of income over the simulation. Income inequality is dramatically reduced, yet not eliminated entirely due to skill heterogeneity. Once the socialist allocation is achieved at $t = 21$, agents' income is proportional to their effective labour, and agents with the highest skills receive the highest incomes—agents get out what they put into the economy. While incomes remains unequal at the socialist allocation has been achieved, the income distribution is not nearly as unequal as at the beginning of the simulation. During the socialist phase post-tax shares of income range from 0.000331 to 0.015995.

9 Socialism, education and skills

The previous simulations raise the important question of how best to resolve the apparent trade-offs faced by socialists and egalitarians. Egalitarian are likely to find the inequalities in wealth and income that persist under Rule 2 problematic. Whereas the persistence of exploitation under Rule 1 is undesirable from a socialist perspective. At the heart of these trade-offs is the heterogeneity of labour, and perhaps the best way to satisfy both sets of concerns is through an education system designed to eliminate, or at least alleviate inequalities in skills. In this section, we consider such a possibility by using wealth taxes according to Rule 2 and diverting a portion of tax

Figure 14: Distribution of income - Model with wealth taxes and socialist allocation



revenue to augmenting agents' skills so that the distribution of s^ν is compressed over t.

Let ϵ_t^ν denote an agent's claim to a share of the social fund available for education. At the end of t,

$$\epsilon_t^\nu = \frac{1}{1+\exp\left(s_t^\nu\right)} \Bigg/ \sum_\nu \frac{1}{1+\exp\left(s_t^\nu\right)},$$

where "exp" denotes the natural exponential function. The above equation uses an inverted logistic function so that as agents' skills increase over t their claim on the education fund decreases, thus education will make skills asymptotically approach uniformity over time, without ever reaching perfect uniformity.

Agents' skills at $t+1$ are updated as follows:

$$s_{t+1}^\nu = s_t^\nu \left(1 + \epsilon_t^\nu \sigma_t \sum_\nu \tau_t^\nu \omega_t^{\prime\nu}\right),$$

where σ_t denotes the portion of overall tax revenue dedicated to education. This algorithm can be added on to the end of Rule 2 so skill factors are updated after tax revenue is collected. Modifying agents' skills in this way raises the skills of all agents over time, but agents who begin the simulation with low skills are prioritised in order to reduce inequalities in effective labour. Thus, agents who begin the simulation with the highest skills benefit from education, albeit less than agents who begin at bottom of the skills distribution.

The education algorithm is added to the model of section 8. The simulation is initialised as in section 4 and sets $\sigma_t = 0.25$ for all t, thus 25% of tax revenue during each t is diverted to augmenting agents' skills. Figure 15 reports the summary results, which show that the continuous augmentation of the aggregate skill endowment s_t prevents the economy from ever becoming labour constrained. There is also a noticeable dip in the accumulation rate g_t at the start of the simulation as a portion of aggregate wealth is used to alter skills, rather than being used for further accumulation. However, this decline is quickly corrected and accumulation resumes its course after roughly $t = 15$. Figure 16 shows the evolution of the Gini coefficient of skills, γ_t^s. The effect of education in reducing skill inequality is immediately apparent in the decline of γ_t^s, which reaches a value of 0.0232483 at $t = 50$.

Figure 15: Summary results - Model with education

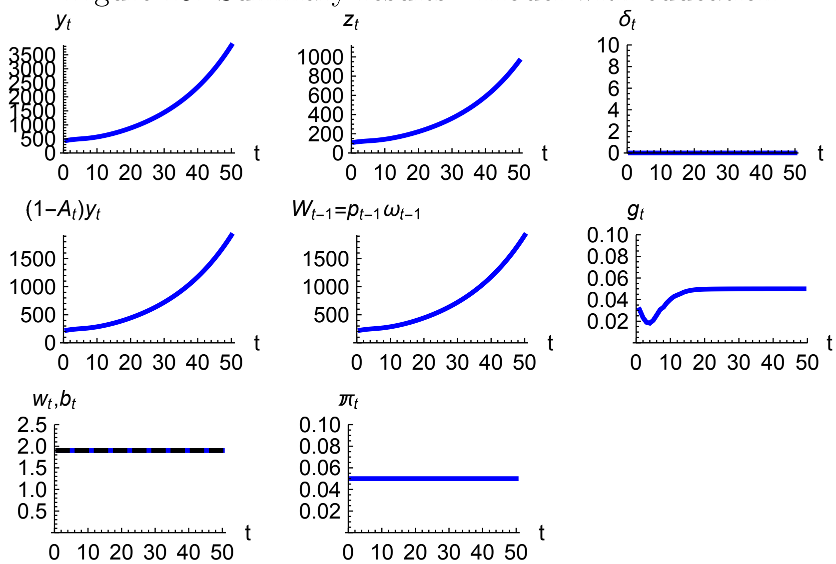


Figure 16: Skill inequality - Model with education

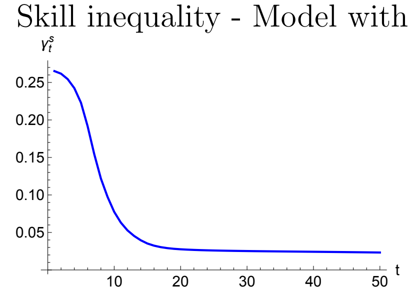


Figure 17 shows the dynamics of post-tax exploitation: over time, exploitation is not completely eliminated (Figure 17(a)), but differences in post-tax exploitation intensity after $t = 20$ are very small (figures17(b)-17(d)). The small inequalities in ε_t^ν during later time periods show that while there is a clear delineation of exploited and exploiter agents, the differences between them are small and driven by the small degree of heterogeneity in skills, with γ_t^ε reaching 9.4709×10^{-6} at $t = 50$.

Figure 17: Exploitation - Model with education

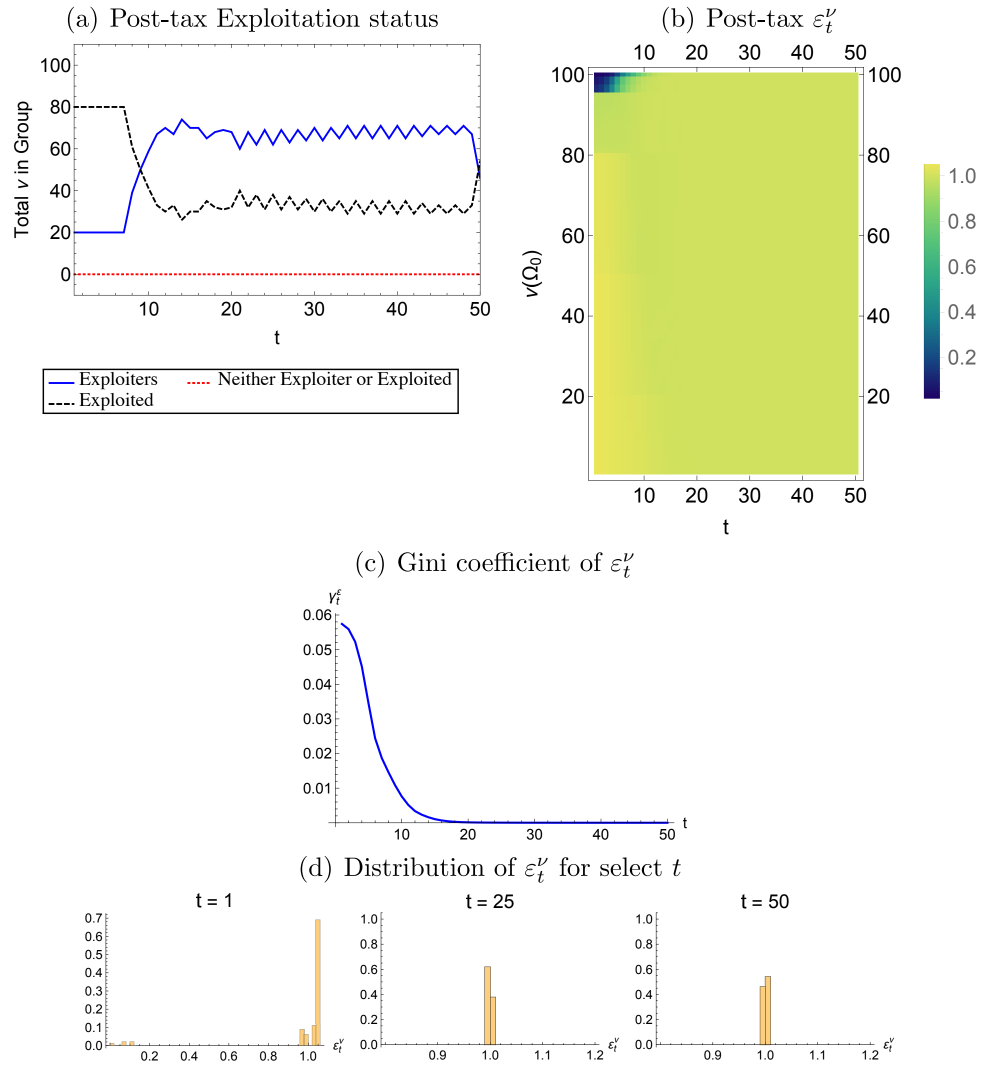
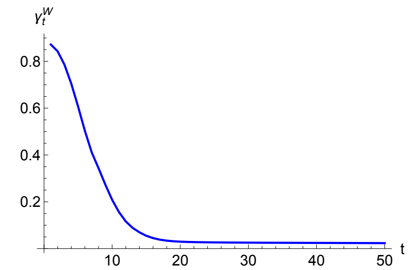


Figure 18 shows the dynamics of the distribution of wealth. Figure 18(a) shows that γ_t^W quickly declines and reaches a value of 0.0234334 at $t = 50$ as a result of wealth taxes. This level of γ_t^W is lower than in section 8 due to the compression of the distribution of skills. At the proportional solution, where wealth is redistributed in proportion to effective labour capacity, the asymptotic convergence of effective labour induces greater wealth equality than earlier iterations of Rule 2.

Figure 19 shows the dynamics of the distribution of income. The rapid reduction of inequalities is apparent in figures 19(a)-19(b): the redistribution of wealth in conjunction with the compression of the skills distribution induces greater income equality with a much lower Gini coefficient of income—0.0233454 at $t = 50$—than in section 8.

Figure 18: Distribution of wealth - Model with education

(a) Gini coefficient of wealth



(b) Distribution of wealth for select t (relative frequency)

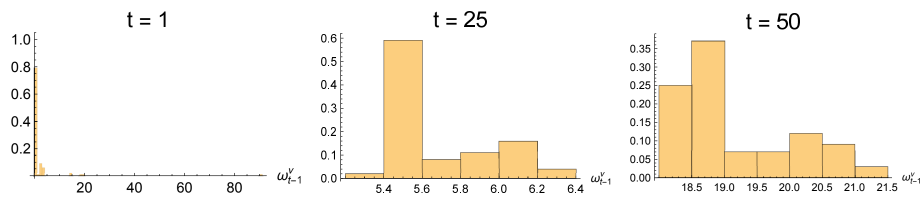
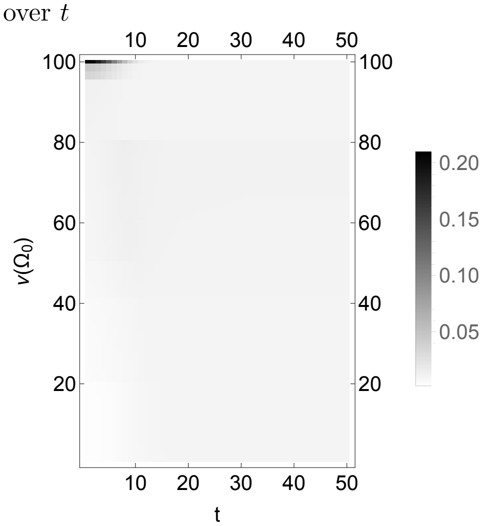
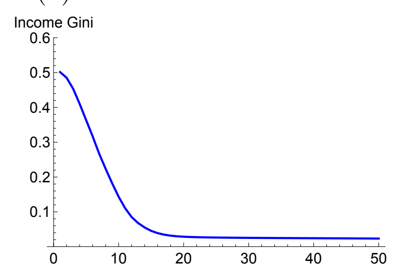


Figure 19: Distribution of income - Model with education

(a) Post-tax distribution of income shares over t



(b) Gini coefficient of income



The pre- and post-tax behaviour of exploitation intensity and wealth are similar to that of the simulation in section 8 and omitted for space concerns.

The incorporation of education leads to a significant reduction in income inequalities in the socialist, non-exploitative allocation. Such a limited degree of inequality may be deemed tolerable for both egalitarians and socialists, especially given that the education system is designed to gradually reduce, and tendentially eliminate, the residual skill heterogeneity. Thus, our results speak to the importance of education as part of what could be considered a socialist project—one aimed at ameliorating social inequities stemming from heterogeneities in skills *in addition to* eliminating wealth inequalities causing systemic problems of exploitation.

10 Robustness

We have run a bevy of alternative simulations to ensure that our results are robust to changes in parameters and specifications of the models. In this section, we briefly summarise the main findings: a detailed description can be found in the Addendum.

10.1 Skills

First, we have analysed the economy under the special case of homogeneous labour, or $l^\nu = 1$ for all $\nu \in \mathcal{N}$. The addition of skills does not significantly affect the macro-behaviour of the economy as shown in the summary results, and the distribution of wealth. However, predictably, the dynamics of the distributions of the exploitation intensity index and income are significantly different, and there is no trade off between the elimination of exploitation and the reduction of wealth and income inequalities.

Second, it may be objected that our results depend on the specific assumptions concerning the distribution of skills. Although we believe that our specification is empirically justified, we have explored alternative assumptions on $(s^\nu)_{\nu\in\mathcal{N}}$. To be precise, we have considered: (i) skills assigned to be increasing in agents' wealth so that the wealthiest agents have the *highest* effective labour capacity; (ii) skills assigned to be decreasing in wealth so that the wealthiest agents have the *lowest* effective labour capacity; and (iii) skills that are normally distributed and ordered according to Ω_0. We have re-run all simulations under assumptions (i)-(iii): our results remain qualitatively unchanged, save for the patterns of exploitation intensity. As expected, the patterns of exploitation intensity in these alternative scenarios are driven by the distribution of skills, with certain similarities to the main simulations reported in previous sections. Specifically, agents with the highest skills relative to their wealth experience the most intense exploitation. The addition of wealth taxes, Rule 1 taxes, and Rule 2 taxes also has the same effect on simulations with alternative skill distributions as the simulations reported above.

Third, we have considered an alternative method of incorporating heterogeneous labour whereby agents have the same skills but different endowments of labour *time*, with agents at the lower end of the wealth distribution having more potential labour time than wealthier ones. In these simulations the population of agents is divided into quartiles according to the initial wealth distribution—i.e. $(1/4)N$ agents per quartile—and assigned different values of l^ν depending on which quartile they fall in. This simpler way of introducing heterogeneous labour does not qualitatively alter any of the results.

10.2 Taxes

First, in all economies with taxation, we have compared the dynamics of all variables in pre-tax as well as in post-tax terms. Given the structure of the optimisation programme, there is no qualitative difference in the evolution of the two sets of variables. Indeed, the relatively low taxation rates imply that in any given period the pre- and post-tax distributions are extremely similar. Second, all results are robust to various perturbation of the taxation schemes, including different tax rates or different rules concerning the distribution of tax proceeds.

10.3 Classes

We have also analysed the equilibrium structure and dynamics of classes, and the relation between class and exploitation status, in all economies. Roemer's [17] definition of classes and his celebrated *Class-Exploitation Correspondence Principle* can be generalised to economies with heterogeneous skills. A remarkably stable class structure emerges in equilibrium until the economy becomes labour constrained and classes disappear. The introduction of wealth taxation does reduce class polarisation but it does not eliminate classes, except in the case of Rule 2.

10.4 Alternative measures of income and exploitation

In all of simulations above, we focus on the distribution of potential income, $(1+\pi_t)\,\omega^{\nu}_{t-1} + \widehat{w}_t\Lambda^{\nu}_t$, because it captures the total income that agents may devote to consumption and accumulation in every period. Our main results and conclusions remain unchanged if one focuses instead on the flow of income deriving from individual endowments and considers $\pi_t\omega^{\nu}_{t-1} + \widehat{w}_t\Lambda^{\nu}_t$.

The exploitation index defined in section 3 measures exploitation intensity according to agents' *effective* labour performed. Alternatively, one may argue that the concept of exploitation is meant to capture some inequalities in the distribution of material well-being and free hours that are—at least *prima facie*—of normative relevance (Fleurbaey [7]). For example, they may be deemed relevant because material well-being and free hours are key determinants of *individual well-being freedom* (Veneziani and Yoshihara [30]). But they are also relevant in approaches that link exploitation and the Marxian notion of alienation in production (Buchanan [2]). From this perspective, the key variable of normative interest is labour *time*.

In constructing an index measuring exploitation intensity according to the amount of *time* agents work we immediately encounter a difficulty. While the numerator of such index can be taken to correspond to the unadjusted labour hours that agents spend in production, there is no obvious way of defining the denominator which should measure the amount of labour hours that agents receive via their (notional) bundle c_t. The amount vc_t provides a skill-adjusted (via v) quantity of labour and this needs to be transformed into an amount of labour *time*. There is no natural, or obvious way of doing this.

One may divide vc_t by s^ν to measure the time that ν receives, given *her* skills. But this is not necessarily equal to the actual amount of time used to produce c_t. Further, in this case, the time-based index would be trivially equal to ε_t.[22]

Following Veneziani and Yoshihara [30], we explore an alternative option by dividing vc_t by the average skill in the economy, so that the exploitation index e_t^ν captures inequalities in the labour hours supplied to obtain one unit of potential consumption. Formally,

$$e_t^\nu = \frac{\Lambda_t^\nu / s^\nu}{vc_t^\nu / \frac{\sum_\nu s^\nu}{N}}.$$

Observe that unlike for ε_t^ν there is no clear threshold to define exploitation status, but the variation in $(e_t^\nu)_{\nu\in\mathcal{N}}$ yields insight into differential relationships between the labour time agents perform and their available resources for potential consumption.

In all simulations, inequalities in the distribution of $(e_t^\nu)_{\nu\in\mathcal{N}}$ increase and eventually stabilise at a pretty high level. The least-skilled agents at the top and bottom of the vertical axis—those with the highest and lowest initial wealth, respectively—experience higher degrees of time-based exploitation intensity. This is due to the low skill levels of these agents relative to their labour endowment $\zeta^\nu = 1$. Because their skills are low, they receive little labour income yet put in the same amount of time as relatively high-skilled agents in the “middle class”. This pattern is even clearer if the economy becomes labour constrained or wealth inequalities disappear, because agents with large amounts of initial wealth experience the most intense time-based exploitation due to their extremely low skills. Significant inequalities in e_t^ν persist even at the socialist allocation.

Inequalities in e_t^ν are only eliminated in economies with education (or with homogeneous labour). The compression of the skill distribution caused by education leads to a convergence of effective labour performed by agents and their labour-time expended—and by extension of ε_t^ν and e_t^ν. Consistent with other results in economies with education, this convergence of effective labour and labour-time is asymptotic, but inequalities in e_t^ν are very small at $t = T$.

10.5 Alternative assumptions on consumption and labour supply

Throughout our analysis, we have modelled individual choices by assuming identical preferences and rational maximising behaviour in the determination of labour

[22]Interestingly, this suggests an alternative and intriguing interpretation of ε_t^ν as measuring the amount of *time* given and received by ν if she was the only productive agent in the economy, or if all agents were alike.

performed, consumption, and savings, *and* by introducing a (uniform) subsistence constraint such that all agents must consume b units of the produced good per unit of effective labour. Both modelling choices can be altered without essentially changing our results.

First, we have explored a version of the model with $b = 0$. This is interesting because with a zero subsistence condition our model reduces to Roemer's [17] accumulating economies. Moreover, the condition $b = 0$ (trivially) obliterates the distinction between the assumption that a certain amount of consumption is required to reproduce individual *skills*, and the polarly opposite assumption that b is only required to reproduce the labour *time* endowment. Finally, by setting $b = 0$, we can more clearly capture individual differences in the capacity to accumulate arising from unequal earnings due to differential skills. These unequal earnings are apparent in the level of γ_t^{ε}, which is above 0.60 during the capital constrained portion of the simulation without taxes. The Gini coefficient of income is also higher, above 0.80, while the simulation is capital constrained. The behaviour of these Gini coefficients is consistent across the simulations with $b = 0$ and different tax schemes, at least at the initial time periods before the redistribution of wealth alters the landscape of inequality.

Compared with the simulations in sections 5-8, the only real difference is the time period at which the economy becomes labour constrained. The lack of consumption and more rapid accumulation typically lead the economy to become labour constrained after only a few time periods. However, during the relatively short capital constrained phase of the simulations, all of our conclusions regarding the determinants of exploitation and the effects of different wealth taxes continue to hold.

We have also explored several versions of the model in which agents do not solve the optimisation programme MP_t^{ν} in order to explore the effects of alternative behavioural rules and of more radical heterogeneity in individual choices.

As a first step, we have considered an economy in which at every t agents' optimal $(x_t^{\nu}, y_t^{\nu}, z_t^{\nu}, \delta_t^{\nu})$ is determined as specified in section 4, so that the agent's revenue is the same as it would be at the solution to MP_t^{ν}. However, this revenue is allocated to consumption and accumulation in a different manner: we assume that agents hold heterogeneous preferences over consumption such that each $\nu \in \mathcal{N}$ consumes $h^{\nu} b \Lambda_t^{\nu}$ in every t, where $h^{\nu} \in (0, 1)$ is randomly determined at the beginning of the simulation and remains constant throughout. By construction, parts (b)-(d) of Definition 1 continue to hold, even though agents do not solve MP_t^{ν}.

In this economy, the determinants of individual exploitation status and the effects of different wealth taxation schemes remain the same compared with the simulations in sections 5-8. The only qualitative differences are, again, the timing at which the economy becomes labour constrained and the structure of exploitation while the economy is capital constrained. These simulations use the same parameters as those in sections 5-8, and similar to the simulations reported above, γ_t^{ε} begins close to 0.06. However, with $h^{\nu} \in (0, 1)$, agents consume less than $b\Lambda_t^{\nu}$, and thus accumulate more.

A more widespread accumulation leads to a reduction in the number of exploited agents when the economy is capital constrained, and the heterogeneity of h^ν induces different experiences of exploitation intensity for different agents, but exploitation does not disappear until the economy is labour constrained. This behaviour holds across different versions of these simulations as the different tax schemes are incorporated.

Next, we have dropped all notions of subsistence, and considered an economy in which consumption is instead a decreasing function of wealth. To be specific, at every t agents' optimal $(x_t^\nu, y_t^\nu, z_t^\nu, \delta_t^\nu)$ is determined as specified in section 4, so that the agent's revenue is the same as it would be at the solution to MP_t^ν. However, at every t, each $\nu \in \mathcal{N}$ consumes a portion $h_t^\nu \in (0, 1]$ of their net income, so that $c_t^\nu = h_t^\nu \left(\pi_t \omega_{t-1}^\nu + \widehat{w}_t \Lambda_t^\nu\right)$.

We capture a well-known empirical feature of capitalist economies by assuming that the parameter h_t^ν depends inversely on wealth, with poorer agents consuming a larger fraction of their income. To be specific, during any t: $h_t^\nu = 1$ for all $\nu \in \mathcal{N}$ with zero wealth; $h_t^\nu = 0.9$ for all $\nu \in \mathcal{N}$ with positive wealth up to and including the median wealth level; $h_t^\nu = 0.8$ for all $\nu \in \mathcal{N}$ with wealth above the median up to and including the 90th percentile; $h_t^\nu = 0.7$ for all $\nu \in \mathcal{N}$ with wealth above the 90th percentile up to and including the 99th percentile; and $h_t^\nu = 0.4$ for all $\nu \in \mathcal{N}$ in the top percentile of the wealth distribution. The coefficients h_t^ν are time-dependent because agents can move up or down the wealth distribution over time, and so their consumption behaviour would change accordingly. By construction, in this economy parts (b)-(d) of Definition 1 continue to hold, even if agents do not solve MP_t^ν.

In these simulations, agents who are initially endowed with at least some wealth experience lower exploitation over time compared with the benchmark scenarios, but exploitative social relations do not disappear until the economy is labour constrained. Wealth-dependent consumption behaviour leads to a greater range of experiences of exploitation, in the sense that every agent with *some* initial wealth experiences varying levels of exploitation intensity over time, but it does not fundamentally alter the key drivers of accumulation and the exploitative dynamics of the economy.[23] Further, as the different tax schemes are introduced into these simulations, all of the same tradeoffs faced in sections 7 and 8 remain relevant.

Lastly, we have explored the impact of even greater heterogeneity in agent behaviour, by combining wealth-dependent consumption with heterogeneous preferences over labour and leisure. To be specific, at every t, agents' labour supply Λ_t^ν is determined as follows: as in the basic model, for each $\nu \in \mathcal{N}$, if $\widehat{w}_t > b$ then $\Lambda_t^\nu = l^\nu$ and if $\widehat{w}_t < b$, then $\Lambda_t^\nu = 0$. If, however, the economy is capital constrained and $\widehat{w}_t = b$, then $\Lambda_t^\nu = \ell^\nu \frac{LA^{-1}\omega_{t-1}}{l} l^\nu$, where $\ell^\nu \in \{0.98, 1.02]$ are randomly assigned to all $\nu \in \mathcal{N}$ at the

[23] These simulations use most of the same parameters as the simulations in sections 5-8, and similar to the simulations above, γ_t^ε begins close to 0.06 and declines as accumulation progress. This pattern is consistent under different tax schemes, but the speed of the decline in γ_t^ε increases as wealth is redistributed.

start of the simulation such that total labour demand is equivalent to what labour demand would be in the absence of ℓ^ν, i.e. the labour demand in the corresponding simulations reported in sections 5-8. This is done by randomly assigning ℓ^ν to all agents and then adjusting the ℓ^ν of the highest-skilled agent such that there is no over- or undershooting of initial labour demand. This sets $\ell^\nu \frac{LA^{-1}\omega_{t-1}}{l}$ as proportions of labour supplied across ν at all t while the simulation is capital constrained. This setup treats ℓ^ν as a randomly assigned, time-invariant parameter which captures heterogeneous preferences over leisure at wages that merely guarantee subsistence. In other words, when the monetary incentive to work is weak, different agents break the indifferences in different ways.

The choice of $\ell^\nu \in \{0.98, 1.02\}$ introduces small disturbances to the standard capital constrained simulation. While these disturbances are small, they do affect the aggregate behaviour of the simulation and agents' individual experiences of exploitation. However, these disturbances are small enough to ensure that no agent's effective labour constraint is violated at any point during the capital constrained phase of the simulation and they are such as to ensure that labour market equilibrium can continue to hold. At every t agents' optimal x_t^ν, y_t^ν, and δ_t^ν are determined as specified in section 4.

Consumption behaviour is determined as in the previous model: at every t, for each $\nu \in \mathcal{N}$, consumption is a portion h_t^ν of ν's income, $h_t^\nu \left(\pi_t \omega_{t-1}^\nu + \widehat{w}_t \Lambda_t^\nu\right)$, where h_t^ν is a function of ν's wealth as described above. Thus, by construction, in this economy parts (b)-(d) of Definition 1 continue to hold.

This extension is meant to be only a first (and admittedly naive) step into the exploration of the kind of broad heterogeneity in agent behaviour that is common in agent-based models (ABM), and it introduces some interesting patterns in the aggregate accumulation rate and in the dynamics of exploitation. The accumulation rate exhibits a rather irregular—and often non-monotonic—pattern reflecting the interaction of agents' consumption behaviour and their willingness to supply labour, ℓ^ν. Although they do not necessarily work up to their endowment, propertyless agents always experience the highest exploitation intensity, but in scenarios where some high-skilled agents possess low ℓ^ν, or wealth is redistributed through taxes to agents with low ℓ^ν, agents may experience lower levels of exploitation intensity compared with the benchmark simulations in the paper.[24] Overall, this extension introduces a much wider range of experiences of exploitation and patterns of accumulation. However, these interesting additions do not change the impact of wealth taxes or the potential trade-offs faced in achieving wealth equality or the end of exploitative social relations.

[24]These simulations use most of the same parameters as the simulations in sections 5-8, and similar to earlier simulations, γ_t^ε begins near 0.06 but begins to decline as agents accumulate and the simulation eventually becomes labour constrained. This pattern largely holds under different tax schemes, except the speed of the decline in γ_t^ε increases as wealth is redistributed.

11 Conclusions

This paper shows that, contrary to the received wisdom, a notion of exploitation exists that is logically coherent, well-defined, and firmly anchored to empirical data. Exploitation can be defined both at the aggregate and at the individual level by means of an *exploitation index* which measures an agent's effective labour per unit of income received. For each individual, this index is a clearly defined magnitude that can be measured based on available empirical data, and its distribution can be analysed with the standard tools of the theory of inequality measurement. Further, the notion of exploitation is normatively relevant, and the analysis of the distribution of the exploitation index yields distinctive insights on the injustices that characterise advanced economies and on the effects of redistributive policies. In short, the news of the death of exploitation theory is greatly exaggerated.

In closing this paper, it is worth mentioning some open questions. In our economies, accumulation eventually drives exploitation and profits to zero: after settling on a growth path that leads to overaccumulation, the economy becomes labour constrained. This seems unrealistic, and exploitation is a persistent feature of capitalist economies. What are the mechanisms that guarantee the persistence of (capital scarcity and) exploitation? And, relatedly, what is the relation between the persistence of exploitation and the cyclical and crisis-prone dynamics of capitalist economies?

One interesting extension of our model would be to consider endogenous consumption dynamics. In our model, agents consume in proportion to the amount of effective labour they supply, and such proportion is fixed and exogenously given. Although we do analyse some versions of the model with alternative behavioural assumptions on consumption choices (see section 10.5), it would definitely be interesting to explore heterogeneous consumption dynamics further, and to analyse consumption inequality jointly with the distribution of inequality and wealth. In our computational framework, possible implementations may rely on the behavioural insights about peer effects and social norms (see, among others, Frank et al. [8]).

Another interesting extension of our analysis would be to explore the role that technological progress may play in guaranteeing the persistence of exploitation. The role of technical change in creating labour unemployment to maintain exploitative social relations has been stressed by a large number of authors (for example, see Skillman [22]), and it is central in Marx's own analysis of the "industrial reserve army of labour". Cogliano et al [5] have analysed a model with exogenous labour-saving technical change, yet it would be interesting to examine a broader range of technical innovations and their interaction with agents' exploitation status.

Finally, we have conducted our analysis focusing on general equilibria with optimising agents. This is an important theoretical benchmark for our normative analysis, and it allows us to compare our results to the relevant literature. Nonetheless, with the richest 5% of the population holding around 70% of the wealth and employing a mass of propertyless agents (50% of the population), and the issues of power and class solidarity that this polarised wealth distribution raises, it seems natural to analyse a more complex model for the determination of the key distributive variables.

Moreover, there is growing empirical and experimental evidence questioning the assumptions underlying standard optimising behaviour. A natural extension of our approach would be to fully exploit the computational techniques and study exploitation by means of an ABM, which would allow us to introduce further sources of heterogeneity in behaviour and/or individual characteristics, and possibly to model disequilibrium dynamics.

We have taken some preliminary steps in this direction in the simulations reported in section 10.5, but it would be interesting, for example, to provide a more complex, and realistic, analysis of the process of creation of skills. In our model, the education system is a black box: the growth rate of skills is determined by the amount of educational investment of an individual. In a more general computational, or ABM approach, it would be interesting to model explicitly the process of production of skills and human capital by means of physical goods *and* skills. This might have relevant implications for the distributional effects of education and in general for the dynamics of exploitation in the economy.

Although it does not provide answers to these questions, this paper provides a conceptual and analytical framework to tackle them.

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