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TIME-INCONSISTENT OUTPUT SUBSIDY/TAX POLICIES IN FREE-ENTRY MIXED MARKETS*

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Abstract

This study considers time-inconsistent output subsidy/tax policies in free-entry mixed markets under different competition modes. In a committed regime where the subsidy is determined before entry, the optimal rate is zero in either Cournot or Stackelberg game with private leadership, while it is negative in Stackelberg game with public leadership. In a non-committed regime where the subsidy is determined after entry, the optimal rate is always positive. We also show that private leadership is the best for social welfare regardless of the timing of subsidy policies, but public leadership might be equilibrium if the timing of entry decisions is endogenous.

Keywords: free-entry mixed market, committed policy, non-committed policy, output subsidy
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I. *Introduction*

It is a well-established fact in economic theory and practice that among many alternative policy measures of a government, the public institution and output subsidy/tax policies are very important instruments to enhance the economic efficiency. The government can improve social welfare by holding public enterprises directly,[1] while it can also indirectly influence activities of firms in the market by providing output subsidy (or imposing tax).[2]

The literature on mixed oligopolies with public and private firms in an imperfect competition market has provided many policy considerations on output subsidy to draw practical policy implications. For example, White (1996), Pal and White (1998) and Poyago-Theotoky (2001) showed that output subsidy policy can achieve the first-best outcomes irrespective of the ownership of a public firm unless there exists political cost.[3] Tomaru and Saito (2010) considered an endogenous timing game, Matsumura and Tomaru (2013) studied the excess burden of taxation and Cato and Matsumura (2013, 2015) examined a long-run analysis of output subsidy and entry tax along with trade policy in free-entry markets. Recently, Lee et al. (2017) compared the different effects between output and R&D subsidies as a policy instrument in mixed oligopolies and showed that social welfare is higher under the output subsidy. Moreover, Haruna and Goel (2017) emphasized that government might have to choose to subsidize output rather than R&D in the case that R&D subsidy is relatively more complicated than output subsidy.

On the other hand, public firm has played a leading role in the mixed industries for many years. For example, many researches believed that lending by public financial institutions had a pump-priming effect on private bank lending.[4] Wang and Mukherjee (2012) and Wang and Lee (2013) showed that public leadership where the public firm takes a leader position benefits social welfare under undesirable competition in free entry market, while Gelves and Heywood (2013) found that mergers of public and private firms can improve welfare under public leadership. However, Pal (1998) and Matsumura and Ogawa (2010) emphasized that mixed oligopoly model with private leadership where the public firm takes a follower position in output competition is more robust and risk-dominant. Ino and Matsumura (2010) also examined the role of public firm in regulated and free-entry industries and showed that private leadership provides the highest social welfare in free-entry mixed markets. In contrast, public leadership provides the lowest level of social welfare.[5]

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[1] In early discussion, De Fraja and Delbono (1989), Cremer, et al. (1989, 1991) and Delbono and Denicolo (1993) showed that a public firm can represent a useful instrument of the government in a mixed market where the public firm coexists with private firms, which makes the social welfare to be higher than that in a private market.

[2] Tax-subsidy policies are widely observed as industry policies in many countries. See Itoh et al (1991). .

[3] It is showed that privatization does not affect welfare regardless of time structure, competition mode, the number of firms, product differentiation, and the degree of privatization under the optimal output subsidy. This has been continuously discussed by Pal and White (1998), Poyago-Theotoky (2001), Hashimzade, et al. (2007) and Matsumura and Okumura (2013).

[4] Bose et al. (2014) emphasized that the public sector worldwide has an important role in lending markets. For example, public financing occupied an important position in Japan's financial markets for over 60 years since 1940s. See Lin and Matsumura (2017).

[5] The role of leadership and its policy implications are extensively examined by Etro (2006, 2007) and Ino and Matsumura (2012).

In this paper, we integrate these two research streams on mixed oligopolies and examine the output subsidy policy and the role of public firm in free-entry mixed markets. Further, we consider the timing of the subsidy policy, which can be decided by the government before or after observing the entry behaviors of private firms, and examine how the government's commitment to the policy affects the welfare under different competition modes. Then, we can emphasize that the ability of a government to commit credibly to the subsidy has significant policy implications in the process of policy-making. However, if the government can not commit credibly, firms have strategic incentives to induce different welfare consequences because the governement has an ex-post possibility to ratchet up the policy.[6]

Specifically, we compare committed and non-committed subsidy regimes under different competition modes between Cournot and Stackelberg games. In the former regime with subsidization-then-entry order, the government chooses the output subsidy level before private firms enter the market. Whereas in the latter regime with entry-then-subsidization order, the government chooses the output subsidy level after private firms enter the market. Thus, under the non-committed regime, when the subsidy is chosen firms would expect the regulator to change it after they have determined their entry into the market. We investigate this time-inconsistency problem in deciding subsidy policy in the free-entry mixed markets.[7]

Our analysis reveals the existence of time-inconsistent output subsidy policies between committed and non-committed subsidy regimes, and the optimal subsidy policy is dependent on the market structure. In particular, we show that the optimal output subsidy rate is zero under the committed regime in both Cournot and Stackelberg games with private leadership, while it is negative in Stackelberg game with public leadership. However, in the non-committed regime where the subsidy is not determined before entry, the optimal rate is always positive. We also show that private leadership is the best for social welfare regardless of the timing of output subsidy policy. Finally, if the timing of decisions made by the firms is endogenous, there exist an equilibrium with public leadership under the non-committed subsidy.

Our findings complement the previous results in free entry mixed markets where the excessive entry exists. Under the committed subsidy regime, there exists a trade-off between over-entry and under-production. In the case that the public firm is a follower in both Cournot and Stackelberg games with private leadership, the output subsidy will not only increase the number of entering firms but increase total market outputs. Thus, the welfare effect of the subsidy is nullified when the market demand is linear (Cato and Matsumura, 2017; Xu and Lee, 2018). It justifies the robustness of previous results in Cato and Matsumura (2017) who shows that output subsidy/tax policies are redundant, while the subsidy is negative in Stackelberg game with public leadership. However, we find that the output subsidies are always positive irrespective of competition mode in the non-committed regime. Further, in the case that the public firm is a leader, the output tax will decrease the number of entering firms, but incurs the loss of total outputs. Thus, private leadership is the best for social welfare. This supports the findings in Ino and Matsumura (2010) who shows that private leadership provides the highest

[6] For related discussions, see Lee (1999), Davidson and Mukherjee (2007) and Ino and Matsumura (2012) for welfare and policy implications for free-entry private markets. Recent works include Cato and Matsumura (2013, 2017), Xu et al. (2017), Leal et al. (2018), Lee et al. (2018) and Xu and Lee (2018) for free-entry mixed oligopolies.

[7] For more discussions on the practical policy implications of the time-inconsistency problem in the mixed markets, see Xu et al. (2017), Garcia et al. (2018), Leal et al. (2018), Lee et al. (2018) and Xu and Lee (2018).

social welfare in the absence of subsidy/tax policies.

On the other hand, under the non-committed subsidy regime, under-production effect prevails in imperfect competition. It implies that output subsidies are always positive irrespective of competition mode. Further, private leadership is the best for social welfare regardless of the timing of output subsidy/tax policies. However, if the timing of decisions made by the firms is endogenous under the non-committed regime, public leadership is an equilibrium, which causes welfare loss.

The structure of this paper is as follows. In section 2, we introduce the basic model of free-entry mixed market with output subsidy. In sections 3, 4 and 5, we analyze the committed and non-committed subsidy policies in Cournot and Stackelberg games with public leadership or private leadership, respectively. In section 6, we provide comparisons between the committed and non-committed subsidy policy regimes. In section 7, we conclude our paper.

II. *The Model*

We consider a mixed oligopoly in which $n+1$ firms produce homogeneous commodities. Firm 0 is a public firm fully owned by a government and firm i ($i=1,2,..., n$) is a private firm which is completely characterized by private property rights. The inverse demand function is denoted by $P=1-Q$, where P is market price and Q is total market outputs, i.e., $Q=q^0+\sum_{i=1}^{n} q^i$ where q^0 is the output of firm 0 and q^i is the output of firm i. Then, consumer surplus can be denoted as $CS=Q^2/2$.

The cost function of the firms is identical and quadratic, $C(q^i)=(q^i)^2/2+F^2$, where F^2 is the entry cost of each firm. According to Matsumura and Kanda (2005), the fixed cost of public firm is a sunk cost to the government, therefore the number of entering firms (or the fixed cost of private firm) matters for its policy decisions. The number of private firms is determined at the point where the profit of the private firms is equal to zero in a free-entry mixed market.

We also assume that the government can provide a uniform output subsidy with the rate of s. Note that s can be positive (subsidy) or negative (tax). The resulting total subsidy is given as $S=sQ$. Then, the profit of firm is

$$\pi^j=Pq^j+sq^j-(q^j)^2/2-F^2 \text{ for } j=0,..., n\ , \tag{1}$$

We will define social welfare as the sum of consumer surplus and firms' profit minus total subsidy, which is given by:

$$W=CS+\pi^0+\sum_{i=1}^{n}\pi^i-sQ \tag{2}$$

Note that the subsidies are financed from taxpayers in a lump-sum manner, so that they do not directly influence welfare.

Finally, the firms' objective functions are subject to their ownership structures. We assume that the private firm maximizes its profits, while the public firm maximizes the social welfare.

In the following sections, we shall consider two alternative policy regimes of output subsidy policy, each featuring a three-stage game between the government and owners of private firms. In particular, we examine the properties of different equilibria between government's committed and non-committed subsidy policies. In the former case, the government chooses the output subsidy in the first stage and then private firms, taking the subsidy rate as given, enter the market until they get zero economic profit. In the latter case, private firms enter the market until they get zero economic profit in the first stage and then the government decides its output subsidy level in the second stage. In the last stage, all firms choose outputs simultaneously or sequentially, depending on the market structure. In the following analysis, we examine three different market structures in the output production stage: Cournot and Stackelberg public leadership or private leadership (public followership) competitions. We solve the subgame perfect Nash equilibrium of these games by backward induction.

III. *Simultaneous-Move Game*

1. Committed Policy

In this game, the government chooses the output subsidy before the private firms enter the market. Note that the government and public firm has the same objective function in a mixed market. Therefore, in the third stage, the public firm chooses q^0 to maximize social welfare while the private firms choose q^i to maximize their profit functions simultaneously and independently. The first order conditions yield the resulting outputs[8]:

$$q^0=\frac{2-ns}{4+n} \tag{3}$$

$$q^i=\frac{1+2s}{4+n} \tag{4}$$

$$Q=\frac{2+n(1+s)}{4+n} \tag{5}$$

In the second stage, the private firms enter the market until they get zero profit. The resulting number of private firms is given as:

$$n=\frac{\sqrt{6}\,(1+2s)-8F}{2F} \tag{6}$$

Note that as the output subsidy increases, the number of firms entering the market increases, i.e., $\partial n/\partial s>0$. It is reasonable, since the government provides output subsidy to firms, firms will prefer to enter the market.

Assuming that $n>0$, we can obtain the equilibrium outputs of public firm and private firms, respectively:

$$q^0=2\sqrt{\frac{2}{3}F}-s \tag{7}$$

[8] In this study, all of the second-order conditions are satisfied.

$$q^i=\sqrt{\frac{2}{3}}F \tag{8}$$

$$Q=1-2\sqrt{\frac{2}{3}}F+s \tag{9}$$

In the case that $q^0>0$, we have (i) $\partial q^0/\partial s<0$. (ii) $\partial q^i/\partial s=0$. (iii) $\partial Q/\partial s>0$. That is, the output subsidy decreases the output of public firm but does not change the output of private firm, while it increases the number of entering firms and thus increases the total market output.[9]

In the first stage, the government chooses the output subsidy that maximizes social welfare, defined in (2). Substituting the equilibrium outcomes, obtained in the previous stages, into W and taking a differentiation with respect to s, yields:

Lemma 1. $s_B^C=0$ *when* $0<F<\frac{\sqrt{\frac{3}{2}}}{4}$.

where we employ the subscript "B" to denote the equilibrium (**before** entry) under the committed subsidy policy. It represents that social welfare will be maximized without output subsidy under the committed subsidy regime. It also implies that if the government provides output subsidy policy, it will distort the number of firms as well as total output production and consequently reduce social welfare.

The resulting number of private firms is given as:

$$n_B^C=\frac{\sqrt{6}-8F}{2F} \tag{10}$$

Note that $n_B^C>0$ when $0<F<\frac{\sqrt{\frac{3}{2}}}{4}$.

The equilibrium outputs are as follows:

$$q_B^{0C}=2\sqrt{\frac{2}{3}}F \tag{11}$$

$$q_B^{iC}=\sqrt{\frac{2}{3}}F \tag{12}$$

$$Q_B^C=\frac{1}{3}(3-2\sqrt{6}F) \tag{13}$$

Finally, the resulting welfare is as follows:

$$W_B^C=\frac{1}{6}(3-4\sqrt{6}F+10F^2) \tag{14}$$

[9] These results are well known in the literature. For example, Lee (1999) considered an environmental tax in free-entry private markets while Cato and Matsumura (2017) considered an output tax in free-entry mixed markets.

2. Non-committed Policy

In this game, the government chooses the output subsidy after the private firms enter the market. The last stage in output choice is the same as in the committed subsidy case, where the equilibrium outputs of the firms are derived in (3) and (4).

In the second stage, the government chooses the welfare maximizing output subsidy level. The first order condition of this problem yields the following optimal output subsidy:

$$s=\frac{1}{2+n} \tag{15}$$

Note that output subsidy is decreasing in the number of private firms, i.e., $\partial s/\partial n<0$.

Then the resulting outputs are given as follows:

$$q^0=\frac{1}{2+n} \tag{16}$$

$$q^i=\frac{1}{2+n} \tag{17}$$

$$Q=\frac{1+n}{2+n} \tag{18}$$

Note, that (i) $\frac{\partial q^0}{\partial n}=\frac{\partial q^i}{\partial n}<0$; (ii) $\frac{\partial Q}{\partial n}>0$. That is, the output of each firm is decreasing in the number of private firms while the total outputs are increasing in the number of private firms. Thus, when the number of private firms increases, the business-stealing effect (between the firms) reduces the output of existing firm but the scale effect (from the number of private firms) outweighs business-stealing effect, which results in the increase of the total outputs.

In the first stage, private firms enter the market until they get zero profit. Therefore, substituting the resulting outputs and output subsidy into (1), we can obtain the profit of private firms. Hereafter, we equalize obtained profit function to zero and solve it for *n*, in order to get optimal number of private firms in the market:

$$n_A^C=\frac{\sqrt{6}-4F}{2F} \tag{19}$$

where the subscript "A" denotes the equilibrium (**after** entry) under the non-committed subsidy policy. Note that $n_A^C>0$ when $0<F<\frac{\sqrt{\frac{3}{2}}}{2}$.

Lemma 2. $s_A^C=\sqrt{\frac{2}{3}}F>0$ *when* $0<F<\frac{\sqrt{\frac{3}{2}}}{2}$.

It represents that social welfare will be maximized with positive output subsidy in a non-committed subsidy policy. Here, the government decided the output subsidy after observing the number of private firms, which can be treated as a fixed number of private firms. This market structure causes under-production because of imperfect competition. Thus, the government has a strong incentive to use an output subsidy policy to achieve the optimal social welfare. Hence, when the government provides an output subsidy policy after the firms enter the market, it will increase total production and social welfare[10].

The resulting outputs are as follows:

$$q_A^{0C}=\sqrt{\frac{2}{3}}F \tag{20}$$

$$q_A^{iC}=\sqrt{\frac{2}{3}}F \tag{21}$$

$$Q_A^{C}=1-\sqrt{\frac{2}{3}}F \tag{22}$$

The resulting welfare is given as:

$$W_A^{C}=\frac{1}{2}-2\sqrt{\frac{2}{3}}F+F^2 \tag{23}$$

3. Comparisons

We compare the committed and non-committed policies.

Proposition 1. *Suppose that* $0<F<\frac{\sqrt{\frac{3}{2}}}{4}$. *Then, we have:*
(i) $q_B^{0C}>q_A^{0C}$. *(ii)* $q_B^{iC}=q_A^{iC}$. *(iii)* $Q_B^{C}<Q_A^{C}$. *(iv)* $n_B^{C}<n_A^{C}$. *(v)* $0=s_B^{C}<s_A^{C}$. *(vi)* $W_B^{C}>W_A^{C}$.

It states that the output of public firm of committed subsidy is higher than that of non-committed subsidy while the outputs of private firms are equal in both subsidy regimes. Thus, the total output of committed subsidy is lower than that of non-committed subsidy. Note that the output subsidy effect can be offset by the effect on the number of private firms which is endogenously chosen in committed subsidy case. This induces a lower output subsidy but a larger number of private firms and total outputs than that in non-committed subsidy. Moreover, a decrease in subsidy stimulates the production of the public firm while the output of private firms is independent with output subsidy because of zero profits. Hence, the social welfare of committed subsidy is higher than that of non-committed subsidy.

IV. *Sequential-Move Game with Public Leadership*

1. Committed Policy

In this game, first the public firm and then the private firms choose their output quantity

[10] These results are also well known in the literature of mixed oligopolies. For example, see, Poyago − Theotoky (2001).

levels sequentially. Further, in the second stage private firms enter the market. Then, the government chooses optimal output subsidy rate in the first stage.

The first-order condition of private firms to maximize profit function given in (1) provides the reaction function, which we put into (2) and maximization procedure yields the equilibrium outputs in the third stage and resulting number of private firms in the second stage as following:

$$q^0=\frac{1}{6}(3+\sqrt{6}F+3s-\sqrt{9(1+s)^2-42F^2+6\sqrt{6}F(3s-1)}) \tag{24}$$

$$q^i=\sqrt{\frac{2}{3}}F \tag{25}$$

$$Q=1-2\sqrt{\frac{2}{3}}F+s \tag{26}$$

$$n=\frac{-10F+\sqrt{6}(1+s)+\sqrt{-28F^2+6(1+s)^2+4\sqrt{6}F(-1+3s)}}{4F} \tag{27}$$

Note that the output subsidy increases the number of entering firms i.e., $\partial n/\partial s>0$.

In the first stage, government chooses the optimal output subsidy level:

Lemma 3. $s_B^L=\frac{1}{72}(-45-36\sqrt{6}F)-\frac{1}{2}\sqrt{a}+\frac{1}{2}\sqrt{b}<0$ *when* $0.27<F<0.306$.

Where "a" and "b" are parts of s_B^L function, which composed by F and described in Appendix. It implies that the output subsidy is negative, which is used as an output tax in the committed subsidy with public leadership. When the public firm is a leader in a free-entry market, it has a strong incentive to reduce its output, which will induce private firms to produce more outputs and more entries. Thus, the government uses an output tax to avoid the loss of social welfare. This results a larger cost to entering firms thus causes a lower number of private firms.

Substituting the optimal output subsidy into (27) yields the equilibrium number of private firms:

$$n_B^L=\frac{1}{32F}(3\sqrt{6}-4\sqrt{6a}+4\sqrt{6b}-104F+\sqrt{2\gamma}) \tag{28}$$

Note that $n_B^L>0$ when $0.27<F<0.306$.

Then the resulting equilibrium outcomes are given as:

$$q_B^{0L}=\frac{1}{48}(9-12\sqrt{a}+12\sqrt{b}-4\sqrt{6}F-\sqrt{3\gamma}) \tag{29}$$

$$q_B^{iL}=\sqrt{\frac{2}{3}}F \tag{30}$$

$$Q_B^L=\frac{3}{8}-\frac{\sqrt{a}}{2}+\frac{\sqrt{b}}{2}-\frac{7F}{\sqrt{6}} \tag{31}$$

$$W_B^L=\frac{1}{12}(18F^2-\frac{1}{64}(-3+4\sqrt{a}-4\sqrt{b}+4\sqrt{6}F)(69+36\sqrt{a}-36\sqrt{b}+36\sqrt{6}F+\sqrt{3}$$
$$\sqrt{(27+48a+72\sqrt{b}+48b-440\sqrt{6}F+96\sqrt{6b}F-1760F^2-24\sqrt{a}(3+4\sqrt{b}+4\sqrt{6}F)})$$
$$-\frac{1}{8}F(32\sqrt{6}+3\sqrt{2}\sqrt{(27+48a+48b-440\sqrt{6}F-1760F^2+24\sqrt{b}(3+4\sqrt{6}F)}$$
$$-24\sqrt{a}(3+4\sqrt{b}+4\sqrt{6}F))))\quad(32)$$

2. Non-committed Policy

The last stage in output choice is the same as in the previous committed subsidy case, where the equilibrium outputs of the firms are derived in (24) and (25). Then, the government chooses optimal output subsidy rate in the second stage. Finally, the private firms enter the market in the first stage.

In the second stage, the government chooses the optimal output subsidy from the differentiation of W in (2) with respect to s, which yields:

$$s=\frac{1}{2+n}\quad(33)$$

Note that the output subsidy is decreasing in the number of private firms, i.e., $\partial s/\partial n<0$. In the first stage, by substituting the resulting outputs and output subsidy we can obtain following number of private firms in the market:

$$n_A^L=\frac{\sqrt{6}-4F}{2F}\quad(34)$$

where $n_A^L>0$ when $0<F<\frac{\sqrt{\frac{3}{2}}}{2}$.

Lemma 4. $s_A^L=\sqrt{\frac{2}{3}}F>0$ *when* $0<F<\frac{\sqrt{\frac{3}{2}}}{2}$.

It implies that the output subsidy of non-committed regime is positive under public leadership, which is similar with that in Cournot model (Simultaneous-move game).

The equilibrium outcomes are given as:

$$q_A^{0L}=\sqrt{\frac{2}{3}}F\quad(35)$$

$$q_A^{iL}=\sqrt{\frac{2}{3}}F\quad(36)$$

$$Q_A^L=1-\sqrt{\frac{2}{3}}F\quad(37)$$

$$W_A^L = \frac{1}{2} - 2\sqrt{\frac{2}{3}}F + F^2 \tag{38}$$

3. Comparisons

Proposition 2. *Suppose* $0.27 < F < 0.306$. *Then, we have:*

(i) $q_B^{0L} > q_A^{0L}$. *(ii)* $q_B^{iL} = q_A^{iL}$. *(iii)* $Q_B^L < Q_A^L$. *(iv)* $n_B^L < n_A^L$. *(v)* $s_B^L < 0 < s_A^L$. *(vi)* $W_B^L > W_A^L$

We obtain the same results with that in Cournot model, which implies that the output of public firm of committed subsidy is higher than that of non-committed subsidy. The outputs of private firms are same in both cases, but the total output as well as the number of private firms is lower in committed subsidy case. Moreover, the optimal output subsidy of committed subsidy is lower than that of non-committed subsidy. However, the social welfare of committed subsidy is higher than that of non-committed subsidy.

V. *Sequential-Move Game with Private Leadership*

1. Committed Policy

In this game, first the private firm and then the public firm choose their output quantity levels sequentially. After calculating the equilibrium outputs and number of private firms respectively, in the third and second stage, we can obtain the optimal output subsidy in the first stage.

Lemma 5. $s_B^F = 0$ *when* $0 < F < \frac{1}{3}$.

It implies that no output subsidy maximizes the social welfare under the committed subsidy with a private leadership. When the public firm is a follower, it decides its behavior after observing the private firms' action in the market. Thus, each firm's behavior in the market gets closer to marginal cost pricing. Similar with that in Cournot model, zero output subsidy/tax is optimal.

Then, the equilibrium outcomes are as follows:

$$n_B^F = -3 + \frac{1}{F} \tag{39}$$

where $n_B^F > 0$ when $0 < F < \frac{1}{3}$.

The equilibrium outcomes are given as:

$$q_B^{0F} = \frac{3F}{2} \tag{40}$$

$$q_B^{iF} = F \tag{41}$$

$$Q_B^F = 1 - \frac{3F}{2} \tag{42}$$

$$W_B^F = \frac{1}{4}(2 - 6F + 5F^2) \tag{43}$$

2. Non-committed Policy

The last stage in output choice is the same as in the previous committed subsidy case; and then we obtain the optimal output subsidy in the second stage.

Lemma 6. $s_A^F = \frac{F}{2} > 0$ *when* $0 < F < \frac{1}{2}$.

Then, substituting the results into (1) provides the number of private firms in the first stage:

$$n_A^F = \frac{1 - 2F}{F} \tag{44}$$

Note that $n_A^F > 0$ when $0 < F < \frac{1}{2}$.

Then, the equilibrium outcomes are given as:

$$q_A^{0F} = F \tag{45}$$

$$q_A^{iF} = F \tag{46}$$

$$Q_A^F = 1 - F \tag{47}$$

$$W_A^F = \frac{1}{2} - \frac{3F}{2} + F^2 \tag{48}$$

3. Comparisons

Proposition 3. *Suppose* $0 < F < \frac{1}{3}$. *Then, we have:*

(i) $q_B^{0F} > q_A^{0F}$. *(ii)* $q_B^{iF} = q_A^{iF}$. *(iii)* $Q_B^F < Q_A^F$. *(iv)* $n_B^F < n_A^F$. *(v)* $0 = s_B^F < s_A^F$. *(vi)* $W_B^F > W_A^F$

We obtain the same results that the optimal output subsidy of committed subsidy is lower than that of non-committed subsidy, but the social welfare of committed subsidy is higher than that of non-committed subsidy.

VI. *Comparison and Discussion*

1. Comparison

First, we compare the results in the committed subsidy regime and obtained the followings:

Proposition 4. *Suppose* $0.27<F<0.306$. *Then, we have:*
(i) $q_B^{0F}<q_B^{0L}<q_B^{0C}$; *(ii)* $q_B^{iL}=q_B^{iC}<q_B^{iF}$; *(iii)* $Q_B^L<Q_B^C<Q_B^F$. *(iv)* $n_B^L<n_B^C<n_B^F$.
(v) $s_B^L<s_B^F=s_B^C=0$. *(vi)* $W_B^L<W_B^C<W_B^F$

It states that the Cournot model provides the highest output of public firm. However, the private leadership provides the highest output of private firms, so is the total output and the number of private firms. These results are consistent to Ino and Matsumura (2010). Further, the output subsidy under public leadership will be used as an output tax and which is lower than that of Cournot and private leadership where there are no output subsidies. Thus, the private leadership provides the best social welfare.

Second, we compare the results in the non-committed subsidy regime and obtained the followings:

Proposition 5. *Suppose* $0<F<\frac{1}{2}$. *Then, we have:*
(i) $q_A^{0L}=q_A^{0C}<q_A^{0F}$; *(ii)* $q_A^{iL}=q_A^{iC}<q_A^{iF}$; *(iii)* $Q_A^F<Q_A^L=Q_A^C$. *(iv)* $n_A^F<n_A^L=n_A^C$.
(v) $0<s_A^F<s_A^L=s_A^C$. *(vi)* $W_A^L=W_A^C<W_A^F$

It states that the private leadership provides the highest output of both public and private firms, however, it provides the lowest total outputs and the number of private firms. Thus, private leadership provides the lowest output subsidy while the public leadership and the Cournot model have the same output subsidy. It is sharply contrast to Proposition 4. Moreover, the private leadership provides a higher social welfare than that of public leadership and Cournot model which provide the same social welfares.

Proposition 6. *The equilibrium outcomes under the committed subsidization with public followership is the best to the society when* $0<F<\frac{1}{3}$.

Proposition 3 and 6 imply that the government will commit the optimal subsidy policy before the firms enter the market if has an ability to provide a credibly commitment and always let the public firm act as a follower without providing any output subsidy or tax to get the best outcomes. It supports the findings in Ino and Matsumura (2010) who didn't consider the subsidy policies. However, due to the political reason, the regulator might not commit credibly to the stringency of the policy instrument. Then, firms have strategic incentives because they expect that the government has an ex-post possibility to ratchet up regulation. In that case, Proposition 5 indicates that the government will provide a positive subsidy and after the firms enter the market and let the public firm act as a follower, if it is strategically feasible, to get the best outcomes.

2. Endogenous Choice Game

We now examine the extensive case where the firms choose the competition mode endogenously and discuss its policy implications. For this, we imagine the following endogenous timing game where both a public firm and private firms decide which to act as a "leader" or a "follower" before the private firms decide to enter the market under the committed or non-committed regime, respectively. Then, if both firms choose the same roles, the equilibrium is a simultaneous-move game. Otherwise, the equilibrium is a sequential move game. Table 1 provides the payoff matrix of the game.

TABLE 1. PAYOFF MATRIX OF THE ENDOGENOUS CHOICE GAME (k=A, B)

Firm 0 \ Firm i	Leader	Follower
Leader	W_k^C, π_k^{iC}	W_k^L, π_k^{iF}
Follower	W_k^F, π_k^{iL}	W_k^C, π_k^{iC}

Then, from Proposition 4 and 5, we can see that choosing to be a follower is the dominant strategy for the public firm irrespective of the commitment regime. That is, if a private firm chooses to be a leader, a public firm chooses to be a follower since $W_k^C < W_k^F$ where k=A, B and if a private firm chooses to be a follower, a public firm also chooses to be a follower since $W_B^L < W_B^C$ and $W_A^L = W_A^C$. Further, the private firms will get zero profit at any market competition and thus they are indifferent at equilibrium. Hence, we can conclude that an equilibrium with public leadership does not occur in the committed subsidy, which might yield worse outcome, while public leadership might exist in the non-committed subsidy.

Proposition 7. *If the timing of decisions made by the firms is endogenous, there exist an equilibrium with public leadership in the non-committed subsidy.*

VII. *Conclusion*

This paper investigated time-inconsistent output subsidy/tax policies in free-entry mixed markets and compared the equilibrium outcomes of committed and non-committed subsidy regimes, respectively, under different competition modes. We found that the output subsidy does not affect the social welfare in both Cournot and Stackelberg games with private leadership, while it is used as an output tax under the Stackelberg game with public leadership in the case of committed subsidy. However, it is always used as an output subsidy under the non-committed case. We also found that the output subsidy in the committed case is always lower than that in the non-committed case. Nevertheless, committed regime provides higher social welfare. We also showed that private leadership is the best for social welfare regardless of the timing of output subsidy/tax policies. Finally, if the timing of decisions made by the firms is endogenous, there exist an equilibrium with public leadership under the non-committed subsidy.

These results complement previous literature such as Cato and Matsumura (2017) and Xu and Lee (2018) that argues the output tax or subsidy is superfluous in the committed regime under Cournot game when a public firm is fully owned by a government. These results also suggest that policymakers have to implement liberalization policy before firms enter the market and induce the public institution (organization) to be a follower. The consequences of implementing these policies in a combined manner are expected to be superior for society.

We believe that our results are robust and its implications are useful in the real world. But, there are still remain many limitations mainly because of its model-specific assumptions. However, extending this model with other influential parameters remains for the future researches. In particular, the analysis with differentiated products market and/or foreign penetration can be promising topics for future research.

Appendix: γ, a and b.

$$\gamma=48a^2+3(3+4b)^2+8\sqrt{6}(-55+12b)F-1760F^2-24a(3+4b+4\sqrt{6}F)$$

$$a=\frac{(297+2952\sqrt{6}F+11808F^2)(H+\sqrt{L})^{1/3}+22^{2/3}(H+\sqrt{L})^{2/3}-12962^{1/3}(2F(F(8F(37\sqrt{6}+155F)-81)-15\sqrt{6})-9)}{1296(H+\sqrt{L})^{1/3}}$$

$$b=\frac{62208\sqrt{6}F^3+32F^2(4212+41\sqrt{M})+4\sqrt{6}F(2511+82\sqrt{M})+\sqrt{M}(33+72Y)-243}{72\sqrt{M}}$$

Where

$$M=\frac{(297+2952\sqrt{6}F+11808F^2)(H+\sqrt{L})^{1/3}+22^{2/3}(H+\sqrt{L})^{2/3}-12962^{1/3}(2F(F(8F(37\sqrt{6}+155F)-81)-15\sqrt{6})-9)}{(H+\sqrt{L})^{1/3}}$$

$$Y=\frac{648(2F(F(8F(37\sqrt{6}+155F)-81)-15\sqrt{6})-9)-2^{1/3}(H+\sqrt{L})^{2/3}}{3242^{2/3}(H+\sqrt{L})^{1/3}}$$

$$H=11664(27-837\sqrt{6}F+6264F^2+14214\sqrt{6}F^3-56736F^4-132672\sqrt{6}F^5-296128F^6)$$

$$L=136048896((592\sqrt{6}F^3+2480F^4-9-30\sqrt{6}F-162F^2)^3+(837\sqrt{6}F-27-6264F^2-14214\sqrt{6}F^3+56736F^4+132672\sqrt{6}F^5+296128F^6)^2)$$

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