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Long-Run Mild Deflation Under Fiscal Unsustainability in Japan

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Abstract: A macroeconomic policy debate has been ongoing in Japan for over the past two decades, with one side proposing drastic fiscal reforms to avoid hyperinflation and the other recommending expansionary policies to escape from a liquidity trap. However, neither side has been able to explain why mild deflation has continued for such a long time, despite primary budget deficits and unprecedented monetary expansion. This paper presents an alternative theory, arguing that fiscal sustainability will be restored in the future not as a result of drastic fiscal reforms, hyperinflation, or continuous mild inflation, but largely through a one-off surge in the price level, such that the price level becomes several times higher than before. Such a price surge is considered a rare event accompanied by catastrophic endowment shocks in the following years. Within this framework, mild deflation coexists with fiscal unsustainability until this sharp surge in the price level occurs.

Key words: the fiscal theory of the price level, fiscal sustainability, mild deflation, price surges, yield curves.
JEL classification: E31, E41, E58, E63.

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1. Introduction

Bitter debates about Japanese macroeconomic policies have been ongoing for over the past two decades between fiscal reformers and demand-siders. On the one hand, citing the compelling evidence of fiscal unsustainability, the fiscal reformers have argued for drastic spending cuts and tax increases to avoid hyperinflation or sovereign default.[3] On the other hand, the demand-siders have interpreted mild deflation as an indicator of feeble aggregate demand and proposed maintaining fiscal and monetary expansions to escape from a liquidity trap.[4] Indeed, they have recommended that expansionary policies be continued to prevent interest rates rising even after escape from a liquidity trap. However, both sides have failed to explain why the mild deflation with near-zero interest rates, which began in the mid-1990s, has continued for such a long time in Japan. In contrast to the theories of the fiscal reformers, hyperinflation has not occurred despite the continuation of primary budget deficits, whereas the theories of the demand-siders have been confounded by the fact that unprecedented expansionary policies have not achieved mild inflation with low interest rates.

Ironically, each side has been able to pursue its own favorable prescription without fear of side effects while mild deflation has continued together with near-zero interest rates. In such a lukewarm macroeconomic environment, the fiscal reformers could easily hedge against hyperinflation without fear of severe deflation, whereas the demand-

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[3] Many papers, including Braun and Joines (2015) and Imrohoroglu et al. (2016, 2018), convincingly argue that fiscal sustainability in Japan is never achieved without any drastic tax increases or spending cuts. Armstrong and Okimoto (2016) survey the literature on fiscal sustainability in Japan.

[4] On the monetary side, Krugman (1998), Eggertsson and Woodford (2003), and Jung et al. (2005), among others, recommend that a central bank strongly commit to a zero-interest rate policy not only before, but also after an economy escapes from a liquidity trap. On the fiscal side, Sims (2016) proposes a fiscal stimulus to yield upward pressure on the price level, following the standard implications of the fiscal theory of the price level. Christiano et al. (2011), Woodford (2011), and others perceive a liquidity trap as a consequence of weak aggregate demand and demonstrate that the fiscal multiplier is much larger in a liquidity trap than during normal times. In the US context, Bianchi and Melosi (2017) demonstrate that the lack of deflation in the US economy at zero-interest rates can be explained by people's believing, with some probability, that aggressive fiscal policies will continue even after an escape from a liquidity trap. As Blanchard (2019) emphasizes, debt rollovers may be feasible even in an economy with much public debt when interest rates continue to be below growth rates. Applying this implication, Blanchard and Tashiro (2019) recommend even more aggressive fiscal policies for the current Japanese economy.

siders could simply bet against mild inflation without fear of sharp interest-rate hikes.

This paper presents an alternative theory in which fiscal unsustainability resulting from undisciplined fiscal policies is permitted temporarily or even persistently, in contrast to the case in standard monetary models, and in which fiscal sustainability will be restored at some point in the future not by drastic fiscal reforms, hyperinflation, or continuous mild inflation with low interest rates, but largely through a one-off price surge, to the extent that price rises to several times its previous level. In other words, in this scenario, a government will repay its own debt largely through a heavy devaluation of nominal public bonds. In the model, the price jump is considered a rare event with a probability of occurrence of less than 5% per year and it is accompanied by adverse, indeed, possibly catastrophic, impacts on endowments in the years that follow it. It is assumed in the model that a moderate fiscal reform is implemented only after such catastrophic shocks disappear completely. Within this framework, mild inflation is achieved only after such price surges and it is accompanied not by low but by relatively high interest rates.

This theoretical framework allows us to explain the above seemingly puzzling phenomena in a consistent manner. First, under undisciplined fiscal policies, the government's intertemporal budget constraint (GIBC) is not tightened but relaxed when a part of public debt is unfunded by future fiscal surpluses. Accordingly, this places downward (not upward) pressure on the current price level, which continues until the price level surges sharply, at which time the bubbles that back the unfunded component of the GIBC burst.

Second, at near-zero interest rates, the expected deflation is almost equal to the real rate of interest, but the continuously realized deflation is larger due to the small possibility of price surges in the next period. The probability that the price level will jump to a level proportional to existing money stocks in the next year is very low, but even a remote possibility of sharper price surges driven by faster monetary expansion makes ongoing deflations even more severe. However, this tendency is offset to some extent by a strong aversion toward the catastrophic risks that follow a price surge, which assists in lowering the real rate of interest, and makes the current deflation milder.

Third, mild deflation may continue for a long time. Because sharp price surges are considered a rare event, they are hardly likely in the next year and not very likely in the next decade, but most likely in the next century. In this model, this probabilistic nature of price surges, with a sharp contrast in likelihood between the near and far future, is reflected in slowly flattening yield curves; (ultra) long-term yields remain relatively high, with the possibility of distant-future price surges, even if short-term yields approach zero. In this way, mild deflation can coexist with fiscal unsustainability and monetary expansion until a one-off price surge occurs.

The key feature of the above framework is that it allows for temporary and even persistent fiscal unsustainability, which results from undisciplined fiscal policies, as well as for the restoration of fiscal sustainability through price surges. Several issues related to this feature have been explored intensively in the existing literature on the fiscal theory of the price level (FTPL). First, LeRoy (2004), Bloise (2005), and Bloise and Reichlin (2008) present a case in which the GIBC is relaxed to the extent that the real valuation of public bonds is sustained partially by the unfunded component (the nonzero terminal condition or the bubble component) and a continuum of equilibria emerges. Bassetto and Cui (2018) demonstrate that, given lower real returns, which are induced by either dynamic inefficiency or liquidity premiums on government bonds, the present value of fiscal surpluses is not well defined and the price level is indeterminable only by its lower bound. Kobayashi (2019) and Sakuragawa (2019) present a case where deflationary equilibria emerge as a consequence of the bubble component in the GIBC. However, neither researcher analyzed explicitly how fiscal sustainability is restored; that is, fiscal policies are assumed to be unsustainable forever.

Second, many studies, including Davig et al. (2010), Bianchi and Ilut (2017), and Bianchi and Melosi (2017), consider policy environments in which an economy switches between the non-Ricardian (active fiscal policies) and the Ricardian (passive fiscal policies) regimes.[5] The current model differs from these papers because a regime switch is triggered not by a fiscal policy shift from a non-Ricardian to a Ricardian regime, but

[5] In the sense of Woodford (1995), a fiscal disturbance is neutralized in the case of Ricardian fiscal policies, but it is not in the case of non-Ricardian policies.

by a one-off price surge, which is a rare and catastrophic event. In our model, a moderate fiscal reform is implemented only after the catastrophic shocks disappear completely. In the existing models, fiscal sustainability is achieved on equilibrium even in the non-Ricardian regime, whereas in our paper, it is not maintained before the occurrence of the one-off price surge. In these respects, the model of Davig et al. (2011) is closest to the model in this paper. Starting from the Ricardian regime (involving an active monetary policy and a stationary transfer process), Davig et al.'s economy hits the fiscal limit as a consequence of a nonstationary transfers process (the non-Ricardian policy), and eventually returns to the Ricardian regime as an absorbing state. In their model, fiscal sustainability may be achieved not by drastic cuts in transfer payments, but by unprecedented inflation, which breaks out when growing public debt is stabilized by a passive monetary policy.

In terms of the relationship between monetary phenomena and fiscal (un)sustainability, Benhabib et al. (2002) employ the possibility of fiscal unsustainability as an instrument to *ex ante* eliminate a liquidity trap from possible equilibrium paths. In the neo-Fisherian model such as Schmitt-Grohe and Uribe (2017), deflationary phenomena result from near-zero interest rates, as in the current model, but fiscal sustainability is always achieved for any path of the price level.

Given that a one-off price surge is unprecedented by its nature and that it is not apparent in any observations from past decades, it is quite difficult to establish empirical relevance for the current model. To overcome this type of Peso problem, two empirical analyses are conducted. First, instead of regarding a price jump itself as a rare catastrophe, a one-off price surge is assumed to be triggered by a rare and catastrophic event. In Section 4, calibration exercises are made under the assumption that a large-scale Tokyo inland earthquake triggers a one-off price surge. According to the calibration, the model can successfully explain not only the occurrence of mild deflation with near-zero interest rates after the mid-1990s, but also the price stability starting in the mid-1980s, the sharp decline in short-term yields in the first half of the 1990s, and the slowly flattening yield curves in the twenty-first century. Second, we search for and examine any episode comparable to a one-off price surge in Japanese monetary history. The sharp price increase following the end of World War II in 1945 is examined and we find that it is empirically convincing to interpret the sharp inflation not as a hyperinflationary phenomenon but as a one-off price surge event.

The remainder of this paper is organized as follows. Section 2 explains briefly the nominal phenomena observed in Japan's long-run mild deflation. Section 3 presents a simple exchange economy in which fiscal sustainability is restored by one-off price surges. In Section 4, the theoretical framework is applied to an examination of the long-run mild deflation experienced in Japan. Section 5 concludes this paper.

2. Three features of Japan's long-run mild deflation

This section briefly presents three features associated with Japan's long-run mild deflation, which commenced in the mid-1990s, together with near-zero interest rates.[6] We emphasize that the price level was stable, despite rapid monetary expansion and heavy fiscal deficits, even before the nominal rate of interest almost reached zero in the mid-1990s, while long-term yields (longer than 10 years) and ultra-long-term yields (longer than 20 years) remained relatively high even after the mid-1990s. Accordingly, the calibration exercises presented in Section 4 focus not only on the nominal behavior after interest rates reached the near-zero level in the mid-1990s, but also on the behavior while they were above zero.

First, the results show that the price level in Japan was quite stable despite rapid monetary expansion. In Figure 2-1, the price level per unit of consumption goods (the private consumption deflator[7]) is compared with the money stocks per unit of output (the outstanding Bank of Japan (BoJ) notes[8] divided by real gross domestic product (GDP)) for the years 1955 to 2018.[9] Both of the time-series are standardized as of 1955. The

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[6] The data sources used in Section 2 include the Ministry of Finance, the Cabinet Office, and the Bank of Japan.

[7] The consumption deflator is adopted because it captures a deflationary trend as a result of the nature of the Paasche index.

[8] Here, the narrowest category of money stocks is chosen.

[9] When a 3% consumption tax was introduced in April, 1989, most of existing indirect taxes were abolished. Thus, its introduction had little impact on the average price level. On the other hand, the overall price level increased by around 2% when the tax rate was raised to 5% in April, 1997, and the level increased by about 2% when the rate was hiked to 8% in April, 2014. The private consumption deflator, reported throughout this paper,

price level and the money stocks moved together up to the late 1970s. However, from the mid-1980s, the price level began to stagnate despite continuing monetary expansion. More precisely, the price level inflated only slightly up to the mid-1990s, and it deflated mildly from then onwards.

Second, the Japanese government bonds (JGBs) were valued highly in real terms despite the continuation of the primary budget deficits. As shown in Figure 2-2, the primary budget balance of the government's general account, relative to nominal GDP, was close to zero, or even negative, except for 1989–1993. More precisely, the primary balance deteriorated from the early 1970s, and reached –5.0% in 1979. In the 1980s, it recovered gradually, reaching 2.2% in 1991. However, it deteriorated again from the early 1990s, reached –5.0% in 2012, and remained negative. As shown in Figure 2-3, on the other hand, the outstanding JGBs, adjusted by a real macroeconomic scale or divided by real GDP, grew much faster than the price level (the private consumption deflator) from the early 1970s. Putting the two figures together, the real valuation of the outstanding (growth-adjusted) JGBs improved considerably from the early 1970s, although the primary balance deteriorated substantially for the same period.[10]

Third, the shape of the yield curves on the JGBs, from 1-year yields to 40-year yields, has experienced a dramatic change since the price level started to stagnate in the mid-1980s. As shown in Figure 2-4, the yield curves were almost flat at relatively high rates in the 1980s. While the short-term (1-year) rate declined quickly in the 1990s, almost reaching zero in 1995, the longer-term rates remained relatively high. Accordingly, the yield curves were upward-sloping even when the short-term rate was close to zero. More concretely, the spreads of 10-year and 20-year yields over 1-year yields were, respectively, 2.5% and 3.1% in 1996, 1.3% and 2.0% in 2001, 1.2% and 1.6% in 2006, and 0.9% and 1.7% in 2011. Only in late 2016 did the yield curves flatten substantially, although the emergence of the flat curves for up to 10-year yields (the 10-year term spread was only 0.2% in 2016) might have been caused by heavy intervention by the

is adjusted by the impact of these consumption tax hikes on the average price level.

[10] According to Ito et al. (2011), Japanese fiscal policy began to lack discipline as early as 1970, and the debt–GDP ratio was nonstationary from then.

BoJ.[11] In 2018, yield curves became slightly steeper.

3. Behavior of the price level in the fiscally sustainable and unsustainable regimes

3.1. Sketching a model of a price surge as a trigger for a regime switch

In this section, we describe the following equilibrium behavior step by step. Initially, the economy is under the regime in which fiscal unsustainability results from undisciplined fiscal policies (the **FU** regime). In the GIBC, the terminal condition may not converge to zero, and the component that is unfunded by future surpluses may emerge. In this model, a one-off price surge, characterized as a rare and catastrophic event, serves as a trigger for a regime switch. That is, a price surge takes place with a small probability in the next year, and it is accompanied by adverse impacts on endowments in the following years. Upon experiencing this price surge, the economy switches from the FU regime to the regime in which fiscal sustainability is restored (the **FS** regime). At the regime switch, the bubble supporting the unfunded component in the GIBC bursts immediately, and nominal public bonds are heavily devalued by the price surge. In addition, a moderate fiscal reform is implemented after the catastrophic shocks disappear completely. In this way, fiscal sustainability is recovered.

The most important part of this section is to show how a one-off price surge is modeled as a trigger for a regime switch. We demonstrate in Section 3.3 that, in the FS regime, the price process is uniquely determined by the quantity theory of money (QTM); both deflationary and hyperinflationary paths are ruled out by several assumptions regarding the FS regime. In the FU regime, on the other hand, as Section 3.4 shows, we prove that a continuum of deflationary equilibria emerges once we relax the assumption that the terminal condition in the GIBC holds tightly. Accordingly, a price jump occurs at the regime switch; this discontinues the deflationary trend in the FU regime, and the price level rises up to the QTM price. Because hyperinflationary paths are infeasible in the FU regime, a discontinuous price drop from the hyperinflationary trend down to the QTM price never occurs.

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[11] The BoJ attempted to flatten the yield curves for up to 10-year yields from September 2016 by carrying out quite generous limit orders for the long-term JGBs.

3.2. A simple monetary model of the exchange economy

3.2.1. Basic setup

We employ a simple monetary model of the exchange economy proposed by Kocherlakota and Phelan (1999) as a basic framework. A representative household has the following preference regarding streams of consumption (c_t) and the real money balance $(\frac{M_t}{P_t})$:

$$\sum_{t=0}^{\infty} \beta^t E_0 \left[u(c_t) + v\left(\frac{M_t}{P_t}\right)\right], \tag{1}$$

where a discount factor β is less than one, and $u(c)$ and $v\left(\frac{M}{P}\right)$ are twice differentiable, strictly increasing, and strictly concave. E_0 is the expectation operator conditional on time-0 information. The sources of uncertainty are described in Section 3.2.3.

The maximization of the objective function (1) is subject to $B_{t+1} + M_{t+1} = P_{t+1}(y_{t+1} - \tau_{t+1} - c_{t+1}) - (R_{1,t} - 1)M_t + R_{1,t}(B_t + M_t)$, where y_t is a real endowment stream in terms of consumption goods, c_t is the real amount of consumption goods, τ_t is a real lump-sum tax, P_t is the price of consumption goods, M_t is the nominal money balance, B_t is the nominal amount of public bonds, and $R_{1,t}$ is the one-period nominal gross rate of interest.

The following functional forms are applied to $u(c)$ and $v\left(\frac{M}{P}\right)$.

$$u(c) = \ln(c), \tag{2}$$

with a unit elasticity of intertemporal substitution, and:

$$v\left(\frac{M}{P}\right) = \frac{\lambda}{1-\frac{1}{\sigma}}\left(\chi + \frac{M}{P}\right)^{1-\frac{1}{\sigma}}, \tag{3}$$

where $\sigma > 0$ is interpreted as part of the interest and income elasticities of money demand, as discussed in Section 4. Both λ and χ are positive. Here, a positive χ represents the existence of an alternative means of exchange to central bank notes M, and it assists in setting an upper bound on nominal interest rates $R_{1,t}$.

3.2.2. Two fiscal policies and two fiscal regimes

In the FU regime, fiscal policy always lacks discipline, and the fiscal surplus is never responsive to the outstanding public bonds, that is:

$$P_t^{FU}(\tau_t^{FU} - g_t) = P_t^{FU}\varepsilon - (M_t - M_{t-1}),$$

where P_t^{FU} is the price level prevailing in the FU regime, and ε is a constant real primary balance. In the context of the FTPL, the above fiscal policy is called non-Ricardian, in the sense that any disturbance in the fiscal surplus is not neutralized. In contrast with standard FTPL models, however, ε may be zero or negative. Any seigniorage $M_t - M_{t-1}$ is reimbursed to households as a lump-sum subsidy.[12] Thus, the nominal balance of the public bonds evolves according to:

$$B_{t+1} = R_{1,t}B_t - P_{t+1}^{FU}(\tau_{t+1}^{FU} - g_t) - (M_{t+1} - M_t) = R_{1,t}B_t - P_{t+1}^{FU}\varepsilon. \quad (4)$$

The nominal value of the public bonds may be unstable if ε is quite small, or negative.

Turning now to the FS regime, it is assumed that a disciplined fiscal policy is not established immediately upon switching, but only after the catastrophic shocks disappear completely. A major reason for this assumption is that it is hard to imagine fiscal reforms being successfully implemented during catastrophic periods in the real world. Given a drastic reduction in the debt–GDP ratio caused by the price surges, any fiscal reform in the FS regime must be moderate.

Under a disciplined fiscal policy, the surplus responds positively to the outstanding public bonds as follows. If $\gamma B_{t-1} > \underline{B} > 0$ with $0 < \gamma < 1$, then:

$$P_t^{FS}(\tau_t^{FS} - g_t) = (R_{1,t-1} - \gamma)B_{t-1} - (M_t - M_{t-1}),$$

and otherwise:

$$P_t^{FS}(\tau_t^{FS} - g_t) = (R_{1,t-1}B_{t-1} - \underline{B}) - (M_t - M_{t-1}),$$

where P_t^{FS} is the price level prevailing in the FS regime. The above fiscal policy is called Ricardian in the sense that the outstanding public bonds are stabilized in nominal terms. A disciplined fiscal policy is called "moderate" if γ is close to one, or $\underline{B}$ is set high.

Again, any seigniorage revenue is reimbursed to households. Thus, the outstanding public bonds evolve according to:

$$B_{t+1} = R_{1,t}B_t - P_{t+1}^{FS}(\tau_{t+1}^{FS} - g_t) - (M_{t+1} - M_t) = \gamma B_t, \quad (5\text{–}1)$$

if $\gamma B_t > \underline{B} > 0$, and otherwise according to:

[12] In the sense that the public bonds are redeemed only by real fiscal surpluses, the assumption of reimbursing seigniorage to households follows the tradition of the FTPL. In a related study, Sargent and Wallace (1981) include seigniorage in the government's budget constraint in determining the price process.

$$B_{t+1} = R_{1,t}B_t - P_{t+1}^{FS}(\tau_{t+1}^{FS} - g_t) - (M_{t+1} - M_t) = \underline{B}. \quad (5\text{–}2)$$

Under the above fiscal policy, the nominal balance of the public bonds converges to its lower bound $\underline{B}$.

In terms of monetary policy, the money stocks grow at a constant rate of $\mu > 0$ in both regimes:

$$M_{t+1} = (1 + \mu)M_t. \quad (6)$$

3.2.3. A discontinuous price jump as a rare and catastrophic event

A price surge takes place with probability π, which is less than 5% per year, and is accompanied by catastrophic endowment shocks in the years that follow. Thus, the regime remains fiscally unsustainable in the next year with probability $1 - \pi$, but fiscal sustainability is restored at a regime switch with probability π. As t years pass, the regime will remain fiscally unsustainable with probability $(1 - \pi)^t$, and will become fiscally sustainable with probability $1 - (1 - \pi)^t$. Thus, the FS regime is regarded as an absorbing state. As discussed in Section 3.2.2, a disciplined (Ricardian) fiscal policy is not actually implemented at the point of time when the economy switches to the FS regime. At first, an undisciplined fiscal policy is maintained under the FS regime, with the introduction of a disciplined policy occurring only several years after the price surge.

During the FU regime, an endowment stream of consumption goods y_t is constant, and real consumption is time-invariant at constant y, net of constant real government expenditure g:

$$c_t = c = y - g. \quad (7)$$

As a rare event, a price surge has catastrophic impacts on endowments in the following years. The endowment available for consumption declines substantially, partly because of the negative endowment shocks $(y_t < y)$, and partly because of the extra public expenditures required to deal with catastrophic events $(g_t > g)$. When the economy switches regimes at time s, consumption declines from c to:

$$c_s = c(1 - d)^L, \quad (8\text{–}1)$$

where $0 < d < 1$, and L is a natural number. Even after switching, consumption remains stagnant at:

$$c_{s+l} = c(1-d)^{L-l}, \tag{8–2}$$

at time $s+l$ ($l = 1,2,\ldots,L-1$). That is, it takes $L+1$ years for consumption to recover to c.

3.3. The QTM in the FS regime

3.3.1. Maximization after switching to the FS regime

Let us begin by solving the maximization problem when the economy switches to the FS regime at time s:

$$\sum_{t=s}^{\infty} \beta^{t-s}\left[\ln(c_t) + \frac{\lambda}{1-\frac{1}{\sigma}}\left(\chi + \frac{M_t}{P_t}\right)^{1-\frac{1}{\sigma}}\right],$$

subject to $B_{t+1} + M_{t+1} = P_{t+1}^{FS}(y - \tau_{t+1} - c_{t+1}) - \left(R_{1,t} - 1\right)M_t + R_{1,t}(B_t + M_t)$. In the FS regime, every variable is deterministic.

Focusing on time t and $t+1$ consumption, the above maximization problem is reformulated as:

$$\max_{c_t, c_{t+1}, M_t}\left\{\beta\left[\ln(c_{t+1}) + \frac{\lambda}{1-\frac{1}{\sigma}}\left(\chi + \frac{M_{t+1}}{P_{t+1}}\right)^{1-\frac{1}{\sigma}}\right] + \left[\ln(c_t) + \frac{\lambda}{1-\frac{1}{\sigma}}\left(\chi + \frac{M_t}{P_t}\right)^{1-\frac{1}{\sigma}}\right]\right\},$$

subject to $B_{t+1} = P_{t+1}^{FS}(y_{t+1} - \tau_{t+1} - c_{t+1}) - (M_{t+1} - M_t) + R_{1,t}\left[(y_{t+1} - \tau_{t+1} - c_{t+1}) - (M_{t+1} - M_t) + R_{1,t-1}B_{t-1}\right]$.

Together with equations (8–1) and (8–2), the first-order conditions with respect to consumption (c) and the money stocks (M_t) are obtained as follows:

$$\beta R_{1,t}\left[\frac{P_t^{FS}}{P_{t+1}^{FS}}(1-d_0)\right]=1, \tag{9}$$

where $d_0 = d$ for $t = s, s+1, s+2, \ldots, s+L-1$, and $d_0 = 0$ for $t \geq s+L$, and:

$$\lambda\left(\chi + \frac{M_t}{P_t^{FS}}\right)^{-\frac{1}{\sigma}} = \frac{1}{c_t}\left(1 - \frac{1}{R_{1,t}}\right). \tag{10}$$

Substituting equation (10) into equation (9) leads to:

$$\frac{P_t^{FS}}{P_{t+1}^{FS}} = \frac{1}{\beta R_{1,t}(1-d_0)} = \frac{1}{\beta(1-d_0)}\left[1 - \lambda\left(\chi + \frac{M_t}{P_t^{FS}}\right)^{-\frac{1}{\sigma}} c_t\right]. \tag{11}$$

When a disciplined fiscal policy (5–1) or (5–2) is implemented at time $s' > s+L$, a period-by-period budget constraint can be solved as the following GIBC:

$$\frac{B_{s'}}{P_{s'}^{FS}} = \sum_{t=s'}^{\infty}\left[\beta^{t-s'}(\tau_t^R - g_t)\right] + \lim_{T\to\infty}\left(\beta^{T-s'}\frac{B_T}{P_T^{FS}}\right).$$

Then, the terminal condition associated with the public bonds is:

$$\lim_{T\to\infty}\left(\beta^{T-s'}\frac{B_T}{P_T^{FS}}\right)=0. \tag{12}$$

3.3.2. The QTM without any catastrophic endowment shock ($d=0$)

Which price path does difference equation (11) yield? Suppose that catastrophic shocks are absent, or that $d=0$ for the moment. Figures 3-1 and 3-2 depict the relationship between the current real money balance and the next-period deflation rate ($\frac{M_t}{P_t^{FS}}$ on the x-axis and $\frac{P_t^{FS}}{P_{t+1}^{FS}}$ on the y-axis). There are three cases, as follows.

First, the economy stays at point **A** in Figure 3-1 forever. There, the price level is completely proportional to the money stocks and the inflation rate is constant at the monetary growth rate μ. Hence, the QTM holds with the constant real money stocks. When $d=0$, $P_t^{QT(d=0)}$ is obtained as follows:

$$\frac{P_t^{QT(d=0)}}{P_{t+1}^{QT(d=0)}}=\frac{1}{1+\mu}, \tag{13}$$

$$R_{1,t}^{QT(d=0)}=\frac{1+\mu}{\beta}, \tag{14}$$

$$P_t^{QT(d=0)}=\frac{1}{\left(\frac{\lambda(1+\mu)}{1+\mu-\beta}c^{1-\frac{1}{\sigma}}\right)^{\sigma}-\frac{\chi}{c}}\frac{M_t}{c}. \tag{15}$$

The below choice of λ is consistent with a constant Marshallian k, or constant relative money stocks ($\kappa=\frac{M}{P^{QT}c}$).

$$\lambda=\left(\frac{\chi}{c}+\kappa\right)^{\frac{1}{\sigma}}\frac{1+\mu-\beta}{1+\mu}c^{\frac{1}{\sigma}-1}>0. \tag{16}$$

Equation (16) implies that Friedman's rule (Friedman 1969) is not feasible because λ turns out to be zero given that $\mu=\beta-1$ with $R_{1,t}=1$.[13]

Given equations (5–1), (5–2), (13), (14), and (15), the terminal condition (12) in the GIBC holds for the QTM:

$$\lim_{T\to\infty}\left[\beta^{T-s'}\frac{B}{(1+\mu)^{T-s'}P_s^{QT}}\right]=\lim_{T\to\infty}\left[\left(\frac{\beta}{1+\mu}\right)^{T-s'}\frac{B}{P_s^{QT}}\right]=0.$$

Note that $\frac{\beta}{1+\mu}<1$.

[13] Buiter and Sibert (2007) prove that Friedman's rule is not available in standard monetary models.

Second, the economy approaches an asymptotic line at $\frac{P_t^{FS}}{P_{t+1}^{FS}} = \frac{1}{\beta}$ in Figure 3-1 as the real money balance goes to infinity. Immediately after switching, $P_s^{FS} < P_s^{QT(d=0)}$ or $\frac{P_s^{FS}}{P_{s+1}^{FS}} > \frac{1}{1+\mu}$; then, the deflationary process is initiated. The real money balance goes to infinity, and $\frac{P_t^{FS}}{P_{t+1}^{FS}}$ converges to $\frac{1}{\beta}$ from equation (11). Accordingly, the terminal condition converges to a positive constant, as below, and equation (12) fails to hold:

$$0 < \lim_{T\to\infty}\left[\beta^{T-s'}\frac{\underline{B}}{\beta^{T-s'}P_s^{FS}}\right] = \frac{\underline{B}}{P_s^{FS}} < \infty.$$

Therefore, the deflationary process is ruled out from the possible equilibrium paths.

Third, when $0 < \lambda\chi^{-\frac{1}{\sigma}}c < 1$, the economy converges to point **B** in Figure 3-1 as the real money balance degenerates to zero. At the start $P_s^{FS} > P_s^{QT(d=0)}$ or $\frac{P_s^{FS}}{P_{s+1}^{FS}} < \frac{1}{1+\mu}$. Then, immediately after switching, the inflationary process is accelerated and the real money balance degenerates to zero. As equation (11) implies, if $0 < \lambda\chi^{-\frac{1}{\sigma}}c < 1$, then $\frac{P_t^{FS}}{P_{t+1}^{FS}}$ converges to a positive $\frac{1-\lambda\chi^{-\frac{1}{\sigma}}c}{\beta}$.[14] Consequently, the terminal condition (12) in the GIBC holds:

$$\lim_{T\to\infty}\frac{\beta^{T-s'}\underline{B}}{\left(\frac{\beta}{1-\lambda\chi^{-\frac{1}{\sigma}}c}\right)^{T-s'}P_s^{FS}} = \lim_{T\to\infty}\left[\left(1-\lambda\chi^{-\frac{1}{\sigma}}c\right)^{T-s'}\underline{B}\right] = 0.$$

In this case, the accelerating inflationary (hyperinflationary) process cannot be ruled out from the possible equilibrium paths, as pointed out by Brock (1975) and Obstfeld and Rogoff (1983), and there emerges a continuum of equilibria with an arbitrary initial value for $P_s^{FS} > P_s^{QT(d=0)}$.

As shown in Figure 3-2, however, equation (11) with $1 < \lambda\chi^{-\frac{1}{\sigma}}c$ implies that positive prices cannot be supported eventually because point **C** in Figure 3-2 is not located in the first quadrant. Hence, the accelerating inflationary process is not feasible in this case.

In what follows, it is assumed that:

$$1 < \lambda\chi^{-\frac{1}{\sigma}}c, \tag{17}$$

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[14] If a means of exchange alternative to central bank money is more readily available, and χ is larger, then $0 < \lambda\chi^{-\frac{1}{\sigma}}c < 1$ is more likely to be satisfied with a lower upper limit on the inflation rate ($\frac{P_{t+1}^{FS}}{P_t^{FS}}$).

thereby eliminating the possibility of the accelerating inflationary (hyperinflationary) process. Accordingly, only the QTM price ($P_t^{QT(d=0)}$) is justifiable as a legitimate equilibrium path in the FS regime. As shown in Section 4, if c is standardized to one, χ is set at one, and κ in equation (16) is chosen by the long-run average of the relative money stocks (Marshallian k) of the Japanese economy; then $\lambda\chi^{-\frac{1}{\sigma}}c$ ($=\lambda$ in this case) is indeed greater than one.

3.3.3. The QTM with catastrophic endowment shocks ($d > 0$)

How does the QTM price (P_t^{QT}) behave in the presence of catastrophic endowment shocks ($d > 0$)?[15] As discussed in detail in Appendix 1, the price level is higher at switching because money demand falls with a decline in output, and interest rates are higher owing to the economic recovery after switching. More specifically, ($P_{1,s}^{QT}$) overshoots $P_s^{QT(d=0)}$ and $R_{1,s}^{QT}$ jumps beyond $R_{1,s}^{QT(d=0)}$ at the time of switching, s. Then, the price level grows more slowly than $P_{s+l}^{QT(d=0)}$ and coincides with $P_{s+L}^{QT(d=0)}$ at time $s+L$. The nominal interest rate, on the other hand, continues to be above $R_{1,s}^{QT(d=0)}$ up to time $s+L-1$.

As shown in Appendix 1, P_t^{QT} is determined in relation to $P_t^{QT(d=0)}$ by a positive parameter η as follows:

$$P_{s+q}^{QT} = \left[\frac{1}{(1-d)^{\eta}}\right]^{L-q} P_{s+q}^{QT(d=0)} > P_{s+q}^{QT(d=0)}, \qquad \text{(18) or (A–3)}$$

for $q = 0,1,2,\ldots,Q-1$. Here, η is determined by equation (A–1) in Appendix 1:

$$\eta = R_{1,s}^{QT(d=0)} \Bigg/ \left[1 + \frac{R_{1,s}^{QT(d=0)}-1}{\sigma} \frac{\frac{M_s}{P_s^{QT(d=0)}}}{\chi + \frac{M_s}{P_s^{QT(d=0)}}}\right] > 0. \qquad \text{(A–1)}$$

Given equations (14) and (15), $R_{1,s}^{QT(d=0)}$ and $\frac{M_s}{P_s^{QT(d=0)}} \Big/ \left[\chi + \frac{M_s}{P_s^{QT(d=0)}}\right]$ are constant.

In what follows, it is assumed that:

$$\frac{M_s}{P_s^{QT(d=0)}} \Big/ \left[\chi + \frac{M_s}{P_s^{QT(d=0)}}\right] > \sigma. \qquad \text{(A–2)}$$

Then, $0 < \eta < 1$ from equation (A–1) and $R_{1,s}^{QT} > R_{1,s}^{QT(d=0)}$ from equation (A–4).

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[15] When $d > 0$, a disciplined fiscal policy (5–1) or (5–2) is assumed to be implemented at time $s' > s + L$, that is, only after the catastrophic shocks disappear completely.

3.4. Possible deflationary processes in the FU regime

3.4.1. The FU regime with $d = 0$

Let us move to the FU regime with an undisciplined fiscal policy (4), which commences at time 0. This regime is called fiscally unsustainable because the terminal condition may not hold, and the bubbles that back the unfunded component of the GIBC may emerge. Given inequality (17), the accelerating inflationary process is infeasible in the FU regime. However, the deflationary process can occur once we relax the assumption that the terminal condition holds. Accordingly, there is a discontinuous jump in the price level, which rises from the deflationary trend in the FU regime up to the QTM price in the FS regime, and the bubble supporting the unfunded component bursts at the time of the regime switch. Detailed descriptions follow.

The maximization of $\sum_{t=0}^{\infty} \beta^t E_0 \left[\ln(c_t) + \frac{\lambda}{1-\frac{1}{\sigma}}\left(\chi + \frac{M_t}{P_t}\right)^{1-\frac{1}{\sigma}}\right]$ is subject to $B_{t+1} + M_{t+1} = P_{t+1}^{FU}(y - \tau_{t+1} - c_{t+1}) - \left(R_{1,t} - 1\right)M_t + R_{1,t}(B_t + M_t)$.

The first-order conditions with respect to consumption and the money stock lead to:

$$E_t\left(\frac{P_t^{FU}}{P_{t+1}}\frac{c}{c_{t+1}}\right) = \frac{1}{\beta R_{1,t}} = \frac{1}{\beta}\left[1 - \lambda\left(\chi + \frac{M_t}{P_t^{FU}}\right)^{-\frac{1}{\sigma}} c\right], \tag{19}$$

where P_{t+1} and c_{t+1} are random variables, the realization of which depends on whether the regime remains fiscally unsustainable ($P_{t+1} = P_{t+1}^{FU}$ and $c_{t+1} = c$) or whether there is a switch to the FS regime ($P_{t+1} = P_{t+1}^{QT} = \frac{P_{t+1}^{QT(d=0)}}{(1-d)^{\eta L}}$ and $c_{t+1} = (1-d)^L c$).

Let us begin with a case in which there is no catastrophic endowment shock ($d = 0$). Substituting $E_t\left(\frac{P_t^{FU}}{P_{t+1}}\frac{c}{c}\right) = E_t\left(\frac{P_t^{FU}}{P_{t+1}}\right) = (1-\pi)\frac{P_t^{FU}}{P_{t+1}^{FU}} + \pi\frac{P_t^{FU}}{P_{t+1}^{QT(d=0)}}$ into equation (19) leads to:

$$\frac{P_t^{FU}}{P_{t+1}^{FU}} = \frac{1}{1-\pi}\left\{\frac{1}{\beta}\left[1 - \lambda\left(\chi + \frac{M_t}{P_t^{FU}}\right)^{-\frac{1}{\sigma}} c\right] - \pi\frac{P_t^{FU}}{P_{t+1}^{QT(d=0)}}\right\}. \tag{20–1}$$

Before exploring the price behavior implied by equation (20–1), we derive the GIBC. Given an undisciplined fiscal policy (4), $B_{t+1} = R_{1,t}B_t - P_{t+1}^{FU}\varepsilon$. Even after the economy switches to the FS regime at time s, the undisciplined fiscal policy continues for at least one period ($s' > s$), and $B_{s+1} = R_{1,s}B_s - P_{s+1}^{QT}\varepsilon$ holds at time $s+1$.

Using the first equality of equation (19), equation (4) is further rewritten as:

$$\frac{B_t}{P_t^{FU}} = \beta\left[E_t\left(\frac{B_{t+1}}{P_{t+1}}\right) + \varepsilon\right], \tag{21}$$

where both P_{t+1} and B_{t+1} are random variables. Equation (21) may be interpreted as an arbitrage condition for the public bond pricing, in which the return consists of the real appreciation of the public bonds as capital gains, and the real fiscal surplus as income gains.

Substituting $E_t\left(\frac{B_{t+1}}{P_{t+1}}\right) = (1-\pi)\frac{B_{t+1}}{P_{t+1}^{FU}} + \pi\frac{B_{t+1}}{P_{t+1}^{QT(d=0)}} = (1-\pi)\frac{B_{t+1}}{P_{t+1}^{FU}} + \pi\frac{R_{1,t}B_t - P_{t+1}^{QT(d=0)}\varepsilon}{P_{t+1}^{QT(d=0)}}$ into equation (21) leads to:

$$\frac{B_t}{P_t^{FU}} = \beta(1-\pi)\frac{B_{t+1}}{P_{t+1}^{FU}} + \beta(1-\pi)\varepsilon + \beta\pi\frac{R_{1,t}B_t}{P_{t+1}^{QT(d=0)}}.$$

The above difference equation is solved in a recursive manner as follows:

$$\frac{B_0}{P_0^{FU}} = \sum_{t=0}^{\infty}\left\{\beta^t(1-\pi)^t\left[\beta(1-\pi)\varepsilon + \beta\pi\frac{R_{1,t}B_t}{P_{t+1}^{QT(d=0)}}\right]\right\} + \lim_{T\to\infty}\left[\beta^T(1-\pi)^T\frac{B_T}{P_T^{FU}}\right]$$

$$= \frac{\beta(1-\pi)\varepsilon}{1-\beta(1-\pi)} + \sum_{t=0}^{\infty}\left(\beta^t(1-\pi)^t\beta\pi\frac{R_{1,t}B_t}{P_{t+1}^{QT(d=0)}}\right) + \lim_{T\to\infty}\left[\beta^T(1-\pi)^T\frac{B_T}{P_T^{FU}}\right]. \tag{22–1}$$

In standard models of the FTPL, the terminal condition is respected strictly, and $\lim_{T\to\infty}\left[\beta^T(1-\pi)^T\frac{B_T}{P_T^{FU}}\right] = 0$ must hold in equilibrium. However, in the FU regime, the terminal condition is relaxed, and a positive but finite $\lim_{T\to\infty}\left[\beta^T(1-\pi)^T\frac{B_T}{P_T^{FU}}\right]$ is admitted on the equilibrium path as long as the FU regime continues:

$$0 \le \lim_{T\to\infty}\left[\beta^T(1-\pi)^T\frac{B_T}{P_T^{FU}}\right] < \infty. \tag{23}$$

When inequality (23) holds, the regime is indeed fiscally unsustainable in the sense that the bubbles that back the unfunded component of the GIBC are present.

Equation (22–1) can serve as an instrument to determine the initial price P_0^{FU}. Here, the initial price is determined according to the present value of the fiscal surpluses (the first term of the right-hand side of equation (22–1)) as in the FTPL, the real valuation of nominal public bonds at a regime switch (the second term), and, in contrast to the situation under the FTPL, the component that is unfunded by the future fiscal surpluses (the third term), if any ($0 < \lim_{T\to\infty}\left[\beta^T(1-\pi)^T\frac{B_T}{P_T^{FU}}\right] < \infty$). The bubbles that back the unfunded component of the GIBC burst on switching to the FS regime, under which the terminal condition is satisfied strictly by equation (12).

Equation (22–1) can be simplified as follows. When $B_t = \prod_{i=0}^{t-1} R_{1,i} B_0$ with $\varepsilon = 0$ in the FU regime, the last two terms on the right-hand side of equation (22–1) amount to:

$$\sum_{t=0}^{\infty}\left[\beta^t(1-\pi)^t\beta\pi\frac{R_{1,t}\prod_{i=0}^{t-1}R_{1,i}B_0}{P_{t+1}^{QT(d=0)}}\right]+\lim_{T\to\infty}\left[\beta^T(1-\pi)^T\frac{\prod_{i=0}^{T-1}R_{1,i}B_0}{P_T^{FU}}\right].$$

If ε is not equal to zero, then the above value decreases with the present value of the fiscal surpluses, or $\frac{\beta(1-\pi)\varepsilon}{1-\beta(1-\pi)}$. Hence, equation (22–1) is rewritten as follows:

$$\frac{B_0}{P_0^{FU}}=\frac{\beta(1-\pi)\varepsilon}{1-\beta(1-\pi)}$$

$$+\left\{\left(\sum_{t=0}^{\infty}\left[\beta^t(1-\pi)^t\beta\pi\frac{R_{1,t}\prod_{i=0}^{t-1}R_{1,i}B_0}{P_{t+1}^{QT(d=0)}}\right]+\lim_{T\to\infty}\left[\beta^T(1-\pi)^T\frac{\prod_{i=0}^{T-1}R_{1,i}B_0}{P_T^{FU}}\right]\right\}-\frac{\beta(1-\pi)\varepsilon}{1-\beta(1-\pi)}\right\}$$

$$=\sum_{t=0}^{\infty}\left[\beta^t(1-\pi)^t\beta\pi\frac{R_{1,t}\prod_{i=0}^{t-1}R_{1,i}B_0}{P_{t+1}^{QT(d=0)}}\right]+\lim_{T\to\infty}\left[\beta^T(1-\pi)^T\frac{\prod_{i=0}^{T-1}R_{1,i}B_0}{P_T^{FU}}\right]. \qquad (22–2)$$

The relaxed terminal condition is now rewritten as:

$$0\le\lim_{T\to\infty}\left[\beta^T(1-\pi)^T\frac{\prod_{i=0}^{T-1}R_{1,i}B_0}{P_T^{FU}}\right]<\infty. \qquad (23')$$

An interesting feature of equation (22–2) is that because B_0 is cancelled out on both sides, the initial price (P_0^{FU}) is independent not only of the real primary balance (ε), but also of the initial nominal balance of the public bonds (B_0). Thus, Ricardian equivalence holds even during the FU regime. A reason for this equivalence result is that the public bonds, which accumulate as a result of the undisciplined fiscal policy in the FU regime, are repaid by the devaluation of nominal bonds that occurs because of the price jump at switching, as well as by a moderate fiscal reform, which is implemented later in the FS regime.

It is easy to prove that the initial price can be the QTM price ($P_0^{QT(d=0)}$) from equation (22–2). Substituting $\frac{P_t^{FU}}{P_{t+1}^{FU}}=\frac{1}{1+\mu}$ and $R_{1,t}=\frac{1+\mu}{\beta}$ into equation (22–2) leads to:

$$\frac{B_0}{P_0^{FU}}=\sum_{t=0}^{\infty}\left[\beta^t(1-\pi)^t\beta\pi\frac{\left(\frac{1+\mu}{\beta}\right)^{t+1}B_0}{(1+\mu)^{t+1}P_0^{QT(d=0)}}\right]+\lim_{T\to\infty}\left[\beta^T(1-\pi)^T\frac{\left(\frac{1+\mu}{\beta}\right)^{T}B_0}{(1+\mu)^{T}P_0^{QT(d=0)}}\right]$$

$$=\sum_{t=0}^{\infty}\left[(1-\pi)^t\pi\frac{B_0}{P_0^{QT(d=0)}}\right]+\lim_{T\to\infty}\left[(1-\pi)^T\frac{B_0}{P_0^{QT(d=0)}}\right]=\frac{B_0}{P_0^{QT(d=0)}}.$$

Hence, $P_0^{FU}=P_0^{QT(d=0)}$. In this case, there is no discontinuity in the price level at switching.

As in the FS regime, there are potentially two more scenarios. When the initial price starts from $P_0^{FU} > P_0^{QT(d=0)}$ given inequality (17), positive prices cannot be supported as the real money balance $(\frac{M_t}{P_t^{FU}})$ converges to zero. In this case, however, even before $\frac{M_t}{P_t^{FU}}$ converges to zero, the inflationary price may fall to the QTM price at switching, and a large $\frac{P_t^{NR}}{P_{t+1}^{QT(d=0)}}$ may make positive prices infeasible in equation (20–1). In any case, the accelerating inflation process is not feasible.

When the initial price starts from $P_0^{FU} < P_0^{QT(d=0)}$, the deflationary process is initiated. As equation (20–1) implies, the deflation rate $(\frac{P_t}{P_{t+1}})$ converges to $\frac{1}{\beta(1-\pi)}$ with growing real money balances. Thus, the terminal condition (23′) converges to a positive constant:[16]

$$0 < \lim_{T\to\infty}\left[\beta^T(1-\pi)^T\frac{B_0}{\beta^T(1-\pi)^T P_0^{NR}}\right] = \frac{B_0}{P_0^{NR}} < \infty. \qquad (23'')$$

Here, the relaxed terminal condition (23′) is still satisfied.

Thus, the initial price in the FU regime could not only be the QTM price $(P_0^{QT(d=0)})$ but also $P_0^{FU} \leq P_0^{QT(d=0)}$. If $P_0^{FU} = P_0^{QT(d=0)}$, then there is no discontinuous price jump at switching. If $P_0^{FU} < P_0^{QT(d=0)}$, then the price level surges from the deflationary trend to the QTM price at switching. In the latter case, the deflationary process is determined by equation (20–1) or:

$$\frac{P_t^{FU}}{P_{t+1}^{FU}} = \frac{1}{\beta}\left[1-\lambda\left(\chi+\frac{M_t}{P_t^{FU}}\right)^{-\frac{1}{\sigma}}c\right] + \pi\left(\frac{P_t^{FU}}{P_{t+1}^{FU}} - \frac{P_t^{FU}}{P_{t+1}^{QT(d=0)}}\right). \qquad (20\text{–}2)$$

One interesting feature of equation (20–2) is that with faster monetary expansion (a higher μ), the price process is more deflationary in the FU regime. Given an upward price jump at switching at time t+1, $\frac{P_t^{FU}}{P_{t+1}^{FU}} > 1 > \frac{P_t^{FU}}{P_{t+1}^{QT(d=0)}}$, and $\frac{P_t^{FU}}{P_{t+1}^{FU}} - \frac{P_t^{FU}}{P_{t+1}^{QT(d=0)}}$ in equation (20–2) is positive. Because the QTM price is proportional to existing money stocks, $P_{t+1}^{QT(d=0)}$ is higher with faster monetary expansion. Then, $\frac{P_t^{FU}}{P_{t+1}^{FU}}$ is higher from equation (20–2).

[16] The unfunded component, or the nonzero terminal condition, has the same structure as the rational bubble proposed by Blanchard and Watson (1982), Weil (1987), and others, in the sense that its real value grows at a discount rate $(1-\beta)$ plus a bursting probability (π).

Accordingly, rapid monetary growth makes ongoing deflation severe. With price surges being more likely (a higher π), the price process is also more deflationary in the FU regime. In addition, the first term on the right-hand side of equation (20–2) implies that higher discount rates $\frac{1}{\beta}$ (lower discount factors) add to deflationary pressures during the FU regime.

A final remark in this subsection regards equation (22–2). This equation is rewritten as:

$$\frac{B_h}{P_h^{FU}} = \sum_{t=h}^{\infty}\left[\beta^{t-h}(1-\pi)^{t-h}\beta\pi\frac{R_{1,t}\prod_{i=h}^{t-1}R_{1,i}B_h}{P_{t+1}^{QT(d=0)}}\right] + \lim_{T\to\infty}\left[\beta^{T-h}(1-\pi)^{T-h}\frac{\prod_{i=h}^{T-1}R_{1,i}B_h}{P_T^{FU}}\right]. \quad (22\text{–}3)$$

As time goes by, the price surge (a jump from P_t^{FU} to $P_{t+1}^{QT(d=0)}$) becomes sharper with growing money stocks. Thus, the first term of the right-hand side of equation (22–3) is devalued more heavily, and the share of the unfunded component in the real valuation of the public bonds is larger.

3.4.2. The FU regime with $d > 0$

Let us move to the case with catastrophic endowment shocks ($d > 0$). Given aversion to catastrophic risks, the magnitude of endowment shocks d is adjusted by the degree of relative risk aversion $\gamma > 1$, or $\delta = \gamma d$. If d is relatively small,[17] then:

$$1 - \delta \approx (1-d)^{\gamma}. \quad (24)$$

If the actual magnitude of shocks d is replaced by the risk-adjusted magnitude γd, then a heavier weight is put on the expected marginal utility at switching $\pi u'((1-d)^L c) = \frac{1}{(1-\gamma d)^L c}\pi \approx \left[\frac{1}{(1-d)^L c}\right]^{\gamma}\pi$; that is, $\frac{1}{(1-d)^L c}\pi < \left[\frac{1}{(1-d)^L c}\right]^{\gamma}\pi \approx \frac{1}{(1-\gamma d)^L c}\pi$ with $\gamma > 1$. Note that the functional form of $u(c)$ remains logarithmic, and that the intertemporal elasticity of substitution is still one, as in equation (2). That is, by introducing the risk-adjusted magnitude of shocks (γd) instead of d, the degree of relative risk aversion can be determined independently of the unit intertemporal elasticity of substitution, as in the preference proposed by Epstein and Zin (1989).

In the presence of endowment shocks, equation (21) is replaced by:

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[17] If d (the size of catastrophic damage per period) is small, but L (the length of the catastrophic period) is long, then the initial catastrophic shock is still large.

$$\frac{B_t}{P_t^{NR}} = \beta E_t\left[\frac{c}{c_{t+1}}\left(\frac{B_{t+1}}{P_{t+1}} + \varepsilon\right)\right].$$

Here, the discount factor is not deterministic (β) but stochastic ($\beta\frac{c}{c_{t+1}}$). On the other hand, equation (22–1) is replaced by:

$$\frac{B_0}{P_0^{FU}} = \frac{\beta(1-\pi)\varepsilon}{1-\beta(1-\pi)} + \sum_{t=0}^{\infty}\left[\beta^t(1-\pi)^t\beta\frac{1}{(1-\gamma d)^L}\pi\frac{R_{1,t}B_t}{P_{t+1}^{QT}}\right] + \lim_{T\to\infty}\left[\beta^T(1-\pi)^T\frac{B_T}{P_T^{FU}}\right].$$

From equations (18) and (24):

$$E_t\left(\frac{P_t^{FU}}{P_{t+1}}\frac{c}{c_{t+1}}\right) = (1-\pi)\frac{P_t^{FU}}{P_{t+1}^{FU}} + \pi\frac{P_t^{FU}}{\left(\frac{1}{1-d}\right)^{\eta L}P_{t+1}^{QT(d=0)}}\frac{c}{(1-d)^{\gamma L}c} = (1-\pi)\frac{P_t^{FU}}{P_{t+1}^{FU}} + \pi\frac{1}{(1-d)^{L(\gamma-\eta)}}\frac{P_t^{FU}}{P_{t+1}^{QT(d=0)}}.$$

Then, equation (20–2) is rewritten as:

$$\frac{P_t^{FU}}{P_{t+1}^{FU}} = \frac{1}{\beta}\left[1-\lambda\left(\chi+\frac{M_t}{P_t^{FU}}\right)^{-\frac{1}{\sigma}}c\right] + \pi\left[\frac{P_t^{FU}}{P_{t+1}^{FU}} - \left(\frac{1}{1-\Delta}\right)^L\frac{P_t^{FU}}{P_{t+1}^{QT(d=0)}}\right], \qquad (20\text{–}3)$$

where $\left[\frac{1}{(1-d)^{\gamma-\eta}}\right]^L\pi \approx \left(\frac{1}{1-\Delta}\right)^L\pi$, and:

$$\Delta = (\gamma-\eta)d. \qquad (25)$$

Note that Δ is always positive given that $\gamma > 1$, and $0 < \eta < 1$ from inequality (A–2).[18]

A comparison between equations (20–2) and (20–3) indicates that with a larger Δ, $\left[\frac{P_t^{FU}}{P_{t+1}^{FU}} - \left(\frac{1}{1-\Delta}\right)^L\frac{P_t^{FU}}{P_{t+1}^{QT(d=0)}}\right]$ in equation (20–3) is smaller than $\left(\frac{P_t^{FU}}{P_{t+1}^{FU}} - \frac{P_t^{FU}}{P_{t+1}^{QT(d=0)}}\right)$ in equation (20–2). This implies that the deflationary pressure is mitigated by either a larger d (larger shocks) or a higher γ (greater risk aversion). A reason for this implication is that, as Rietz (1988) and others demonstrate, real interest rates decline substantially with aversion to catastrophic risks. Accordingly, the expected deflation also decreases because it is approximately equal to the real rate of interest at a nominal interest rate of zero.

Another interesting implication is that an inflationary phase may even emerge in the FU regime if Δ is large, either as a result of high-risk aversion or large catastrophic shocks. If P_t^{FU} is smaller than $P_{t+1}^{QT(d=0)}$, but the former does not differ much from the latter just after the economy starts at time 0, then $\left[\frac{P_t^{FU}}{P_{t+1}^{FU}} - \left(\frac{1}{1-\Delta}\right)^L\frac{P_t^{FU}}{P_{t+1}^{QT(d=0)}}\right]$ in equation (20–3) may be negative with a larger Δ. Thus, mild inflationary pressures could be created

[18] If the initial price P_0^{NR} is equal to $P_0^{QT(d=0)}$, then it is assumed that the price process follows the QTM without any catastrophic shocks.

initially in the FU regime. If P_0^{FU} starts from $P_0^{QT(d=0)}$, then the price level behaves as if catastrophic shocks were absent ($d = 0$).

3.5. On the term structures of interest rates during the FU regime

Let us derive the term structures of interest rates that emerge during the deflationary FU regime. From the Euler equation appearing in the first equality of equation (19), an n-period nominal yield ($R_{n,t}$) is obtainable as follows:[19]

$$\left(\frac{1}{R_{n,t}}\right)^n = \beta^n E_t\left[\frac{P_t^{FU}}{P_{t+n}}\frac{c_t}{c_{t+n}}\right].$$

Using notation Δ defined in equation (25), the above Euler equation is developed as:

$$\left(\frac{1}{R_{n,t}}\right)^n = \left\{\beta^n(1-\pi)^n\frac{P_t^{FU}}{P_{t+n}^{FU}}\right\} + \left\{\beta^n\frac{P_t^{FU}}{P_{t+n}^{QT(d=0)}}\sum_{i=1}^n\left[\pi(1-\pi)^{i-1}\left(\frac{1}{1-\Delta}\right)^{L-n+i}\right]\right\}, \quad (26\text{–}1)$$

for $n = 1,2,\ldots,L+1$, and:

$$\left(\frac{1}{R_{n,t}}\right)^n = \left\{\beta^n(1-\pi)^n\frac{P_t^{FU}}{P_{t+n}^{FU}}\right\}$$

$$+\left\{\beta^n\frac{P_t^{FU}}{P_{t+n}^{QT(d=0)}}\left[1-(1-\pi)^{n-L-1}+(1-\pi)^{n-L-1}\sum_{i=1}^{L+1}\left[\pi(1-\pi)^{i-1}\left(\frac{1}{1-\Delta}\right)^{i-1}\right]\right]\right\}, \quad (26\text{–}2)$$

for $n \geq L+2$.

With $d = 0$ or $\Delta = 0$:

$$\left(\frac{1}{R_{n,t}}\right)^n = \left\{\beta^n(1-\pi)^n\frac{P_t^{FU}}{P_{t+n}^{FU}}\right\} + \left\{\beta^n\frac{P_t^{FU}}{P_{t+n}^{QT(d=0)}}\left[1-(1-\pi)^n\right]\right\} \quad (26\text{–}3)$$

The first term on the right-hand side of equations (26–1), (26–2), and (26–3) represents the negative impact on yields caused by the ongoing deflationary expectations, whereas the second term on the right-hand side represents the positive impacts on yields resulting from the expectation of future price surges. Thus, the yield curves are

[19] Here, longer-term public bonds are redundant assets and they can be replicated from the one-period public bonds. Thus, the yield curve is neutral with respect to the maturity structure of the public bonds. In this regard, our model differs from that of Cochrane (2001), where the maturity structure of the public debt has effects on current and future inflation.

determined by these competing expectations.

As discussed in Sections 3.4.1 and 3.4.2, on the one hand, a higher switching probability generates stronger deflationary pressures and helps to flatten yield curves. On the other hand, lower discount rates, larger catastrophic shocks, and greater risk aversion mitigate deflationary pressures and work to steepen yield curves. In addition, higher monetary growth contributes to larger price surges in the future and increases the dominance of inflationary expectations, thereby resulting in steeper yield curves.

Figure 3-3 illustrates these competing effects on yield curves. In the absence of catastrophic endowment shocks ($\Delta = 0$), a downward-sloping path (line $P_t^{FU}B$) depicts the deflationary path that emerges in the FU regime, and a steeper upward-sloping path (line $P_t^{QT}A$) depicts the QTM price path in the FS regime. At some point in the future (for example, time s_1 or time s_2), the price level jumps from the deflationary path to the QTM price path. Reflecting the possibility of such a price surge, the expected price path (line $P_t^{FU}C$) is drawn as a less steep but still upward-sloping line between the deflationary and QTM price path lines. The resulting expected price path makes the yield curve upward-sloping.

Given catastrophic endowment shocks together with risk aversion ($\Delta > 0$), the price level overshoots line $P_t^{QT}A$ at some point in the future (for example, time s_3), thereby increasing inflationary expectations. At the same time, deflationary pressures are mitigated to some extent in the FU regime, and the deflationary path becomes less downward-sloping (from line $P_t^{FU}B$ to $P_t^{FU}B'$). Consequently, the expected price path, as well as the yield curve, is even more upward-sloping (shifting from line $P_t^{FU}C$ to $P_t^{FU}C'$).

3.6. Real risk-free rates and real growth rates

Finally, real risk-free rates are compared with real growth rates in the deflationary FU regime. As Blanchard (2019) emphasizes, debt rollovers may be feasible even in an economy with much public debt, when interest rates continue to be below growth rates. However, the current model suggests that the feasibility of debt rollovers differs essentially from fiscal sustainability. As shown below, interest rates may be below growth rates in real and nominal terms in the FU regime, and the public bonds are valued highly thanks to the bubbles that back the unfunded component of the GIBC. However, the bubbles eventually burst, with price surges and interest-rate hikes at switching.

Let us demonstrate that the real safe rate of interest is quite low in the deflationary FU regime, and that it may even be below the real growth rate. In the current setup, real values are never influenced by fiscal or monetary policies. The real growth rate in the FU regime is determined by $(1-\pi)+\pi(1-d)^L$, and its net rate is approximated by $-\pi dL$. The real safe one-period rate $r_{1,t}$ satisfies the following Euler equation:

$$(1+r_{1,t})\beta E_t\left(\frac{c_t}{c_{t+1}}\right)=(1+r_{1,t})\beta\left[(1-\pi)+\pi\frac{1}{(1-\gamma d)^L}\right]=1.$$

Then, $r_{1,t}$ is approximated by $1-\beta-\gamma\pi dL$.

If $(\gamma-1)\pi dL$ dominates a discount rate $1-\beta$, then the real safe rate is short of the real economic growth rate. With stronger risk aversion (when γ is higher than one) and larger catastrophic risks (a larger dL), the real risk-free rate is lowered substantially, as Rietz (1988) demonstrates. Given the expected inflation, the order of interest rates and growth rates does not change that much in real and nominal terms.

4. Calibration exercises for Japan's long-run deflation

In this section, several calibration exercises are presented to mimic Japan's long-run mild deflation. As discussed in the introduction, there is no price surge event observed during the previous decades, and it is virtually impossible to specify an occurrence probability (π) or the size of catastrophic shocks (d and L). Here, instead of considering a price surge a catastrophic event, a large-scale Tokyo inland earthquake is regarded as triggering a regime switch and causing price surges. The goal of this section is to describe not only the mild deflation with near-zero rates that occurred from the mid-1990s, but also the price stagnation that commenced in the mid-1980s, a drastic decline in short-term yields that occurred in the first half of the 1990s, and the slowly flattening yield curves observed in the twenty-first century.

According to the Headquarters for Earthquake Research Promotion, there is a 4% chance of a Tokyo inland earthquake occurring in any coming year, and around a 70% chance of one occurring in the next three decades ($1-(1-0.04)^{30}\approx 0.706$). The Cabinet

Office predicts possible damage from such an earthquake at around 20% of GDP in the first year, and considers that complete recovery would take several years. Given the above catastrophic event, $\pi \leq 0.04$, $L = 3$, and $d = 0.072$, where $(1-d)^3 \approx 1 - 0.2$. Here, normal consumption c is detrended at one. It is assumed that the economy starts from 1980 with a switching probability ($\pi > 0$), and the catastrophic possibility ($\Delta > 0$) is introduced from 1986 onwards. The timing for this setup accords with the observed stagnation of the price level that commenced from the mid-1980s, despite rapid monetary expansion, as demonstrated in Section 2.

A set of parameters associated with money demand is chosen, as explained below. Given that c is detrended at one, a sequence of monetary growth μ_t for the years 1980 to 2017 is computed from the growth rate of the outstanding BoJ notes, adjusted by a real economic scale or divided by real GDP. In addition, μ is set at 0.02 from 2018 onwards, given that the long-run inflation target is 2%. In the FS regime, the money stock relative to a nominal economic scale $(\frac{M_t}{P_t c})$ is constant, and it is accordingly set at the 1955–1970 average of the Marshallian k (the ratio of the BoJ notes relative to nominal GDP), or $\kappa = 0.078$. In this context, Japan's fiscal policy was considered Ricardian before the early 1970s. λ is determined by equation (16) together with $\beta = 0.98$, $\chi = 1$, $\mu = 0.02$, $c = 1$, and $\kappa = 0.078$. $P_t^{QT(d=0)}$ is approximated by equation (15) with a constant μ (= 0.02) and a time-varying M_t up to 2017, and M_t growing at 2% from 2018.

How should σ in equation (3) be determined? Given the preference specification in equations (2) and (3), the money demand function is linearized as follows:

$$\frac{\Delta(M_t/P_t)}{M_{t-1}/P_{t-1}} = -\sigma\frac{\chi + M_{t-1}/P_{t-1}}{M_{t-1}/P_{t-1}}\frac{\Delta R_t}{R_{1,t-1}-1} + \sigma\frac{\chi + M_{t-1}/P_{t-1}}{M_{t-1}/P_{t-1}}\frac{\Delta c_t}{c_{t-1}}. \tag{27}$$

Thus, $\sigma\frac{\chi + M_{t-1}/P_{t-1}}{M_{t-1}/P_{t-1}}$ can be interpreted as either interest elasticity or income elasticity.

In terms of interest elasticity, σ may be set at a rather low value. More concretely, we choose a value of $\sigma = 0.01$, as we explain below. Above, it is assumed that $\chi = 1$ and $\frac{M_t}{P_t} = \frac{M_t}{P_t c} = 0.078$ for the FS regime.[20] Thus, interest elasticity is computed as -0.13, given $\sigma = 0.01$. This interest elasticity value is quite comparable with the estimates of

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[20] Even in the FU regime, $\frac{M_t}{P_t c}$ remains close to the level that holds in the FS regime for a relatively long time.

interest elasticity in the existing empirical literature: -0.174 by Nakashima and Saito (2012), -0.107 to -0.115 by Fujiki and Nakashima (2019), and -0.0466 to -0.0999 by Watanabe and Yabu (2018).

On the other hand, in the case of income elasticity, Fujiki and Watanabe (2004), Fujiki and Nakashima (2019), and others, estimate that it is close to one, but the above elasticity $\sigma\frac{\chi+M_{t-1}/P_{t-1}}{M_{t-1}/P_{t-1}}$ is much less than one. However, in the current context, income elasticity is associated with temporary deviations from the detrended level $c = 1$. Thus, lower short-run elasticity may not be inconsistent with long-run unit elasticity. In addition, the property whereby $\sigma\frac{\chi+M_{t-1}/P_{t-1}}{M_{t-1}/P_{t-1}}$ decreases in a liquidity trap when $\frac{M_t}{P_t}$ increases is empirically consistent with the finding that money demand became less responsive to aggregate output as the real money balance grew (Nakashima and Saito, 2012; Fujiki and Nakashima, 2019).

With inequality (19) satisfied in all the exercises,[21] the accelerating inflation process is not feasible in either of the two regimes. Thus, the QTM price of 1980 is considered the only legitimate equilibrium price prevailing in the FS regime. The initial money stock M_0 is set at 100.[22] The initial price is set at a level slightly less than the QTM price ($P_{1980}^{FU} = 1280 < P_{1980}^{QT(d=0)} \approx 1282$), thereby initiating the deflationary process.

In sum, the evolution of the price level and the yield curves, which is observed for the years 1986 to 2017, is simulated by assuming only three time-specific exogenous components: (i) the realized monetary growth from 1980 to 2017 (μ_t), (ii) a slight downward deviation of the initial price from the QTM price in 1980 ($P_{1980}^{FU} < P_{1980}^{QT(d=0)}$), and (iii) the small possibility of a catastrophic event, such as a Tokyo inland earthquake ($\pi \leq 0.04$, $L = 3$, and $d = 0.072$). As discussed in Section 3.4.1, the price level prevailing during the FU regime is independent of the fiscal surplus (ε) and the initial holdings of public bonds (B_0). Thus, no assumption is made for ε or B_0.

As a baseline case (**Case 1**), we assume that Δ is 0.144, given $(\gamma - \eta)d = 2 \times$

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[21] Given the above set of parameters, $\lambda\chi^{-\frac{1}{\sigma}}c$ ($= \lambda$ in this case) is greater than one; it is 71.7 in Cases 1 and 2, 1.67 in Case 3, and 53.8 in Cases 4, 5, 6, and 7.

[22] As implied by equation (22–2), the initial price level P_{1980}^{NR} is independent of the initial balance of the public bonds B_{1980}.

0.072.[23] As shown in Figure 4-1, the predicted price level is more deflationary than the observed level. In the absence of catastrophic shocks ($\Delta = 0$ or $d = 0$ in **Case 2**), it is even more deflationary. Conversely, it is relatively inflationary with a higher σ ($\sigma = 0.02$ in **Case 3**). More empirically plausible cases are presented later.

As demonstrated in Figure 4-2, the 1-year yield predicted in Case 1 can capture the downward trend in short-term yields that appeared between the mid-1980s and the mid-1990s, and the near-zero interest rate situation that commenced from the mid-1990s. In the early 1990s, the 1-year yield temporarily increased owing to a temporary decrease in the money stock per output. Comparing Case 1 ($\Delta > 0$) and Case 2 ($\Delta = 0$), such a downward trend in short-term yields is driven not by endowment shocks, but by a switching possibility ($\pi > 0$). In comparison with Case 3 ($\sigma = 0.02 > 0.01$), an extremely low value for σ is necessary to generate near-zero interest rates in the mid-1990s. Note that the assumption of a constant discount factor β limits our ability to trace the observed 1-year yield. With a lower β (< 0.98) in the 1980s, the model would fit better with higher interest rates.[24]

Figure 4-3 presents four more cases for the predicted price level. Compared with Case 1, in which the price level is a little too deflationary, the deflationary pressures are mitigated with a higher β (from 0.98 to 0.99 in **Case 4**), and a larger Δ (from $0.072 \times 2 = 0.144$ to $0.072 \times 3 = 0.216$ in **Case 6**). A comparison between Case 4 ($\pi = 0.04$) and **Case 5** ($\pi = 0.02$) indicates that a higher π may even induce an inflationary trend for the initial decade (up to the mid-1990s), given $\frac{P_t^{FU}}{P_{t+1}^{FU}} - \left(\frac{1}{1-\Delta}\right)^M \frac{P_t^{FU}}{P_{t+1}^{QT(d=0)}} < 0$ in equation (20″). The price level predicted in Case 6 is quite consistent with the observation of initial mild inflation followed by mild deflation for the years 1986 to 2017. With weaker deflationary pressures, the 1-year yield declines more slowly, as shown in Figure 4-4. In **Case 7**, which is the same as Case 6 except that π is higher ($\pi = 0.03 > 0.02$), the price

[23] As we assume a low value for σ, inequality (A–2) and $0 < \eta < 1$ are satisfied in all the cases. Concretely, η is computed as 0.8035 in Cases 1 and 2, 0.9069 in Case 3, and 0.8450 in Cases 4, 5, and 6.

[24] According to Okazaki and Sudo (2018), the natural rate of interest was 4% in the 1980s, but it decreased to 0.3% in the 2010s.

process is even less deflationary, as shown in Figure 4-3, and short-term yields decline much more slowly, as Figure 4-4 shows. However, Case 7 produces more realistic upward-sloping yield curves, as shown below.

How do the term structures of interest rates behave in each case? In Figures 4-5-1 through 4-5-4, the predicted yield curves are compared with the observed one; that is, the observed 1-year yield is adjusted by the predicted one, on which the observed term spreads are added. As shown in Figure 4-5-1,[25] the predicted yield curves are all upward-sloping in Case 1, reflecting more inflationary pressure from future price surges, and less deflationary pressure caused by catastrophic endowment shocks Δ ($\Delta = 0.144$). The predicted yield curve in Case 2 is much less upward-sloping, with the deflationary expectations dominating in the absence of the catastrophic endowment shock (see Figure 4-5-2). Conversely, in Case 6, the yield curve is more upward-sloping with the deflationary expectations being less dominant as a result of the larger endowment shocks (see Figure 4-5-3). However, even in Case 6, the predicted yield curves are less upward-sloping than the observed yield curves. In particular, the predicted 10-year over 1-year yield (0.5%) is much smaller than the observed one (2.5%) in 1996. Increasing inflationary pressures through a higher π ($\pi = 0.03 > 0.02$), Case 7, as shown in Figure 4-5-4, yields predicted upward-sloping yield curves that are more consistent with the observed curves in the 2000s and 2010s. Nevertheless, the predicted yield curve is still flatter than the observed one in 1996.

In either case, a discontinuous nominal adjustment following a catastrophic event would be immense. As an example, as Figure 4-6 demonstrates for Case 6, if a switch occurred in 2025, then the price level would be multiplied by 5.2 from 2024 to 2025, decline at a rate of 6.1% from 2025 to 2028 (from equation (18) or (A–3)), and then commence on a 2% inflation path in 2028. In the same case, 1-year yields would immediately leave the zero level, and they would overshoot a long-run rate of 3% by 1.2% from 2025 to 2027, based on equation (A–4).

[25] The yield curve starting from 1981 is computed by assuming the absence of endowment shocks ($\Delta = 0$) because it is assumed that catastrophic endowment shocks are present from 1986 onwards.

In all seven cases, the share of the unfunded component in the real valuation of the public bonds, which is computed from equation (22–2), is significant during the FU regime.[26] As reported for Case 6 in Table 4-1, for example, the share of the unfunded component amounts to 34.2% in 1980, 58.4% in 2000, 69.9% in 2010, and 77.2% in 2017. As discussed in Section 3.4.1, as time goes by, the price surge is steeper at switching, and the unfunded share is larger as a consequence of heavier devaluations. Compared with other cases, the unfunded share increases with more deflationary pressure caused by either a lower β or a smaller Δ, and it decreases with higher initial inflation caused by a combination of a higher π and a larger Δ.

5. Conclusion

One of the most important implications from this paper is that the current mild deflation is tightly linked with a (far) future price surge. Thus, the present mild deflation with near-zero interest rates cannot be controlled completely independently of such a long-run equilibrium context. More concretely, mild deflation cannot be remedied by the current monetary/fiscal expansion. It can be dissolved only at the cost of one-off price surges. In this context, fiscal sustainability will be restored not by drastic fiscal reforms, hyperinflation, or continuous mild inflation with near-zero interest rates, but largely through a heavy bond devaluation caused by such a one-off price surge. One caveat of these implications is that those living in the pre-surge period are assumed to be identical to those living in the post-surge period in the current representative agent framework. However, intergenerational effects on the price level may emerge in an overlapping generations framework.[27]

Given that a one-off price surge is unprecedented by its nature, and is absent from any observations of the past decades, it is a type of Peso problem that raises the following questions. Is any prediction based on the current model unrealistic or a mere theoretical abstraction? Is there any episode comparable to the one-off price surge phenomena in

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[26] As implied by equation (22–2), the share of the unfunded component is independent of the initial value of the public bonds (B_{1980}).

[27] For example, Aiyagari and Gertler (1985) examine the intergenerational impacts on nominal variables in the context of overlapping generations models.

the Japanese monetary history? Our answer is no for the first question, and yes for the second.

The sharp price increase emerging immediately after the end of World War II in 1945 is often interpreted as a typical hyperinflationary phenomenon, but it can be construed more consistently as a price surge event. If it were a hyperinflationary phenomenon, the real money balance, or the money balance adjusted by a nominal macroeconomic scale, would degenerate to zero. According to Figure 5-1, however, the relative outstanding BoJ notes (divided by nominal gross national expenditure (GNE)) declined significantly following 1945; although they did not fall to 0%, they fell from close to 50% in 1945 to just below 10% in 1950, which was the prewar average. Taking a closer look at the price behavior during the same period, the GNE deflator multiplied 32.4 times, while the outstanding BoJ notes multiplied 6.6 times. Thus, the money-stock-adjusted price level multiplied only 4.9 times (32.4 over 6.6); it was far from a hyperinflationary phenomenon. As a result of this sharp price surge, the Japanese government could repay its public debt practically without any sovereign default; the debt–GNE ratio indeed declined from 174% in 1945 to 14% in 1950.[28] Consistent with the setup of our model, fiscal discipline was finally established by the Dodge Line in 1949, 4 years after the war ended.

As shown in Figure 5-1, the relative size of the outstanding proportion of BoJ notes was quite stable during the pre- and postwar periods; it stayed at around 10% from 1890 to 1937, and at around 8% from 1950 to 1995. Given such a long-run trend in the relative money stocks, it is likely that the prediction demonstrated in Section 4 is not only theoretically consistent, but also empirically plausible. Based on the model, at some point in the future, the relative proportion of outstanding BoJ notes, which was already greater than 20% in 2018, will revert to the post-war average (around 8%) as a result of a one-off price surge, to a level probably several times as high as before. Then, the government could repay the public debt largely without any sovereign default. Given the 1945–1950 experience, the above prediction is not so unrealistic as it may appear.

[28] See Saito (2017) for the data sources.

As a final remark, it is again worth reminding readers that one-off price surges are totally different from hyperinflation;[29] that is, the real money balance is reduced from an excessively high level to a normal level in the former case, whereas under hyperinflation it degenerates to zero, resulting in a chaotic monetary situation.

Appendix 1: The price behavior during the FS regime with $d > 0$

Suppose that the economy switches to the FS regime in time s. The price level coincides with $P_{s+L}^{QT(d=0)}$ when catastrophic shocks disappear completely in time $s+L$. From equation (11), the following holds between time $s+L-1$ and $s+L$:

$$\frac{P_{s+L-1}^{QT}}{P_{s+L}^{QT(d=0)}} = \frac{1}{\beta(1-d)}\left[1-\lambda\left(\chi+\frac{M_{s+L-1}}{P_{s+L-1}^{QT}}\right)^{-\frac{1}{\sigma}}(1-d)c\right].$$

Taking $P_{s+L}^{QT(d=0)}$ and M_{s+L-1} as fixed, and marginally increasing d from zero and P_{s+L-1}^{QT} from $P_{s+L-1}^{QT(d=0)}$, the following total differential is obtainable by a first-order approximation:

$$\eta = \frac{\Delta P_{s+L-1}^{QT}}{P_{s+L-1}^{QT(d=0)}}\Big/\Delta d = R_{1,s}^{QT(d=0)} \Bigg/ \left[1+\frac{R_{1,s}^{QT(d=0)}-1}{\sigma}\frac{\frac{M_s}{P_s^{QT(d=0)}}}{\chi+\frac{M_s}{P_s^{QT(d=0)}}}\right] > 0, \tag{A–1}$$

where $R_{1,s}^{QT(d=0)}$ and $\frac{M_s}{P_s^{QT(d=0)}}\Big/\left[\chi+\frac{M_s}{P_s^{QT(d=0)}}\right]$ are constant, given equations (14) and (15). If:

$$\frac{M_s}{P_s^{QT(d=0)}}\Big/\left[\chi+\frac{M_s}{P_s^{QT(d=0)}}\right] > \sigma, \tag{A–2}$$

then $0 < \eta < 1$.

Equation (A–1) is approximated as:

$$P_{s+L-1}^{QT} = \frac{1}{(1-d)^{\eta}} P_{s+L-1}^{QT(d=0)}.$$

Using the same approximation technique leads to:

$$P_{s+l}^{QT} = \left[\frac{1}{(1-d)^{\eta}}\right]^{L-l} P_{s+l}^{QT(d=0)}, \tag{A–3}$$

for $l = 0, 1, 2, \ldots, L-1$.

From equation (9):

[29] Given a set of parameters for Japan's economy, $\lambda\chi^{-\frac{1}{\sigma}}c > 1$ holds, and hyperinflationary equilibria are ruled out in both regimes.

$$R_{1,t}^{QT} = \frac{1}{\beta(1-d)} \frac{P_{t+1}^{QT}}{P_t^{QT}}.$$

Together with equation (A–1), the total differential is derived by a first-order approximation:

$$\frac{\Delta R_{1,t}^{QT}}{R_{1,t}^{QT(d=0)}} / \Delta d = 1 - \eta. \tag{A–4}$$

Thus, if $0 < \eta < 1$, then $R_{1,t}^{QT} > R_{1,t}^{QT(d=0)}$.

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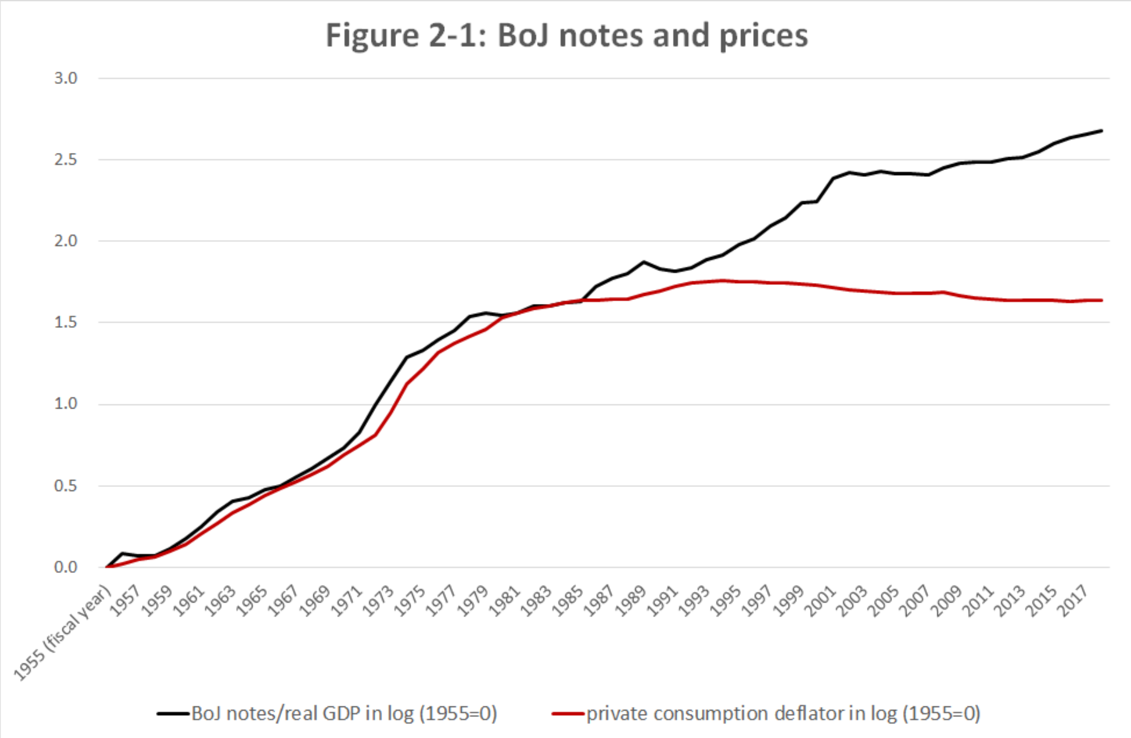
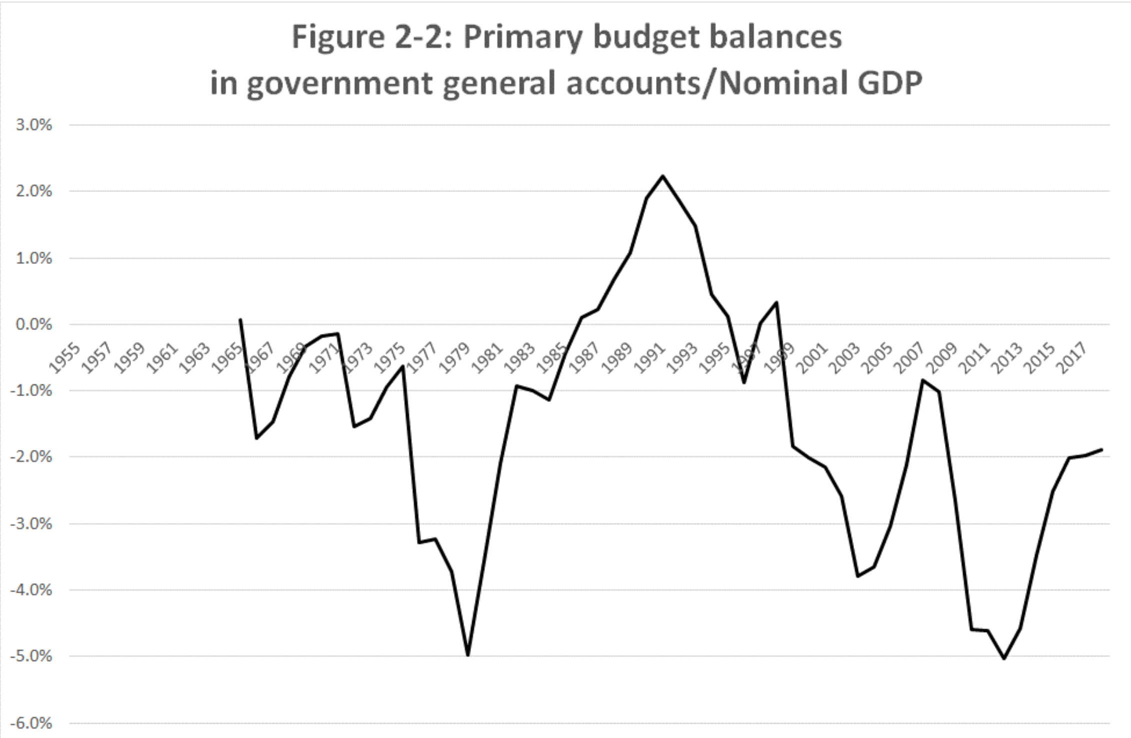
Table 4-1: The share of the unfunded component relative to the real valuation of the public bonds during the FU regime.

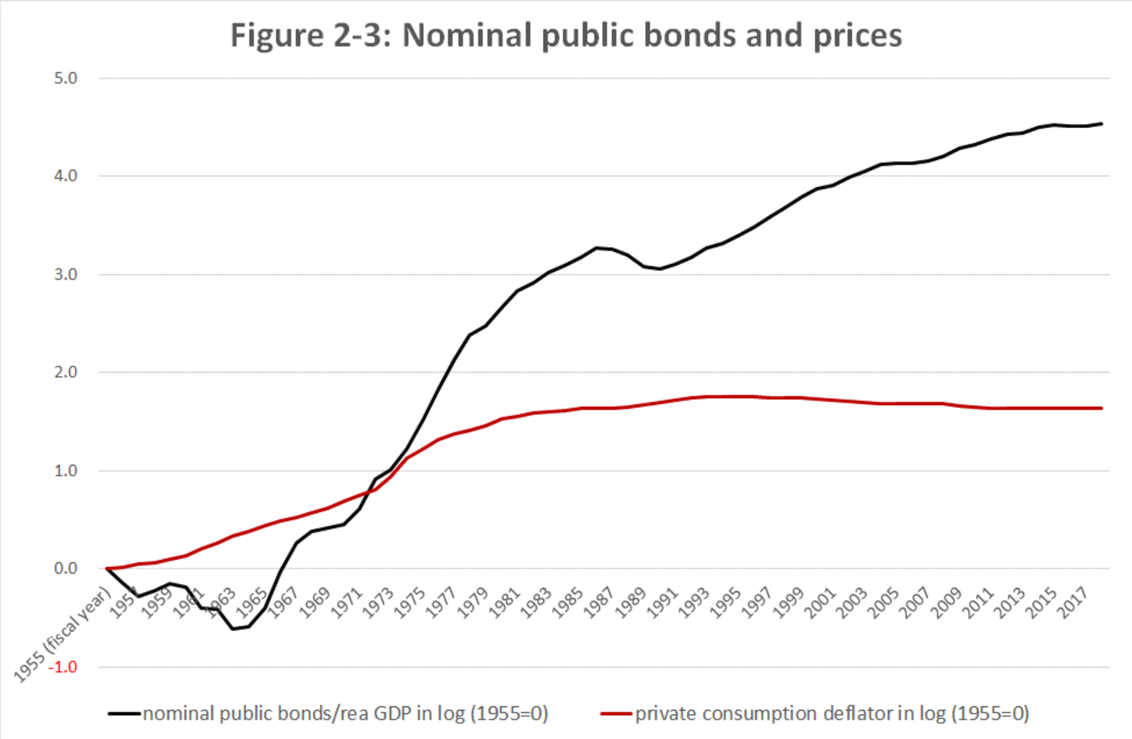
	1980	2000	2010	2017
Case 1: β =0.98, π =0.04, Δ =0.144	26.7%	64.8%	80.3%	87.9%
Case 2: β =0.98, π =0.04, Δ =0.000	45.1%	83.1%	91.6%	95.1%
Case 3: β =0.98, π =0.04, Δ =0.144, σ =0.02	18.3%	49.1%	68.0%	79.2%
Case 4: β =0.99, π =0.04, Δ =0.144	19.1%	49.4%	66.3%	76.6%
Case 5: β =0.99, π =0.02, Δ =0.144	44.3%	69.3%	78.9%	84.5%
Case 6: β =0.99, π =0.02, Δ =0.216	34.2%	58.4%	69.9%	77.2%
Case 7: β =0.99, π =0.03, Δ =0.216	14.8%	35.0%	49.5%	60.4%



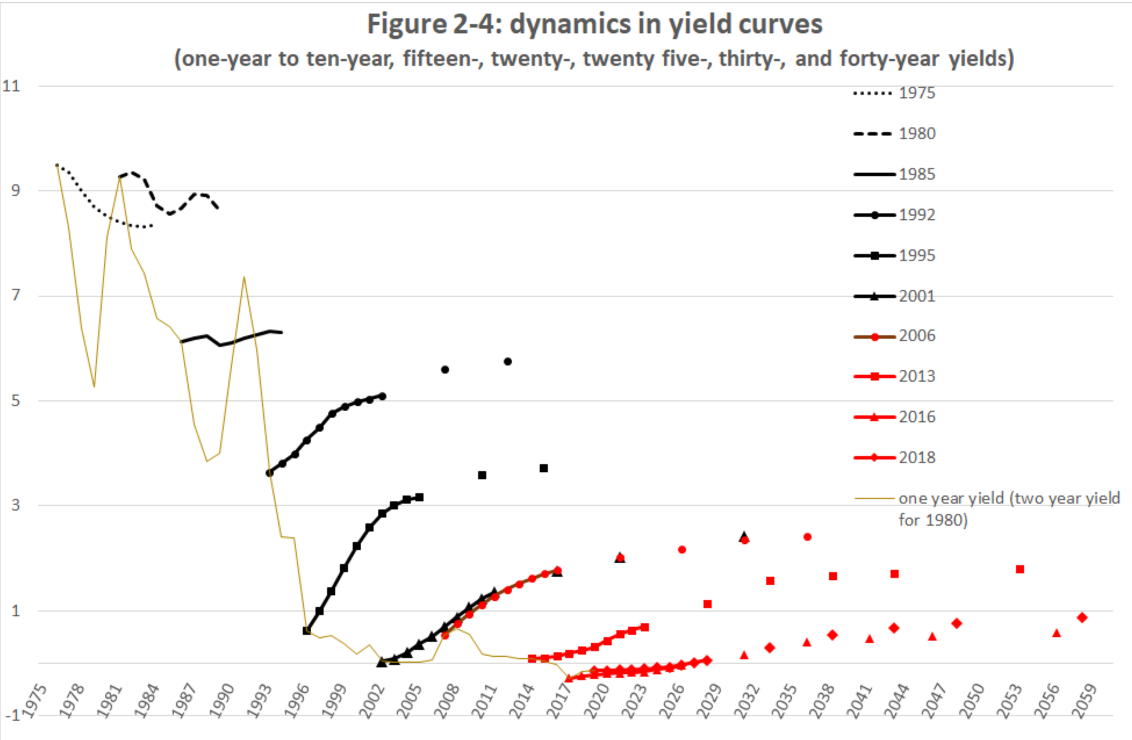
Sources: Bank of Japan, and Cabinet Office.



Sources: Cabinet Office, and Ministry of Finance.



Sources: Bank of Japan, Cabinet Office, and Ministry of Finance.



Sources: Ministry of Finance, and Hamacho SCI GP.

Figure 3-1: The dynamics of $\frac{P_t}{P_{t+1}}$ implied by equation (14) when $\mathbf{0 < \lambda\chi^{-\frac{1}{\sigma}}c < 1}$.

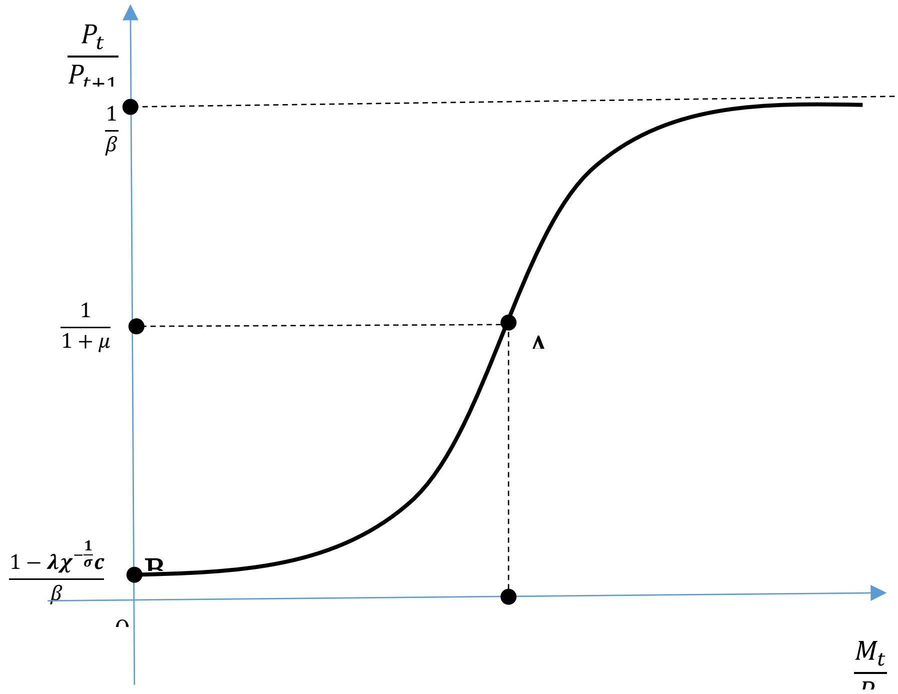


Figure 3-2: The dynamics of $\frac{P_t}{P_{t+1}}$ implied by equation (14) when $\mathbf{1 < \lambda\chi^{-\frac{1}{\sigma}}c}$.

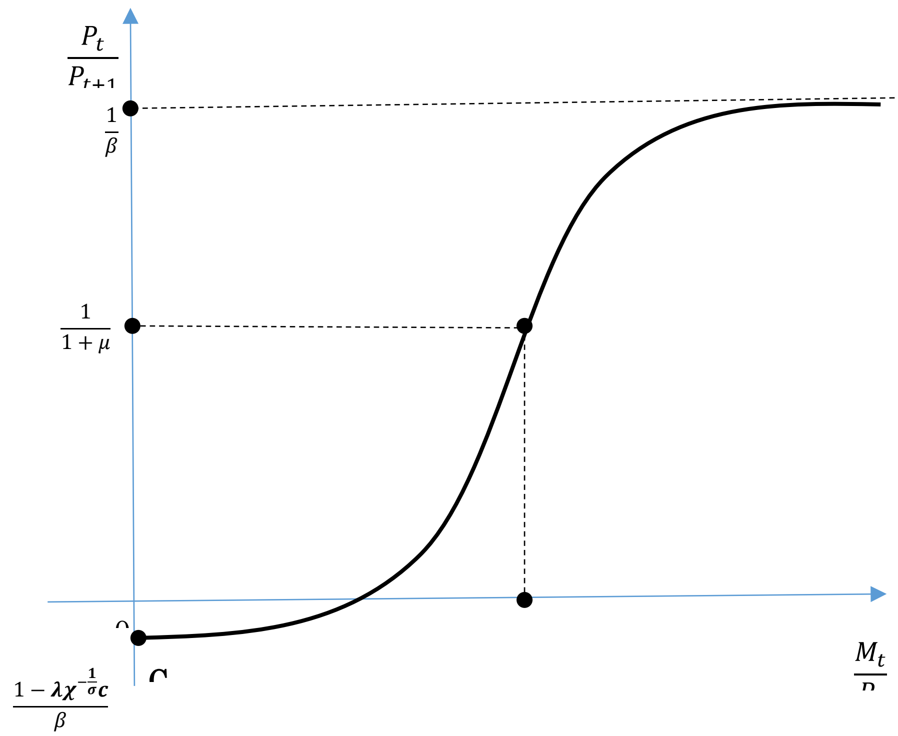


Figure 3-3: Interpretations by switching possibilities from the FU regime to the FS regime

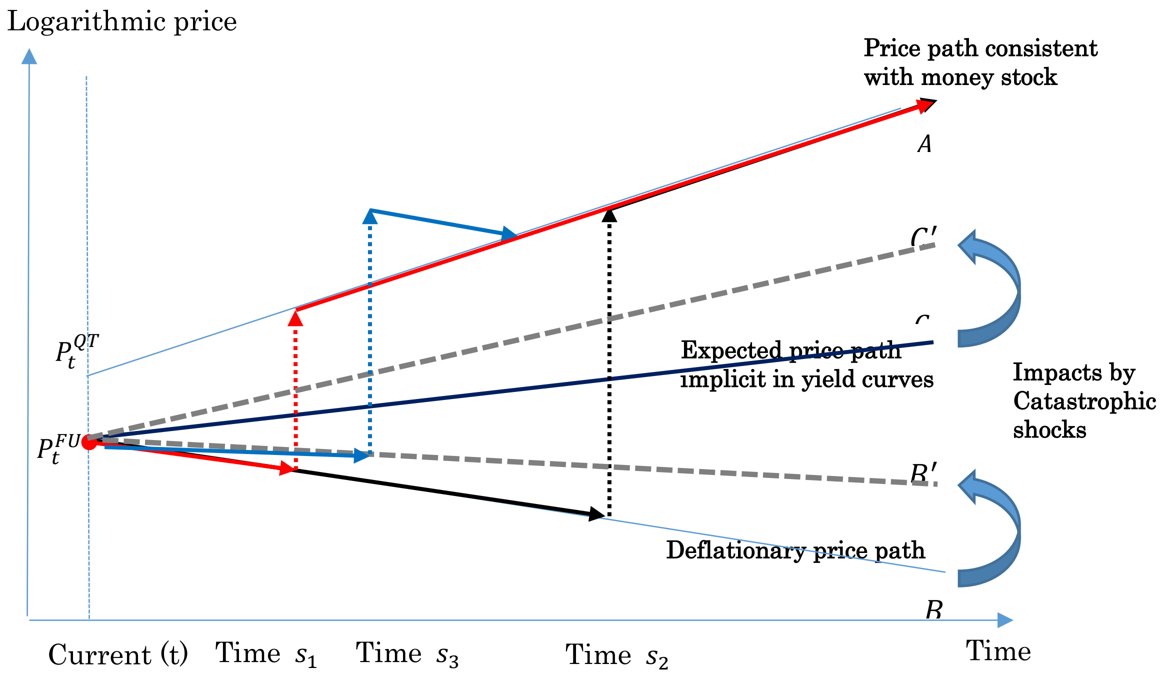


Figure 4-1: Predicted price path

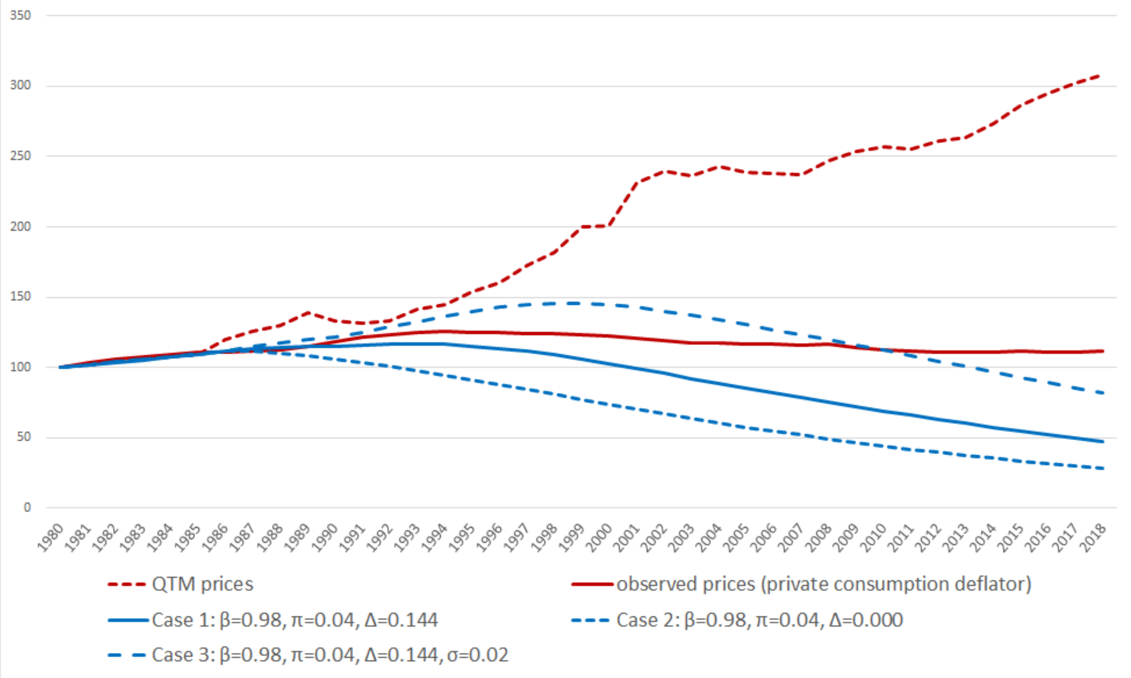
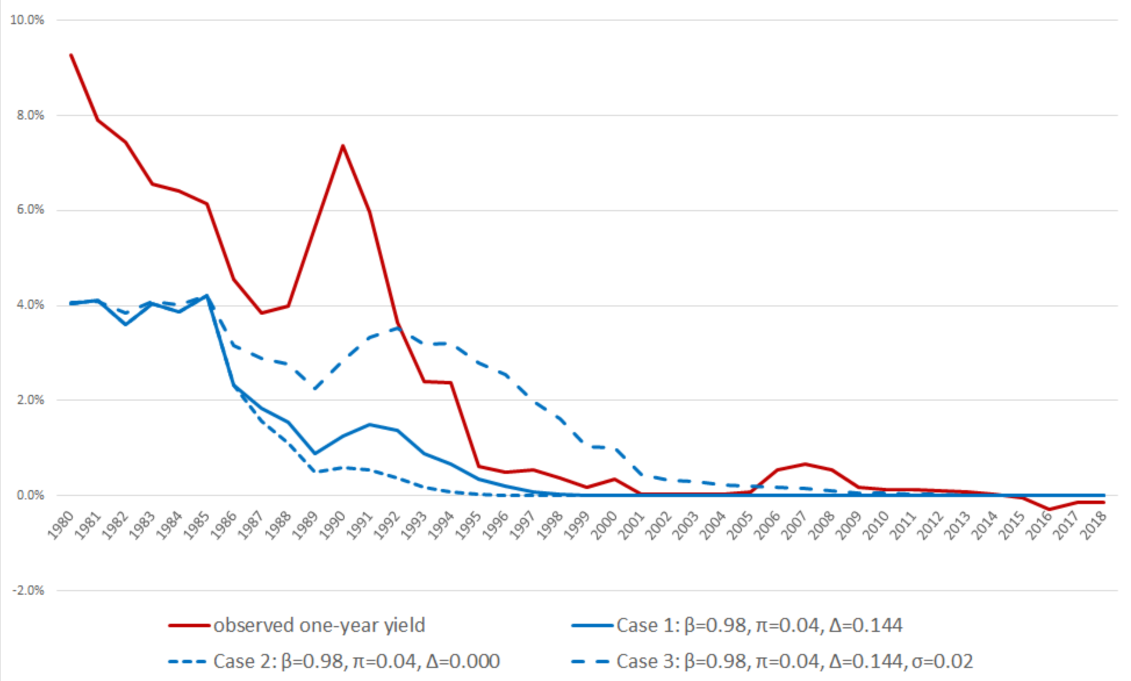


Figure 4-2: Predicted one-year yield



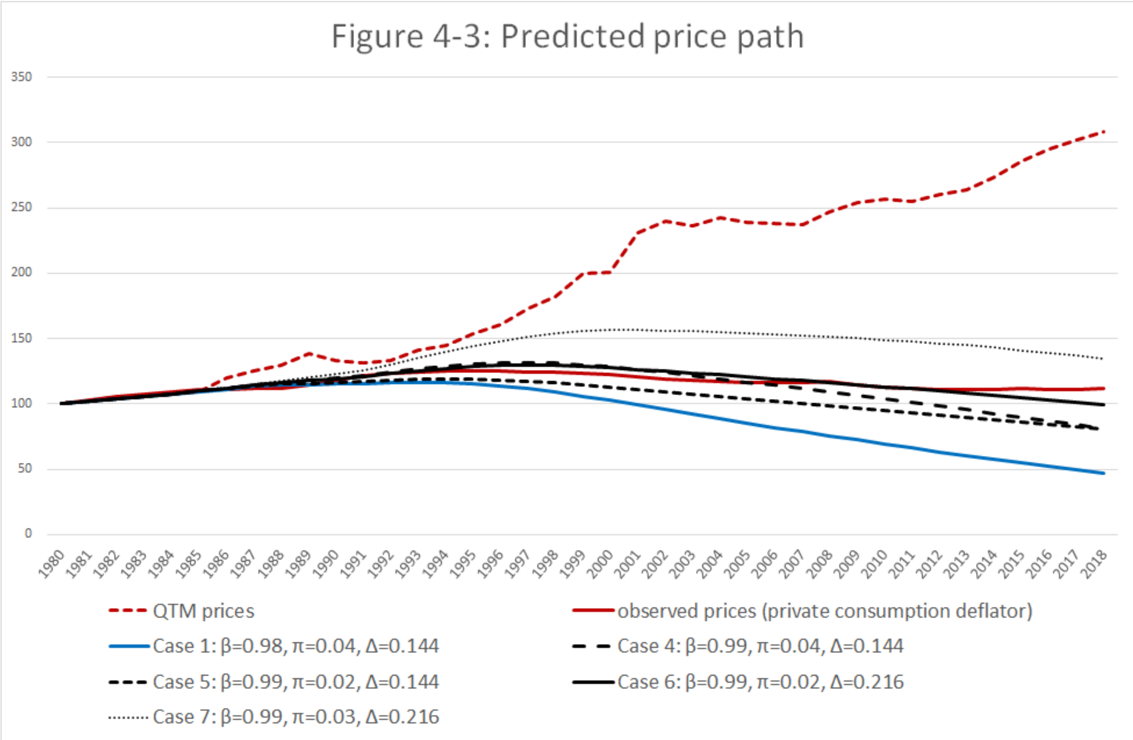


Figure 4-3: Predicted price path
QTM prices
observed prices (private consumption deflator)
Case 1: β=0.98, π=0.04, Δ=0.144
Case 4: β=0.99, π=0.04, Δ=0.144
Case 5: β=0.99, π=0.02, Δ=0.144
Case 6: β=0.99, π=0.02, Δ=0.216
Case 7: β=0.99, π=0.03, Δ=0.216

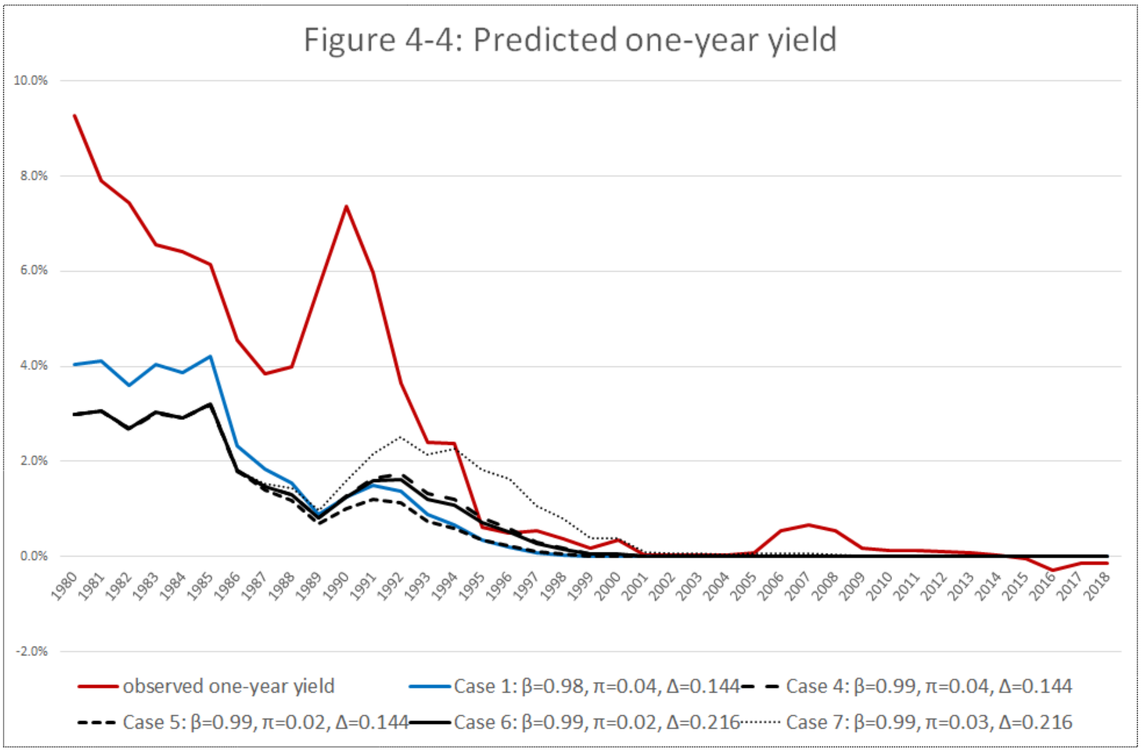


Figure 4-4: Predicted one-year yield
observed one-year yield
Case 1: β=0.98, π=0.04, Δ=0.144
Case 4: β=0.99, π=0.04, Δ=0.144
Case 5: β=0.99, π=0.02, Δ=0.144
Case 6: β=0.99, π=0.02, Δ=0.216
Case 7: β=0.99, π=0.03, Δ=0.216

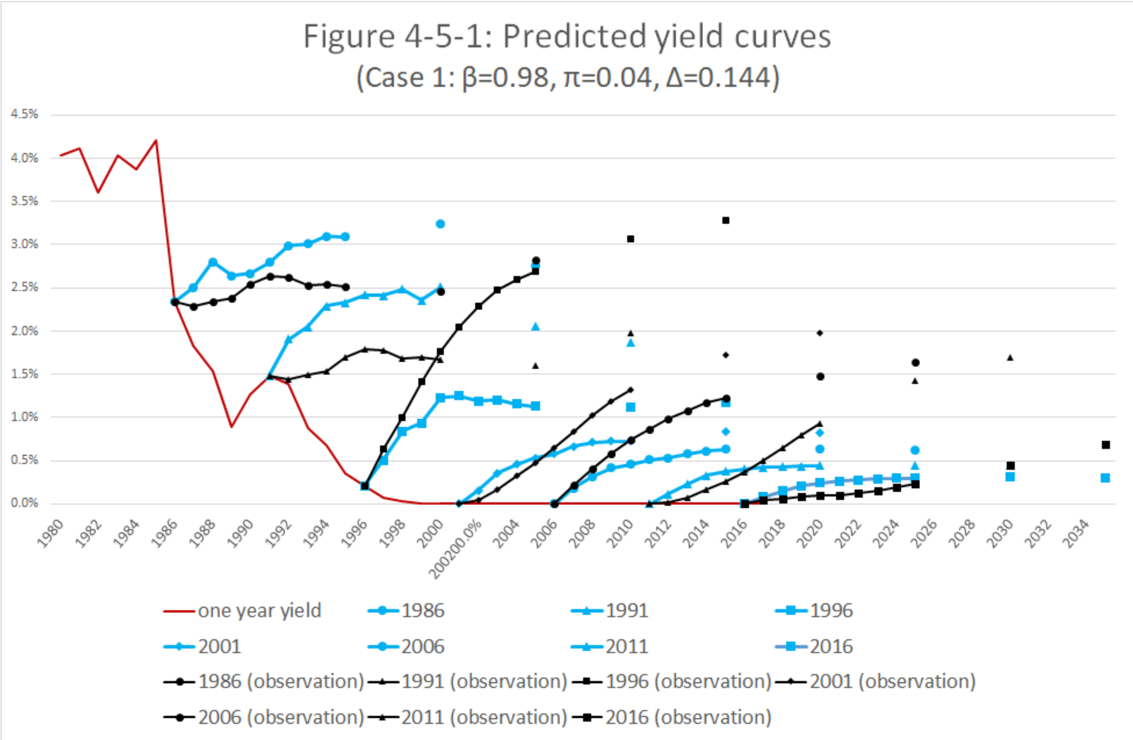


Figure 4-5-1: Predicted yield curves
(Case 1: β=0.98, π=0.04, Δ=0.144)
one year yield
1986
1991
1996
2001
2006
2011
2016
1986 (observation)
1991 (observation)
1996 (observation)
2001 (observation)
2006 (observation)
2011 (observation)
2016 (observation)

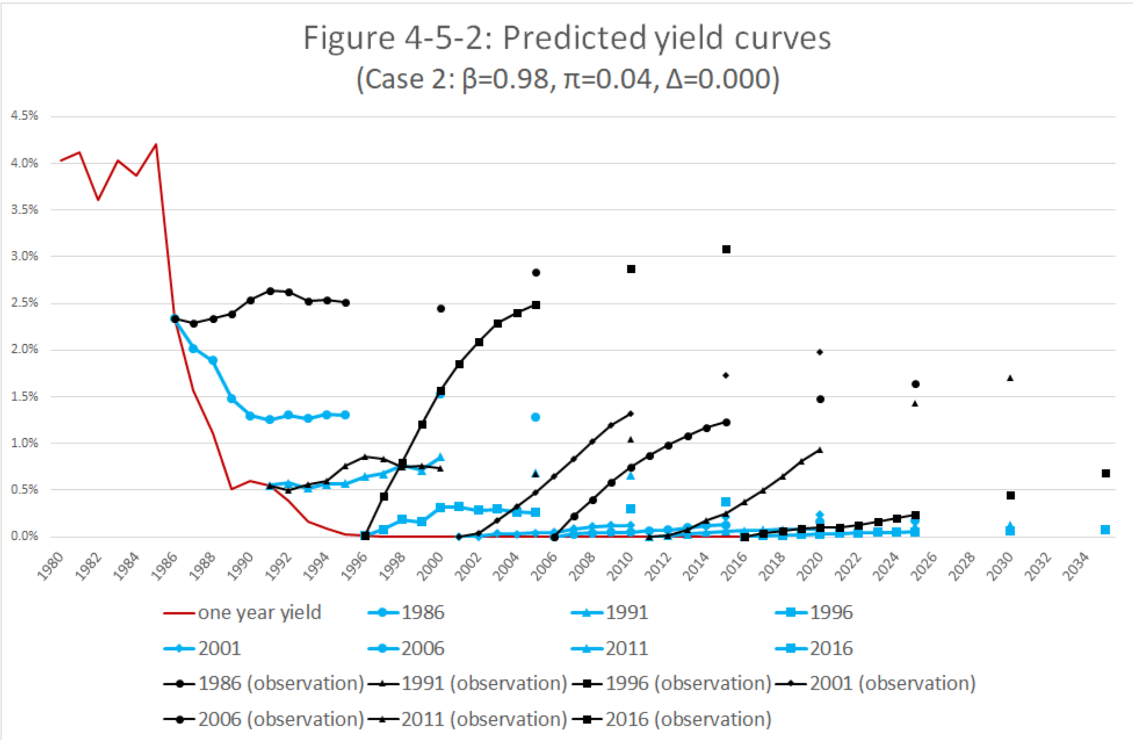


Figure 4-5-2: Predicted yield curves
(Case 2: β=0.98, π=0.04, Δ=0.000)
one year yield
1986
1991
1996
2001
2006
2011
2016
1986 (observation)
1991 (observation)
1996 (observation)
2001 (observation)
2006 (observation)
2011 (observation)
2016 (observation)

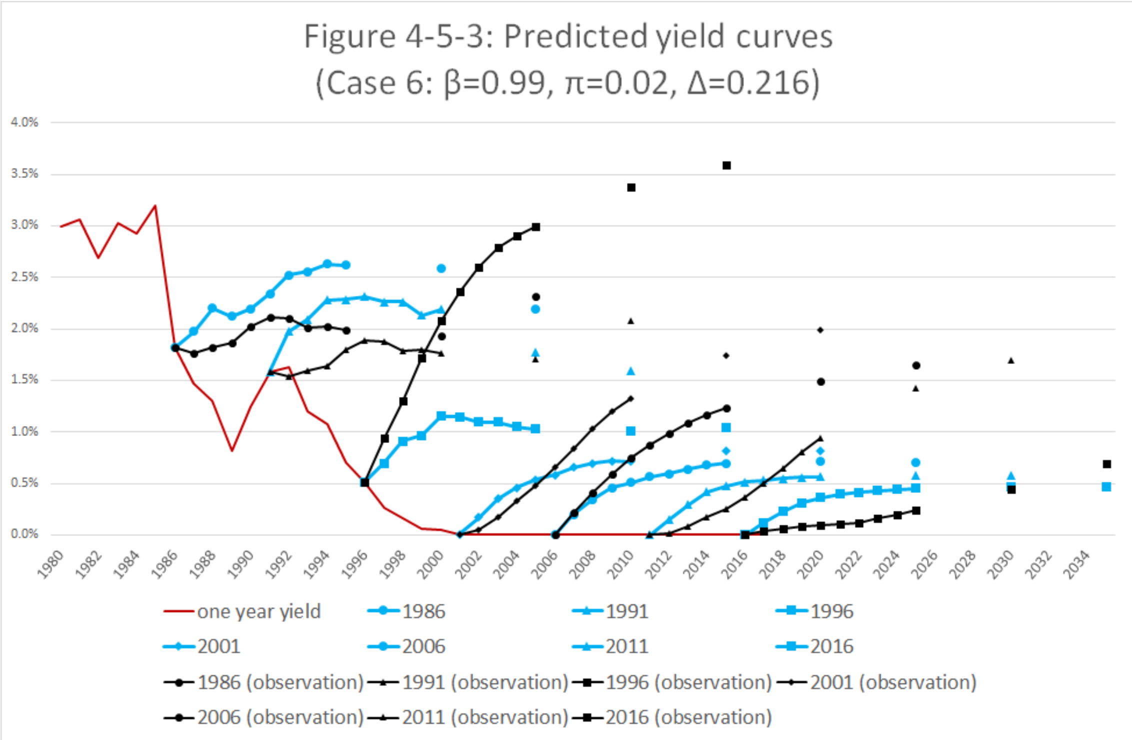


Figure 4-5-3: Predicted yield curves
(Case 6: β=0.99, π=0.02, Δ=0.216)
one year yield
1986
1991
1996
2001
2006
2011
2016
1986 (observation)
1991 (observation)
1996 (observation)
2001 (observation)
2006 (observation)
2011 (observation)
2016 (observation)

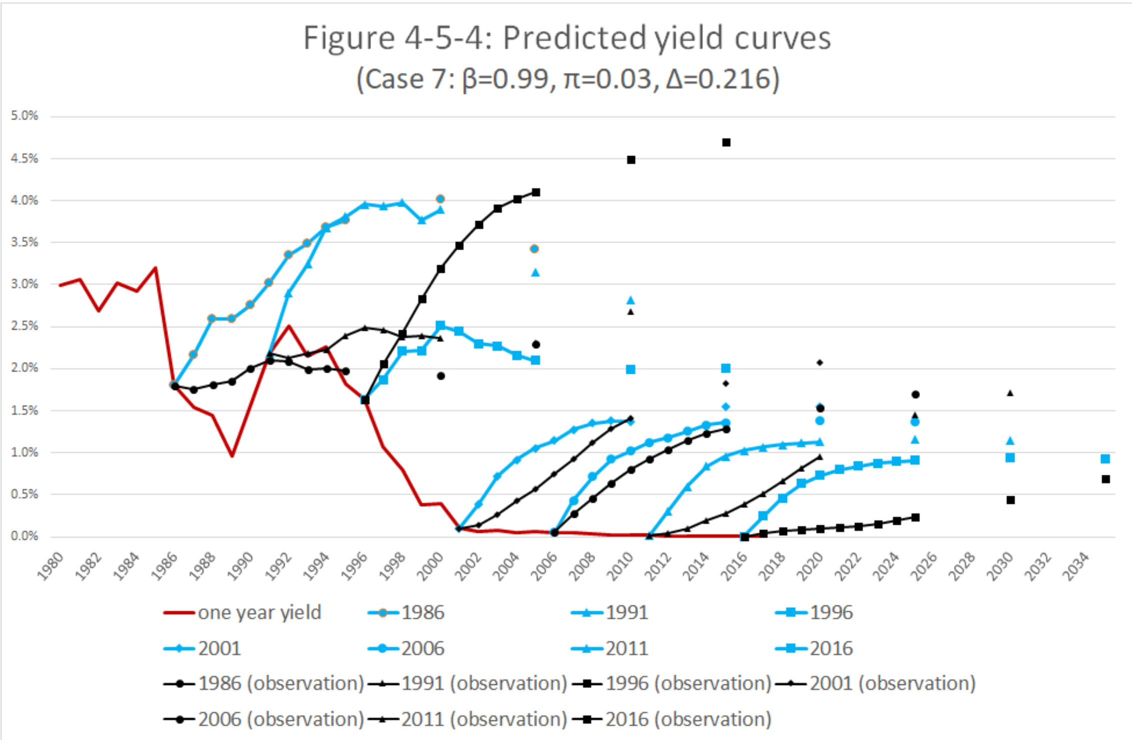
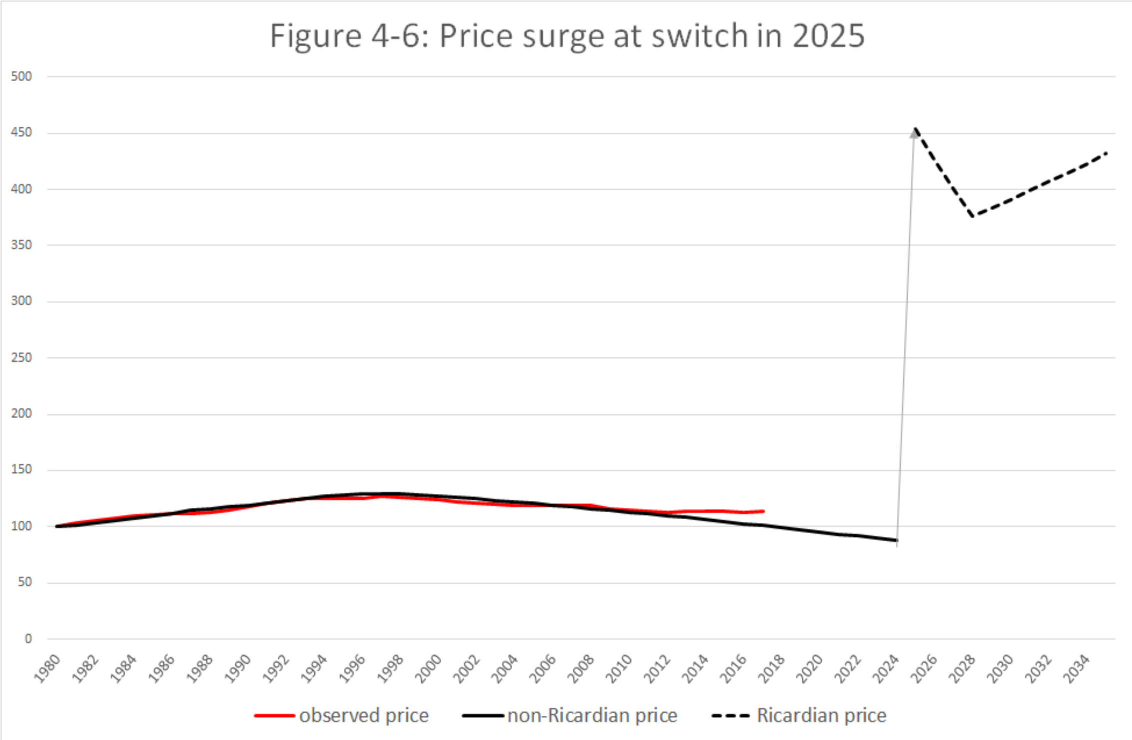
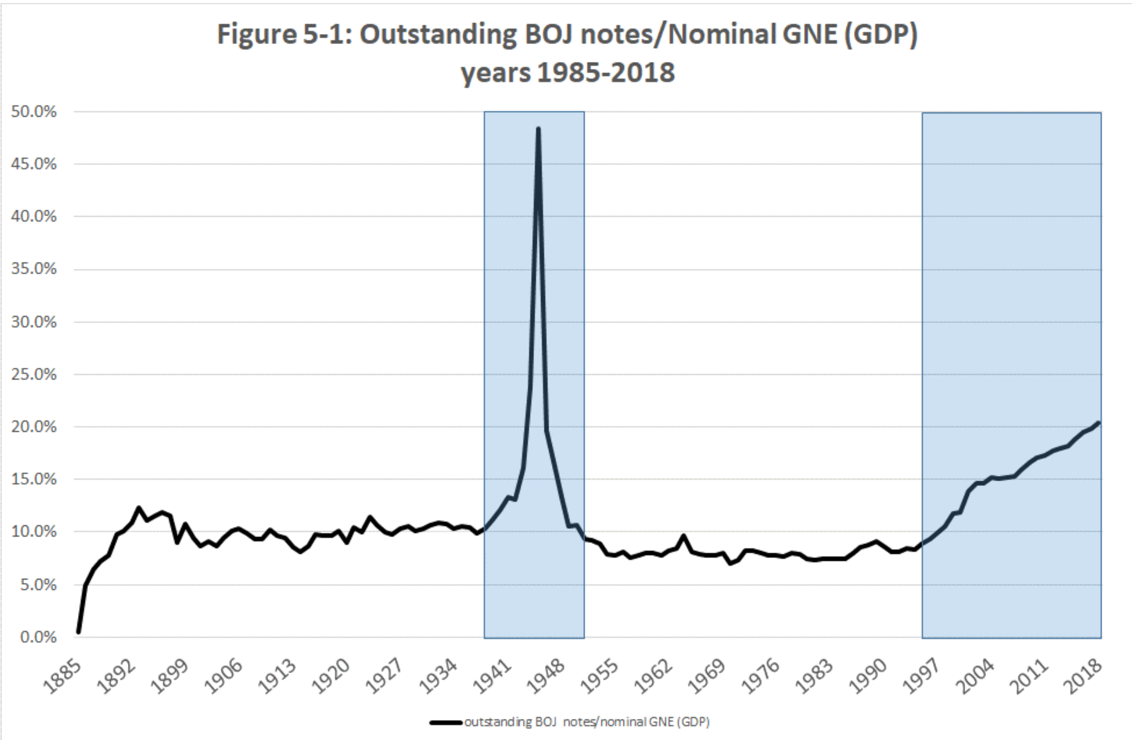


Figure 4-5-4: Predicted yield curves
(Case 7: β=0.99, π=0.03, Δ=0.216)
one year yield
1986
1991
1996
2001
2006
2011
2016
1986 (observation)
1991 (observation)
1996 (observation)
2001 (observation)
2006 (observation)
2011 (observation)
2016 (observation)





The source of data is described in Saito (2017).