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DOES DIVERSIFICATION STRATEGY CREATE OR DESTROY VALUE?*

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Abstract

The authors examine the financial structure of diversified corporations in Japan and measure diversification discount/premium. First, we find that the more diversified a business is, the lower its profitability, turnover, and business risk, and the higher its financial leverage. Second, using business segment data, the valuation effect of diversification is examined for large samples of firms in Japan for 1998-2001. We find a 5% discount for unrelated diversified firms on average. By dividing the discount/premium into several factors, the overall discount is due to a large discount for unrelated diversification strategy, which cannot be offset by the premium from a tax-savings effect from high financial leverage.

I. Introduction

Recent empirical research suggests that corporate diversification strategies have not been beneficial for US firms over the past three decades. During the 1980s and 1990s, many Japanese firms undertook substantial diversification programs. This process reached its climax with the M&A wave during the ‘bubble economy,’ from 1986 to 1990, and the accompanying rise to prominence of huge, diversified corporations. In the last ten years the trend has reversed. The push toward focus or specialization strategy apparently resulted from the view that unrelated diversification decreases firm value.

In recent years, Japanese capital market participants have begun to whisper about the phenomenon of ‘conglomerate discount’ or ‘diversification discount.’ Recent empirical research based on event studies, such as Desai and Jain [1999], show that focused strategy has a positive impact on corporate value. Berger and Ofek [1995] find a 13% to 15% average value loss from diversification during 1986-1991 in the US. The value loss is smaller for related diversification. Lins and Servaes [1999] gives an international comparison — while finding no

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significant diversification discount in Germany, there were significant ones of 10% in Japan and 15% in the UK during 1992-1994.

However, 'diversification discount' is still an open question in Japan. Do diversified firms have different financial characteristics compared with single business firms? Is there a diversification discount phenomenon in Japan? What are the sources of that phenomenon, if any?

Theoretical arguments suggest that diversification has both value-creation and value-destruction effects. The potential benefits of operating different lines of business within one firm include operating synergies, the improved ability to take advantage of the tax benefits of debt financing, and the coinsurance effect. The potential costs of diversification include poor cross-subsidies that lead to the use of inefficient internal capital markets. There is no clear prediction about the overall value effect of diversification strategy.

Most of the relevant research on value in accounting focused on overall numbers, such as book value and net income. In this paper, we use segment data to estimate the value effect of diversification and to examine the potential sources of value gains or losses. We compare the sum of the imputed, stand-alone values of the segments of diversified firms to the actual values of those firms. We find that related diversified firms have values that average, during 1998-2001, from 1.1% to 1.8% above the sum of the imputed values of their segments. In contrast, unrelated diversified firms have values that average 1.6% to 7.9% below the sum of the imputed values. These results suggest that type of diversification, related or unrelated, is one determinant of the value effect.

Section II describes the sample and Section III compares profitability, turnover, business risk, and financial leverage between diversified and single-segment firms. Section IV assesses the overall value effect of diversification using imputed business segment values. Section V investigates factors affecting EXVAL (excess value) and sheds light on the sources of value gain and loss, and Section VI concludes. The Appendix provides additional details on our empirical method.

II. Data

1. Data

The NIKKEI NEEDS Segment Information database provides segment information on Japanese companies. The file contains information on variables by segment. First, we collected data for all publicly-traded companies from 1998 to 2001. Financial institutions were then removed from consideration, as applying this particular valuation method is problematic for such institutions. To be included in the final sample, diversified firms must have had both financial data and market price data. These procedures resulted in a sample of 13,310 firm-years and 27,640 business segments. We analyzed stock price data at the end of June because financial data of most companies is available three months after the fiscal year-end (March). Industry classification depended on Japanese SIC (Standard Industrial Classification 10th revised version). Following Berger and Ofek [1995], we used the narrowest SIC grouping

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See Kothari [2001] for a review of empirical research on the relation between capital markets and financial statements.
to measure discount or premium unless sample size was too small.

2. **Degree of diversification**

Table 1 shows the percentage of sample firms by number of reported business segments, and by type of diversification. Diversified firms are those reporting two or more business segments. We classify firms as ‘related diversified’ when more than 90% of total sales, operating income, and total assets of all segments are within a one-digit SIC code level. Others are classified as ‘unrelated diversified.’

Single-segment firms account for approximately half of the total, or 52.8%. Among the multi-segment firms, firms with three segments account for about 16.8%. The ‘Type of diversification’ column indicates that 21.7% are related diversified firms, and 25.5% of the firms are classified as unrelated diversified. Regarding size, larger firms tended to report more business segments and to be classified as unrelated diversified.

From rows four to six, almost all firms without subsidiaries and affiliated companies have only one segment. In contrast, firms with more than 11 subsidiaries tend to have multisegments.

Table 2 presents mean and median numbers for business segments by industry. The real estate industry has three segments (median), which is the largest. This is mainly due to two reasons. First, Japanese real estate companies often report ‘real estate trading’ and ‘management of real estate’ separately. Second, they often run businesses in industries such as construction, leisure, and finance. Utilities and transportation industries also report three segments (median). Those industries have been under regulation, which promises area monopoly, for a long period. They have been severely restricted in expanding their core business into other geographic areas. Instead, they have diversified into various industries,
which has led to multi-segment reporting.

III. **Financial Characteristics of Diversified Firms**

In this section, we look at financial characteristics of diversified Japanese firms. Table 3 presents several financial ratios that may be affected by diversification strategy. Lower row data are industry-adjusted numbers.\(^2\) Theoretical arguments suggest that the potential benefits of operating different lines of business within one firm include operating synergies, greater debt capacity, and coinsurance. The potential costs of diversification include entering into an industry in which management has no experience or little skill, and power struggles between divisions which result in poor profitability (see Rajan et al. [2000]).

The top panel of Table 3 indicates that the more diversified firms are, the lower the operating profit margin (\(^1\)), total assets turnover (\(^2\)), and ROA (\(^3\)). This trend is the same for industry-adjusted ratios as well. Non-operating asset ratios (\(^4\)) are lower for diversified firms. This suggests that diversified firms manage excess cash efficiently, and that single-segment firms are required to hold excess cash to prepare for contingencies.

One of the potential benefits of diversification arises from combining business with imperfectly correlated earnings streams. This coinsurance effect gives diversified firms greater debt capacity than single-line firms. The middle panel of Table 3 indicates that the more diversified firms are, the lower business risk (\(^5\)) is. The same is true when comparing single-segment, related diversification, and unrelated diversification. In addition, the more diversified a firm is, the higher the financial leverage. This suggests that diversified firms enjoy lower business risk and it is possible for them to use high financial leverage.

According to Yoshihara et al. [1981], companies with mature core businesses often try to diversify. These companies strive to grow through diversification because they cannot expect much growth from existing business lines. Five-year revenue growth (\(^6\)) is lower for multi-segment firms. In contrast, multi-segment firms have higher capital expenditure (\(^7\)), which supports the above conjecture.

Finally, the bottom panel of Table 3 clearly illuminates the financial characteristics of

\(^2\) See Appendix for details of industry adjustment calculation.
diversified Japanese firms. In general, ROE can be decomposed as follows:

\[
\text{ROE} = (\text{NOPAT return}) + (\text{Financial leverage effect})
\]

What this formula means is that there are two determinants of ROE. The first is business strategy performance, and the second, capital structure. The NOPAT return\(^3\) declines monotonically as degree of diversification increases (1). It is expected a priori that diversified firms enjoy a financial leverage effect because of their high debt ratio. However, when we look at the industry-adjusted leverage effect, such a trend is not found. It should be noted that although

\^[3\] ‘NOPAT’ = Net operating profit after tax.
unrelated-diversification firms have high financial leverage, they do not enjoy a financial leverage effect. What is the reason for this phenomenon?

To answer this question, the financial leverage effect is decomposed as follows:

Financial leverage effect = \((\text{NOPAT return} - \text{After-tax debt cost}) \times \text{Financial leverage}\)

Unrelated diversified Japanese firms earn a lower NOPAT return and a lower spread (NOPAT return - After-tax debt cost) than do single-segment firms. Diversified firms, therefore, cannot create a financial leverage effect even though they have higher financial leverage.  

IV. Valuation of Diversified Firms

1. EXVAL approach

To investigate whether diversified firms are valued differently than single-segment ones, we employ the valuation methodology proposed by Berger and Ofek [1995]. Berger and Ofek developed a method based on the ratio of total market value of equity to three accounting items: sales, assets, and earnings (operating profit) after adjusting for inter-segment eliminations and unallocated amounts. Each segment of a diversified firm is assigned the valuation ratio of the median of the single-segment firms that operate in the same industry. The imputed values of all the segments of a company are summed up to compute the imputed value (theoretical value) of that company. The natural logarithm of the ratio of the actual market value to imputed value is called the ‘excess value (EXVAL)’ of the firm, and EXVAL is used to determine whether diversified firms are trading at a discount or a premium.

\[
IV_{UL} = \sum_{i=1}^{n} AI_i \times \left( \frac{V_{UL}}{AI_i} \right) \\
EXVAL_{UL} = \ln \left( \frac{V_{UL}}{IV_{UL}} \right)
\]

\(V_{UL}\): Unlevered enterprise value  
\(IV_{UL}\): Unlevered imputed value  
\(EXVAL_{UL}\): Excess value  
\(AI_i\): Accounting item (Sales, operating profit, assets)  
\(\left( \frac{V_{UL}}{AI} \right)_i\): Median multiple of segment i

Our approach is similar to Berger and Ofek [1995], with the following two exceptions. First, instead of enterprise value (‘EV’ = Market value of equity + Book value of debt), we employ unlevered enterprise value (Market value of equity + Book value of debt – Tax effect). The reason we use unlevered EV will be explained in a later section. Second, for measurement accuracy, we only employ market-to-assets ratios. We measured three multiples:

\[4\] See Appendix for definition and details of ROE decomposition.
market-to-sales (‘PSR’: Price-to-sales ratio), market-to-assets (‘PAR’: Price-to-assets ratio), and market-to-earnings (‘PER’: Price-to-earnings ratio). Among these three multiples, PAR gives the most accurate and stable results. Sales and earnings are volatile because they are flow data. In the case of a poorly-performing firm, those multiples become higher. That leads to higher valuation, which is misleading.

2. Discount or premium?

Table 4 shows year-by-year EXVAL by diversification type. We find significant and consistent discounts for unrelated diversification firms. The diversification discount is 1.6% for 1998, 7.9% for 1999, 7.1% for 2000, and 4.9% for 2001. The results are statistically significant at the 1% level, except for 1998. In contrast, we find premiums for related diversification firms, but which are not always statistically significant. The result clearly shows that the effect of diversification on firm value varies between related and unrelated diversification strategy. Diversification does not always create or destroy value.

V. Analysis of Excess Value

1. Factors Affecting Excess Value

Diversification is not the only factor that affects value. In this section, EXVAL is regressed on several fundamental variables such as size, growth, profitability, and efficiency.

Table 5 describes regression results. First, look at two basic control factors: size (natural logarithm of total assets) and growth (two-year revenue growth). According to Damodaran [1999], when using the multiples method for valuation, growth rate and risk should be considered. Both size and growth coefficients are positive and statistically significant at the 1% level. Size in this regression is the proxy for risk.

In addition to two basic factors (size and growth), coefficients of profitability (operating profit margin) and efficiency (total asset turnover) are also positive and statistically significant. It should be interpreted that high profitability and high turnover are positively related to excess value. The last two columns present coefficients of dummy variables: related diversification and unrelated diversification. Regarding related diversification, we find no consistent pattern in the sign of dummy variables. They are not statistically significant. In short, when compared with...
single-segment firms, we find no discount or premium for related diversified firms. In contrast, we find a discount of approximately 5% for unrelated diversified firms. That leads us to conclude that we find a discount only for unrelated diversified firms during 1999-2001 in Japan.

2. Sources of Value Gain and Loss

In this final section, we shed light on sources of excess value gain and loss, especially focusing on tax benefit. As mentioned in an earlier section, much of the prior research (Berger and Ofek [1995], Lins and Servaes [1999]) employs enterprise value (book value of debt plus market value of equity) in calculating excess value. In this paper, we employ unlevered enterprise value. The relationship between the two EXVALs is as follows:

\[ \text{EXVAL}_{EV} - \text{EXVAL}_{UL} \equiv \text{TSGap} \]

- \( \text{EXVAL}_{EV} \) : Excess value based on ‘enterprise value’
- \( \text{EXVAL}_{UL} \) : Excess value based on ‘unlevered enterprise value’
- \( \text{TSGap} \) : Tax-savings gap

Prior research does not distinguish the diversification effect on EXVAL from tax savings. In the analysis of Berger and Ofek [1995], tax benefits do not affect the empirical results, because diversified firms and single-segment firms have a similar debt ratio in US sample firms. Lins and Servaes [1999] examined the valuation effect for Germany, Japan, and the UK for 1992-1994 and do not distinguish tax-benefit effect either. However, during 1998-2001 in Japan, it is clear from Table 3 that financial leverages vary among single-segment firms, related diversified firms, and unrelated diversified firms. Ignoring this gap would misrepresent the effect of diversification on firm value. Some parts of EXVAL may be due to diversification strategy, but the rest is due to capital structure.

To highlight the tax-benefit effect, we decompose EXVAL as follows.

### Table 5. Regression Results of EXVAL

<table>
<thead>
<tr>
<th>Observations</th>
<th>AdjustedR²</th>
<th>Intercept</th>
<th>Ln (Total assets)</th>
<th>Expected 2-year revenue growth</th>
<th>Operating profit margin</th>
<th>Ln (Total assets turnover)</th>
<th>Related diversification dummy</th>
<th>Unrelated diversification dummy</th>
</tr>
</thead>
<tbody>
<tr>
<td>FY 1999</td>
<td>0.182</td>
<td>-0.06%***</td>
<td>0.042***</td>
<td>1.83***</td>
<td>1.97***</td>
<td>0.129***</td>
<td>0.008</td>
<td>-0.049***</td>
</tr>
<tr>
<td>FY 2000</td>
<td>0.194</td>
<td>-0.60***</td>
<td>0.046***</td>
<td>1.43***</td>
<td>1.91***</td>
<td>0.133***</td>
<td>-0.002</td>
<td>-0.050***</td>
</tr>
<tr>
<td>FY 2001</td>
<td>0.190</td>
<td>-0.63***</td>
<td>0.051***</td>
<td>1.48***</td>
<td>1.51***</td>
<td>0.099***</td>
<td>0.003</td>
<td>-0.051***</td>
</tr>
<tr>
<td>Pooled</td>
<td>0.079</td>
<td>-0.242***</td>
<td>0.77***</td>
<td>1.21***</td>
<td>0.127***</td>
<td>-0.022</td>
<td>-0.034***</td>
<td></td>
</tr>
</tbody>
</table>

Data for FY 1998 is eliminated due to the lack of expected 2-year revenue growth rate in the database. ( ) : t-statistics. * statistically significant at the 10% level, ** 5% level, *** 1% level. Cross-sectional regression is OLS. Pooled analysis is based on fixed-effect model.
EXVALEV based on EV

TSGap

DivE

#ect

others

The tax-savings factor is calculated as EXVALEV minus EXVALUL.

Table 6. Sources of Value Gain and Loss

<table>
<thead>
<tr>
<th>EXVAL Based on EV</th>
<th>Tax savings effect</th>
<th>Diversification effect</th>
<th>Other effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Related diversification</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FY 1999</td>
<td>4.35%</td>
<td>3.26%</td>
<td>0.77%</td>
</tr>
<tr>
<td>FY 2000</td>
<td>3.65%</td>
<td>2.26%</td>
<td>−0.22%</td>
</tr>
<tr>
<td>FY 2001</td>
<td>3.36%</td>
<td>1.61%</td>
<td>0.26%</td>
</tr>
<tr>
<td>Pooled</td>
<td>3.29%</td>
<td>2.20%</td>
<td>−2.23%</td>
</tr>
<tr>
<td>Unrelated diversification</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FY 1999</td>
<td>−1.63%</td>
<td>6.23%</td>
<td>−4.92%</td>
</tr>
<tr>
<td>FY 2000</td>
<td>−1.43%</td>
<td>5.71%</td>
<td>−5.00%</td>
</tr>
<tr>
<td>FY 2001</td>
<td>−0.66%</td>
<td>4.22%</td>
<td>−5.12%</td>
</tr>
<tr>
<td>Pooled</td>
<td>−1.79%</td>
<td>4.92%</td>
<td>−3.44%</td>
</tr>
</tbody>
</table>

‘Tax-savings effect’ is calculated as ‘EXVAL based on EV’ minus unlevered EXVAL.

‘Diversification effect’ is the coefficient of the diversification dummy presented in Table 5. ‘Other effect’ includes the effect of basic variables and control variables in Table 5. ‘EXVAL based on EV’ = Tax savings effect + Diversification effect + Other effect.

\[ EXVAL_{EV} = TSGap + DivEffect + others \]

\[ EXVAL_{EV} : EXVAL based on EV \]
\[ TSGap : Tax-savings gap \]
\[ DivEffect : Diversification effect \]
\[ others : Other effect \]

The tax-savings factor is calculated as EXVAL_{EV} minus EXVALUL.

The diversification strategy effect is the coefficient of the diversification dummy presented in Table 5. This coefficient shows the impact of the diversification strategy itself on EXVAL with other things being equal. The coefficient sign varies between related and unrelated diversification. Other effect includes that of basic variables (total assets and two-year revenue growth) and control variables (operating profit margin and total assets turnover). EXVAL_{EV} of related diversified firms is positive, 4.35% for 1999, 3.65% for 2000, and 3.36% for 2001. For the pooled sample, we find a 3.29% premium. What is the source of value gain? Both tax-savings effect and other effects are positive. We find no consistent pattern in the diversification strategy effect. The bottom panel of Table 6 contains the results for unrelated diversification. The tax-savings effect is more favorable for the unrelated diversification group, which has high financial leverage. However, both diversification strategy effect and other effects are largely negative.

To summarize, related diversified firms enjoy a premium. In contrast, although unrelated diversified firms have high tax benefits, they lose in terms of strategy effect, and end up with an overall discount.

VI. Conclusion

Many large, modern companies have multiple business segments across different industries. This is called corporate diversification strategy (see Barney [2002]). In some cases, this diversification strategy creates value through ‘operating synergies,’ ‘internal capital market,’
and ‘coinsurance effect.’ In contrast, diversification sometimes reduces firm value due to ‘cross-subsidies’ and other reasons. Stultz [1990] argues that diversified firms will invest too much in lines of business with poor investment opportunities. An unprofitable business cannot have a value below zero if operated on its own, because the capital market will not provide any funds to that firm. But a failing business continues to have a negative net present value if it is part of a conglomerate that provides cross-subsidies. Most of the value-relevance research in accounting has focused on overall accounting numbers, such as book value and net income. In this paper, using business segment reporting data, we shed light on the effect of diversification on firm value. Our sample covered all publicly-traded companies in Japanese securities markets from 1998 to 2001. The sample size is 13,310 firm-years and 27,640 business segments.

First, we examined the financial structure of diversified corporations in Japan. There were several findings. Diversified corporations tended to have less profitability, lower turnover, lower business risk, and higher financial leverage.

Second, using business segment reporting data, we estimated the effect of diversification on Japanese firm value by imputing stand-alone values for individual business segments, as if they were operated as a separate company. We calculated the imputed value of each segment by multiplying the median ratio of single-segment firms in the same industry. This is called the ‘multiplier approach.’ Diversification discount/premium was measured based on EXVAL measurement. In doing so, we found that diversification increased the value of related diversified companies by 1.1-1.8%. On the other hand, we estimated that the value loss of unrelated diversified companies ranged from 1.6-7.9%. Those results were robust after controlling other fundamental factors.

Third, we analyzed from where discount/premium comes. By dividing the source of the discount/premium into several factors, we found that the overall discount of unrelated diversification was due to a large discount for the ‘unrelated diversification strategy factor,’ which could not be offset by the premium from a tax-savings effect from high financial leverage.

This study, of course, has limitations. Other value gain or loss factors should be examined. Rajan et al. [2000], Lamont and Polk [2002], and Nakano and Yoshimura [2004] analyzed the effect of the internal capital market function. Lins and Servaes [1999] examined the effect of corporate governance on excess value. These are possible areas for future research.

**APPENDIX**

**Industry-Adjusted Ratios**

We use some financial ratios adjusted by industry as follows. First, we calculate the median of the industry’s financial ratios ($X_j$) from single-segment firms in the industry only. Then, the industry-adjusted ratios ($X_{adj}$) are calculated by subtracting the weighted average of the median of the industry financial ratio from the original data ($X_{org}$). $j$ denotes business segment of each firm. That is,

$$X_{adj} = X_{org} - \frac{\sum_{j=1}^{J} w_j X_j}{J}$$  \hspace{1cm} (A-1)

We use appropriate weight ($w_j$) of equation (A-1) for every financial ratio. For example,
we use weight of sales for operating profit margin, and weight of assets for total assets turnover. In addition, we use a narrow SIC industry grouping which includes at least five single-segment firms for the purpose of accurate analysis.

Section III The Alternative Method of ROE Decomposition (Palepu et al (2000))

In general, ROE is analyzed by the ‘DuPont-style ROE decomposition’ method. That is, 
\[
\text{ROE} = \text{Net profit after tax margin} \times \text{Total assets turnover} \times \text{Financial leverage multiplier.}
\]
While this method is generally used, it contains a fault in that operating activities and financial activities are not distinguished.

Instead, we use the following method following Palepu et al [2000]. First, we define ROE as NOPAT return × Financial leverage effect. The first term, ‘NOPAT return,’ is derived by dividing NOPAT by Net assets (Total assets – Non-operating assets), and is never affected by capital structure. The second term, ‘Financial leverage effect’ is calculated as follows: Financial leverage effect = Spread × Net financial leverage = (NOPAT return – After-tax debt cost) × Net financial leverage. Net financial leverage, Net debt (Interest-bearing liabilities – Cash & short-term investment securities) / Equity, is affected by firms’ financing strategies but not by changes in operating receivables. When the NOPAT return is above the cost of debt, ROE rises with the multiplication of net financial leverage.

Handling outliers

We set the upper boundary and the lower boundary for every variable. If the variables are above (below) the upper boundary (lower boundary), we change the values of the variables to the boundaries as follows.

- **Operating profit margin** lower boundary -20%, upper boundary 50%.
- **Natural logarithm of total assets turnover** lower boundary -200%, upper boundary 200%. We remove the firms whose natural logarithm of total assets turnover is below the lower boundary from the row data.
- **Excess value** lower boundary ln(1/4), upper boundary ln(4). We remove the firms whose excess value is above (below) the upper boundary (lower boundary) from row data.
- **Expected two-year revenue growth** lower boundary -30%, upper boundary 30%.

Section V Separation of Tax-Savings Effect

We set up the difference between excess value based on enterprise value (EXVAL\textsubscript{EV}), and excess value based on unlevered enterprise value (EXVAL\textsubscript{UL}).

First, we define the tax-savings ratio as the ratio of value gained by tax savings.

\[
TS = \frac{EV}{EV - tD} = \frac{V_{EV}}{V_{UL}}
\]  

(A-2)

Second, we define (\overline{TS}) as the average of the tax-savings ratios of single-segment firms. When we use (\overline{TS}), the difference between excess value based on enterprise value and excess value based on unlevered enterprise value is expressed as below:

\[
EXVAL_{EV} - EXVAL_{UL} = \ln \left[ \frac{V_{EV}}{IV_{EV}} \right] - \ln \left[ \frac{V_{UL}}{IV_{UL}} \right] = \ln \left[ \frac{V_{EV}}{IV_{EV}} \right] - \ln \left[ \frac{IV_{UL}}{V_{UL}} \right]
\]
\[
\ln \left[ \frac{V_{EV}}{V_{UL}} \cdot \frac{IV_{UL}}{IV_{EV}} \right] = \ln \left[ TS \cdot \frac{\sum_{j=1}^{J} AI_j \cdot \left( \frac{V_{UL}}{AI} \right)_j}{\sum_{j=1}^{J} AI_j \cdot \left( \frac{V_{UL}}{AI} \frac{V_{EV}}{V_{UL}} \right)_j} \right]
\]

\[
\approx \ln \left[ TS \cdot \frac{\sum_{j=1}^{J} AI_j \cdot \left( \frac{V_{UL}}{AI} \right)_j}{\sum_{j=1}^{J} AI_j \cdot \left( \frac{V_{UL}}{AI} \right)_j \cdot TS_j} \right]
\]

\[
= \ln \left( \frac{TS}{\sum_{j=1}^{J} w_j TS_j} \right) = TSGap \tag{A-3}
\]

The denominator of the last equation, the weighted average of tax-savings effects of the industries to which the segments belong, can be called ‘imputed tax-savings effect.’ In comparison, the numerator is the tax-savings effect of multi-segment firms. Thus the difference between both excess values is the tax-savings gap, which is a firm’s actual tax-savings effect divided by ‘imputed tax-savings effect.’

In this paper, there is a small margin of error between the results of data analysis and the equation (A-3) due to using median rather than mean, and we use an approximation in equation (A-3). This is because the mean of the product does not equal the product of the mean, in general. If we do not use this approximation, the equation (A-3) is:

\[
\left( \frac{V_{UL}}{AI} \right)_j \cdot \left( \frac{V_{EV}}{V_{UL}} \right)_j = \left( \left( \frac{V_{UL}}{AI} \right)_j \cdot \left( \frac{V_{EV}}{V_{UL}} \right)_j \right)
\]

The second RHS term of equation (A-4) is an approximation error. Although the margin of error is different by the industry j, it is about -0.2 to -0.1 times the value of the LHS. As this error exists in the denominator of (A-3), the tax savings effect in this paper is overvalued by about 10% to 20% of the tax-savings gap. The absolute value of this error is small when the coefficient of the variation of the multiples of single-segment firms in the industry for evaluation is low, or the coefficient of variation of the tax-savings ratio in the industry is low. Thus categorizing industries appropriately makes both the valuation model by multiple methods and the evaluation of tax-savings effect fair.
REFERENCES


