An Essay on the Priority Growth of Department I
——A Marxian Long-term Growth Model——

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I. Introduction

The problem of the priority growth of department 1 has become controversial as an important thema of the reproduction theory since Lenin's "On so-called Market Problem" (1893). Lenin himself argued against the Russian Narodniks that not only in the period of transition from feudalism to capitalism but also under the "general and exclusive domination of the capitalist mode of production",(1) it is possible for Russia to develop by generating markets. And when he proved this assertion in the case of the exclusive domination of capitalism, he presented the proposition of the so-called priority growth of department 1. According to him, sector 1 (production goods producing sector) grows more rapidly than sector 2 (consumption goods producing sector) by technical progress in the long run. Since this Lenin's pamphlet it has been questioned whether this proposition is proved as a law.(2) Also in Japan some models were built about this problem in the 1970's as a development of the analysis of reproduction theory.(3)

Recently Prof. Glombowski approached to this problem.(4) He proved under many restrictive assumptions (e. g. equal organic compositions of capital between the two sectors, constant rates of accumulation, etc) that the sectoral composition of outputs can rise ad infinitum. But it is due to the constant rates of accumulation that he can reach this conclusion. Prof. Takasuka has already revealed in the 1960's that Lenin's schema depends decisively upon the hidden assumption, i. e. the constancy of accumalation rates.[5] Glombowski's model is not free from Takasuka's criticism.

The purpose of this paper is to present a model from a different angle. Especially the relation between the problem of long-term tendency and a cycle will be made clear. Though many models have been built hitherto, this point has not been necessarily apparent.[6] The author is of opinion that trend can be regarded as long-term average growth path which a cycle realizes post festum, and when we consider the transition of this long-term path, we can see the movement of sectoral composition of output in the long run. In this way we can reach a conclusion of this problem in the long-term tendency. But what is the long-term average path in Marxian economics?

We will consider the problem in the following order. First of all in section II we examine the growth possible area in the two-dimensional output space. It is considered where balanced growth or unbalanced growth is possible. In section III we consider the short-term unbalanced growth which is the mechanism that realizes the long-term average growth path. In other words, it is an attempt to ascertain the long-term path in incessant unbalanced growth. Finally, in section IV we study how the long-term path changes with technical progress. And remaining problems and prospects will be pointed out.

The framework of our analysis is the Marx-Leontief economy with two types of goods (production goods & consumption goods), i.e. no fixed capital, no joint-production, homogeneous labour, no altenative techniques. We do not take technical progress into account until section IV. Assumptions of the model are as follows.

1. Classical saving function, i. e. workers do not save and

capitalists do not consume.

2. Wage is payed in advance.[7]

3. $a_1/l_1 > a_2/l_2$, i. e. the technical composition of capital in sector 1 is greater than that in sector 2.

4. Matrix of input coefficient A is productive and nonnegative.

(Symbols of the model)

x_i : output, a_i : input coefficient, l_i : labour coefficient, w : real wage rate, g_i : rate of accmulation (rate of growth), Q : sectoral composition of output, $X=\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$: output column vector, $A=\begin{pmatrix} a_1 & a_2 \\ wl_1 & wl_2 \end{pmatrix}$: input coefficient matrix,[8] t_i : value of commodity i, K_i : quantity of capital input, L_i : quantity of labour input

(subscript i presents each sector 1, 2)

II. Divisions of the growth possible cone

First of all we investigate where proportionate growth or disproportionate growth is possible in growth possible areas.[9] This consideration is necessary when we want to know what kind of path is realized through the fluctuations of economies.

(I) Reproducible Area (RA)

Def. The area in the output space, in which the economy is possible to generate surplus, is called the reproducible area (RA).

$$\text{i. e.} \quad RA=\{X \in R_+^2 \mid X > AX\} \qquad \cdots\cdots (1)$$

By developing the inequality (1) componentwise

$$Q > \frac{a_2}{1-a_1}, \quad Q < \frac{1-wl_2}{wl_1}$$

In this area

$$Q_{min} < Q < Q_{max} \quad \left(Q_{min} = \frac{a_2}{1-a_1}, \quad Q_{max} = \frac{1-wl_2}{wl_1}\right) \qquad \cdots\cdots (2)$$

In fact by assumption 4, there exists an $X > 0$ which satisfies the inequality (1). This condition is equivalent to Hawkins-Simon's (H—S) condition and when the H—S condition holds we can prove the inequlity (2) by $Q_{max}-Q_{min}>0$.

(II) Growth Possible Area (GPA)

Def. The area in which demand-supply equation is possible under the condition of $g_1>0$ and $g_2>0$ is called the growth possible area (GPA).

i. e. $GPA=\{X \in R_+^2 | X=AGX, \quad G>I\}$ ...... (3)

where $G=\mathrm{diag}(1+g_1, 1+g_2)$

Prop. *GPA* is a convex cone existing in the interior of *RA*.

(proof)

From the matrix above

$$g_1=\frac{wl_2Q-a_2}{(a_1l_2-a_2l_1)wQ}-1$$

$$g_2=\frac{a_1-wl_1Q}{(a_1l_2-a_2l_1)w}-1$$

To satisfy the condition of GPA, i. e. $g_1>0$ & $g_2>0$, next two inequalities must be satisfied.

$$g_1>0 \quad \rightleftarrows \quad Q>\frac{a_2}{wl_2-(a_1l_2-a_2l_1)w}(=Q_\beta)$$

$$g_2>0 \quad \rightleftarrows \quad Q<\frac{a_1-(a_1l_2-a_2l_1)w}{wl_1}(=Q_\alpha)$$

So,

$$Q_\alpha-Q_\beta=\frac{(a_1l_2-a_2l_1)\{(1-a_1)(1-wl_2)-a_2l_1w\}}{wl_1\{l_2-(a_1l_2-a_2l_1)\}} \quad \cdots\cdots (4)$$

$$Q_{max}-Q_\alpha=\frac{(1-a_1)(1-wl_2)-a_2l_1w}{wl_1} \quad \cdots\cdots (5)$$

$$Q_\beta-Q_{min}=\frac{a_2\{(1-a_1)(1-wl_2)-a_2l_1w\}}{w(1-a_1)\{l_2-(a_1l_2-a_2l_1)\}} \quad \cdots\cdots (6)$$

The positivity of the above differences (4), (5), (6) is guaranteed by assumption 3 and the H—S condition. So $Q_{max}>Q_\alpha>Q_\beta>Q_{min}$. This means that GPA is in RA. (Q. E. D.)

(III) Equilibrium Growth Possible Area (EGPA)

Def. The area, in which the condition $g_1 = g_2 = g > 0$ is satisfied and also demand-supply equation is possible, is called the Equilibrium Growth Possible Area (EGPA).

i. e. $EGPA = \{X \in R_+^2 \mid X = (1+g)AX,\ g>0\}$ …… (7)

The proportionate growth is possible only on the von Neumann ray by assumption 1. The rate of growth is determined by the reciprocal of the Frobenius root subtracted by 1. And we also have the sectoral composition of output from the Frobenius vector.

i. e. $Q_N = \frac{1}{2wl_1}\{a_1 - wl_2 + \sqrt{(a_1 - wl_2)^2 + 4a_2wl_1}\}$ …… (8)

Prop. EGPA (i. e. von Neumann ray) divides GPA into two areas.

In fact, it is evident from the definition of two sets (3), (7) that EGPA is in the GPA. This is also immediately proved by the subtraction $Q_\alpha - Q_N$ and $Q_N - Q_\beta$.

We have defined three growth possible areas. Illustrating these areas in the output space will be helpful to understand their locational relation.(10)

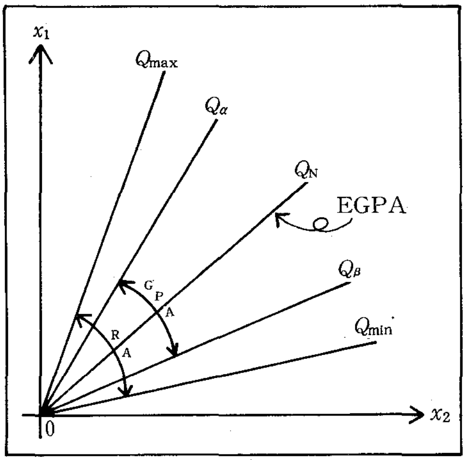


Fig. 1

III. Disproportionate growth and the Marxian golden age

We have introduced in the preceding section the divisions of growth possible areas. The consequence of the section is that the proportionate growth is possible only on the von Neumann ray. So off the von Neumann ray the economy must grow disproportionately. But we have only considered the possible areas in which the economy grows proportionately or disproportionately. The purpose of this section is to consider how the economy grows actually in the short run.

First of all we explain the period analysis of our reproduction model.

In the market which is held between one period (symbolized by t) and the next period (symboliyed by $t+1$), the output of each sector 1, 2 and also the sectoral composition of output Q is given as the result of production in this period. The output of production goods produced in this period is equated to the volume of production material which will be used in both sectors in the next period.(11)

i. e. $K_{1(t)}=K_{1(t+1)}+K_{2(t+1)}=a_1x_{1(t+1)}+a_2x_{2(t+1)}$
$$=a_1(1+g_{1(t)})x_{1(t)}+a_2(1+g_{2(t)})x_{2(t)} \qquad \cdots\cdots (9)$$

The output of consumers' goods produced in this period is on the other hand, equated to the quantities that workers will consume in the next period by assumption 1 and 2.

i. e. $x_{2(t)}=w(L_{1(t+1)}+L_{2(t+1)})=w(l_1x_{1(t+1)}+l_2x_{2(t+1)})$
$$=w\{l_1(1+g_{1(t)})x_{1(t)}+l_2(1+g_{2(t)})x_{2(t)}\} \qquad \cdots\cdots (10)$$

The matrix form of the equation (9) and (10) has been already shown in the analysis of growth possible area.

As the sectoral composition of output is given in the market, these equations can be interpreted as the one to determine the rates of accumulation.(12) Fig. 2 illustrates that the rates of accumulations of both sectors are determined at the point where two equations intersect each other. When the rates of accumulation are determined, capital goods (which include also the increments in capital) are allocated to the both sectors and equipped at the beginning of next period $t+1$. The laborers necessary for the capital are employed in the labour market and the production of both goods in the period $t+1$ starts. Fig. 3 illustrates this process mentioned above.

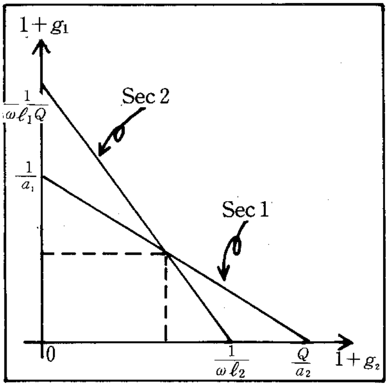


Fig. 2

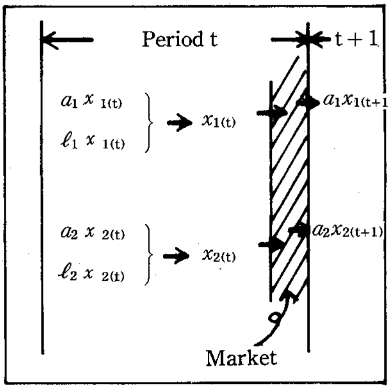


Fig. 3

From equations the rates of accumlations are

$$g_1 = \frac{l_2 - a_2/wQ}{a_1 l_2 - a_2 l_1} - 1$$

$$g_2 = \frac{a_1/w - l_1 Q}{a_1 l_2 - a_2 l_1} - 1$$

Here we define the ratio of gross accumulation rates as follows.

$$\frac{G_1}{G_2} = \frac{1+g_1}{1+g_2} = \left(l_2 - \frac{a_2}{wQ}\right) / \left(\frac{a_1}{w} - l_1 Q\right) = \frac{Qwl_2 - a_2}{Q(a_1 - Qwl_1)}$$

We prove next two lemmas for the proposition.

Lemma 1. Given the sectoral composition of output of this period, the sectoral composition of output of next period is determined by the ratio of gross accumulation rates.

(proof)

The conclusion soon follows by

$$Q_{(t+1)} = \frac{x_{1(t+1)}}{x_{2(t+1)}} = \frac{(1+g_{1(t)})x_{1(t)}}{(1+g_{2(t)})x_{2(t)}} = \frac{G_{1(t)}}{G_{2(t)}} \cdot Q_{(t)} \qquad \text{(Q. E. D.)}$$

Lemma 2. When the sectoral composition of output rises (falls), the ratio of accumulation rates also rises (falls).[13]

(proof)

Differentiating the proportion of accumulation rates,

$$\frac{d(G_1/G_2)}{dQ}=\frac{wQl_1(wQl_2-a_2)+a_2(a_1-Qwl_1)}{Q_2(a_1-Qwl_1)^2} \quad \cdots\cdots (11)$$

By the way, the gross accumulation rates of both sectors is positive by assumption 4.

$$G_1>0 \quad \rightleftarrows \quad wQl_2-a_2>0$$
$$G_2>0 \quad \rightleftarrows \quad a_1-Qwl_1>0$$

So the numerator of the above (11) is positive and this lemma is proved. (Q. E. D.)

By these two lemmas above we can immediately derive the next proposition.

Prop. 〈Cumulativeness of Disproportionate Growth〉

Once the rate of accumulation of the sector 1 becomes higher (lower) than that of the sector 2, this state tends to be continued.

In fact, suppose $G_{1(t)}/G_{2(t)}>1$. Then from Lemma 1 and 2,

$$\frac{G_{1(t)}}{G_{2(t)}}>1 \quad \to \quad Q_{(t+1)}>Q_{(t)} \quad \to \quad \frac{G_{1(t+1)}}{G_{2(t+1)}}>\frac{G_{1(t)}}{G_{2(t)}} \quad \to \quad Q_{(t+2)}>Q_{(t+1)}$$

We have shown the instability mechanism of the economy. Though there are some problems in the above analysis from the view of crisis theory, it is enough for us here to understand where the von Neumann ray is situated in relation to the cyclical fluctuation. And when we concentrate on the quantity system, disproportionate growth which we have considered above plays an important role to cyclical fluctuations. (See also Appendix)

Let us suppose for example the initial position is under the von Neumann ray (see Fig. 4). The proportional growth is impossible in this area. Once a boom begins

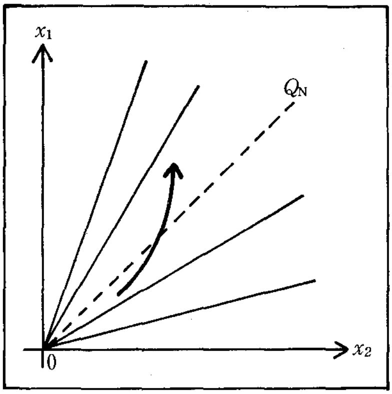


Fig. 4

and the rate of accumulation of sector 1 becomes higher than that of sector 2, the sector 1 grows disproportionately in comparison to sector 2 by the proposition and will cross the von Neumann ray. The proportional growth is not possible also in this area. If this disproportionate growth continues, sooner or later the economy reaches the area in which reproduction is not possible with positive rate of accumulation. But before reaching that area, a crisis may break out. Why crises must occur is the subject of crisis theory. So we can not ask about it here. Once a slump begins and the rate of accumulation of sector 1 becomes lower than that of sector 2(15), the sectoral composition of output decreases and the economy may return to the area below the von Neumann ray. The proposition includes only the logic of disproportionate growth, so it does not explain a crisis. But anyway the meaning of the von Neumann ray has become apparent. The von Neumann ray is situated at the center of a cycle and the economy fluctuates around it. According to Marx, the buisiness cycle is the mechanism that realizes the golden age.(16) In other words, it is the incessant disproportionate growth that realizes the Marxian golden age post festum or as the long-term trend.

Now we can regard the von Neumann growth ray as the long-term average growth path or the trend in the Marxian economic theory.

IV. Effects of technical progress on the long-term growth path

In the previous section we have made clear why the von Neumann ray can be regarded as the long-term growth path for the Marxian economic theory. Now by investigating how this long-term path is changed by the technical progress, we can understand what kinds of effect the technical progress has in the long run.

We have already seen in the section II that the sectoral composition of output of this long-term path is given by (8).

i. e. $Q_N=\frac{1}{2wl_1}\{a_1-wl_2+\sqrt{(a_1-wl_2)^2+4a_2wl_1}\}$

First of all differentiate Q partially by the technical coefficients, and it follows that

$$\frac{\partial Q}{\partial a_1}=\frac{Q}{S}, \quad \frac{\partial Q}{\partial l_1}=\frac{-wQ^2}{S},$$

$$\frac{\partial Q}{\partial a_2}=\frac{1}{S}, \quad \frac{\partial Q}{\partial l_2}=\frac{-wQ}{S}, \text{ where } S=\sqrt{(a_1-wl_2)^2+4a_2wl_1}$$

Summing up the signs of partial differential coefficients,

	a_1	a_2	l_1	l_2
Q	+	+	−	−

The effect of the changes in input coefficients on the sectoral composition of output contrasts to that of labour coefficients. Next we consider the synthesis of both effects by differentiating Q totally by both coefficients.

$$\frac{dQ}{dt}=\sum_{i=0}^{2}\left(\frac{\partial Q}{\partial a_i}\cdot\frac{da_i}{dt}+\frac{\partial Q}{\partial l_i}\cdot\frac{dl_i}{dt}\right) \quad \cdots\cdots (12)$$

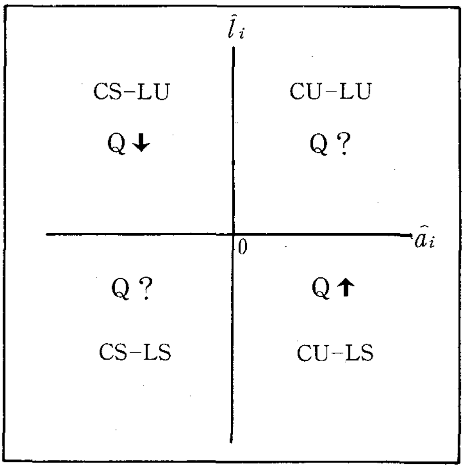


Fig. 5

Since the rate of change $\hat{a}_i$ is $\frac{da_i}{dt}$ divided by a_i (and also $\hat{l}_i=\frac{\dot{l}_i}{l_i}$), the change of the sectoral composition of output, i. e. (12) can be considered in $\hat{a}_i-\hat{l}_i$ plane (We assume here for simplicity that the rates of change are same.)

CU-LU : capital using & labor using thchnical change

CS-LU : capital saving & labor using technical change

CS-LS : capital saving & labor saving technical change

CU-LS : capital using & labor saving technical change

From the above consideration the next proposition is evident.

Prop. The sectoral composition of output rises when the technical change is CU-LS type, but it falls when the technical change is CS-LU type. And we can not say anything definitely when the other types of technical change (i. e. CU-LU, CS-LS) occur.

But our task is to consider the change of Q not in relation to the technical change in general but to the technical progress. Then how is the technical progress defined and how is it expressed in the technical change plane, i. e. $\hat{a}_i - \hat{l}_i$ plane ?

There are some definitions or criteria of technical progress in the Marxian economic theory. For example,[(18)]

(1) rise of output to current labor. i. e. $\widehat{(x_i/L_i)} = -\hat{l}_i > 0$

(2) rise of output to total labor including past labor (i. e. the present estimation of past labor embodied in capital goods). This is equivalent to the value revolution or the fall of value by technical change.[(19)]

$$\text{i. e. } \hat{t}_i < 0 \text{ where } (t_1, \ t_2) = (t_1 \ 1)\begin{pmatrix} a_1 & a_2 \\ l_1 & l_2 \end{pmatrix}$$

(3) rise of organic composition of capital. This does not mean the rise of labor productivity itself as (1) or (2), but is often adopted concept. According to Marx's definition

$$\hat{r}_i > 0 \text{ where } r_i = \frac{t_1 K_t}{t_2 w L_i} = \frac{t_1}{t_2} \cdot \frac{1}{w} \cdot \frac{a_i}{l_i}$$

Which of these did Marx set as a main criterion when he considered the problem of technical progress ? Certainly Marx pointed out e. g. the value lessening effect of technical progress. But Marx evidently attached importance. to the organic composition of capital as we con see the argument of the law of falling tendency of profit rate. So it is appropriate to define Marx's technical progress index by the increase in organic composition of capital.

But instead of it we shall use here the index of the technical composition of capital. The reason is that the technical composition of capital is more fundamental. According to Marx, the organic composition of capital is determind by the technical composition of capital and also mirrors the changes of the latter.[20] In addition, the technical composition of capital is easy to treat mathematically. The area in which the technical composition of capital rises in $\hat{a}_i - \hat{l}_i$ plane is given by the next inequality and Fig. 6.

$$\widehat{\left(\frac{K_i}{L_i}\right)} = \widehat{\left(\frac{a_i}{l_i}\right)} = \hat{a}_i - \hat{l}_i > 0$$

We exclude the CU-LU part of this area, because it is evidenty not the technical progress. In comparison to that the CS-LS part certainly expresses a technical progress, but it is different from Marx's supposition. So we suppose here that the Marxian technical progress area is given by the Quadrant IV, i. e. CU-LS area of this Fig. 6. When the technical progress of this type occurs, the technical composition and also the organic composition of capital rises. And then the sectoral composition of output rises as we have already considered. In this way we can prove the priority growth of the department 1 in the long run when we suppose the Marxian type of technical progress by definition (3).[21]

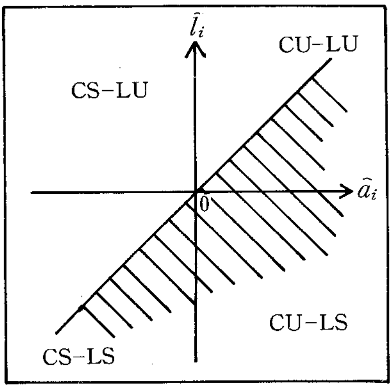


Fig. 6

But there are some problems in the above consideration.

They are related to the technical progress and the real wage rate.

The first problem is the technical progress. If we adopt Marx's criterion (3) on technical progress and assume that type of technical change, certainly we can prove the proposition. But what will become of the consequence when we consider the other types of technical progress ? For example, let us consider the CS-LS type of technical progress. Then we can not say anything about the change of the sectoral composition of output. We have only found that the proposition can be proved in Marx's CU-LS case. Marx evidently knew however that there could be various types of techniques.(22) He asked what the dominant or characteristic technique is in capitalism among them. And he regarded it as the capital-using & labour-saving type. So we must question whether his supposition is valid also at the present day. Moreover is the organic composition of capital or the technical composition of capital is suitable for understanding the technical progress today ? One of the subjects necessary for Marxian economics today is theoretical and empirical studies of technique or technology itself.

The second problem is related to real wages. When we have analysed the change of the long-term growth path, we have only taken the changes of technical coefficient into consideration. But the long-term path also depends on the real wage rate. So we can only speak of the effect of technical progress on the long-term path until we take the movement of the real wage rate into our consideration. It has not been questioned however how the technical progress effects the growth path, but how it will be actually with technical progress as I have mentioned in the introduction of this paper.(23) In order to know what the sectoral composition of output will be actually we must consider not only the technical coefficients but also the real wage rate in our framework.

But when we come to consider also the real wage rate, the relation between real wage and technical coefficient comes into question. Is it possible to assume that technical changes are independent of the movement of the real wage rate? ′ If so, we can consider such as the next total differentiation.

$$\frac{dQ}{dt}=\sum_{i=1}^{2}\left(\frac{\partial Q}{\partial a_i}\cdot\frac{da_i}{dt}+\frac{\partial Q}{\partial l_i}\cdot\frac{dl_i}{dt}\right)+\frac{\partial Q}{\partial w}\cdot\frac{dw}{dt}$$

But if we must consider the relation between them, we can not derive such a total differentiation. And we must take also the relation between them into consideration. Is technical change introduced by capitalists because wage share or money wage and also real wage change (e. g. rise)? Or is wage share or real wage adjusted because technical progress brings about the change of it? (24) Moreover we must ask these questions not in the short run but in the long run.

These are difficult but important problems which I can not discuss here easily.

V. Tentative Concluding Remarks

The problem of the priority growth of department 1 is obviously different from the problem of the disproportionate growth which can be observed in booms. Because it has been questioned whether there exists a long-term tendency of priority growth or how can it be expressed in a model? In this paper we have first of all attempted to pursue the long-term growth path in the Marxian economics. In section II we have investigated what position the von Neumann ray occupies in the growth possible area and proved that it lies in the interia of the disproportionate growth possible area. Next in section III it has been considered how the economy grows actually in the disproportionate growth possible area. And we have concluded that the long-term growth path, i. e. the von Neumann ray is realized as the trend or the tendency through the short-term cumulative disproportionate growth. So if we consider the change of this path with technical progress, we can comprehend the transition of the sectoral composition of output in the long-term tendency. We have done this analysis in section IV and found that at least in the case of capital-using & labor-saving technical progress which Marx seems to have thought, the priority growth of sector 1 occurs. But it does not mean necessarily that the analysis is satisfactory as we have pointed out at the last section. It must be further questioned by considering the mechanism of technical progress or the relation between technical progress and other factors.

«Appendix» : An Empirical Study of the Movement of Two Sectors

Next Figures show the movement of the sectoral composition of output and the sectoral net growth rates. The basic data, i. e. the sectoral output were derived from Shaw's data [14] [15] by the following way.(25)

Department I		Department II		
Producer Durable	Construction Materials	Consumer Durable	Consumer Semidurable	Consumer Perishable

(I)

Aggregating the social output by the above classification and expressing the sectoral composition of output by the index (1913=100),

Year	Q	Year	Q	Year	Q	Year	Q	Year	Q
1890	125	1900	96	1910	100	1920	96	1930	83
91	113	01	94	11	86	21	70	31	63
92	132	02	106	12	96	22	75	32	41
93	112	03	102	13	100	23	88	33	44
94	106	04	95	14	88	24	85	34	55
95	103	05	100	15	88	25	86	35	68
96	97	06	104	16	94	26	85	36	81

97	93	07	108	17	107	27	82	37	86
98	90	08	97	18	109	28	83	38	75
99	83	09	95	19	96	29	92	39	82

(II)

Form the aggregated sectoral outputs the sectoral net growth rates (%) can be derived.[26]

Year	g_1	g_2	Year	g_1	g_2	Year	g_1	g_2	Year	g_1	g_2	Year	g_1	g_2
1890	−2	8	1900	9	12	1910	−9	5	1920	−26	0	1930	−3	−8
91	22	5	01	13	0	11	15	4	21	22	15	31	−43	−13
92	−14	1	02	1	5	12	9	4	22	31	11	32	10	3
93	−9	−3	03	−7	0	13	−13	−1	23	−3	0	33	36	10
94	10	13	04	10	4	14	−2	−2	24	9	8	34	32	8
95	−6	−1	05	15	11	15	20	12	25	6	7	35	34	11
96	4	8	06	5	1	16	6	−6	26	−4	−1	36	12	6
97	−2	1	07	−18	−8	17	8	6	27	5	4	37	−22	−10
98	3	11	08	10	12	18	−4	10	28	15	4	38	22	11
99	15	0	09	8	2	19	0	0	29	−20	−11			

The following figure 7 and 8 show the numerical values of (I) and (II) respectively. Though we can see apparently the disproportionate growth of sector 1 in booms, the long-term tendency of the priority growth of sector 1 cannot be confirmed.

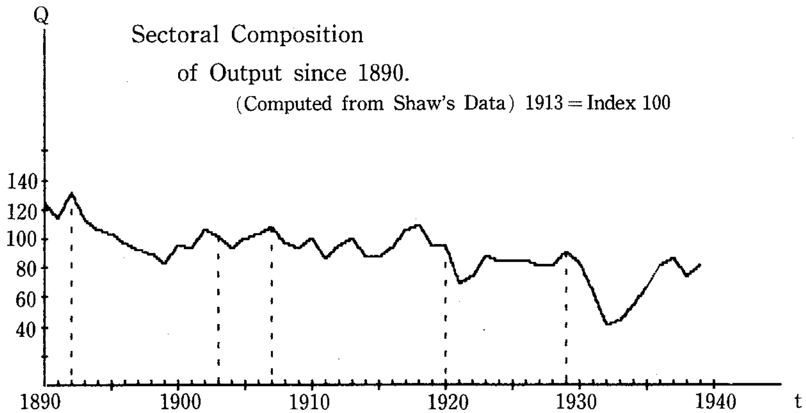
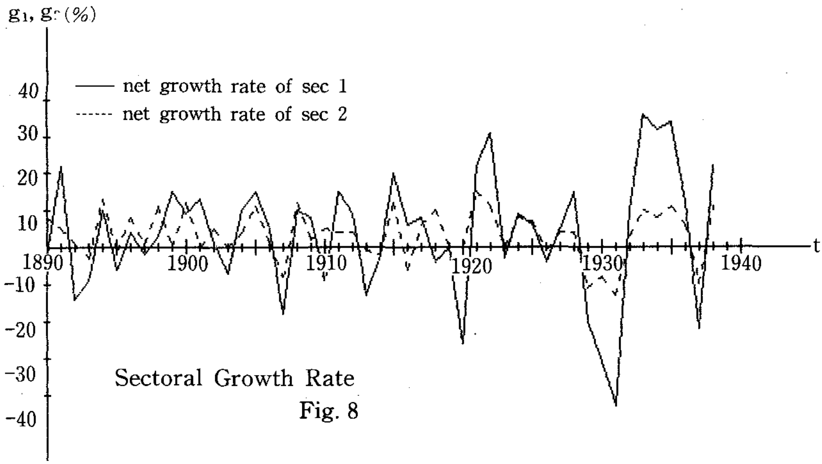


Fig. 7



Sectoral Growth Rate
Fig. 8

(Footnote)

The author thanks Prof. J. Glombowski for sending his papers and also thanks Prof. T. Seki (Hitotsubashi Univ) and Prof. Y. Fujimori (Josai Univ) for helpful and valuable comments. Needless to say, the author alone is responsible for the idea and remaining errors in this paper.

(1) Lenin [6], p. 89.

(2) In the Soviet Union the problem gave rise to much debate in the 1950's and 1960's. It had been questioned whether this proposition could be the ground of actual "Priority Policy". See for example Schönekess [13] and Turban [17].

(3) Yoshinaga [18] is a critical survey of the controvercy in Japan.

(4) In detail, refer to Glombowski [4].

(5) Y. Takasuka [16].

(6) Most of the model built on this problem assume that long-term tendency can be regarded as a continuation of short-term. (e. g. Glombowski [4]) But the author has a different opinion which will be made clear in this paper.

(7) This is not so important analytically but conceptually. (See D. J. Harris [5]).

(8) Abraham-Frois & Berrebi call this matrix socio-technological. (See [1]).

(9) The analysis in this section is the modified version of Prof. Fujimori [3], pp. 53-58.

(10) If we include capitalist's consumption into consideration, these areas become as the next figure 9.

(11) This equlization including consumption goods case is assumed to be done quickly.

(12) However rapidly capitalists intend to accumulate, the rate of accumulation must be determined at the intersection of two equations in the end if we take the equation approach. So this approach itself does not exclude capitalist's intention.

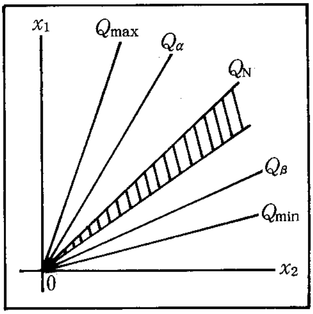


Fig. 9

(13) This lemma itself is also proved in a reduced reproduction case such as $0>g_2>g_1>-1$.

(14) The logic of this explanation is a Marxian version of Leontief dynamics. The similar explanation is also found in Roemer [12] and Nikaido [11]. (But Prof. Nikaido treats only simple reproduction case.) In order to apply this logic to business cycles, we have to consider not only the quantity system but also the price system. For exemple we have not taken the movement of the real wage rate into consideration in the analysis. The rael wage rate is determined by money wage rate and the price of consumer's goods. So the real wage rate is a point of contact between two systems and has to be introduced into the analysis to make a thorough study of a situation of cycles. It is also necessary to consider not only from structural angle but also from behavioral one. But we are not aiming at the study of crisis theory here.

(15) This includes of course the next case, i. e. $0>g_2>g_1$.

(16) The Marxian golden age does not necessarily include full employment. The meaning of this Marxian golden age to the Marxian economic theory is very important especially in relation to the value or price theory. For example it is in this golden age that the equlization of profit rates is realized and also the aggregated profits become equal to the mass of surplus value. See e. g. [1], [2], [3].

(17) Roemer used similar plane when he considered the problem of

falling rate of profit. (See [12], p. 102).

(18) We do not refer here to the technical selection considered by Roemer, Okijio. They say that capitalists select the technique only if it is cost-reducing at initial prices. Roemer [12] call this viable technical change. (p. 97).

(19) Roemer call this progressive technical change. (p. 100).

(20) "I call the value composition of capital, in so far as it is determined by its technical composition and mirrors the changes in the latter, the organic composition of capital." (Marx [7], p. 762).

(21) Morishima proved the problem of falling tendency of profit rate on the value constant line in CU-LS area. (Morishima [10], pp. 142-144) So on this line Marxian two propositions are both proved. This is an interesting fact.

(22) Mâtyâs also pointed out this fact. See Mâtyâs [9], p. 507.

(23) Lenin concluded after his analysis, "...... in capitalist society, the production of the means of production increases faster than the production of means of consumption". (Lenin [6], p. 88) It is obvious that he intended to argue what the sectoral composition of output would be actually.

(24) Roemer considered the problem of falling tendency of profit rate from this angle. (See [12], chap. 6).

(25) Shaw's data is very useful from the Marxian point of view, because we can easily derive the annual sectoral output from 1890 to 1939. But a weak point of it is that the unfinished goods are not fully recorded. Prof. Matsuishi also pointed out this fact. (Matsuishi [8], p. 252).

(26) In relation to the analysis of this paper the sectoral net growth rates is defined as follows. $g_{i(t)}=(x_{i(t+1)}-x_{i(t)})/x_{i(t)}$

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