Another Root of Disequilibrium

A Preliminary Note

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1. Introduction

Thinking of equilibrium theory, one could consider, as its ways of approaching, two worlds called the Walrasian economy and the Edgeworthian economy. Correspondingly the two approaches will also naturally be taken when one intends to describe still indefinite phenomena so-called 'disequilibrium': e. g. see Weintraub(1977). However, it seems that, even considering the two ways of setting, there have been indeed only one-sided trials with respect to 'disequilibrium' description.

The logic, which has developed since the enlightenment made by Clower-Leijonhufvud, has been almost consistently of description of disequilibrium by means of 'fixed prices' and so of detection of alternative concept of equilibrium under such background: e. g. see trials by Dreze, Grandmont-Laroque, Benassy, etc. On the other hand, apart from such a way of 'given', one would find a development such as to illustrate endogenously disequilibrium phenomena, that is, to explain the 'fixity' as results of agents' voluntary choice: e. g. see Negishi and Hahn.

This note would respect the latter way of thinking but radically differ with it in the respect of description of 'disequilibrium'. Its difference may be connected with a descriptive way of economy

itself: here, considering especially the Edgeworthian world, there will rather be provided another root of disequilibrium, so connected with the concept of 'time'. It should, however, be noticed that its stream never springs from the difference between the Walrasian economy and the Edgeworthian economy, which rather belong to the same class.

2. Price-Core with Production

Consistently let us consider an Edgeworthian economy characterized with recontracting. We consider not an economy characterzed with single blocking but one characterized with particular blocking mediated by prices.[1] The particular blocking is defined as follows:

Definition. A coalition $S \subset A$ is called to price-mediatedly block (say, p-block) an allocation x, if the following conditions hold;

1) $x'_i \gtrsim_i x_i$ all $i \in S$,
$x'_i >_i x_i$ some $i \in S$;

2) $\sum_{i \in S} x'_i = \sum_{i \in S} \bar{x} + y$,
$y \in Y(S)$ where $Y(S)$ is the production set peculiar to S;

3) $px'_i = p\bar{x} + d_i py$, for each $i \in S$,
provided $\sum_{i \in S} d_i = 1$;

A stands for the set of agents; $\bar{x}_i$ the endowment of agent i; $\gtrsim_i, <_i$ preference relations of i; $x \equiv (x_i)_{i \in S}$; x_i, x'_i and $p \in \overset{n}{+}$.

This definiton is more restrictive in the respect of the condittion (3) than the original (prototype) one.[2] Therefore the 'core' derivatively generated from it does not coincide with the original core. Let us denote by C the original core and by C_p the set of coalitions not p-blocked (say **price-core**) ; and then

$$C \subset C_p$$

from Definition.

This implies the inclusiveness of C_p and, considering a large economy (a replica economy or an economy with a continuum of agents), C_p would be, as ordinarily, approaching quasi-competitive equilibrium, as far as the difference between C_p and C is not so large.

Let us prove it briefly according to Arrow-Hahn (1971). Let$(\tilde{x}, \tilde{y})$ be an allocation of C_p; $\tilde{x}$ stands for a consumption allocation and $\tilde{y}$ production allocation. $Y \equiv \{(x_i, -k) | y \in Y(k)\}$; k is an m-dimensional vector denoting the number of agents in each production type (whose kinds are m); k is also called profile, e. g. $k(s)$ represents the profile of a coalition S. $Y(k)$ stands for the production set according to the profile k. $X_i \equiv \{(x_i, o) | x_i \gtrsim_i \tilde{x}_i\}$, $X'_i \equiv X_i \cup \{(\bar{x}_i, e^{t(i)})\}$, $X_i'' \equiv \{x_i'' | x_i'' \geq x_i' \text{ for some } x_i' \in X_i'\}$ and $Z_i'' \equiv X_i'' - Y$; e^t is the m-dimensional unit vector whose t th coordinate is one and the others are zeros, and $t(i)$ denotes the production type of agent i.

If one observes $r(Z_i'') \leq \tilde{L}$ for all i [2], where $\tilde{L}$ is finite, one can apply Arrow-Hahn (1971, Chapter 8, Theorem 5) to a price-core allocation $(\tilde{x}, \tilde{y})$. That is, there is a vector $(p^*, w^*) \geq 0$ such that

a) $\sum_{i \in A} | p^*(\tilde{x}_i - \bar{x}_i) - w^*{}_{t(i)} | \leq \tilde{M}$,

b) $p^* \sum_{i \in A} \tilde{x}_i - \sum_{i \in A} \min_{x_i \gtrsim_i x_i} p^* x_i \leq \tilde{M}$,

c) $w^* k(A) - \tilde{M} \leq p^* \tilde{y} \leq w^* k(A)$,
$p^* y \leq w^* k(S)$, for all $y \in Y(k(S))$, $S \subset A$,

d) $p^* > o$ if $k_t(A) > \tilde{M}$ for all t,

where $\tilde{M} \equiv 2\tilde{L}\sqrt{m+n}$ and $(p^*, w^*)e = 1$.[3]

Consider a replica economy where the number of agents of the same type is k for all types i. e., $i = 1, \ldots, k$ and $t = 1, \ldots, m$;

agents of the same type have the same preferences and endowments. The income of agent i of type t, say M_{it}, is defined $M_{it} \equiv p^* \bar{x} + d_t p^* y$, where d_t stands for the share of type t. Thus one can apply Arrow-Hahn (1971, Chapter 8, Theorem 6) to this economy. That is, if $(\tilde{x}^k, \tilde{y}^k)$ is a price-core allocation over all k, i. e. $k=1, 2, \ldots$, it coincides with a quasi-competitive allocation; in other words, there exists a price vector p^* such that $(p^*, \tilde{x}^t, \tilde{y}^k)$ is a quasi-competitive equilibrium.

Last it remains unsolved whether the above vector p^* is coincident with price vectors $\{p\}$ attached to a price-core allocation; its prices may not be unique. It is not sufficiently derived from the convexity of preferences.[4] One sufficient condition is that each agent's preference is strictly convex. Then the prices of quasi-competitive equilibrium would uniquely be determined. Thus so far as price-core allocation and quasi-competitive allocation are mutually coincident in a large (replica) economy, the corresponding prices $\{p\}$ and p^* become also coincident. This result is effective to competitive equilibrium when assuming positive incomes or resource relevancy. It is also conjectured that this holds even in an atomless economy with a continuum of agents.[5]

3. Another Root of Disequilibrium

Let us put concentration on the departure from the so-called competitive economy. When 'disequilibrium' could be interpreted in a wider sense as the deviation from competitive economy, it would be possible to consider disequilibrium more generally from properties of 'deviation'.

The viewpoint is put on reconsideration of p-blocking mentioned before. P-blocking has implied, although not explicitly, the follow-

ing properties:

1) **Myopic Behavior**: recontracting is not possible unless there exists another feasible allocation preferable to an existing allocation; it must, however, be noted that this process does not always imply a Pareto-improving process :
2) **Cooperative Price-setting**: prices are uniquely determined cooperatively within each coalition:
3) **Static Environments**: preferences, production technology and initial endowments are constant during the period of recontracting:
4) **Costless Transaction and searching**: there is no cost of transaction and searching.

Elimination or modification of each of them would bring about a source of 'disequilibrium'. The logic of the ready-made disequilibrium theory has been penetrately on 'price-rigidity'. It has primarily been regarded as the manna from heaven and later tried so as to explain endogenously. It has indeed been done on the basis of incomplete information and moreover of monopolistic actions of agents.

However, there is another root. It stems from 1), 3) or 4). Firstly there are changes of preferences. Over recontracting preference systems might change themselves, which would result in indeterminacy of (quasi-)competitive equilibrium. Yet it might only imply the coming of disorder as a theoretical consequence. More reasonable supposition is to modify 1) from a point of view of preference. That is to assume that recontracting depends on not only myopic behavior but propensity for stability of contract itself. Insofar as contracts are not realized stably, actual enjoyment of the allocations is impossible. It seems reasonable to suppose that the preference for stable contracts would be superior to myopic behavior in some

elapsing from an initial contract. Thus it leads to modification of 1). As a proposition one can consider contracts with time-limit of recontracting; within a limit any recontracting is possible but after it existing contracts must immediately be practiced into transaction. Possibly price-core allocations in such an economy would depend upon agents' ability of searching and the extent to which agents participate in markets.

Secondly 4) is raised. It does not follow from the simple proposition that transaction or searching costs bring about the change of characterization of core allocations. Logically core with such transaction costs could be considered and furthermore such equilibrium concepts in Walrasian economies have already been exposed: e. g. see Foley, Hahn, Kurz etc. The point is rather on the range of contracting. Restrictions to transaction or searching are factors fixing this range.

So ordinarily the range of exchange is potentially covered over all participating agents and all sorts of commodities. For any i, j of A, i and j can bargain with each other over exchange as to any commodities. The idea can be applied, as it is, to an economy with production. Production processes are certainly known and production plans are previously determined in the form of contracts. When time-relevancy as to recontracting and production is independent of preferences, exchange plans will be cancelled and reformed as far as the contracts are not in (price—)core. In this sense the space of exchange is synchronized for all participants.

This is, however, only to suppose a particular world. In the case that barriers as to information and management keep uncertainty existing, the space of exchange would not always be synchronized.

Nevertheless, we can think that the space of exchange is easier

to synchronize in a pure exchange economy. So long as every agent possesses an endowment enough to exchange and takes voluntary (myopic) actions he will bargain with one another and make as much recontracting as he desires. Immediate (or certain) existence of commodities for purpose of exchange will bring about certain feasibility of exchange whenever bargains are proposed. When time-elements as to recontracting are permitted as irrelevant to preferences for each agent, then it will easily be conjectured that the synchronized space of exchange is naturally formed.

On the other hand, things will differ in the case of an economy with production. It brings different things such as existence of production processes and time-preferences of participants. Considering a contract in a coalition, the input and output on production must be consistent. For example, if some members supply labor, they must receive in turn the output on production. It must be, however, delivered with some time lapse, however certain it may de. In addition, what will be produced must be taken into account in a coalition which agents will participate in. Of course, it is likely that, if there are joint productions whose processes are certain, 'double coincidence' is more probable than in a pure exchage economy. Yet there is no doubt a time lapse that is enough to have an effect on time-preferences.

Incidentally, suppose that there exist exclusively dominant media of exchange; say, m-goods.[6] If each agent holds a positive time-preference, there will appear some agents who immediately m-goods in contracts of exchange, instead of outputs, since they must be waiting to obtain the latter. If there are some producers possessing m-goods to some extent, they can dominantly call for suppliers of inputs. The suppliers can also potentially have a wider range of ex-

change with such producers since they can sell the inputs (endowments) to the producers without consideration of the corresponding outputs.

Thus let us suppose the following situation: almost all producers purchase inputs with m-goods and sell the outputs in exchange for m-goods as many as possible. Furthermore, the sellers of inputs also sell the inputs in exchange for m-goods. In such a situation direct trades between input and output not mediated by m-goods will almost probably be excluded.

Notably uncertainty rather works to enforce this situation. Uncertainty in production processes brings about increasing transaction-searching costs and makes direct trades with time-elapsing less and less attractive, and moreover m-goods are more and more demanded because of its immediate exchangeability. While consumers (input suppliers) desire m-goods as media of exchange from their time-preferences and furthermore because of the easiness of exchange, producers have to practice, in consequence, trades mediated by m-goods such as input-purchases with m-goods and output-sales against m-goods.

This situation provides a remarkable property. Direct trades with production (which belong to the space of exchange $\sum_{i\in S} Z_i$, where $\sum_{i\in S} Z_i \equiv \sum_{i\in S}(R^n{}_+ - \bar{x}_i) - Y(S)$, $S \subset A$) are almost excluded and simultaneous trades may be limited, if possible, to pure exchange: of course including one with m-goods. It implies that there are hardly contracts that bring about allocations characterized by consistent inputs and outputs. Exchange processes are separated from production processes and, in consequence, feasible allocation plans including certain production plans are hardly found and *ex-post* modification (or adjust-

ment) of plans will be frequently made in each exchange process by producers. Failures of plans take place, more or less, necessarily.

In addition, when exchange mediated by m-goods is generalized (say, m-good economy), markets of goods other than m-goods become established and existing independently of one another. The markets are mutually related through m-goods but the relation must be through at least two exchange processes such as i-good→m-good→j-good. They themselves are irreversible processes and also accompany time-elapsing. In this case it is sufficient that recontracting is perfectly made not in markets as a whole but in individual markets. To sum up, with respect to agents and goods, in an m-good economy, there exists no simultaneous space in both vertical (exchange) and horizontal (production) senses.(7)

4. An Example

Let us consider an economy; in which there are m-goods and two kinds of goods; and there are the producers and consumers(who are also suppliers of labor). The production function is as follows: $y^i{}_{t+1}=a\,\ell^i{}_t$, $a^i>0$, $i=1, 2$, which implies that for each good ($i=1, 2$)the outputs $(y^1{}_{t+1}, y^2{}_{t+1})$ will be produced in one production period (i. e. at $t+1$) if the inputs $(\ell^1{}_t, \ell^2{}_t)$ are used at t. Consumers' preferences at t are expressed such as $u(x_t)\equiv b(\log x^1{}_t+\log x^2{}_t), b>0$, where $x_t\equiv (x^1{}_t, x^2{}_t)$ means a consumption allocation. Remark that a^i and $u(.)$ are all the same for all producers and consumers, respectively, and that they behave as if they were united.

Imagine the following situation: the time is fixed, i. e. t, and the producers are divided into the two classes for each industry; the producers in the first class only supply the outputs, say $(\bar{y}^1{}_t, \bar{y}^2{}_t)\gg 0$ and have no intention to employ labor, while those in the second

only empoly labor (input) without supplying the outputs. The second will employ labor by expecting the future demand and price, $(\bar{x}^i{}_{t+1}, \bar{p}^i{}_{t+1}) \gg 0, i=1, 2.$

The derived demand of labor is $\bar{\ell}^i{}_t = \bar{x}_{t+1}/a^i, i=1, 2.$ Assume $\bar{\ell}^1{}_t + \bar{\ell}^2{}_t \leq L_t$ and $\bar{p}^i{}_{t+1} a^i > w_t$, $i=1, 2$, where w_t is a wage rate and L_t the available labor power at t. This is profitable for the producers. Further assume that they have enough m-goods, i. e. $w_t \bar{\ell}^i{}_t < m^i{}_t, i=1, 2,$ where $m^i{}_t$ is the m-good possessed by i.

First of all (conveniently) suppose that the producers in the first class will demand m-goods in exchange for their outputs. Then consumers will have to desire m-goods in exchange for labor; they will not otherwise purchase their desirable goods $(x^1{}_t, x^2{}_t)$ since they have extremely strong time-prefernces.[8] Let $(p^1{}_t, p^2{}_t)$ be the equilibrium prices of goods at t; they are $(p^1{}_t, p^2{}_t) = (w_t(\bar{\ell}^1{}_t + \bar{\ell}^2{}_t)/2^{-1}y_t, w_t (\bar{\ell}^1{}_t + \bar{\ell}^2{}_t)/2\bar{y}^2{}_t)$. Also remark that this situation is consistent with the preceding assumption that the first class of producers should demand m-goods in exchange for the outputs, as long as this is continued over time; their actions are rational in this sense.

Uncertainty reinforces this situation: even if time-preferences are not so extremely strong, uncertainty heightens searching costs and so consumers more demand m-goods that give convenience of exchange; the existence of m-goods will help them to prefer present goods not contracted to future goods contracted by means of diminishing costs. Correspondingly producers will also desire m-goods to call for more suppliers of input in such a situation.

5. Concluding Remarks

The original Edgeworthian economy has been characterized by perfect enforcement of recontracting, so to speak, by 'perfect rever-

sibility'. This feature is also found, as it is, in the Walrasian economy in the name of 'tâtonnement' process. On the other hand, the phenomena of 'disequilibrium' may be constructed by imperfect reversibility or 'irreversibility' under the common term of the deviation from competitive equilibrium. Here the roots of disequilibrium have been detected in the Edgeworthian economy in such forms as 1) fixed or institutionally predetermined prices, 2) modification of contracting actions and 3) non-synchronization from points of view of time preferences and production processes. Existence of uncertainty or transaction-searching costs has been shown as enforcing factors of (3).

This result, it is thought, be almost applied to the Walrasian economy. It is described by characterization of price rigidity or non-tâtonnement process. The former may correspond to (1) but the latter has not yet been reasonably justified: can we well explain what moves prices and stops them? This question is applied in the same way to an economy without auctioneer, i. e. with dealers. Indeed, the setting so as to move from the reversible process of 'tâtonnement' to the irreversible process (practices of trades) is still put as given from an individualistic point of view. We have considered the propensity for stable contracts as one of its justification. But it remain doubtful whether it sufficiently justifies the non-tâtonnement.

Last it remains unsolved whether disequilibrium phenomena can be found when perfect reversibility holds, that is, all of 1) 2) and 3) are excluded. The answer is no. One has already found that price-core allocations coincide with (quasi) competitive allocations in a perfect reversible world (which implies a static world from definition) and a large economy. Thus, so far as each agent takes myopic behavior and recontracting processes play a role to lower entolopy even

when a situation is initially uncertain, it would generate competitive states. Correspondingly it will easily be conjectured that the so-called 'conjectural equilibrium' consequently approaches competitive equilibrium in a reversible and large economy in consideration of such information-cumulative processes. That is also to remark that, even though producers take monopolistic actions, they would have to change, step by step, their action patterns because of reciprocal provision of information in the name of 'recontracting' with partners (consumers) and in consequence would have to materialize actions such as bringing about competitive equilibria in a large economy, as far as they consistently behave myopic-wise.(9)

Notes.

(1) Coalitions are here interpreted as groups agreeing with a single price system.

(2) With respect to r(.), see Arrow-Hahn (1971. Chapter 8 or Appendix).

(3) $k \equiv (k_1, \ldots, k_n)'$ and $e \equiv (1, \ldots, 1)'$.

(4) In the case that demands are insensitive to the change of prices within some range, it may take place that there exist more than one system of prices corresponding to the same allocation.

(5) Though in a pure exchange case, this proof will be provided if required.

(6) Let them be goods such as minimizing (expected) searching costs.

(7) This non-simultaneous view can also be grasped by means of remarking Walras' law. Its detailed discussion is seen in Akashi (1979).

(8) If $x_{t+1} \equiv (x^1_{t+1}, x^2_{t+1})$ is additionally considered in the preferences, they may be expressed such as $u(x_t, x_{t+1}) \equiv b_1(\log x^1_t + \log x^2_t) + b_2(\log(x^1_{t+1} + x^1_0) + \log(x^2_{t+1} + x^2_0))$, $b_1, b_2 > 0$; in this case b_2 is small enough relative to b_1 and $(x^1_0, x^2_0) \gg 0$,

(9) Discussion with respect to the change of action-patterns is also made in Akashi (1978).

References.

Akashi, S. (1978), "Competitive Equilibrium from Viewpoints of Processes

and Knowledge", mimeographed.

Akashi, S. (1979), "Walras' Law Reconsidered", mimeographed.

Arrow, K and Hahn, F. (1971), *Competitive Equilibrium Analysis*, Holden-Day Inc., San Francisco, California.

Weintraub, E. R. (1977), "The Microfoundations of Macroeconomics: A Critical Survey", *J. Econ. Lit.*, 15, 1—23.

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