IMPERFECT INFORMATION AND CONTRACTS BETWEEN TWO FIRMS

By AKIMITSU SAKUMA*

I. *Introduction*

There are many cases where a manufacturer of materials or intermediate goods supplies its product not to anonymous buyers but to a specified one. For example, in the Japanese automobils industry almost all accessories and parts manufacturers supply only one assembly maker, though the formers are not perfectly integrated into the latter. Another example is found in the international iron ore trades where the mining company in Australia sells its products to the specified steel manufacturing firm or the group of firms in Japan for a long time.

If all markets concerning the buyer and seller were perfect, then no benefit would be yielded by internalizing the transactions. But if each pair of parties has different transaction costs, an agent will select as his partner the one who enable him to minimize these costs. Search and transportation costs are the instances of transaction costs. One may continue to be in contact with the present partner due to the existence of these costs, even if he is open to other chances.

Economic theories of contracts have recently been developing in those areas of labor contracts (G. A. Akerlof and H. Miyazaki (1980), C. Azariadis (1975), M.N. Bailey (1974)), insurance contracts (M. Rothchild and J. Stiglitz (1976), M. Spence and R. Zeckhauser (1971) and the principal and agent relationship (M. Harris and A. Raviv (1979), S. Shavell (1979), M. Weitzman (1980)). These theories have common structures so that one side of a contract (an employer, insurer and principal) sellects a payoff schedule so as to maximize his (exployer and principal) expected utility or that of his partner (the insured) on condition that he guarantees at least a given level of expected utility to his partner which the latter may secure by other means.

In this paper I shall analyze some problems of the long-term contracts between two firms under uncertainty by using the above theoretical framework. In Section 2, the analytical framework will be shown. In Section 3 and 4, I shall examine some fundamental features of an optimal contract. In Section 5, two types of contracts are compared. The one is a usual optimal contract and the other is a contract through two parties' negotiation. In section 6, the effects of both parties' attitudes to risk and increasing risk on the contents of a contract are analyzed by the comparative static methods. Through these analyses I shall make clear how risk attitude, uncertainty and bargaining power affect a form and content of a contract. And in the last section 7, I shall refer to why in practice a fixed price-quantity contract is more prevailing than a contingent one.

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II. *Analytical Framework*

The buyer produces his products by the use of the intermediate goods purchasing from the seller and the other inputs (for example, labor) and sells them to the outside market. Here let us assume both buyer and seller are competitive in their factor and product markets. And assume both of them make a contract *ex ante* without certainly knowing the market price of the buyer's output. But they share some common stochastic knowledge about it.

Let p be a market price of the buyer's output, and P be the set of p and $P \equiv \{p\} = [p_1, p_2]$, $0 < p_1 < p_2 < \infty$. It is assumed that all states of nature concerning this problem is completely described by the set P. As already stated, both firms share a common probability dense function $\varphi(p)$ on the set P. Assume that $\varphi(p) > 0$, $\forall p \in P$.

The buyer and seller make a contract in which they arrange the price and quantity of the traded intermediate goods. There are four possible contract forms depending on whether or not the price or quantity is contingent on the states of nature. Let us consider the contract whose price and quantity are contingent on the states of nature p and denote it as $(\pi(p), Q(p))$, then this contract means that the seller shall supply $Q(p)$ units to the buyer at the price $\pi(p)$ when the market price of the buyer's output is identified as p *ex post*. Another exreme form is represented (π, Q) and this means that the seller supplies Q units at some price π whatever the market price of the buyer's output may be *ex post*. There are other two contract forms, $(\pi(p), Q)$ and $(\pi, Q(p))$ whose meanings are self evident. In this paper the first form of a contract will mainly be analyzed because it provides the buyer with the maximum expected utility and because the other contract forms are considered to be its variations.

The buyer produces its output by using the intermediate goods supplied by the seller and the other inputs. Among these inputs, only labor is taken into consideration here for simplicity. From the competitive labor market the buyer can purchase labor $L(p)$ at the fixed price w contingent on a state p. Let F be the buyer's production function, the output at a state p will be represented as $F(Q(p), L(p))$, where

$Q(p)$——the purchased quantity of the intermediate goods at a state p.

$L(p)$——the purchased quantity of labor at a state p.

Concerning this function F, let us assume

(2.1) F: strictly concave, $F_Q, F_L > 0$, $F_{QQ}, F_{LL} < 0$, $F_{QL} > 0$.

where $F_Q = \partial F / \partial Q$, $F_{QQ} = \partial^2 F / \partial Q^2$, *etc.* $F_{QL} > 0$ in (2.1) means the intermediate goods and labor are completely complementary in the buyer's production.

Then the buyer's profit $x(p)$ at a state p is shown as follows,

(2.2) $x(p) = pF(Q(p), L(p)) - \pi(p)Q(p) - wL(p)$.

where $\pi(p)$ is the contract price of the intermediate goods at a state p and w is a fixed wage rate.

Further let $G(\cdot)$ be a utility function of the buyer and assume.

$G'(x(p)) > 0, \quad G''(x(p)) \leq 0.$

where $G = dG / dx$, $G'' = d^2G / dx^2$. These mean that the buyer's marginal utility of profit is always positive and he is risk neutral or risk averse.

For simplicity let us assume that the seller produces the intermediate goods using labor only. If his production function is ϕ and labor input l, then $Q=\phi(l)$, where it is assumed $\phi'>0$, $\phi''<0$. Further let us define f as $l=\phi^{-1}(Q)\equiv f(Q)$, then f satisfies

(2.3) $f'>0, f''>0,$ for all Q.

Namely the seller's marginal cost is positive and is a increasing function of Q.

Then the seller's profit at a state p is shown as follows,

(2.4) $y(p)=\pi(p)Q(p)-wf(Q(p))$.

Further let U(y) be a utility function of the seller and assume,

$$U'(y(p))>0,\ U''(y(p))\leq 0.$$

Based on the above preparation, each expected utility of both parties deriving from the contract ($\pi(p)$, $Q(p)$) is given by (2.2) and (2.4) as follows,

$$EG(x(p))=\int_{p_1}^{p_2}G[pF(Q(p),\ L(p))-\pi(p)Q(p)-wL(p)]\varphi(p)dp,$$

$$EU(y(p))=\int_{p_1}^{p_2}U[\pi(p)Q(p)-wf(Q(p))]\varphi(p)dp,$$

where E is the expectation operator.

Here we can define the optimal contract ($\pi(p)$, $Q(p)$) as the solution of the following optimal problem.

(2.5) $\max\limits_{\pi(p),\ Q(p),\ L(p)} E[G(x(p))]$, subject to $E[U(y(p))]=\bar{u}$,

where $\bar{u}$ is the seller's maximum level of expected utility which he would gain if he made a contract not with the buyer in question but with another buyer. In other words, the optimal contract is the schedule that maximizes the buyer's expected utility guaranteeing the seller with what he would secure by other means.

III. *Some Features of the Optimal Contract*

In this section I shall show some fundamental features of the optimal contract. Let λ be a Lagrange multiplier concerning the constraint of (2.5), then the necessary conditions for the maximal problem are given by the Euler's theorem as follows,

$$G'[x(p)]=\lambda U'[y(p)],\ \forall p,$$

(3.1) $$(pF_Q(Q,\ L)-\pi)G'[x(p)]+\lambda(\pi-wf'(Q))U'[y(p)]=0,\ \forall p.$$

$$(pF_L(Q,\ L)-w)G'[x(p)]=0,\ \forall p,$$

where the assumption $\varphi(p)>0$, $\forall p$ is used and the state variable p is ommitted in $\pi(p)$, $Q(p)$ and $L(p)$.

The mutiplier λ appearing in the first equation of (3.1) is the shadow price concerning the garanteed utility level of the seller ($\bar{u}$) and it is a constant real number independent of a state. Further (3.1) gives the sufficient conditions for the problem (2.5) as well by the assumptions of the production functions and the utility functions.

Let us rewrite the second equation by using the first one in (3.1) and apply $G'[x(p)]>0$ to the third one, then three equations of (3.1) are rewritten as follows,

$$G'(x)=\lambda U'(y),\ \forall p,$$

(3.2) $$pF_Q(Q,\ L)=wf'(Q),\ \forall p,$$

$$pF_L(Q,\ L)=w,\ \forall p.$$

The second and third equations in (3.2) show that the input levels of both intermediate goods and labor at any state are determined in the optimal contract so that they may maximize the joint profits $(x(p)+y(p))$ at that state. (Remak: the contract quantity $Q(p)$ and labor input $L(p)$ are determined independent of both parties' utility functions when labor is purchasable depending on a state.[1]

Formally, all unknown function $\pi(p)$, $Q(p)$ and $L(p)$ and the constant multiplier λ are determined by the equation (3.2) and the constraint $U(y(p))=\bar{u}$. More precisely, the functions $Q(p)$ and $L(p)$ are simultaneouly determined by the second and third equations of (3.2). The contract price function $\pi(p)$ and the constant number λ are determined by these functions $Q(p)$ and $L(p)$, the first equation of (3.2) and the constraint equation $U(y(p))=\bar{u}$.

First of all, let us examine some properties of the contract quantity function $Q(p)$ and the buyer's labor input function $L(p)$. Since the second and third equations in (3.2) hold true for any state p, let us differentiate both sides of these equations by p, and calculate dQ/dp and dL/dp, then we have

$$(3.3)\quad \begin{aligned} dQ/dp &= (F_{LQ}F_L - F_{LL}F_Q)/|D|, \ \forall p, \\ dL/dp &= (F_{LQ}F_Q - XF_L)/|D|, \ \forall p, \end{aligned}$$

where $X=F_{QQ}-wf''$, $|D|=p^2(XF_{LL}-F^2{}_{QL})$. By the assumptions (2.1) and (2.3), we have $X<0$, $|D|>0$. Therefore we conclude $dQ/dp>0$, $dL/dp>0$. Namely, the higher the market price of the buyer's output is, the more the contract quantity of intermediate goods and labor inputs will be required.

If we differentiate both sides of the first equation in (3.2) by p, we have $G''(x)(dx/dp) = \lambda U''(y)(dy/dp)$. Divide both sides by $G'(x)(=\lambda U'(y))$, and use the definition of the absolute risk aversion, $A_G(x)=-G''(x)/G'(x)$ (for the buyer) and $A_U(y)= -U''(y)/U'(y)$ (for the seller), then the above equation may be rewritten as follows,

$$(3.4)\quad A_G(x)(dx/dp)=A_U(y)(dy/dp),\ \forall p.$$

By the second and third equations of (3.2), we have $d(x(p)+y(p))/dp=F(Q, L)$. By combining this result and (3.4), we have

$$(3.5)\quad \begin{aligned} dx/dp &= A_U(y)F(Q, L)/(A_G(x)+A_U(y)),\ \forall p, \\ dy/dp &= A_G(x)F(Q, L)/(A_G(x)+A_U(y)),\ \forall p. \end{aligned}$$

If the buyer is risk neutral ($A_G(x)\equiv 0$) and the seller is risk averse ($A_U(y)>0$), two equations of (3.5) become $dx/dp=F(Q, L)$ and $dy/dp=0$. In this case, the seller's profit becomes constant independent of a state in the optimal contract. Further the buyer's profit increases according to $dx/dp=F(Q, L)$ as the market price of the buyer's output becomes higher. If the buyer is risk averse and the seller is risk neutral, the above results are reversed.

If both parties are risk averse, two equations of (3.5) show that their profits at each state increase as the market price of the buyer's output become higher. Further these two equations show that both parties share the incremental joint profits at each state according to the ratio $(A_U/(A_G+A_U))$ and $(A_G/(A_G+A_U))$, respectively. In other words, the relative size of one's absolute risk aversion to their sums plays a critical role in the joint profit sharing.

That is, if one of both parties is risk neutral and the other is risk averse, then the former bears all risks resulting from an unknown market price. If both parties are risk averse, they share these risks according to the relative size of one's absolute risk aversion.

[1] A. Sakuma (1982) deals with the case where the buyer has to purchase the input other than the intermediate goods *ex ante* whatever a state may be.

The above results may be summized as follows.

PROPOSITION 1. In the optimal contract, (1) at any state (the market price level of the buyer's output), the contract quantity of intermediate goods and the buyer's labor input are determined so that the joint profits at that state may be maximized. (2) The contract quantity of the intermediate goods and the buyer's labor input increase as the market price becomes higher. (3) If both parties are risk averse, then they will share the incremental joint profits according to the relative size of one's absolute risk aversion to their sums. Their absolute shares of joint profits increase as the market price becomes higher. (4) If one of both parties is risk neutral and the other is risk averse, then the former may guarantee the latter a constant level of profit independent of a state and he bears all risks.

IV. *Contract Price in the Opitimal Contract*

In this section the behavior of the contract price will be examined. From the relation $d(x(p)+y(p))\,/\,dp=F(Q,L)$, we can easily show two equation of (3.5) are equivalent. So the second equation of (3.5) may be rewritten by using (2.4),

$$(4.1)\quad Q(d\pi\,/\,dp)=-(\pi-wf'(Q))(dQ\,/\,dp)+\frac{A_G(x)F(Q,L)}{A_G(x)+A_U(y)},$$

where Q and L are the solution of the last two equations of (3.2), and x, y are defined by (2.2) and (2.4), respectively.

In the following, it is assumed that if one is risk averse, then his absolute risk aversion decreases as his profit is increasing (the decreaing absolute risk aversion hypothesis). Further it is assumed that the elasticity of the buyer's absolute risk aversion with respect to his profit is so small that it satisfies the following inquality.

$$(4.2)\quad -\frac{A_G'(x)(dx\,/\,dp)}{A_G(x)}<\frac{dF(Q,L)\,/\,dp}{F(Q,L)},\ \forall p,$$

where $A'_G(x)=dA_G(x)\,/\,dx$. Later it is shown that this assumption gives a stable property to the behavior of the contract price.

Let us examine the gloval behavior of the solution $\pi(p)$ of (4.1). For that purpose, extend the set P to $(0,\infty)$ and assume,

$$(4.3)\quad \begin{array}{l}\lim\limits_{p\to 0} Q(p)=0,\quad \lim\limits_{p\to 0}\pi(p)>0,\\ \lim\limits_{Q\to 0} F(Q,L)=0,\quad \lim\limits_{Q\to 0} f'(Q)=0.\end{array}$$

LEMMA 1. (1) If both of the buyer and seller are risk averse, their absolute risk aversions are decreasing and (4.2), (4.3) are satisfied, then the following two equations hold good,

$$(4.4)\quad \begin{array}{l}\sup\{p|d\pi(p)\,/\,dp>0\}=+\infty,\\ \sup\{p|d\pi(p)\,/\,dp<0\}<+\infty.\end{array}$$

(2) If one of the parties is risk neutral and the other is risk averse, and (4.2) and (4.3) are satisfied, then (4.4) is also true.

Proof. The statement (2) is the special case of (1). So it is enough to prove (1) only.

For simplicity, $d\pi(p) / dp$ is denoted by $\dot{\pi}(p)$ and other derivatives with respect to p are equally denoted in the following.

The set $\{p|\dot{\pi}(p) < 0\}$ is non-empty because of the assumption (4.3) and the continuity of the related functions.

Next let us assume the set $\{p|\dot{\pi}(p) > 0\}$ is empty, namely $\dot{\pi}(p) < 0$ for all $p \in (0, +\infty)$. Then there exists some state $\bar{p}$ so that the term $(\pi - wf'(Q))$ becomes negative for any p larger than $\bar{p}$ since $f''(Q(p)) > 0$. So $\dot{y}(p) = \dot{\pi}(p)Q(p) + (\pi - wf'(Q))\dot{Q}(p)$ becomes negative, it contradicts the result of Proposition 1 (3). Therefore the set $\{p|\dot{\pi}(p) > 0\}$ is non-empty. Non-emptiness of $\{p|\dot{\pi}(p) > 0\}$ and the assumption (4.3) imply the second equation of (4.4) holds good. Let $\bar{p}$ be defined as follows,

$$sup\{p|\dot{\pi}(p) < 0\} \equiv \bar{p}.$$

Finally, let us show the first equation of (4.4) holds good. As already shown the set $\{p|\dot{\pi}(p) > 0\}$ is non-empty. Let $sup\{p|\dot{\pi}(p) > 0\}$ be finite and denote it as $p_0 (> \bar{p})$. Then by the continuity of $\dot{\pi}(p)$,

$$(4.5)\quad \begin{array}{l} \dot{\pi}(p_0) = 0, \\ \dot{\pi}(p) < 0, \text{ in the right neighborhood of } p_0. \end{array}$$

The second inequality of (4.5) means the following,

$$(4.6)\quad \pi(p) < \pi(p_0) \text{ in the right neighborhood of } p_0.$$

Let p be arbitarily near to p_0, then the following equation holds good approximately by (4.1).

$$(4.7)\quad \pi(p) - \pi(p_0)$$
$$= \left\{wf''(Q)\dot{Q}^2 - \frac{A'_U A^2_G F^2}{2(A_G + A_U)^3} + \frac{A_G' \dot{x} A_U F + (A_G + A_U) A_G \dot{F}}{2(A_G + A_U)^2}\right\} \frac{(p - p_0)^2}{2Q}.$$

where all functions in the R.H.S. of (4.5) except $(p - p_0)^2$ are evaluated at p_0 and $\ddot{Q}(p_0)\ (p - p_0)^2 \fallingdotseq 0$ is used.

The first two terms is the R.H.S. of (4.5) are positive since $f''(Q) > 0$ and $A'_U < 0$. Further the third term is also positive by the assumption (4.2). Therefore $\pi(p) > \pi(p_0)$ in the neighborhood of p_0. But this contradicts (4.6). This implies the first equation of (4.4).

Q.E.D.

Lemma 1 immediately leads to the following proposition.

PROPOSITION 2. (1) If both of the buyer and seller are risk averse and the conditions of Lemma 1 (1) are satisfied, then the contract price of the intermediate goods declines (rises) as the market price of the buyer's output is rising when it is lower (higher) than a certain level $\bar{p}$, and there uniquely exists such a price as $\bar{p}$. (2) Even if one of the parties is risk neutral and the other is risk averse, when the assumption (4.2) and (4.3) are satisfied, then the behavior of the contract price has the same pattern as (1).[2] (3) If the buyer is risk neutral and the seller is risk averse, then the contract price of the intermediate goods is higher than the seller's marginal cost at that state when the market price of the selling firm's output is lower than a certain level, and *vice versa*.

Proof. (1) and (2) are only the restatements of Lemma 1 (1) and (2), respectively. (3) Let A_G be equal to zero ac every state in the equation (4.1) and use the relationship

[2] I. Nakatani (1981) shows the same result by using a graph.

$\dot{\pi}(p) \gtreqless 0$ as $p \gtreqless \bar{p}$ in (2).

Q.E.D.

Proposisition (1) and (2) shows that the contract price as the function of the market price of the buyer's output takes a U-shape form. Namely, at a state where the market price of the buyer's output is relatively low, the contract price of the intermediate goods is declining as the market price is increasing and *vice versa.*

It may be understandable that the contract price is a increasing function of p in a boom phase, but it requires some explanations concerning why the contract price is a decreasing function of p in a slump phase. It directly results from two assumptions. The one is that the buyer has to guarantee a certain level of expected utility to the seller. The other is that the income elasticity of the buyer's absolute risk aversion is small enough. Proposition 1(3) and (4) show that the seller's profit does not decrease as the market price of the buyer's output becomes higher in order for the seller to secure a constant level of expected utility throughout the contract. For this reason, the buyer has to set a higher contract price even in the state of a relatively lower market price. Further a sufficient condition that makes the above contract pricing feasible is given by the assumption that the income elasticity of the buyer's absolute risk aversion is small enogh. Otherwise, the buyer would avoid to raise the contact price because doing so amplifies the decrease of its profit in the state of a low market price.

V. *Bargaining vs. Optimal Contract*

First of all, let us examine the effects of the changes of guaranteed expected utility to the seller on the contract price, quantity and the labor input of the buyer. For this purpose, differentiate both sides of all three equations of (3.2) with respect to λ respectively, noting that $\pi = \pi(\lambda, p)$, $Q = Q(\lambda, p)$ and $L = L(\lambda, p)$,

$$\begin{aligned} & -A\rho(\partial Q / \partial\lambda) + Q\rho(\partial\pi / \partial\lambda) = 1 / \lambda, \\ (5.1) \quad & X(\partial Q / \partial\lambda) + pF_{QL}(\partial L / \partial\lambda) = 0, \\ & pF_{LQ}(\partial Q / \partial\lambda) + pF_{LL}(\partial L / \partial\lambda) = 0, \end{aligned}$$

where $A = pF_Q - \pi$, $\rho = A_G + A_U$, $X = pF_{QQ} - wf''$.

As $pXF_{LL} - p^2F_{LQ}{}^2 = p^2(F_{QQ}F_{LL} - F_{LQ}{}^2) - pwf''F_{LL} > 0$, the last two equations of (5.1) give the solution $\partial Q / \partial\lambda = \partial L / \partial\lambda = 0$. From the first equation of (5.1), we have $\partial\pi / \partial\lambda = 1 / (\lambda\rho Q)$. Further $\partial x / \partial\lambda = -Q(\partial\pi / \partial\lambda) < 0$, $\partial y / \partial\lambda = Q(\partial\pi / \partial\lambda) > 0$.

On the other hand, the point $(\lambda, \bar{u})$ satisfies the relation $EU(y) = \bar{u}$, where λ is the shadow price of $\bar{u}$ and $\bar{u}$ is a guaranteed expected utility level to the seller and both λ and $\bar{u}$ are non-stochastic variables. So $\partial\lambda / \partial\bar{u} = 1 / (\partial\pi / \partial\lambda)EU'Q > 0$. Therefore all partial devivatives with respect to $\bar{u}$ have the same signs as those with respect to λ.

These results are summarized as follows.

PROPOSITION 3. Even if the guaranteed expected utility level to the seller rises by some cause, (1) the contract quantity and the buyer's labor input at each state are not affected,[3] (2) but the contract price rises at any state, (3) the profit of the buyer (seller) decreases (increases) at any state so that both increment and decrement may be compensated.

Until now, we have been considering the contract form that the buyer sellects a price-quantity schedule so as to maximize his expected utility on condition that he guarantees a given level of expected utility to the seller. But there may be other contract forms. Among them, let us consider a contract through bargaining where both parties negotiate a payoff schedule on their own interests. J.F. Nash (1950) and J.C. Harsanyi (1956) showed an elegant solution to the bargaining problem. Nash's bargaining solution which satisfies four reasonable axioms is formally given by a point in the two dimensional expected utility plane so that it maximizes the product of two person's expected utility from the threat point which means one's secured expected utility level when he does not make a contract with his partner.

And let the point $(\bar{g}, \bar{u})$ be a threat point. Then Nash's bargaining solution is given by a point so that it maximizes the following product,

(5.2) $(E[G(x)]-\bar{g})\,(E[U(y)]-\bar{u})$

where x and y are given by (2.2) and (2.3), respectively.

The necessary conditions for the above maximal problem are given as follows,[4]

$$G'(x)=\mu U'(y),$$
$$(5.3)\quad pF_Q - wf' = 0,$$
$$pF_L - w = 0,$$

where $\mu = E[G(x^*(p)]-\bar{g})/E[U(y^*(p)]-\bar{u})$, and $(x^*(p), y^*(p))$ is the point that maximizes the product (5.2).

Compare (5.2) with (3.1) which gives the necessary and sufficient condition for the optimal contract. If λ is replaced by μ, then we have (5.2) instead of (3.2). They differ in the parameters λ and μ only. So all the properties in the optimal contract stated in Proposition 1 and 2 are reserved in the bargaining contract as well.

It may be reasonable to assume that the seller's threat point in bargaining is equal to its guaranteed expected utility level in the optimal contract. Because both are the expected utility level that the seller can secure even when he does not make a contract with the buyer in question.

Let us compare the levels of the endogenous variables in the bargaining contract with a threat point $(\bar{g}, \bar{u})$ and those of the optimal contract with the constraint $EU(y)=\bar{u}$. If a bargaining problem is converted to some form of an optimal contract, such a compraison may easily be treated. First, let us confirm the following fact.

LEMMA 2. The bargaining solution (x^*, y^*) with a threat point $(\bar{g}, \bar{u})$ is equal to the

[3] This conclusion depends on how the buyer punchases the input other than the intermediate goods. In this paper we analyse the case where the buyer's production function takes a form $F(Q(p), L(p))$ in which two factors are dependent on a state p. But we can pressume a form $F(Q(p), L)$ in which L is not dependent on p. See A. Sakuma (1982) in such a case.

[4] Let x, φ and ϕ be mappings so that $x: S \to X(\subset R^n)$, $\varphi: X \to R$, $\phi: X \to R$ and assume they satisfy proper differentiability conditions. Then the $x(s)$ that maximizes the product $(\int_{s_1}^{s_2}\varphi(x(s))ds)\,(\int_{s_1}^{s_2}\phi(x(s))ds)$ satisfies the follwing necessary condition,

$$grad\ (\varphi(x))\int_{s_1}^{s_2}\phi(x(s))ds + grad(\phi(x))\int_{s_1}^{s_2}\varphi(x(s))\ \mathrm{ds}.$$

Proof is omitted.

optimal contract solution (x^{**}, y^{**}) with the constraint $E[U(y)] = E[U(y^*)] \equiv \bar{\bar{u}}$.

Proof. Since the solution of the optimal contract (x^{**}, y^{**}) has to satisfy its constraint, *i. e.*,

(5.4) $E[U(y^{**})] = E[U(y^*)] = \bar{\bar{u}}$.

Let λ^* be defined as follows,

$$\lambda^* = (E[G(x^*)] - \bar{g}) / (E[U(y^*)] - \bar{u}).$$

and let λ^{**} be the shadow price of the optimal contract problem with the constraint $EU(y)=\bar{\bar{u}}$.

Then if $\lambda^* > \lambda^{**}$, from Proposition 3(3),

(5.5) $y^* = y(p, \lambda^*) > y(p, \lambda^{**}) = y^{**}, \forall p$.

But (5.5) leads to the following inequality since $U' > 0$.

$$E[U(y^*)] > E[U(y^{**})].$$

this contradicts with (5.4). The assumption $\lambda^* < \lambda^{**}$ leads to the similar contradiction. So $\lambda^* = \lambda^{**}$. This result and the uniqueness of the optimal contract solution lead to,

$$x^*(p) = x^{**}(p), \; y^*(p) = y^{**}(p), \; \forall p.$$

Q.E.D.

After all, by Lemma 2 the comparison of the bargaining problem with a threat point $(\bar{g}, \bar{u})$ and the optimal contract with the constraint $EU(y) = \bar{u}$ is equivalent to those of two optimal contracts each of which has the constraint $EU(y) = \bar{\bar{u}}(\equiv EU(y^*))$ and $EU(y) = \bar{u}$, respectively, where y^* is defined in Lemma 2. Let $\bar{\lambda}$ and $\bar{\bar{\lambda}}$ be the shadow prices corresponding to $\bar{u}$, $\bar{\bar{u}}$, respectively, then $\bar{\bar{\lambda}} > \bar{\lambda}$ since $\bar{\bar{u}} > \bar{u}$. By applying this result to Proposition 3, we have the following one.

PROPOSITION 4. If one of parties is risk averse at least and it is reasonable to assume the equality of the guaranteed expected utility level to the seller and its threat point, then (1) the contract quantity and the buyer's labor input are equal at each state between the bargaining and optimal contract. But (2) the contract price of bargaining at each state is higher than that of the optimal contract.

Finally, let us examine how the bargaining solution changes as a threat point $(\bar{g}, \bar{u})$ is changing. From the last two equations of (5.3), we have $\partial q / \partial \bar{u} = \partial L / \partial \bar{u} = 0$ for any p. Next, set $\mu = (E[G(x)] - \bar{g}) / (E[U(y)] - \bar{u})$ in the first equation of (5.3), then we have,

$$G'(x) / (E[G(x)] - \bar{g}) = U'(y) / (E[U(y)] - \bar{u}).$$

Let us take the logarithmic forms of both sides in the above equation and differentiate partially with respect to $\bar{u}$, then we have,

(5.6) $(A_G + A_U)Q(\partial\pi / \partial\bar{u}) + 2\phi_U E[QU'(\partial\pi / \partial\bar{u})] = \phi_U$.

And by the similar procedures, we have,

(5.7) $(A_G + A_U)Q(\partial\pi / \partial\bar{g}) + 2\phi_G E[QU'(\partial\pi / \partial\bar{g}] = -\phi_G$,

where $\phi_G = (EG - \bar{g})^{-1}$, $\phi_U = (EU - \bar{u})^{-1}$.

On these preparations the following is shown.

PROPOSITION 5. If one of parties is risk averse at least, the change of both parties' threat point does not affect the contract quantity and the buyer's labor input at any state.

But the contract price rises (lowers) at any state as the seller's (buyer's) threat point is rising.

Proof. The former part is apparent from $\partial Q / \partial s = \partial L / \partial s = 0\,(s = \bar{g}, \bar{u})$.

Let us assume only the seller's threat point is changing. It is enough to show that $(\partial \pi / \partial \bar{u})$ is positive for all $p \in P$ in the equation (5.6).

There exists $p^+ \in P$ so that $\partial \pi(p^+, \bar{u}) / \bar{u} > 0$, since $\phi_U = (EU - \bar{u})^{-1} > 0$ in (5.6). Let us assume that there exists $p^- \in P$ so that $\partial \pi(p^-, \bar{u}) / \partial \bar{u} < 0$ where it can be assumed $p^+ > p^-$ without loss of generality. Then by the continuity of the function $\partial \pi / \partial \bar{u}$, there exists p^0 in the interval (p^-, p^+) so that $\partial \pi(p^0, \bar{u}) / \partial \bar{u} = 0$.

The equation (5.6) becomes $EQU'(\partial \pi / \partial \bar{u}) = 2^{-1}$ at the point p^0 since $\phi_U > 0$. Since this value is independent of p, the equation (5.6) becomes $(A_G + A_U)Q(\partial \pi(p^+, \bar{u}) / \partial \bar{u}) = 0$ at the point p^+. Namely $\partial \pi(p^+, \bar{u}) / \partial \bar{u} = 0$ since $(A_G + A_U)Q > 0$. But this contradicts the assumption. Therefore $\partial \pi(p, \bar{u}) / \partial \bar{u} > 0$ for all $p \in P$.

By the similar procedures, it is shown $\partial \pi(p, \bar{u}) / \partial \bar{u}$ is negative for all $p \in P$.

Q.E.D.

VI. *Changes in Absolute Risk Aversion, Increasing Risk and Optimal Contract*

In this section we shall examine the problem how the contents of the contract are transformed as both parties' absolute risk aversion and the extent of risk are changing.

First, let us examine the effect of changing absolute risk aversion on the contents of a contract. Now introduce two pairs of utility functions (G_1, G_2) and (U_1, U_2) for the buyer and seller, respectively. For these functions, the buyer's (seller's) absolute risk aversion is higher in $G_1(U_1)$ than $G_2(U_2)$, when the following two inequalities are satisfied [J.W. Pratt (1964)].

$$(6.1)\quad \begin{aligned} &- G''_1(x) / G'_1(x) > - G''_2(x) / G'_2(x).\ \forall x. \\ &- U''_1(y) / U'_1(y) > - U''_2(y) / U'_2(y),\ \forall y. \end{aligned}$$

Let us consider the impacts of the buying firm's utility function change from G_2 to G_1 satisfying the first inequality of (6.1). Let $(x_1(p), y_1(p))$ and $(x_2(p), y_2(p))$ be the solutions in terms of profits of the following two optimal problems, respectively,

$$(6.2)\quad \begin{aligned} &\max EG_1(x), \text{ subject to } EU(y) = \bar{u}, \\ &\max EG_2(x), \text{ subject to } EU(y) = \bar{u}. \end{aligned}$$

Further let $(\lambda_1, \pi_1(p))$ and $(\lambda_2, \pi_2(p))$ be the pairs of the multiplier and the optimal contract price in the first and second optimal problems in (6.2), respectively.

Next consider two optimal problems where only the seller's utility function changes.

$$(6.3)\quad \begin{aligned} &\max EG(x), \text{ subject to } EU_1(y) = \bar{u}, \\ &\max EG(x), \text{ subject to } EU_2(y) = \bar{u}, \end{aligned}$$

where two utility functions satisfy the second relationship of (6.1).

LEMMA 3. (1) For two solution tuples $(\pi_1(p), x_1(p), y_1(p))$ and $(\pi_2(p), x_2(p), y_2(p))$ corresponding to two optimal in (6.2) where G_1 and G_2 satisfy the first inequality of (6.1), there uniquely exists $\bar{p} \in P$ so that it satisfies the following inequalities,

$$\text{(6.4)}\quad \begin{array}{l} \pi_1(p) \lesseqgtr \pi_2(p) \\ x_1(p) \gtreqless x_2(p) \\ y_1(p) \lesseqgtr y_2(p) \end{array} \quad \text{as} \quad p \lesseqgtr \bar{p}.$$

(2) For two solution tuples $(\bar{\pi}_1(p), \bar{x}_1(p), \bar{y}_1(p))$, and $(\bar{\pi}_2(p), \bar{x}_2(p), \bar{y}_2(p))$ corresponding to (6.3) *mutatis mutandis*, there uniquely exists $\bar{\bar{p}} \in P$ so that it satisfies,

$$\text{(6.5)}\quad \begin{array}{l} \bar{\pi}_1(p) \gtreqless \bar{\pi}_2(p) \\ \bar{x}_1(p) \lesseqgtr \bar{x}_2(p) \\ \bar{y}_1(p) \gtreqless \bar{y}_2(p) \end{array} \quad \text{as} \quad p \lesseqgtr \bar{\bar{p}}.$$

Proof. The statement (2) is proven by the same ways as (1), so it is enough to prove only (1).

Since two optimal problems in (6.3) have the same constraint, the solution functions $y_1(p)$ and $y_2(p)$ have to satisfy the relationship $EU(y_1(p)) = EU(y_2(p))$. If two functions satisfy $y_1(p) > y_2(p)$ (or $y_1(p) < y_2(p)$) for all $p \in P$, the above equality does not hold good. So there exists $p \in P$ at least so that it satisfies $y_1(p) = y_2(p)$. For such p, $\pi_1(p) = \pi_2(p)$ and $x_1(p) = x_2(p)$ must be true since both of the contract quantity and the buyer's labor input at any state are independent of the functions G, U and a constant number λ. (See the last two equations in (3.2)).

First, it is shown that there uniquely exists such $p(\in P)$. Let $\bar{p}$ be the minimum value that satisfies the relation $y_1(p) = y_2(p)$. Let us assume there exists another $\bar{\bar{p}}(\in P)$ that satisfies the relation $y_1(p) = y_2(p)$.

For simplicity, let us denote $x_1(\bar{p}) = x_2(\bar{p}) = \bar{x}$, and $x_1(\bar{\bar{p}}) = x_2(\bar{\bar{p}}) = \bar{\bar{x}}$.
Then we have the following relationship by the first equation of (3.1).

(6.6) $\lambda_2 / \lambda_1 = G_2'(\bar{x}) / G_1'(\bar{x}) = G_2'(\bar{\bar{x}}) / G_1'(\bar{\bar{x}})$.

But $\bar{\bar{x}} > \bar{x}$ as $\bar{p} < \bar{\bar{p}}$ by the first equation of (3.5). Then since the first inequality of (6.1) holds true, by the (20) inequality of J.W. Pratt (1964),

$$G_2'(\bar{x}) / G_1'(\bar{x}) < G_2'(\bar{\bar{x}}) / G_1'(\bar{\bar{x}}),\ \bar{x} < \bar{\bar{x}},$$

this result contradicts (6.6). Therefore there uniquely exists $p \in P$ so that it satisfies the relationship $y_1(p) = y_2(p)$. Such p is expressed as $\bar{p}$ in the following.

Secondly, let us show that $x_1(p) > x_2(p)$ for all p in the interval $[p_1, \bar{p})$. For this purpose, conversely let us assume $x_1(p) < x_2(p)$ at some p in this interval. Then by the first equation of (3.5) and the (22) of J.W. Pratt's (1964), the following inequality holds true at the point p.

$$\text{(6.7)}\quad \frac{\lambda_2}{\lambda_1} = \frac{G'_2(\bar{x})}{G'_1(\bar{x})} > \frac{G_2(x_2(p)) - G_2(x_1(p))}{G_1(x_2(p)) - G_1(x_1(p))},\ x_1(p) < x_2(p) < \bar{x}.$$

Since the functions $G_1(\cdot)$ and $G_2(\cdot)$ are strictly concave, and $G_2'(x_2(p)) = \lambda_2 U'(y_2(p))$, $G_1'(x_1(p)) = \lambda_1 U'(y_1(p))$ by the first equation of (3.2), the inequality of (6.7) continues as follows,

$$\frac{\lambda_2}{\lambda_1} = \frac{G_2'(\bar{x})}{G_1'(\bar{x})} > \frac{G_2'(x_2(p))}{G_1'(x_1(p))} = \frac{\lambda_2 U'(y_2(p))}{\lambda_1 U'(y_1(p))}.$$

By the assumption $x_1(p) < x_2(p)$, then $y_1(p) > y_2(p)$. Namely we have $U'(y_1(p)) < U'(y_2(p))$. Therefore the most right hand side of the above inequality is strictly larger than (λ_2 / λ_1). This leads to a contradiction. Therefore, it is shown that $x_1(p) > x_2(p)$, $y_1(p) < y_2(p)$ and $\pi_1(p) > \pi_2(p)$ for all p in the interval $[p_1, \bar{p})$.

By the same ways, it is shown $\pi_1(p) < \pi_2(p)$ *etc.* for all p in the interval $(\bar{p}, p_2]$,

Q.E.D.

FIG. 1 THE EFFECTS OF THE BUYER'S INCREASING RISK AVERSION

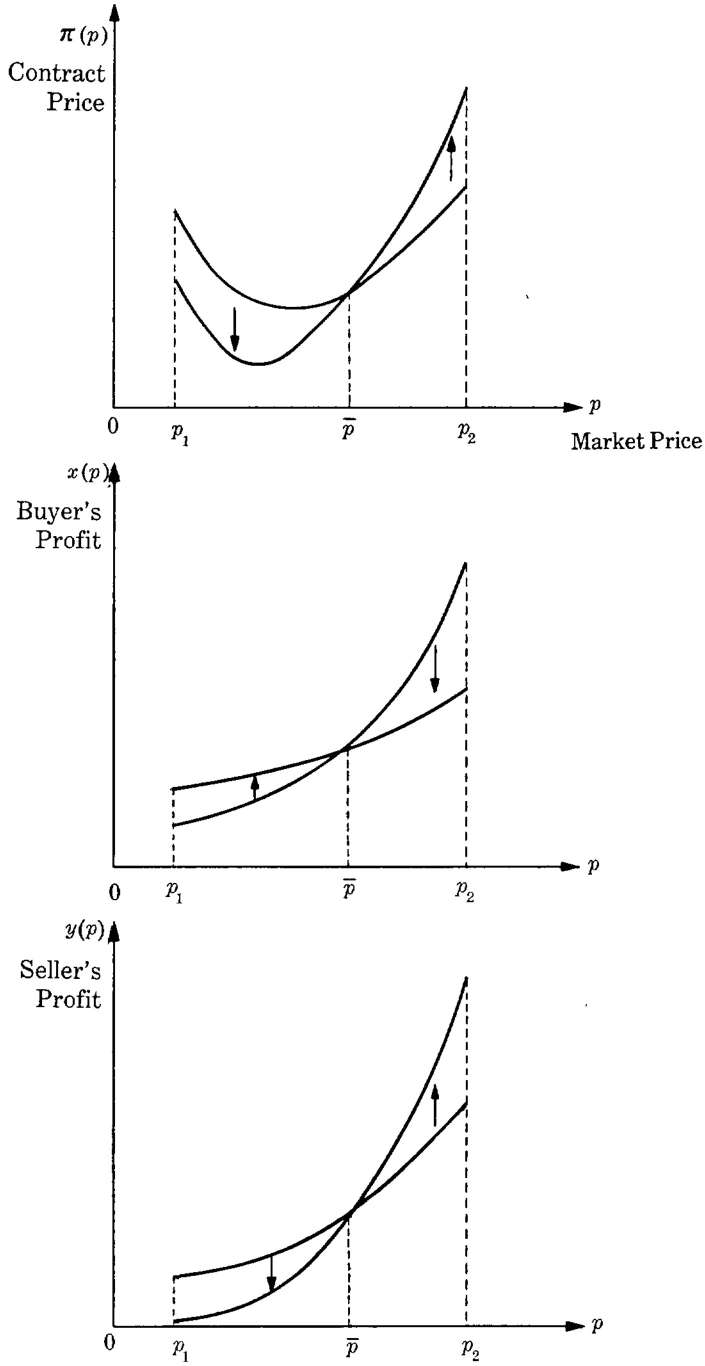


Figure 1 shows how the contract price $\pi(p)$, and the buyer's and seller's profits are changed as the buyer's absolute risk aversion is rising.

Lemma 3 immediately leads to the following proposition.

PROPOSITON 6. (1) The buyer (seller) prefers the contract whose profit variations all over the states are less when the buyer's (seller's) absolute risk aversion becomes higher. (2) For any pair of the buyer's utility functions which satisfies the first inequality of (6.1), there uniquely exists a state $\bar{p}$ where two contract price coincides with each other. In any state where the market price of the buyer's output is less than $\bar{p}$, the contract price corresponding to the lower buyer's risk aversion is higher than the one to the higher risk aversion. Conversely in any state where the price is higher than $\bar{p}$, the former is lower than the latter (see Figure 1). And the changes of the seller's absolute risk aversion has the converse effect on the contract price at each state. (3) The contract quantity and the buyer's labor input are not affected at all by the changes of both parties' absolute risk aversions.

The increase of the buyer's risk aversion makes his profit variation less but makes the seller's more. The same is true concerning the seller's risk aversion change. But those effects on the contract price variation are ambiguous since the contract price depicts a *U*-shaped form as the market price of the buyer's output is rising.

If the contract situations are confined to the subset of $[p_1, \bar{p})$ or $(\bar{p}, p_2]$, then the effects on the contract price variation is unique. For example, if the contract situations are confined to the slump phases of the buyer's output market which correspond to a sub-interval of $[p_1, \bar{p})$, then the contract price variation is less as the buyer's (seller's) absolute risk aversion becomes higher (lower). The conclusion coincides with that of J.R. Markusen (1979).[5] The extreme case is obstained by setting $A_U \equiv 0$ (i. e., the seller is risk neutral), but even in this case the contract price is changing according to the differential equation $Q(d\pi / dp) = -(\pi - wf(Q))(dQ / dp) + F(Q, L)$ so that the contract price is not fixed all over the states.

Next, I shall examine the effect of increasing risk on the contents of a contract. Let us introduce a new stochastic variable $(\gamma p + \theta)$ instead of p according to *A*. Sandomo (1971). Two parameters γ and θ are assumed to be changed so that the means of two stochastic variables $(\gamma p + \theta)$ and p are equal. That is, there is the relationship $d\theta / d\gamma = -\mu$ between two parameters θ and γ, where μ is the mean of p.

Let us set $(\gamma p + \theta)$ in three equation (3.1) instead of p, partially differentiate both sides of each equation with respect to γ and use the relationship $d\theta / d\gamma = -\mu$. Then we have,

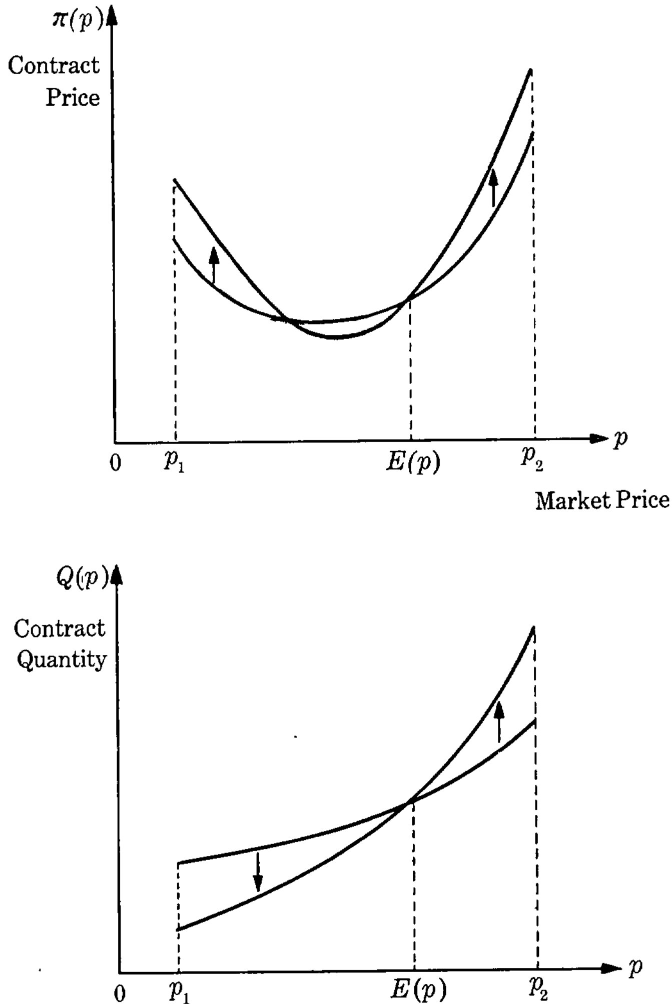
$$
\begin{aligned}
&\partial Q / \partial \gamma = (p-\mu)(F_L F_{LQ} - F_Q F_{LL}) / |D|, \\
(6.8)\quad &\partial L / \partial \gamma = (p-\mu)(F_Q F_{LQ} - X F_L) / |D|, \\
&(\partial \pi / \partial \gamma) Q = -(\pi - wf')(\partial Q / \partial \gamma) + (p-\mu) A_G F / (A_G + A_U),
\end{aligned}
$$

where γ and θ are evaluated at $\gamma = 1$ and $\theta = 0$, respectively and X and $|D|$ are defined in the same ways as (3.3).

The first two equations in (6.8) show that the increase of risk raises the contract quantity

[5] In his firm vs. employee wage contract theory, J.R. Markusen assumes two states case, p_a and p_b ($p_a > p_b$) each of which is the market price of the firm's output and shows that the wage rate w_a in the state p_a is lower than w_b in the state p_b. He also shows that the difference $(w_b - w_a)$ becomes less as the employee's absolute risk aversion becomes smaller when the firm is risk neutral. And it is shown that the difference $(w_b - w_a)$ is less as the firm becomes more risk averse.

FIG. 2. THE EFFECTS OF THE INCREASING RISK



and the buyer's labor input at each state where the market price of the buying firm is higher than its mean, and *vice versa*. This in turn shows that the decrease of risk makes less the variations of these variables and allows the contract to come near to the fixed quantity one (see Figure 2).

Finally, let us examine the effect of increasing risk on the contract price at each state. Set $\tilde{p} = \gamma p + \theta$, then the third equation of (6.8) is rewritten when $p \neq \mu$.

(6.9) $Q(\partial\pi \,/\, \partial\tilde{p}) = -(\pi - wf')(\partial Q \,/\, \partial p) + A_G F \,/\, (A_G + A_U)$.

Since the equation (6.9) is evaluated at $\gamma = 1$, $\theta = 0$, it is identical to (4.1). Let p_0 be the point at which the $d\pi \,/\, d\tilde{p}$ of (6.9) becomes zero. By Proposition 2 (2) and (3), we have,

(6.10) $\partial\pi \,/\, \partial\tilde{p} \gtreqless 0$ as $p \gtreqless p_0$.

Since $\partial\pi \,/\, \partial\gamma = (p - \mu)(\partial\pi \,/\, \partial\tilde{p})$, the sign of $(\partial\pi \,/\, \partial\tilde{p})$ is determined by (6.10) as follows,

$$
\begin{aligned}
&\partial\pi / \partial\gamma > 0, \text{ if } p_1 \leq p < \min(p_0, \mu), \\
(6.11)\quad &\partial\pi / \partial\gamma < 0, \text{ if } \min(p_0, \mu) \leq p < \max(p_0, \mu), \\
&\partial\pi / \partial\gamma > 0, \text{ if } \max(p_0, \mu) < p \leq p_2.
\end{aligned}
$$

The inequalities of (6.11) shows that the increase of risk raises the contract price at each state where the market price of the buyer's output is relatively high or low, but it lowers the contract price where the market price is in the middle range (see Figure 2). Therefore, the increase of risk makes larger the contract price variation. In other words, the decrease of risk makes the variation smaller.

The above results are summarized as follows.

PROPOSITION 7. (1) The variations of the contract price and quantity become smaller as risk decreases, and the contract approaches a fixed one. (2) The decrease of risk makes smaller the profit variations of both parties.[6]

When the buyer can purchase labor after knowing what state has happened, the changes of both parties' absolute risk aversion does not affect the contract quantity at any state. But the decrease of risk has the effect of making smaller the contract quantity variation.

Though it is not clear how the changes of the absolute risk aversion affect on the contract price variation, the decrease of risk makes its variations smaller. It may be concluded that whether or not the fixed price-quantity contract is made is more relevant to the extent of risk than to the parties' attitudes towards risk.

VII. *Conclusion*

In this paper, some aspects of the contracts between two competitive firms under the situations where they have only imperfect imformations about the market price of the buyer's output and they have different transaction costs to each other party. Here we have concentrated on a type of contracts that the parties arrange the contract price and quantity at each state because this type of a contract theoretically gives the buyer the maximum expected utility on condition that he guarantees the seller a given level of expected utility.

The main results are summarized as follow.

(1) The contract quantity and the buyer's labor input at each state is determined so as to maximize their joint profits at that state. They become larger as the market price of the buyer's output rises.

(2) If both parties are risk averse, then each of them shares the joint incremental profits at each state according to the relative size of their absolute risk aversions.

(3) If one is risk neutral and the other is risk averse, the former guarantees the latter a given profit independent of a state.

(4) The contract price curve generally shows a U-shape form as the market price of the buyer's output is rising.

(5) The increase of a guaranteed expected utility level to the seller raises only the contract price level at each state without affecting the contract quantity level at each state.

(6) If the buyer's (seller's) absolute risk aversion becomes larger (smaller) the contract

[6] It is shown by the following two equations,
$\partial x/\partial\gamma = A_U F(p-\mu)/(A_G + A_U)$, $\partial y/\partial\gamma = A_G F(p-\mu)/(A_G + A_U)$.

price level goes down at each state where the market price of the buyer's output is relatively low, on the contrary the level goes up where the market price is relatively high. But the changes do not affect the contract quantity level at any state.

(7) The less risky the situations are, the less both contract price and quantity variations become.

These conclusions are concerning the optimal contract that the buyer sets the contract price and quantity schedules so as to maximize his expected utility on condition that he guarantees the seller a given expected utility level. Another contract form is through bargaining in which both parties use their threat strategies in order to decide the outcomes.

(8) The contract price level is higher at each state in the bargaining contract than in the optimal contract. But the contract quantity of both forms is identical with each other at each state.

It seems to be common in a contract between the firms to fix the contract price and quantity independent of a state. First, in practice it may be nearly impossible to calculate precisely the contract price at each state. Secondly, it may be ordinal to make a contract in a relatively less risky condition, as a result it comes to a fixed price and quantity contract. Thirdly, if the buyer is not a competitive firm, but it has more or less market powers, these may enable it to make less the variations of contract prices and quantities.

Finally, the adjustment costs are concerned with it. Both parties have to prepare their productive facilities in advance whatever quantity may be produced in case that the contract is arranged at each state. But this contract type may be very inefficent where it requires much adjustment costs to change the productionn scales.

For these reasons, many contracts between the firms may be arranged in the form of a fixed price and quantity. But on the other hand, there are some examples in which the price and quantity at each state are taken into consideration though they are founding on the fixed ones. For example, in some international iron ore trade contracts, though their foundamental forms are fixed ones, it is allowed to arrange the articles in response to the changes of circumstances because the contract periods extend over ten or fifteen years.[7] they may be regarded as a kind of contingent contracts. Both parties have to take contingencies into consideration when uncertainty increases as the contract periods extend over long years.

As the conclusions (1)~(8) are derived from the restrictive asumptions, it is needless to say that these are not immediately applied to the real contract situations. But in such case of the above instance, the conclusions concerning a contingent price-quantity contract in this paper will offer some effective analytical view points.

[7] I. Yamazawa (1981, p. 176).

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