THE NOMINAL RATE OF INTEREST AND INFLATIONARY EXPECTATIONS

By TOSHIYA HANAWA* and YASUO KOFUJI**

1. Introduction

One of the major debates in monetary theory in recent years has been concerned with the analysis of the Fisherian proposition about the Gibson paradox. According to the Fisherian proposition, the inflationary expectations effect raises the nominal rate of interest by exactly the expected rate of price change, the real rate of interest remaining unchanged. Using their respective models, D. Karnovsky and W. Yohe (1969), W. Gibson (1972), T.J. Sargent (1973), K. Lahiri (1976), V. Tanzi (1980) and others have examined the effects of inflationary expectations on the nominal rate of interest.

The first purpose of this paper is to develop a model of the formation of the nominal rate of interest which can account for the Fisherian proposition about the Gibson paradox. The second purpose is to empirically test the extent to which the nominal rate of interest incorporates inflationary expectations. The outline of the study is as follows: In section 2, the basic model is presented and the dynamics of our system in terms of the nominal rate of interest and inflationary expectations are derived. We place more emphasis on analyzing the differential equations describing the dynamics of the system in section 3. By examining these equations and tracing the time paths of the system, an attempt is made to explain the Fisherian proposition. Our empirical results from testing the proposition in Japan for the period 1965–1979 are presented in section 4, followed by the conclusion.

2. The Basic Model

Our model can be described by the following set of equations

\[ \log Y = a \left( p - p^* \right) \quad a > 0 \quad (2-1) \]
\[ p = p^* + \theta \left( \log I - \log S \right) \quad \theta > 0 \quad (2-2) \]
\[ \log I = -b \left( r - p^* \right) \quad b > 0 \quad (2-3) \]
\[ \log S = s \log Y \quad s > 0 \quad (2-4) \]
\[ \log M/P = -c \left( r - p^* \right) + d \log Y \quad c > 0, \quad d > 0 \quad (2-5) \]
\[ \dot{p}^* = \gamma \left( p - p^* \right) \quad \gamma > 0 \quad (2-6) \]
\[ p = \dot{P}/P \quad (2-7) \]

* Professor (Kyojū) of Monetary Theory.
** Ph. D. candidate (Daigakuin-gakusei), Department of Commerce, the Graduate School of Hitotsubashi University.
where \( Y \) = real output (income)
\( p \) = actual rate of price change
\( p^* \) = expected rate of price change
\( I \) = investment
\( S \) = savings
\( s \) = constant savings rate
\( M \) = nominal stock of money
\( r \) = nominal rate of interest
\( P \) = price level.

Equation (2-1) is an aggregate supply schedule relating real output directly to the gap between the actual rate of price change and the expected rate of price change. In this formulation, only the unexpected rate of price change, that is, \( p - p^* \), affects real output. An increase in the expected rate of price change by itself leaves real output unaffected. This is essentially the kind of formulation that Lucas (1973) has used to explain the Phillips curve.

Equation (2-2) states that the rate of price change is determined on the basis of both the state of excess demand (\( \log I - \log S \)) and the expected rate of price change \( p^* \). Investment and savings are specified by equation (2-3) and (2-4), respectively.

Equation (2-5) describes the condition for portfolio balance, where the demand for real money balance depends upon the real rate of interest \( (r - p^*) \) and real income (\( \log Y \)).

In a dynamic analysis it is necessary to endogenize the expected rate of price change. We assume that the expected rate of price change is revised according to the process of adaptive expectations. Equation (2-6) states that the evolution of inflationary expectations is generated by the process of adaptive expectations.

Equation (2-7) defines the rate of price change.

Thus, these equations define seven relationships involving the seven endogenous variables \( Y, P, p, p^*, I, S, r \), the exogenous variables \( M, s \), and the constant parameters such as \( \gamma, \theta, a, b, c, d \). However, this model which we have presented is a rather complex one. In order to comprehend the Fisherian proposition more readily, the system must be reduced to the two equations which describe the interactions between the nominal rate of interest and inflationary expectations: the equation for the nominal rate of interest and the equation for the expected rate of price change. Let us derive these two equations from equations (2-1)–(2-7).

First, substituting equation (2-3) and (2-4) into equation (2-2) and using equation (2-1), we can readily obtain
\[
 p = p^* - \frac{\theta b}{1 + \theta a s} (r - p^*) \tag{2-8}
\]
or
\[
 p - p^* = - \frac{\theta b}{1 + \theta a s} (r - p^*). \tag{2-8}'
\]
Differentiating equation (2-8) with respect to time and substituting equation (2-6) yields
\[
 \dot{p} = - \frac{\theta b}{1 + \theta a s} \dot{r} + r \left( 1 + \frac{\theta b}{1 + \theta a s} \right) (p - p^*). \tag{2-9}
\]

Similarly, substituting equation (2-1) into equation (2-5), the money market equili-


The equilibrium condition can be written as
\[ \log \frac{M}{P} = -c(r - p^*) + ad(p - p^*). \] (2-10)

Differentiating equation (2-10) with respect to time and substituting equation (2-6) and (2-7) yields
\[ r = \frac{1}{c} p - \frac{1}{c} \mu + \tau \left( 1 - \frac{ad}{c} \right)(p - p^*) + \frac{ad}{c} \dot{p}. \] (2-11)

Therefore, substituting equation (2-8), (2-8)', and (2-9) into equation (2-11), the differential equation for the nominal rate of interest can be expressed in terms of the expected rate of price change, the nominal rate of interest and the constant rate of monetary expansion
\[ r = \frac{(1 + \theta as)(1 + \theta as + \gamma \theta ab + \theta b) + \gamma \theta^2 ab^2 d}{(1 + \theta as)(\theta abd + \theta acs + c)} p^* - \frac{(1 + \theta as)(\theta b + \gamma \theta bc) + \gamma \theta^2 ab^2 d}{(1 + \theta as)(\theta abd + \theta acs + c)} r \]
\[ = \frac{1 + \theta as}{\theta abd + \theta acs + c} \mu \] (2-12)

where, \( \mu = \dot{M}/M \).

Next, by substituting equation (2-8)' into equation (2-6) we can obtain the differential equation for the expected rate of price change in terms of the expected rate of price change and the nominal rate of interest
\[ \dot{p}^* = \frac{\gamma \theta b}{1 + \theta as} p^* - \frac{\gamma \theta b}{1 + \theta as} r. \] (2-13)

Thus, the complete system is described by the differential equation for the nominal rate of interest (2-12) and the differential equation for the expected rate of price change (2-13), which can be written in matrix form as
\[ \begin{bmatrix} \dot{p}^* \\ r \end{bmatrix} = \begin{bmatrix} \alpha & -\alpha \\ \beta + \tau & -\beta \end{bmatrix} \begin{bmatrix} p^* \\ r \end{bmatrix} - \begin{bmatrix} 0 \\ \tau \end{bmatrix} \mu \] (2-14)

where,
\[ \alpha = \frac{\gamma \theta b}{1 + \theta as} > 0, \]
\[ \beta = \frac{(1 + \theta as)(\theta b + \gamma \theta bc) + \gamma \theta^2 ab^2 d}{(1 + \theta as)(\theta abd + \theta acs + c)} > 0, \]
\[ \tau = \frac{1 + \theta as}{\theta abd + \theta acs + c} > 0. \]

As we have described the dynamic system (2-14), we will now consider the necessary and sufficient conditions for stability of the system.

The characteristic equation of (2-14) is
\[ \lambda^2 - (\alpha - \beta)\lambda + \alpha \tau = 0 \]
with characteristic roots \( \lambda_1 \) and \( \lambda_2 \).
The necessary and sufficient conditions for stability of (2-14) are that the sum of the characteristic roots be negative and their product be positive, that is,

\[ \lambda_1 + \lambda_2 = \alpha - \beta \]
\[ = -\frac{\theta b}{\theta abd + c(1 + \theta sa)} < 0 \]
\[ \lambda_1 \lambda_2 = \alpha \tau \]
\[ = \frac{\gamma \theta b}{\theta abd + c(1 + \theta sa)} > 0. \]

Therefore, as these stability conditions are satisfied with values of the parameters, the system will be stable.¹

3. The Dynamics of the Model

The macroeconomic model that we have described in the previous section constitutes a dynamic model in two endogenous variables: the nominal rate of interest and the expected rate of price change. In this section, using this dynamic model (2-14) we analyze the effects of a rise in the rate of monetary expansion on the nominal rate of interest and the expected rate of price change.

It can be seen that steady state equilibrium requires

\[ \hat{\rho}^* = \tau = 0 \]

so that in a steady state the system (2-14) reduces to

\[
\begin{bmatrix}
\alpha \\
\beta + \tau \\
-\alpha \\
-\beta
\end{bmatrix}
\begin{bmatrix}
\hat{\rho}^* \\
\hat{\tau}
\end{bmatrix} =
\begin{bmatrix}
0 \\
\tau
\end{bmatrix} \mu. \tag{3-1}
\]

This steady state equilibrium condition (3-1) can now be used to calculate the steady state changes in the endogenous variables. The effects of a change in the rate of monetary expansion on the nominal rate of interest and the expected rate of price change can then be driven by taking differentials of the system (3-1), namely

\[
\begin{bmatrix}
\alpha \\
\beta + \tau \\
-\alpha \\
-\beta
\end{bmatrix}
\begin{bmatrix}
\frac{d\hat{\rho}^*}{d\mu} \\
\frac{d\hat{\tau}}{d\mu}
\end{bmatrix} =
\begin{bmatrix}
0 \\
\tau
\end{bmatrix} \mu. \tag{3-2}
\]

These are obtained from (3-2) and are simply

\[
\frac{d\hat{\rho}^*}{d\mu} = \frac{d\hat{\tau}}{d\mu} = 1. \tag{3-3}
\]

¹ The condition for a nonoscillatory approach is

\[ (\alpha - \beta)^2 - 4\alpha \tau > 0 \]

that is

\[ \left( -\frac{\theta b}{\theta abd + c(1 + \theta sa)} \right)^2 - \frac{4\gamma \theta b}{\theta abd + c(1 + \theta sa)} > 0. \]

In what follows, we assume that this condition is satisfied.
That is, the steady state equilibrium effect of a rise in the rate of monetary expansion by one unit is to raise both the nominal rate of interest and the expected rate of price change by exactly one unit.

The relation between the nominal rate of interest and the expected rate of price change for a rise in the rate of monetary expansion can be also analyzed graphically using a phase diagram. From (3–1), the slopes of the loci along which $\ddot{p}^* = 0$ and $\ddot{r} = 0$ are located, respectively,

$$\frac{dp^*}{dr} \bigg|_{\ddot{p}^* = 0} = -\frac{\partial \ddot{p}^*/\partial r}{\partial \ddot{p}^*/\partial p^*} = 1$$

$$\frac{dp^*}{dr} \bigg|_{\ddot{r} = 0} = -\frac{\partial \ddot{r}/\partial r}{\partial \ddot{r}/\partial p^*} = \frac{\beta}{\beta + \tau} > 0.$$  

(3-4)

Note that the slope of $\ddot{r} = 0$ locus is positive and smaller than unity. From (3–4) and (3–5) we find that the $\ddot{p}^* = 0$ locus must be steeper than the $\ddot{r} = 0$ locus, as shown in Figure 1. The loci along which $\ddot{p}^* = 0$ and $\ddot{r} = 0$ are located in Figure 1 are drawn for a given rate of monetary expansion $\mu_0$. The directions of the arrows are based on the signs of the partial derivatives of $\ddot{p}^*$ and $\ddot{r}$ with respect to $p^*$ and $r$, that is,

$$\frac{\partial \ddot{p}^*}{\partial p^*} = \alpha > 0$$

FIG. 1

The relation between the nominal rate of interest and the expected rate of price change for a given rate of monetary expansion $\mu_0$. 

![Phase Diagram](image-url)
The paths of the expected rate of price change and the nominal rate of interest for a rise in the rate of monetary expansion can now be analyzed graphically using a phase diagram and the Fisherian proposition about the Gibson paradox can be explained. Suppose that $E_0$ is the initial steady state corresponding to $\mu_0$. Let there be a monetary policy which raises once and for all the rate of monetary expansion from $\mu_0$ to $\mu_1$. The rise in the rate of monetary expansion will shift the $r=0$ locus from $r=0 (\mu_0)$ to $r=0 (\mu_1)$ in Figure 2, while no change will occur in the $p^*=0$ locus since it does not contain $\mu$ as an argument. The path to the new equilibrium $E_1$ is indicated by the dotted line. Initially, the expected rate of price change is revised upwards while the nominal rate of interest is revised downwards. This process continues until point $A$ at which $r$ reaches its minimum value. From point $A$, however, both $p^*$ and $r$ continue to increase towards equilibrium $E_1$.

A time profile of the nominal rate of interest can be deduced from the phase diagram Figure 2. The letters in Figure 3 correspond to those in Figure 2. Initially, the interest rate is equal to $\mu_0$ at the equilibrium level $E_0$. The rise in the rate of monetary expansion tends to decline the interest rate from $E_0$ to $A$. From $A$, however, the interest rate continues to increase monotonically towards its steady state level $E_1$.

![Figure 2](image-url)

The effects of changes in the rate of monetary expansion $\mu$ upon the nominal rate of interest $r$ and the expected rate of price change $p^*$.
The path of the nominal rate of interest $r$ following changes in the rate of monetary expansion $\mu$, corresponding to Figure 2.

According to the Fisherian proposition, the expectations effect raises the nominal rate of interest by exactly the expected rate of price change, the real rate of interest remaining unchanged. Our model developed here can explain the Fisherian proposition. For it can be seen from our model that a rise in the rate of monetary expansion from $\mu_0$ to $\mu_1$ tends to raise not only the nominal rate of interest but also the expected rate of price change from $\mu_0$ to $\mu_1$.

4. Empirical Results

We have analyzed the relation between the nominal rate of interest and the expected rate of price change theoretically within the context of our macroeconomic model developed in section 2. There we showed that, according to the Fisherian proposition, the nominal rate of interest fully incorporates inflationary expectations. Thus, the real rate of interest remains unchanged.

Next, let us empirically test the relation between the nominal rate of interest and inflationary expectations.

In dealing with the question of empirically determining the extent to which the nominal
rate of interest incorporates inflationary expectations, the equation of the form

\[ r_t = a + b p^* t \]  (4-1)

has generally been utilized. According to Irving Fisher's famous proposition, the nominal rate of interest fully adjusts to inflationary expectations, in which case the coefficient of the inflationary expectation variable is unity. Accordingly, the estimated coefficient of \( p^* \) can be used as a test for that hypothesis. Thus, Fisher's famous explanation of the Gibson paradox is expressed in terms of equation (4-1). However, before this equation is estimated by ordinary least squares regression, it should be noted that direct observations on inflationary expectations are not generally available. Therefore, we will use a Koyck transformation to eliminate the unobservable variable.

As previously mentioned, the generally accepted device called the adaptive formation of expectations is used for the mechanism of expectations formation. It is expressed as

\[ p^*_t - p^*_{t-1} = \gamma (p_t - p^*_{t-1}) \]

or

\[ p^*_t = \gamma \sum_{i=1}^{\infty} (1-\gamma)^{i-1} p_{t-i}. \]  (4-2)

Substituting (4-2) into (4-1), we obtain

\[ r_t = a + b \gamma \sum_{i=1}^{\infty} (1-\gamma)^{i-1} p_{t-i}. \]  (4-3)

Similarly, \( (1-\gamma)r_{t-1} \) is given by

\[ (1-\gamma)r_{t-1} = a(1-\gamma) + b \gamma \sum_{i=2}^{\infty} (1-\gamma)^{i-1} p_{t-i}. \]  (4-4)

Therefore, subtracting (4-4) from (4-3) yields

\[ r_t - (1-\gamma)r_{t-1} = \gamma a + b \gamma \sum_{i=1}^{\infty} (1-\gamma)^{i-1} p_{t-i} - b \gamma \sum_{i=2}^{\infty} (1-\gamma)^{i-1} p_{t-i} \]

\[ = \gamma a + \gamma bp_{t-1}. \]  (4-5)

Thus, we can measure the effect of inflationary expectations on the nominal rate of interest by estimating equation (4-5). However, before this equation is estimated, it is necessary to determine the value of \( \gamma \). We will show the least squares regression estimates and their t-statistics for \( \gamma = 0.1, 0.2, \ldots, 1.0 \) and use the coefficient of determination adjusted for degrees of freedom and t-statistics as a criterion for determining the value of \( \gamma \).

Table 1 shows the results of estimating equation (4-5) over the period January 1965–December 1979, the period January 1965–December 1969, the period January 1970–December 1974 and the period January 1975–December 1979, using monthly data, for \( \gamma = 0.1, 0.2, \ldots, 1.0 \). We find that, except for the period January 1965–December 1969, the higher the value of \( \gamma \), the better in terms of the coefficient of determination adjusted for degrees of freedom and t-statistics: \( \gamma = 1.0 \) yields the best result. Therefore, from Table 1 the relation between the nominal rate of interest and the expected rate of price change can be obtained. Table 2 summarizes the empirical results for the case of \( \gamma = 1.0 \).
## Table 1: Regression Results of Equation (4.5)

<table>
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<tr>
<td></td>
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<tr>
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<tr>
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<td>$R^2$</td>
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<tr>
<td>$r$</td>
<td>$\gamma a$</td>
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<td>0.2</td>
<td>1.38</td>
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<tr>
<td></td>
<td>(7.45)*</td>
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<td>0.3</td>
<td>2.06</td>
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<td></td>
<td>(10.66)*</td>
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<td></td>
<td>(13.03)*</td>
</tr>
<tr>
<td>0.5</td>
<td>3.42</td>
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<tr>
<td></td>
<td>(14.71)*</td>
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<td>4.10</td>
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<td>(15.87)*</td>
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<td></td>
<td>(17.64)*</td>
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<tr>
<td>1.0</td>
<td>6.82</td>
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<tr>
<td></td>
<td>(17.94)*</td>
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### Notes:
- We used the percentage change of the consumer price index between the current month and the same month in the previous year as the rate of price change ($p_t$) and the average yield of N.T.T. bonds as the nominal rate of interest ($r_t$). The consumer price index and the yields of N.T.T. bonds were obtained from *Economic Statistics Monthly*, published monthly by the Bank of Japan.
- $R^2$ shows the coefficient of determination adjusted for degrees of freedom.
- The numbers inside parentheses under estimated coefficients stand for t-statistics.
- * Denotes significant at the 0.01 level.
- The numerical calculations were performed on a FACOM 230-25 system at the Hitotsubashi University.
TABLE 2. THE RELATION BETWEEN THE NOMINAL RATE OF INTEREST AND INFLATIONARY EXPECTATIONS JANUARY 1965-DECEMBER 1979 BY SUBPERIOD

<table>
<thead>
<tr>
<th>Subperiod</th>
<th>( r_t = a + b p^*_t )</th>
<th>( a )</th>
<th>( b )</th>
<th>( R^2 )</th>
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<td>January 1965 – December 1979</td>
<td></td>
<td>6.82</td>
<td>0.20</td>
<td>0.643</td>
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<tr>
<td>January 1965 – December 1969</td>
<td></td>
<td>7.91</td>
<td>0.06</td>
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<td>January 1970 – December 1974</td>
<td></td>
<td>6.42</td>
<td>0.22</td>
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<tr>
<td>January 1975 – December 1979</td>
<td></td>
<td>6.35</td>
<td>0.23</td>
<td>0.518</td>
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</tbody>
</table>

Notes: We used the percentage change of the consumer price index between the current month and the same month in the previous year as the rate of price change (\( p_t \)) and the average yield of N.T.T. bonds as the nominal rate of interest (\( r_t \)). The consumer price index and the yields of N.T.T. bonds were obtained from Economic Statistics Monthly, published monthly by the Bank of Japan.

\( R^2 \) shows the coefficient of determination adjusted for degrees of freedom.

The numbers inside parentheses under estimated coefficients stand for \( t \)-statistics.

* Denotes significant at the 0.01 level.

The numerical calculations were performed on a FACOM 230-25 system at the Hitotsubashi University.

The equations in Table 2, except for the period January 1965–December 1969, support the relation between the nominal rate of interest and the expected rate of price change: in three periods, the \( t \)-statistics for \( p^* \) are high and significant at the one percent level. However, the coefficients of \( p^* \) for each period are 0.20, 0.06, 0.22 and 0.23, which are considerably less than unity, indicating that the nominal rate of interest only minimally incorporates inflationary expectations. We, therefore, conclude that the Fisherian proposition about the Gibson paradox cannot be supported by our data.

5. Conclusion

In this paper we have developed a model of the formation of the nominal rate of interest which can account for the Fisherian proposition about the Gibson paradox and empirically tested the extent to which the nominal rate of interest incorporates inflationary expectations. By applying the equation of the form \( r_t = a + bp^*_t \) to the Japanese economy for the period 1965–1979, we have found that the degree of the response of the nominal rate of interest to inflationary expectations is considerably less than the coefficient of unity which is the value required for a confirmation of the Fisherian proposition. This result indicates that the Fisherian proposition cannot be supported by our data.

As indicated in section 2, we have assumed that the expectations of price changes are formed by adaptive expectation schemes. Indeed, the representation of expectations based on adaptive expectation schemes is used by many researchers who have explored the relationship between the inflationary expectations variable and other variables. But, many researchers who have used adaptive expectation schemes suffer from the criticism that the
behavior they assume for the process of inflationary expectations is incredibly simple and does not resemble the actual pattern of the process of inflationary expectations during any historical period. This implies that it is probably inadequate to hypothesize that inflationary expectations are formed by adaptive expectation schemes.

Recent researchers have broadly applied John Muth's concept of rational expectations. This is because they explicitly recognize that the rational expectations hypothesis is truer to the actual pattern of the process of inflationary expectations. Therefore, in the future, we should empirically test the extent to which the nominal rate of interest incorporates inflationary expectations under the rational expectations hypothesis.

REFERENCES