ON EXPECTATION IN DECISIONS

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Generally speaking there are two types of decisions, that is, an objective type and a subjective one. The purpose of this paper is to show one approach by which we analyse the subjective decisions.

We have no general methods in the treatment of the subjective decision, but we can distinguish a reasonable subjective decision from an unreasonable subjective one according to the problems which we take up.

In section 1 we shall explain a basic idea. In other words we shall interprete the subjective decision as a mapping from X to $\mathfrak{B}\times\mathfrak{M}$ and call it an EXPECTATION, where X and Y are respectively sets of behaviors of two individuals and $\mathfrak{B}$ is a completely additive class of Y and $\mathfrak{M}$ is the set of all probability measures on the measurable space $(Y, \mathfrak{B})$. And we shall define a REASONABLE EXPECTATION by means of two requirements which will be explained in section 1.

In section 2 we shall show an example which explains a basic idea about the EXPECTATION in section 1. In this example two individuals are supposed to be an enterprise and a consumer and in this way we specialize reasonable subjective decision, which we shall call the REASONABLE EXPECTATION.

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Mrs. Y. Matsue pictures four figures from figure 2. to 5. in this paper using FACOM 230-25 at Hitotsubashi University and Mrs. M. Nagano typewrites for me. I will express my heartfelt thanks to them.

1. *Basic idea.*

Let us consider the following non-zero-sum two-person game. Namely we shall denote two individuals as A and B and use the following notations.

$X=\{x, x_0, x^*, \cdots\}$; a set of behaviors of A,
$Y=\{y, y_0, y^*, \cdots\}$; a set of behaviors of B,
$f : X\times Y\to \boldsymbol{R}$; a pay-off function of A,
$g : X\times Y\to \boldsymbol{R}$; a pay-off function of B,

where $\boldsymbol{R}$ is the set of all real numbers.

Let us introduce the assumptions that A knows X, Y and f but not g, while B knows

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X, Y and g but not f. In the following several points our two-person game is different from the ordinary games in which two individuals A and B simultaneously exhibit their own behaviors; i.e. at first an individual A exhibits his behavior $x_0 \in X$ to an individual B, and after that the individual B exhibits his behavior $y_0 \in Y$ to the individual A in consideration of the behavior x_0 of A. Of course B will decide his behavior $y_0 \in Y$ as is defined

$$\operatorname*{Max}_{y \in Y} g(x_0, y)=g(x_0, y_0).$$

Therefore the value of the pay-off function of A is $f(x_0, y_0)$. Henceforward we shall call the behavior $y_0 \in Y$ of B a response to the behavior $x_0 \in X$ of A.

The problems which should be considered in our two-person game above explained are what kind of methods is natural and reasonable when the individual A decides his behavior $x_0 \in X$. If the individual A knew the pay-off function g of the individual B, then A would know the mapping G which would be defined as follows:

$$G : X \ni x^* \rightarrow y^* \in Y \text{ where } \operatorname*{Max}_{y \in Y} g(x^*, y)=g(x^*, y^*).$$

Here we assume the existence and uniqueness of $y^* \in Y$. Then the natural and reasonable method of decision of A will be given by solving the maximization problem: maximize $f(x, G(x))$ on X. But by our assumptions A does not know the pay-off function g of B. Accordingly let us form a new concept, an EXPECTATION of the individual A whose definition will be given as below.

We shall use the following notations.

$\mathfrak{B}$ will denote a completely additive class of Y.

$\mathfrak{M}$ will denote the set of all probablity measures on the measurable space $(Y, \mathfrak{B})$.

Let us call the mapping from X to $\mathfrak{B}\times\mathfrak{M}$ the EXPECTATION of the individual A, and $\varepsilon_{\mathrm{A}}(X, Y; \mathfrak{B})$ or ε_{A} will denote the set of all EXPECTATIONS, i.e.

$$\varepsilon_{\mathrm{A}}(X, Y; \mathfrak{B})=\{\omega : X \rightarrow \mathfrak{B}\times\mathfrak{M}\}.$$

The meaning of the EXPECTATION is given as follows. Let ω be the EXPECTATION of the individual A and π_1 (respectively π_2) be a projection from $\mathfrak{B}\times\mathfrak{M}$ to $\mathfrak{B}$ (respectively $\mathfrak{M}$). At this time the property of the EXPECTATION of the individual A is characterized by the following two requirements;

$$\begin{tikzcd} & & \mathfrak{B} \\ X \arrow[r, "\omega"] & \mathfrak{B}\times\mathfrak{M} \arrow[ur, "\pi_1"] \arrow[dr, "\pi_2"'] & \\ & & \mathfrak{M} \end{tikzcd}$$

(i) When the individual A decides his behavior $x \in X$, A has the expectation that B must decide his behavior in $(\pi_1 \circ \omega)(x)$.

(ii) When the individual A decides his behavior $x \in X$, A estimates that the probability of the decision of behavior of B in $(\pi_1 \circ \omega)(x)$ will be $(\pi_2 \circ \omega)((\pi_1 \circ \omega)(x))$.

In this frame work we replace the decision of the behavior of the individual A in X by the decision of the EXPECTATION of the individual A in $\varepsilon_{\mathrm{A}}(X, Y; \mathfrak{B})$.

One of the advantages which the decision of the EXPECTATION has in comparison with the decision of the behavior is that though the individual A can not decide his behavior in X under the informations such as X, Y and f, the decision of the EXPECTATION of the individual A can be restricted to some subset of $\varepsilon_{\mathrm{A}}(X, Y; \mathfrak{B})$ in accordance with the property of X and Y. Let us define the subset of $\varepsilon_{\mathrm{A}}(X, Y; \mathfrak{B})$ above mentioned as follows; at first let us assume that X is a metric space, and then let us introduce the following two requirements:

(I) boundedness of ω

$$\{x \in X\,;\ (\pi_2 \circ \omega)((\pi_1 \circ \omega)(x))=1\} \text{ is bounded.}$$

(II) effectiveness of ω

$$\{x \in X\,;\ (\pi_2 \circ \omega)((\pi_1 \circ \omega)(x))>0\} \text{ is non-empty.}$$

The first requirement, boundedness of ω, shows that there exist not so many such decisions x of A that when the individual A decides his behavior x, A has the EXPECTATION that B must decide his behavior in $(\pi_1 \circ \omega)(x)$ with probability 1.

The second requirement, effectiveness of ω, shows that such EXPECTATION ω of A is the UNREASONABLE EXPECTATION that when the individual A decides his behavior $x \in X$, A estimates that the probability of the decision of the behavior of B in $(\pi_1 \circ \omega)(x)$ will be zero.

This EXPECTATION of A to B satisfying the above two requirements is called the REASONABLE EXPECTATION of A to B, and the set of all the REASONABLE EXPECTATIONS of A to B is denoted by $\mathfrak{R}_A(X,\ Y;\ \mathfrak{B})$ or $\mathfrak{R}_A$.

Let us consider $\mathfrak{R}_A(X,\ Y;\ \mathfrak{B})$ in $\mathcal{E}_A(X,\ Y;\ \mathfrak{B})$ using an example in the following section.

2. *An example.*

At first let us define several notations which will be used in this section. The individual A described in section 1 is an enterprise which produces only one produces only one product P.

a will denote the price per unit of the product P.

b will denote the fixed cost in producing P.

A set X of the behaviors of A in this case is $(a,\ +\infty)$, i.e.

$$X=(a,\ +\infty)=\{x \in \boldsymbol{R}\,;\ a<x\}$$

and its element, the behavior of A, which we shall denote by x, is the selling price of P. The individual B described in section 1 is a group of the consumers who contemplate buying P. A set Y of the behaviors of B in this case is $[0,\ +\infty)$; i.e.

$$Y=[0,\ +\infty)=\{y \in \boldsymbol{R}\,;\ 0 \leqq y\}$$

and its element, the behavior of B, which we shall denote by y, is the volume of his demand. We assume that the selling price of P is equal to the buying price.

Let $\mathfrak{B}$ be the completely additive class of Y which is generated by the family of all such intervals as $[y,\ +\infty)$ where $y \in Y$. Then it is plain that

$$[y,\ +\infty) \in \mathfrak{B} \quad \text{for all} \quad y \in Y.$$

In this section we are only in need of the element of $\mathfrak{B}$ whose type is as above.

Let $\mathfrak{M}$ be the set of all probability measures on the measurable space $(Y,\ \mathfrak{B})$.

Now using these X, $\mathfrak{B}$ and $\mathfrak{M}$, let us define the EXPECTATIONS of A. At first let us assume that the enterprise A sets such goal that the proceeds of the sale should be more than g, where g is a positive real constant. Then for a decision $x \in X$ of A we call $(b+g)/(x-a)$ the goal point, because when A decides $x \in X$, the attainment of the object of A is equivalent to the fact that the response of B belongs to $[(b+g)/(x-a),\ +\infty)$.

So let us define the first mapping $\phi : X \rightarrow \mathfrak{B}$ such that

$$X \ni x \longmapsto \phi(x)=[(b+g)/(x-a),\ +\infty) \in \mathfrak{B}. \tag{1}$$

Now let us define the second mapping $\psi: X \to \mathfrak{M}$ such that

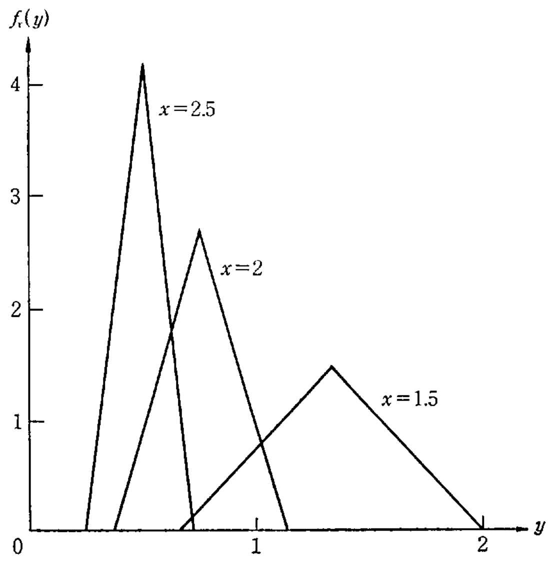
$$\psi(x)(M)=\int_M f_x(y)dy,\ M\in\mathfrak{B}, \tag{2}$$

where for every $x\in X$, f_x is a continuous mapping from Y to $\boldsymbol{R}$ which is defined as follows;

$$f_x:\ Y\ni y\longmapsto f_x(y)=\begin{cases} 0 & ;\ y\in\left[0,\ \dfrac{\beta}{2x^\alpha}\right), \\ \dfrac{4x^{2\alpha}}{\beta^2}\left(y-\dfrac{\beta}{2x^\alpha}\right) & ;\ y\in\left[\dfrac{\beta}{2x^\alpha},\ \dfrac{\beta}{x^\alpha}\right), \\ -\dfrac{4x^{2\alpha}}{\beta^2}\left(y-\dfrac{3\beta}{2x^\alpha}\right) & ;\ y\in\left[\dfrac{\beta}{x^\alpha},\ \dfrac{3\beta}{2x^\alpha}\right), \\ 0 & ;\ y\in\left[\dfrac{3\beta}{2x^\alpha},\ +\infty\right), \end{cases} \tag{3}$$

where α and β are positive real constants. Namely f_x is a probability density function on the measurable space $(Y,\ \mathfrak{B})$. See figure 1.

FIG. 1. THE GRAPH OF f_x, THE CASE OF $\alpha=2,\ \beta=3$



It is plain that

(i) $\psi(x)(M)\geqq 0$ for all $x\in X$ and for all $M\in\mathfrak{B}$,

(ii) $\psi(x)(Y)=1$ for all $x\in X$,

(iii) $M_1,\ M_2,\ M_3,\ \cdots\in\mathfrak{B}$ such that $M_i\cap M_j=\phi$ for all distinct i and j, then $\psi(x)(\bigcup_{n=1}^{\infty}M_n)=\sum_{n=1}^{\infty}\psi(x)(M_n)$ for all $x\in X$.

so $\psi(x)\in\mathfrak{M}$ for all $x\in X$. Using ϕ and ψ which are defined in accordance with (1) and (2), we can define the EXPECTATION $\omega_{\alpha,\beta}: X\to\mathfrak{B}\times\mathfrak{M}$ of A which is defined by two palameters α and β, that is to say

$$\omega_{\alpha,\beta}: X \in x \longmapsto \omega_{\alpha,\beta}(x) = (\phi(x),\ \psi(x)) \in \mathfrak{B} \times \mathfrak{M}. \tag{4}$$

Hereupon let us explain the meaning of the EXPECTATION $\omega_{\alpha,\beta}$ in this example. Since we had explained the meaning of the first component ϕ of $\omega_{\alpha,\beta}$, our notice should be taken of the second component ψ of $\omega_{\alpha,\beta}$. For a behavior $x \in X$ of A which is the selling price of P, the response $y \in Y$ of B which is a volume of consumer's demand should be expressed by a demand function $D(x)$ which is a monotone decreasing with respect to x. Let us consider only the case in which $D(x)$ can be expresssed such as $\beta \cdot x^{-\alpha}$, where α and β are two positive real constants which just characterize the EXPECTATION $\omega_{\alpha,\beta}$ of A. But in order to consider our model introducing the theory of probability, we define f_x by (3) and we define a probability measure by (2). Therefore let us consider $\mathcal{E}_A(X,\ Y;\ \mathfrak{B})$ is $\boldsymbol{R}^2_+$ where $\boldsymbol{R}_+$ is the set of all positive real numbers.

In accordance with the definition of X and Y, the two requirements which are showed in the section 1 must be natural and reasonable, and in other words being based on our model the REASONABLE EXPECTATION $(\alpha,\ \beta)$ is the EXPECTATION which satisfies the following two requirements;

(I)′ $\{x \in X;\ \psi(x)(\phi(x)) = 1\}$ is bounded in X,

(II)′ $\{x \in X;\ \psi(x)(\phi(x)) > 0\}$ is non-empty.

(I)′ and (II)′ correspond to (I) and (II) in the section 1.

Now let us specialize α and β to satisfy the two requirements. It is plain that

$$\frac{3\beta}{2x^\alpha} \leqq \frac{b+g}{x-a} \text{ i.e. } \frac{3\beta}{2(b+g)} \leqq \frac{x^\alpha}{x-a} \Leftrightarrow \psi(x)(\phi(x)) = 0,$$

and

$$\frac{\beta}{x^\alpha} < \frac{b+g}{x-a} < \frac{2\beta}{2x^\alpha} \text{ i.e. } \frac{\beta}{b+g} < \frac{x^\alpha}{x-a} < \frac{3\beta}{2(b+g)} \Leftrightarrow 0 < \psi(x)(\phi(x)) < \frac{1}{2}.$$

In this case

$$\psi(x)(\phi(x)) = \int_{\frac{b+g}{x-a}}^{+\infty} f_x(y)dy = \frac{2(b+g)^2}{\beta^2} \cdot \left(\frac{x^\alpha}{x-a}\right)^2 - \frac{6(b+g)}{\beta} \cdot \frac{x^\alpha}{x-a} + \frac{9}{2},$$

and let us denote this value ξ. It is plain that

$$\frac{\beta}{2x^\alpha} < \frac{b+g}{x-a} \leqq \frac{\beta}{x^\alpha} \text{ i.e. } \frac{\beta}{2(b+g)} < \frac{x^\alpha}{x-a} \leqq \frac{\beta}{b+g} \Leftrightarrow \frac{1}{2} < \psi(x)(\phi(x)) < 1.$$

In this case

$$\psi(x)(\phi(x)) = \int_{\frac{b+g}{x-a}}^{+\infty} f_x(y)dy = -\frac{2(b+g)^2}{\beta^2} \cdot \left(\frac{x^\alpha}{x-a}\right)^2 + \frac{2(b+g)}{\beta} \cdot \frac{x^\alpha}{x-a} + \frac{1}{2},$$

and let us denote this value η. It is plain that

$$\frac{b+g}{x-a} \leqq \frac{\beta}{2x^\alpha} \text{ i.e. } \frac{x^\alpha}{x-a} \leqq \frac{\beta}{2(b+g)} \Leftrightarrow \psi(x)(\phi(x)) = 1.$$

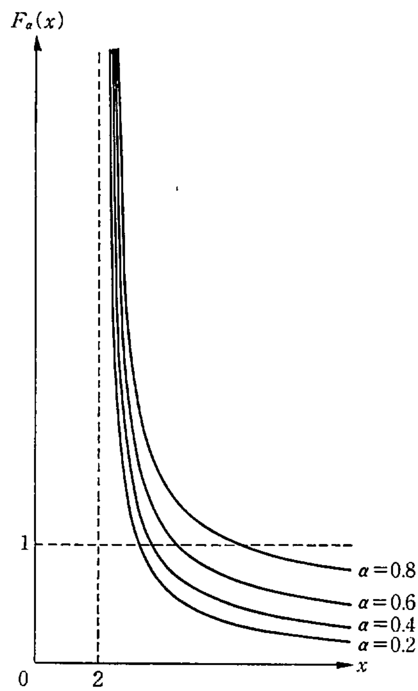
From the above results we will analyse the real valued function $F_\alpha(x) = x^\alpha/(x-a)$ classifying the case into three divisions, i.e. $0 < \alpha < 1$, $\alpha = 1$ and $1 < \alpha$. Since

$F_\alpha'(x) = \{(\alpha-1)x - a\alpha\} x^{\alpha-1}/(x-a)^2$,

$0 < \alpha \leqq 1 \Rightarrow F_\alpha$: monotone decreasing on $(a,\ +\infty)$,

(see figure 2. and 3.)

FIG. 2. $F_\alpha(x)=x^\alpha/(x-a)$ THE CASE OF $a=2$



and $\quad 1<\alpha \Rightarrow \begin{cases} F_\alpha : \text{monotone decreasing on } \left(a, \dfrac{a\alpha}{\alpha-1}\right], \\ F_\alpha : \text{monotone increasing on } \left[\dfrac{a\alpha}{\alpha-1}, +\infty\right), \end{cases}$

and $\quad \underset{x\in(a,\ +\infty)}{\mathrm{Min}} F_\alpha(x)=a^{\alpha-1}\cdot\dfrac{\alpha^\alpha}{(\alpha-1)^{\alpha-1}}.$ (see figure 4.)

Case 1: the case of $0<\alpha<1$.

In this case there exist the real numbers x_1, x_2 and x_3 uniquely such that

$a<x_1<x_2<x_3$,

$F_\alpha(x_1)=\dfrac{3\beta}{2(b+g)}$, $F_\alpha(x_2)=\dfrac{\beta}{b+g}$ and $F_\alpha(x_3)=\dfrac{\beta}{2(b+g)}$.

Then we have

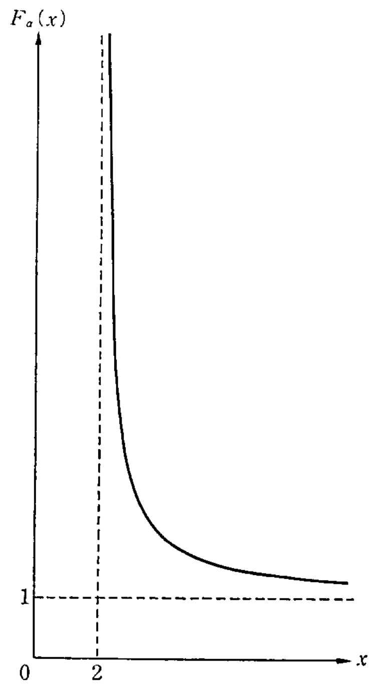
$$\psi(x)(\phi(x))=\begin{cases} 0 & ;\ x\in(a,\ x_1], \\ \xi & ;\ x\in(x_1,\ x_2), \\ \eta & ;\ x\in[x_2,\ x_3), \\ 1 & ;\ x\in[x_3,\ +\infty). \end{cases}$$

The last relation contradicts with the requirement (I)$'$.

Accordingly all the EXPECTATIONS

$\omega_{\alpha,\beta}$ is unreasonable; $0<\alpha<1$, $0<\beta$. (5)

FIG. 3. $F_\alpha(x)=x^\alpha/(x-a)$ THE CASE OF $\alpha=1$, $a=2$



Case 2: the case of $\alpha=1$.
Let us classify this case into four divisions.

2-1: the case of $1<\frac{\beta}{2(b+g)}$ i.e. $2(b+g)<\beta$.

In this case there exist the real numbers x_1, x_2 and x_3 uniquely such that

$$a<x_1<x_2<x_3,$$

$$F_\alpha(x_1)=\frac{3\beta}{2(b+g)},\quad F_\alpha(x_2)=\frac{\beta}{b+g}\quad\text{and}\quad F_\alpha(x_3)=\frac{\beta}{2(b+g)}.$$

Then we have

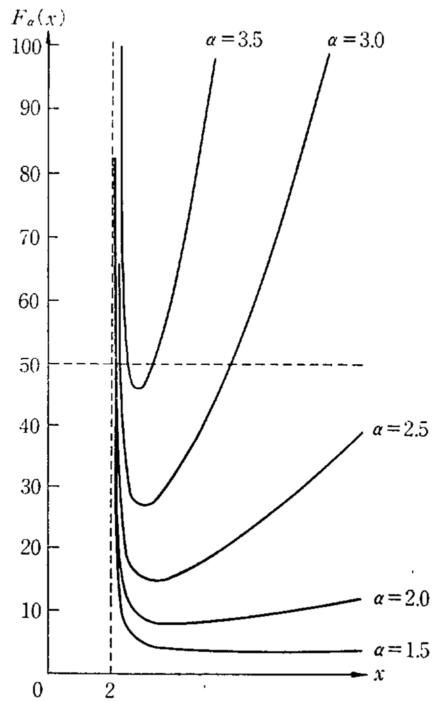
$$\psi(x)(\phi(x))=\begin{cases}0 \ ; \ x\in(a,\ x_1],\\ \xi \ ; \ x\in(x_1,\ x_2),\\ \eta \ ; \ x\in[x_2,\ x_3),\\ 1 \ ; \ x\in[x_3,\ +\infty).\end{cases}$$

The last relation contradicts with the requirement (I)$'$.

2-2: the case of $\frac{\beta}{2(b+g)}\leqq 1<\frac{\beta}{b+g}$ i.e. $b+g<\beta\leqq 2(b+g)$.

In this case there exist the real numbers x_1 and x_2 uniquely such that

FIG. 4. $F_\alpha(x)=x^\alpha/(x-a)$ THE CASE OF $a=2$



$$a<x_1<x_2,$$
$$F_\alpha(x_1)=\frac{3\beta}{2(b+g)} \text{ and } F_\alpha(x_2)=\frac{\beta}{b+g}.$$

Then we have

$$\phi(x)(\phi(x))=\begin{cases}0 \ ; \ x\in(a,\ x_1],\\ \xi \ ; \ x\in(x_1,\ x_2),\\ \eta \ ; \ x\in[x_2,\ +\infty).\end{cases} \tag{6}$$

2-3: the case of $\frac{\beta}{b+g}\leqq 1<\frac{3\beta}{2(b+g)}$ i.e. $\frac{2}{3}(b+g)<\beta\leqq b+g$.

In this case there exists the real number x_1 uniquely such that

$$a<x_1 \text{ and } F_\alpha(x_1)=\frac{3\beta}{2(b+g)}.$$

Then we have

$$\phi(x)(\phi(x))=\begin{cases}0 \ ; \ x\in(a,\ x_1],\\ \xi \ ; \ x\in(x_1,\ +\infty).\end{cases} \tag{7}$$

2-4: the case of $\frac{3\beta}{2(b+g)}\leqq 1$ i.e. $\beta\leqq\frac{2}{3}(b+g)$.

In this case we have

$$\psi(x)(\phi(x))=0\ ;\ x\in(a,\ +\infty).$$

This relation contradicts with the requrement (II)$'$.

Accordingly the EXPECTATION

$$\omega_{\alpha,\beta}\ \text{is}\begin{cases}\text{unreasonable};\ \alpha=1,\ 2(b+g)<\beta,\\ \text{reasonable};\quad \alpha=1,\ \frac{2}{3}(b+g)<\beta\leqq 2(b+g),\\ \text{unreasonable};\ \alpha=1,\ \beta\leqq\frac{2}{3}(b+g).\end{cases}\tag{8}$$

Case 3: the case of $1<\alpha$.

Let us classify this case into four divisions.

3-1: the case of $a^{\alpha-1}\cdot\dfrac{\alpha^\alpha}{(\alpha-1)^{\alpha-1}}<\dfrac{\beta}{2(b+g)}$ i.e. $2(b+g)a^{\alpha-1}\cdot\dfrac{\alpha^\alpha}{(\alpha-1)^{\alpha-1}}<\beta$.

In this case there exist the real numbers x_1, x_2, x_3, x_3', x_2', and x_1' uniquely such that

$a<x_1<x_2<x_3<x_3'<x_2'<x_1'$ and

$F_\alpha(x_1)=F_\alpha(x_1')=\dfrac{3\beta}{2(b+g)}$, $F_\alpha(x_2)=F_\alpha(x_2')=\dfrac{\beta}{b+g}$ and $F_\alpha(x_3)=F_\alpha(x_3')=\dfrac{\beta}{2(b+g)}$.

Then we have

$$\psi(x)(\phi(x))=\begin{cases}0\ ;\ x\in(a,\ x_1]\cup[x_1',\ +\infty),\\ \xi\ ;\ x\in(x_1,\ x_2]\cup[x_2',\ x_1'),\\ \eta\ ;\ x\in(x_2,\ x_3)\cup(x_3',\ x_2'),\\ 1\ ;\ x\in[x_3,\ x_3'].\end{cases}\tag{9}$$

3-2: the case of $\dfrac{\beta}{2(b+g)}\leqq a^{\alpha-1}\cdot\dfrac{\alpha^\alpha}{(\alpha-1)^{\alpha-1}}<\dfrac{\beta}{b+g}$

i.e. $(b+g)a^{\alpha-1}\cdot\dfrac{\alpha^\alpha}{(\alpha-1)^{\alpha-1}}<\beta\leqq 2(b+g)\,a^{\alpha-1}\cdot\dfrac{\alpha^\alpha}{(\alpha-1)^{\alpha-1}}$.

In this case there exist the real numbers x_1, x_2, x_2' and x_1' uniquely such that

$a<x_1<x_2<x_2'<x_1'$,

$F_\alpha(x_1)=F_\alpha(x_1')=\dfrac{3\beta}{2(b+g)}$ and $F_\alpha(x_2)=F_\alpha(x_2')=\dfrac{\beta}{b+g}$.

Then we have

$$\psi(x)(\phi(x))=\begin{cases}0\ ;\ x\in(a,\ x_1]\cup[x_1',\ +\infty),\\ \xi\ ;\ x\in(x_1,\ x_2]\cup[x_2',\ x_1'),\\ \eta\ ;\ x\in(x_2,\ x_2').\end{cases}\tag{10}$$

3-3: the case of $\dfrac{\beta}{b+g}\leqq a^{\alpha-1}\cdot\dfrac{\alpha^\alpha}{(\alpha-1)^{\alpha-1}}<\dfrac{3\beta}{2(b+g)}$

i.e. $\dfrac{2}{3}(b+g)a^{\alpha-1}\dfrac{\alpha^\alpha}{(\alpha-1)^{\alpha-1}}<\beta\leqq(b+g)a^{\alpha-1}\cdot\dfrac{\alpha^\alpha}{(\alpha-1)^{\alpha-1}}$.

In this case there exist the real numbers x_1 and x_1' uniquely such that

$a<x_1<x_1'$,

$F_\alpha(x_1)=F_\alpha(x_1')=\dfrac{3\beta}{2(b+g)}$.

Then we have

$$\psi(x)(\phi(x))=\begin{cases}0\ ;\ x\in(a,\ x_1]\cup[x_1',\ +\infty),\\ \xi\ ;\ x\in(x_1,\ x_1').\end{cases}\tag{11}$$

3-4: the case of $\frac{3\beta}{2(b+g)} \leqq a^{\alpha-1} \cdot \frac{\alpha^{\alpha}}{(\alpha-1)^{\alpha-1}}$ i.e. $\beta \leqq \frac{2}{3}(b+g)^{\alpha-1} \cdot \frac{\alpha^{\alpha}}{(\alpha-1)^{\alpha-1}}$.

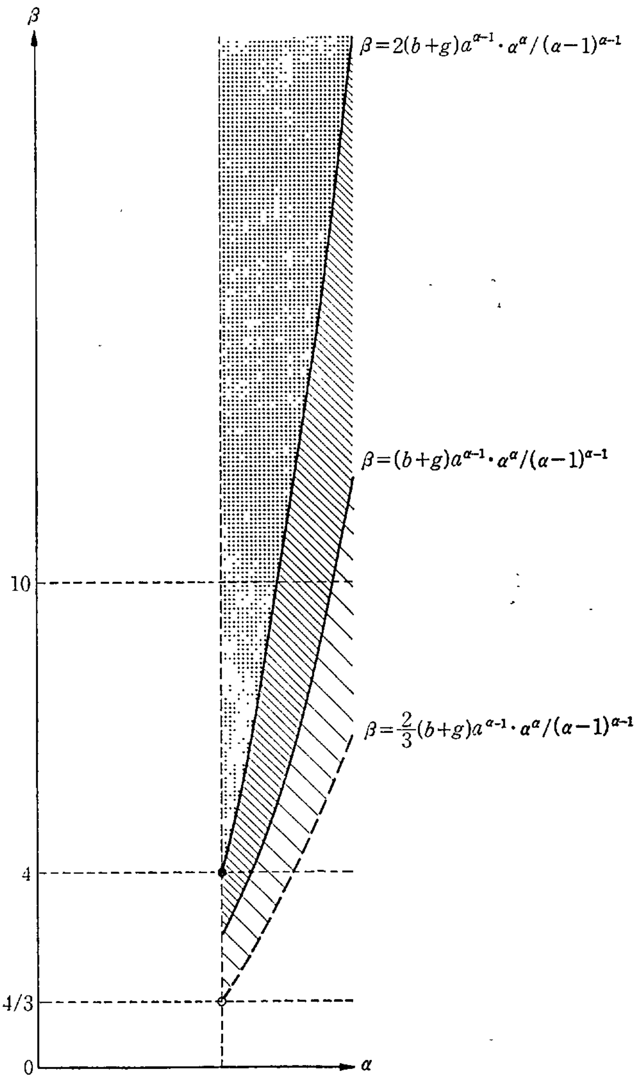
In this case we have

$$\phi(x)(\phi(x))=0\ ;\ x \in (a, +\infty).$$

This relation contradicts with the requirement (II)$'$.

Accordingly the EXPECTATION

FIG. 5. THE CASE OF $a=2$, $b=g=1$



$$\omega_{\alpha,\beta} \text{ is} \begin{cases} \text{reasonable} \quad ; \ 1<\alpha, \ \frac{2}{3}(b+g)a^{\alpha-1}\cdot\frac{\alpha^{\alpha}}{(\alpha-1)^{\alpha-1}}<\beta, \\ \text{unreasonable}; \ 1<\alpha, \ 0<\beta\leqq\frac{2}{3}(b+g)a^{\alpha-1}\cdot\frac{\alpha^{\alpha}}{(\alpha-1)^{\alpha-1}}. \end{cases} \tag{12}$$

In accordance with the results above described, we get the following results; when the individual A, who is an enterprise, has the REASONABLE EXPECTATION $\omega_{\alpha,\beta}$, the probability of the attainment of his goal is given as follows;

(6) , when $\alpha=1$, $b+g<\beta\leqq 2(b+g)$,

(7) , when $\alpha=1$, $\frac{2}{3}(b+g)<\beta\leqq b+g$,

(9) , when $1<\alpha$, $2(b+g)a^{\alpha-1}\cdot\frac{\alpha^{\alpha}}{(\alpha-1)^{\alpha-1}}<\beta$,

(10) , when $1<\alpha$, $(b+g)a^{\alpha-1}\cdot\frac{\alpha^{\alpha}}{(\alpha-1)^{\alpha-1}}<\beta\leqq 2(b+g)a^{\alpha-1}\cdot\frac{\alpha^{\alpha}}{(\alpha-1)^{\alpha-1}}$,

(11) , when $1<\alpha$, $\frac{2}{3}(b+g)a^{\alpha-1}\cdot\frac{\alpha^{\alpha}}{(\alpha-1)^{\alpha-1}}<\beta\leqq(b+g)a^{\alpha-1}\cdot\frac{\alpha^{\alpha}}{(\alpha-1)^{\alpha-1}}$.

We have completed the specialization of two positive real constants α and β which characterize the EXPECTATION in our model. From these results in (5), (8) and (12) we can find the REASONABLE EXPECTATION space $\mathfrak{R}_{A}(x, Y; \mathfrak{B})$ and it is pictured in Figure 5.

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