MULTI-FACTOR FLEXIBLE BUDGETING

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I. *Introduction*

For the purpose of cost control (through evaluating the coat performance of the manager), the standard cost and flexible budgets system of direct costs and overhead cost have been developed in the area of cost accounting over many years. These systems are considered to be integral parts of the broader system, the system of responsibility accounting. Or, in other words, standard costs and flexible budgets are supposed to be outcomes of operational translations of the principle of responsibility accounting. The major question that is asked in this paper is whether these translations and their outcomes are perfect or good enough under all or most circumstances. Our answer on which we shall elaborate in this paper is, "No, in some important circumstances." We further ask here whether it is possible or desirable to have a new operational translation of the principle of responsibility accounting, and if so what that new system should look like as a new operational system of cost control.

Considering that the standard cost system for control of direct costs is a part of the over-all flexible budgeting system, we propose here a multi-factor flexible budgeting system, as opposed to a single-factor flexible budgeting in the traditional standard cost and flexible budgets system. In this way we believe we can serve better or translate better the principle of responsibility accounting for cost control. We have to add that our proposed system does not concern with the product costing aspect which is one of the important aspects of the standard cost system. Our main concern is the cost control through better (or more appropriate or fairer) performance evaluation.

To further pursue this goal, we also attempt to search for new concepts of analysis of variances which will be helpful in evaluating different aspects of abilities that are required of the manager. Thus operational concepts of planning and supervisory variances emerge.

II. *Flexible Budgeting in the Responsibility Accounting Framework*

In the cost accounting literature, flexible budgeting (or budgets as its products) is usually, though not necessarily, associated with the control of overhead cost and defined as

——a set of different budgets which is keyed to different levels of operations, (Horn-

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gren [4], p. 199.)

or

——Flexible budgets reflect the amount of cost that is reasonably necessary to achieve each of several specified volumes of activity. (Shillinglaw [8], p. 373.)

Underlying these definitions of flexible budgets is the implicit assumption that factors other than levels of operations or volumes of activity remain approximately the same or change, if any, only as the level of operations change, or do not affect the cost behavior throughout the period flexible budgets are supposed to serve. Under these circumstances we can concentrate our attention *only* to levels of operations in order to get the right kind of budgets. (See Heckert and Wilson [3], p. 83.) It is this implicit assumption of conventional and widely used flexible budgets that we would like to question in this paper. We hasten to add that our research is not an empirical one to see whether there are any other substantially influencial factors of cost behaviors on top of the levels of operations, but rather a normative one to propose extentions of flexible budgeting with a single dominant factor to multi-factor flexible budgeting in the framework of responsibility accounting, when there are other factors to be considered in arriving at budget standards. Although flexible budgets can be used as a tool for planning, their major uses seem to be for cost control. In particular, we would like to stress and explore to its fullest extent the role of flexible budgets as a tool for performance evaluation of the manager in charge of operations for which flexible budgets are prepared. As such the guiding principle for its preparation is clearly laid down in the philosophy of responsibility accounting. To quote Horngren again,

Each organization unit of *responsibility* is budgeted on its *controllable* costs. Each phase of operations is evaluated, and a prediction is made of how much cost should be incurred *under efficient conditions*. The total of controllable costs is the line manager's budgets. ([4], p. 31. Underlined by the author.)

Key words here are clearly responsibility, controllable costs, and efficient conditions. With these key concepts in mind, let us now examine whether traditional flexible budgets measure up to this guiding principle. When we speak of flexible budgets in this paper, we take them rather broadly to include not only overhead costs but also direct labor and material costs, which are usually considered to be completely variable with levels of operations. One of the reasons for doing so is that in traditional standard cost system direct labor and material costs can be considered completely flexible (linearly changing) parts of over-all flexible cost budgets, the factors of proportionality being the standard costs per unit output and the level of operation being the number of the output produced. Another reason is our suspicion that 'standard unit labor cost or material cost' may not be that standard and constant. Quite often these standard costs are determined as the 'best attainable' costs assuming some normal level of activity under normal circumstances. In those cases, 'best attainable' unit costs for labor and material might change as surrounding conditions change. This implies that we may not take direct labor and material costs just as linear functions of output levels. The factors of proportionality of these linear functions themselves may well change if we apply the idea of 'best attainable' unit cost rather stringently under every circumstance. The following example might help clarify some points.

Suppose the manager has two processes (or activities) to produce product A. Process

1 uses 5.5 units of labor and 7 units of material and Process 2 consumes 4 and 8 of each, respectively. Each process produces a ton of product A. By machine-hour limitations, the manager cannot operate Process 1 more than 500 units. The similar limitation for Process 2 is 300. Suppose that normal conditions mean that the output level is 600 tons of Product A, and prices are $1 for one unit of labor and $2 for one unit of material. Technologically speaking, there is no dominance between Process 1 and Process 2. Cost condition will decide which or what combinations of two is the best under normal conditions. Unit process costs for 1 and 2 are $19.5 and $20 respectively. Therefore it is easy to see that the best combination of two processes under normal conditions is 500 units of Process 1 and 100 units of Process 2, thus making standard cost for labor per ton of output $(5.5\times 500+4\times 100)/600=\$5.25$ and standard cost for material per ton of output 2 $(7\times 500+8\times 100)/600=\$14.33$. Standard labor requirement per ton of output is 5.25 and standard material requirement is 7.17 per ton of output. If the actual production volume is 700 tons of Product A, similar calculations show the best attainable cost for labor per ton of output is $5.07 and for material $14.57.

This example clearly shows that even if the underlying production processes are linear processes, the cost structure might not be linear with respect to output levels when there are alternative production processes and some limitations on the scale of each process.

If we are to consider these cases carefully, we have to incorporate direct labor and material costs into our flexible budgeting scheme as total costs, not as unit costs. In this way we can allow nonlinearity to creap even into direct costs. Let us now see there are situations where we need more factors other than production levels as the determinants of suitable budgets for performance evaluation even under this rather broad concept of traditional flexible budgeting.

Suppose, in the previous example, the material price changes rather frequently from period to period and the manager can know the price for the material he is going to use for a period at the beginning of the period. Suppose the material price happens to be $1.4 at the beginning of a particular period. Then, unit process costs for Process 1 and Process 2 are $15.3 and $15.2. Now it is better to use Process 2 up to the limit, 300, and then use Process 1 for the rest of production. Thus, changes in material prices can cause the change of minimum-cost technology and therefore cause a change in the whole structure of flexible budgets, if we take them as best attainable budgets under changing circumstances. To make these points clear, the following table is set up for best attainable costs under various circumstances and corresponding standard costs calculated using standard labor and material cost per unit of production, $5.25 and $14.33. x denotes the output level, and c_2 denotes material price. Standard labor and material requirements are calculated by using 5.25 and 1.77, standard requirements at normal conditions.
In Table 2, case 3, standard material cost is calculated using $c_2=1.4$, not $c_2=2$, so that we

TABLE 1. LABOR

Case	Labor quantity	Standard quantity	Labor cost	Standard labor cost
1. $x=600$, $c_2=2$	3150	3150	3150	3150
2. $x=700$, $c_2=2$	3550	3675	3550	3675
3. $x=600$, $c_2=1.4$	2850	3150	2850	3150

TABLE 2. MATERIAL

Case	Material quantity	Standard quantity	Material cost	Standard material cost
1. $x=600$, $c_2=2$	4300	4300	8600	8600
2. $x=700$, $c_2=2$	5100	5019	10200	10031
3. $x=600$, $c_2=1.4$	4500	4300	6300	6020

do not have to bother with the material price variance which is assumed to be out of the manager's responsibility.

In case 3, we have the following cost variances reported in the traditional standard cost control system, even when the manager did his best for over-all cost minimization.

Labor cost	Favorable variance	\$300
Material cost	Unfavorable variance	\$280
Total cost	Favorable variance	\$20

These variances are reported only because it is better, from cost-minimization standpoint, to use more material-consuming Process 2 than the standard technology under the given circumstance. If these variance reports are to be used in performance evaluation of the manager by higher management, it is clearly defective. And if we consider the motivational effects of standard budgets on the manager's behavior and efforts and if we consider possible detrimental behavioral influence on the manager's attitude by cost variance investigation when there should be no unfavorable variances to be investigated, deficiencies of the traditional system as a tool of performance evaluation become more apparent.

Thus, under the traditional flexible budgeting system (standard cost control system in this case), budgeted cost may not be the best attainable cost under efficient conditions, because it keeps all factors other than the production level at normal level in determining standards. To follow the guiding principle of responsibility accounting, we have to set up such a flexible budgeting system in which all the factors to which the manager can either exercise some control or can respond if uncontrollable are flexible, including the production level to which the manager has to respond. In the previous example, this means to set up such a budgeting system in which the production level and the material price are flexible factors.

III. *Multi-factor Flexible Budgeting System*

Having seen through examples the need of something more than single-factor flexible budgeting system, let us now turn to rather general and conceptual discussions of how the flexible budgeting system does and should work.

Any flexible budgeting system has to tell what are the appropriate budgets under some specified circumstances. Let us denote the budgets component as u, which is a $l\times 1$ vector, whose typical components whould be direct labor cost, direct material cost, maintenance cost, supervision cost, etc. Obviously, costs are incurred only because we use some kinds of inputs, y, a $m\times 1$ vector. Typically, y consists of labor quantity, raw material quantity, the level of maintenance activity and so on. Once the input mix is determined (or the amounts of input are known) then budget components vector, u, is immediately available as the function of input amounts, y, and their prices, c, $1 \times m$ vector. That is,

(1) $u=f\ (c,\ y)$

, where f is a vector-valued function.

Given the production output quantities the manager has to produce x, a $n \times 1$ vector, the input mix required to produce x depend on many parameters of production process(es) and input prices, c, when the cost minimization is sought. Let us divide the totality of these parameters into two subsets, α, β. Division into two subsets in done considering the manager's responsibility and controllability with respect to each of parameters.

First, α is a vector of those parameters to which the manager cannot exercise any control, like material price in the previous example, or unknown yield on some raw materials. β is a vector of those parameters to which he can maintain some degree of control, like labor productivity which he can influence, for example, through efforts for good human relations. Since the input mix, y, is treated as the basic decision vector here, which essentially covers the area of production planning, kinds of control the manager has over β might be termed supervisory control, most of them of behavioral nature. Given the production goal x, the values of uncontrollable parameters, α, the manager has to decide and implement some production plan, y, in one way or other, assuming that levels of controllable parameters, β, are determined somewhere independently from input mix planning. Thus, input levels, y, can be considered a function (vector-valued) of x, α, β as follows.

(2) $y=\phi\ (x,\ \alpha,\ \beta)$

By substituting y for (1), we get

$$u=f\ (c,\ (x,\ \alpha,\ \beta))$$

Since c is considered to be included in α or / and β,

(3) $u=\phi\ (x,\ \alpha,\ \beta)$

The implicit assumption in the traditional flexible budgeting that we mentioned earlier in the paper is the same thing as a particular assumption about the functional form of (2) and (3) used in establishing the standards. It implicitly says that in arriving at standards of performance evaluation, we can set α and β to their standard values $\bar{\alpha}$, $\bar{\beta}$ and then find out y's and u's for changing x's. By so doing it actually omits α and β from the significant arguments in the function ϕ and therefore φ.

Since, the purpose of flexible budgets is just to establish the standards, not to calculate the actual amount of y's and u's necessary in various situations, this simplication might be good enough in some circumstances. For example, when the production process is rigid enough so that we do not have any choice in production planning other than just to obey the technology of *the* production process in producing a given x, it might be better to account non-standard cost behavior due to deviations of α and β from the standards as variance due to α and variances due to β and then focus our attention on variances due to β, since it is these latter controllable variances that are the focal points of cost control efforts under this circumstance.

In other circumstances, like in the previous example, where there are several alternative production processes and levels of some of α should have substantial effects on the production planning itself (like different choices of processes depending on the material price in the previous example) the approach of the traditional flexible budgeting will leave very important part of the manager's performance evaluation obscured, that is, the evaluation of his planning ability, by not taking into account those responses deemed necessary on the part of the manager to changes of some of α from their standards.

It seems important here to distinguish two different kinds of uncontrollable parameters, α. One group consists of those uncontrollable parameters to which the manager can respond in the production planning phase, called α, here, and the others are those to which the manager cannot respond in the production planning phase but just has to accept their consequences passively in the production implementation phase, α_2.

Differences between α_1, and α_2, might become clear if we consider the availability of information with respect to levels of α_1, and α_2, in a particular period. Should the manager be able to know what value an uncontrollable parameter will take in a particular period before he starts implementing the production plan for the period, thus enabling him to make plans suitable for the known value of the parameter, this parameter is one of α_1. If the value of a parameter is not available beforehand but available only after the fact, this is one of α_2.[1]

With this refinement, (2) and (3) can be rewritten as follows.

(4) $y=\psi\ (x,\ \alpha_1,\ \alpha_2,\ \beta)$

(5) $u=\psi\ (x,\ \alpha_1,\ \alpha_2,\ \beta)$

(4) is a quantity budget and (5) is a cost budget. Any flexible budgeting system can be considered as a special case of (4) and (5), differing in the functional form of ψ and φ and the treatment of x, α_1, α_2, β. For example, in the traditional single-factor flexible budgets, x is somehow reduced to a scalar and the nature of ψ and φ is determined considering 'efficient' cost behavior under normal conditions and effects of different values of α_1, are ignored. Also all the α_1, α_2 and β, are set to their standard values.

What we would like to propose here is a new flexible budgeting scheme, flexible not only over x,[2] but also over α_1.

Making budgets flexible over some parameters implies that the manager is released from the responsibility about the values of those parameters themselves but in return is charged of responding in an appropriate manner to changes of these parameter values. Release of responsibility for parameter values is justified because α_1 is assumed to be uncontrollable, and new charge for appropriate response is exactly in line with the philosophy of responsibility accounting and would be helpful in the evaluation of the manager's planning ability. Coupled with the evaluation of the manager's supervisory abilitythr ough cost variances due to β, new dimensions of performance evaluation would be possible by establishing new variance analysis approach under this multiple-factor flexible budgeting system. These topics will be treated in later sections in detail.

After setting down the general framework of a multi-factor flexible budgeting system, two critical problems remain to be solved.

First of all we have to determine, in each particular situation for which we apply this general framework, what α_1, α_2 and β are.

It is difficult to discuss in general about this other than the definitions of α_1, α_2 and β which are given in the previous pages. Higher management is able to set α_1, α_2 and β as

[1] It is possible that the line of demarkation between α_1, and α_2 may not be clear-cut. In that case it is up to the particular designing principle of the evaluation system to decide between α_1, and α_2.

[2] In case when x is multi-dimensional, flexibility over x itself is already an extension over the single-factor flexible budgeting system.

it sees fit for performance evaluation of lower management. In particular, a parameter would have to satisfy at least the following two conditions to be included in α_1.

1. The manager should be held responsible for responding to changes in the parameter, and can do so in his production planning.
2. The parameter value changes rather frequently *and* its changes are believed to have significant effects on the cost behavior under 'efficient conditions'.

Particularly interesting are the various factors which can be incorporated in a multi-factor flexible budgeting system as x or α_1, and which are often neglected completely in the traditional flexible budgeting system.

For example, when the activity and costs of the department in question has some interdependencies with those of other departments, cost standards for the department should take into these interdependencies into account. As a more concrete example, suppose department A supplies a part of its outputs as an input into department B. If department A has to adjust its production plan rather substantially as department B's request for its intermediate goods changes, then the output level of this intermediate goods should be included as an important component of x (maybe α_1, depending on the model formulation).

Another case in point is when there may exist a substantial "borrowing from the future". Borrowing from the future is said to occur, for example, when the manager defers his maintenance work to the future periods so that he can have less cost in the present period than otherwise, although the manager eventually has to pay for this deferment. Whenever the borrowing from the future can be big enough to warrant careful attention in the performance evaluation in the present period, intertemporal considerations have to be made either through suitably defining and relating some paraments of α, to the present cost or defining some of x as the state of the system which will restrict future alternatives.

The second critical problem of a multi-factor flexible budgeting system is the derivation of the functional form φ, or how to get the value of $u=\varphi(x, \alpha_1, \alpha_2, \beta)$ for each $(x, \alpha_1, \alpha_2, \beta)$ vector. One thing which is clear from the guiding principle of responsibility accounting is that the nature of φ has to incorporate some kind of efficiency criteria. In this paper we consider φ should represent the minimized (or optimum) cost, given x, α_1. Thus we postulate some cost minimizing production planning model behind φ and consider φ to trace optimal solutions for changing x, α_1.

As for the treatment of α_2 and β in setting standards, we discuss it in the next and subsequent sections. It is the purpose of the next section to see how we can determine φ or how we can build a production planning model suitable for a multi-factor flexible budgeting and then find u for each x and α_1 under *linear* production processes.

IV. *Linear Production Process: Activity Analysis Model of Manufacturing Operations*

Linear programming techniques have already been applied in budgeting in Stedry [9], and Ijiri et al. [6] and so on. In this section we would like to propose to use a special type of linear programming models as a planning and budgeting model, especially in manufacturing operations. Such a model is based on the concepts of activity analysis which has been developed mainly in the field of economics to treat the linear production processes[3] and can be used as a way of formulating the production planning model in linear programming context. As we shall see, this activity analysis model seems to be particularly suitable as a basic model in a multi-factor flexible budgeting framework. We have to add here that applications of activity analysis-type models in budgeting should not be limited to manufacturing budgets but can be made in other areas of budgeting, financial budgets etc.

One of the central concepts in activity analysis is 'activity' or 'process' with fixed technological coefficients. Process 1 and Process 2 in the previous example are examples of activities in this sense. Each activity consumes, at unit level of operation, certain amounts of certain inputs, and produces certain amounts of certain outputs. These amounts are assumed not to change as the level of the activity changes. In this framework, the problem of production planning is the problem of how to plan the activities levels so that we can minimize the cost under certain constraints on activity levels. In the previous example, inputs are labor, material, and the output is Product A. Constraints exist for machine-hours. We call a vector of technological coefficients a technology vector. For example,

$$\begin{array}{cc} \text{Process 1} & \text{Process 2} \\ \begin{pmatrix} 1 \\ 5.5 \\ 7 \\ 1 \\ 0 \end{pmatrix} & \begin{pmatrix} 1 \\ 4 \\ 8 \\ 0 \\ 1 \end{pmatrix} \end{array}$$

The first elements are output coefficients. The foruth elements of two vectors are machine-hour requirements for machine 1 which is exclusively used for Process 1, thus zero entry in Process 2's vector. The fifth elements are similar machine-hour requirement for machine 2.

If we denote the activity levels of Process 1 and 2 as z_1, z_2 and the total labor quantity as y_1, the total material quantity as y_2, and the output level as x, the basic production relationship is expressed by:

$$\begin{pmatrix} 1 \\ 5.5 \\ 7 \end{pmatrix} z_1 + \begin{pmatrix} 1 \\ 4 \\ 8 \end{pmatrix} z_2 = \begin{pmatrix} x \\ y_1 \\ y_2 \end{pmatrix}$$

$$z_1 \leq 500$$

$$z_2 \leq 300$$

$$z_1,\ z_2 \geq 0$$

In the previous example, we assumed that costs would be incurred only for labor and material, not for machine-hours. (Machine cost is a fixed or uncontrollable cost in this example.) Therefore, the production planning model with cost minimization objective under normal conditions is

$$\text{min.} \quad y_1 + 2y_2$$

subject to above conditions

[3] See, for example, Koopmans (7).

and $x=600$

In general, an activity analysis model of manufacturing operations with cost minimization objectives and the given output goals is

(6) min. $\theta=cy$

(7) $Az+d=y$

(8) $Pz\geq x$

(9) $Bz\leq b$

(10) $z\geq 0$

where z is a $k\times 1$ vector of activity levels, y is a $m\times 1$ vector of input requirements, x is a $n\times 1$ vector of the given output goals. A is a $m\times k$ matrix of input coefficients whose (i, j) element denotes the amount required of the i-th input to operate the j-th activity at unit level. d is a $m\times 1$ vector of fixed input requirements (if any). P is a $n\times k$ matrix of output coefficients whose (i, j) element denotes the amount of the i-th output produced by unit level operation of the j-th activity. B is a $h\times k$ matrix and b is a $h\times 1$ vector. (9) represents whatever constraints the manager has in his production planning. c is a $1\times m$ vector of input prices.

We call this model (6)-(10), PPM (Production Planning Model). We think PPM formulation is general enough to capture basic characteristics of many manufacturing operations and flexible enough to cope with particular characteristics of each production situation. Furthermore, by shifting our basic concept of production from the input-output relationship[4] to the concept of activity, we think we would have less conceptual difficulties in formulating any manufacturing operation. This is apparent in the case with which we can handle the joint products case which might present some difficulties in model formulation if we stick to the input-output concepts. In PPM it is simply a case of some activities having more than one positive elements in their columns of P matrix.

When some parts of the total production process *consume*, not produce, some of x, which are considered as final outputs, that can be treated as the existence of negative elements in columns of P corresponding to some activities. If some activity *produces*, not consumes, some input to other activities, then it means the column of A corresponding to that activity contains negative element(s). Likewise, the case of intermediate goods is also easy to handle.

We can also handle typical overhead cost situation by devising suitable activities. For example, maintenance activity can be considered as an activity with positive coefficients only for maintenance labor and maintenance material in its column of A matrix, and zero coefficients in its column of P matrix (meaning no direct contribution to outputs), and some non-zero coefficient in its column of B matrix corresponding to the constraint that says the maintenance has to be done at such and such level as the other production activities increase their levels. A simple example may help clarify these points.

To the example that we have been using let us add several sophistications. We add two more processes, 3 and 4. At unit level of operation Process 3 *uses* .5 of Product A, 2 units of labor and 1 unit of raw material to produce 1 unit of Product B. The machine-hour limitation for Process 3 is 200. Process 4 is a maintenance activity which uses, at unit level of operation, 1 unit of maintenance labor and .2 unit of maintenance raw mate-

[4] Not to be confused with the input-output analysis in economics and cost accounting.

rial. It does not contribute to the production of any output directly, but let us suppose that z_4, maintenance activity level, has to be such that

$$.1z_1+.15z_2+.05z_3\leq z_4,$$

that is, the maintenance level has to increase as levels of Processes 1, 2 and 3 increase at least at the rate of .1, .15, .05 respectively.

Now the formulation of this expanded example is as follows, assuming $1.2 for maintenance labor price and $1 for maintenance material price.

$$\text{min.}\quad y_1+2y_2+1.2y_3+y_4$$

$$\begin{pmatrix}5.5\\7\\0\\0\end{pmatrix}z_1+\begin{pmatrix}4\\8\\0\\0\end{pmatrix}z_2+\begin{pmatrix}2\\1\\0\\0\end{pmatrix}z_3+\begin{pmatrix}0\\0\\1\\.2\end{pmatrix}z_4=\begin{pmatrix}y_1\\y_2\\y_3\\y_4\end{pmatrix}$$

$$\begin{array}{l} z_1+z_2-.5z_3\geq x_1 \\ \qquad\qquad z_3\geq x_2 \\ z_1 \qquad\qquad \leq 500 \\ \quad z_2 \qquad\quad \leq 300 \\ \qquad\qquad z_3\leq 200 \\ .1z_1+.15z_2+.05z_3-z_4\leq 0 \\ z_1,\ z_2,\ z_3,\ z_4\geq 0 \end{array}$$

y_3 denotes the amount of maintenance labor, y_4 the amount of maintenance material, x_1 the given output goal for Product A, and x_2 the given output goal for Product B.

As can be seen in this example, input variables, y, in PPM can be of substantial variety. It is not difficult at all to have many different kinds of labor, many different kinds of material, and other kinds of kinputs. This another capability of PPM formulation is additional strength in budgeting where it might be desirable to split master budgets (e.g., master budgets for labor) into smaller segments. For example, after knowing optimal levels of z, y is just linear functions of z ($y=Az+d$) and,

(11) $\sum\limits_{i\varepsilon L} c_i y_i$, $\quad L=$a set of indices for all the labor inputs

is the master labor budgets. Each $c_i y_i$, $i\ \varepsilon\ L$, will give its breakdown.

Although PPM is formulated as a linear programming problem, nonlinearity of input consumption or output production or in the constraints can be handle approximately by using a piece-wise linear approximation of non-linear functions. For details of this formulation, see Ijiri [5]. In corporation of non-linearity into PPM is very important if we are to consider such cost elements as overhead-type costs, like step costs and semi-variable costs.

Trivially, the case when the manager does not have any technological choice in alternative production processes in producing the given output goal x can be considered as a special case of PPM. It is the case when the constraints (8) and (9) are strict enough to allow only one solution or there is only one activity in the model (that is, $k=1$).

From the above arguments about the versatility of PPM as a basic production planing model, we consider we can use PPM in determining multi-factor flexible budgets.

That is, we think PPM has three necessary characteristics as a basic model to be used in a multi-factor flexible budgeting system. First of all, most importantly, PPM can be expected to represent manufacturing operations reasonably well, if not perfect. Secondly, flexible factors or candidates for them in a multi-factor flexible budgeting system are explicitly incorporated into PPM as its parameters. Parameters c, A, P, B, b can be divided into three subsets α_1, α_2, and β, by the judgement of higher management.[5] Flexible factors x also appear as parameters in PPM.[6] Thirdly, the concept of 'efficiency' is made into more operational concept of cost minimization in the framework of linear programming, thus making the determination of budgets for each x and α_1 technically feasible.

This last point touches on the topic of the next section, how to get φ function in PPM framework, or how to get budgets for each x and α_1.

V. *Budgeting: Determination of Evaluation Standards*

In this section we would like to show how we can get budgets which will serve the basic purposes of multi-factor flexible budgets as evaluation standards by using PPM presented in the previous section. Before proceeding further we have to warn the reader that using PPM in flexible budgeting does not necessarily imply that the manager under evaluation *actually is using* the same PPM as being used in budgeting. Of course he might be, but not necessarily. On the contrary, using PPM in budgeting does imply that higher management think that budgets obtained by using PPM will represent better performance evaluation standards than others.

The reasons could be either because higher management think PPM approximates the production process reasonably well and the minimized costs from PPM would be a good approximation for φ under general principles of responsibility accounting, that is, kinds of costs which higher management think 'should be incurred under efficient conditions', or because higher management think the manager is rightly using PPM in his planning and therefore the higher management can measure the manager's supervisoty ability by obtaining such standards as will give only those cost variances due to β, or due to supervisory efficiency or inefficiency. Or it could be a combination of both reasons.

In either case, obtaining budgets through PPM has to be considered an operational way of getting φ function.

In general when we obtain budgets through φ (or in an operational version through PPM) there is a question of how to treat α_2 and β. α_2 are uncontrollable and unrespondable parameters and β are controllable parameters independent of the efficiency of production planning.

Probably the observed costs are a mixture of supervisory inefficiencies (maybe efficiencies sometime), that is, deviations of β from its standard, $\bar{\beta}$, planning inefficiencies, that is, not selecting the optimal technology under the given circumstance, and deviation of α_2

[5] Actually there would be some parameters which do not belong to any of α_1, α_2, and β. They are parameters whose values do not change at all, like coefficients in a definitional relationship or technological coefficients with very high degree of certainty. Since they do not change and hence do not present any problem in budgeting or variance analysis, they are just left out of explicit mention here.

[6] This property of PPM has a very interesting implication, not in flexible budgeting, but in marginal costing or opportunity costing of products. In solving PPM for a particular level of x, say $\bar{x}$, we can get dual evaluators for outputs. These evaluators mean how much the minimized cost would increase if we increase the output goal by one unit (for any of n outputs) from the present level, $\bar{x}$. This matches exactly the definition of managerial cost of outputs.

from its standards, $\bar{\alpha}_2$. Therefore, to get the right kind of evaluation standards that will be capable of evaluating supervisory efficiency, it is clear that we have to set β to $\bar{\beta}$ so that we can detect the deviation of β from $\bar{\beta}$ and its effects on costs by observing the actual costs and comparing them with appropriate standards. As for α_2, we set them to $\bar{\alpha}_2$ in obtaining standards and then try to separate the uncontrollable part of observed cost variances contributed by the deviation of α_2 from $\bar{\alpha}_2$, because α_2, are uncontrollable and *unrespondable.* This implies that the manager may plan his production by using the standard values of α, and then adjust his plan, rather passively because of the definition of α_2 as the realized values of α_2 become known after starting the implementation of the original plan. The problem of variance analysis will be treated in detail in the next section.

Thus the problem of a multi-factor flexible budgeting becomes how to get

$$(12) \qquad u=\varphi\ (x,\ \alpha_1,\ \bar{\alpha}_2,\ \bar{\beta})$$

operationally for different values of x and α_1.

In PPM, this is to solve a parametric programming problem, parametric in x, which is a part of the stipulation vector, and α_1, which may scatter over c, A, P, B, b, with α_2 and β set to α_2 and β.

Let us denote by $A\ (\alpha_1,\ \bar{\alpha}_2,\ \bar{\beta})$ A matrix in which α_2 part and β part are set to $\bar{\alpha}_2$, $\bar{\beta}$ and α_1 are left as parameters. Similar meanings for $c\ (\alpha_1,\ \bar{\alpha}_2,\ \bar{\beta})$, $P\ (\alpha_1,\ \bar{\alpha}_2,\ \bar{\beta})$ and so on. Now the parametric programming problem to be solved is

$$\begin{array}{lll} (13) & \text{min.} & =c\ (\alpha_1,\ \bar{\alpha}_2,\ \bar{\beta})\ y \\ (14) & & A\ (\alpha_1,\ \bar{\alpha}_2,\ \bar{\beta})\ z+d\ (\alpha_1,\ \bar{\alpha}_2,\ \bar{\beta})=y \\ (15) & & P\ (\alpha_1,\ \bar{\alpha}_2,\ \bar{\beta})\ z\geq x \\ (16) & & B\ (\alpha_1,\ \bar{\alpha}_2,\ \bar{\beta})\ z\leq b\ (\alpha_1,\ \bar{\alpha}_2,\ \bar{\beta}) \\ (17) & & z\geq 0. \end{array}$$

Let us call (13)-(17) BDM (Budget Determination Model).

Unfortunately techniques of parametric programming in linear programming are not advanced enough to treat such a complicated parametric program as BDM. Therefore, the best we can hope is to actually solve BDM for each x and α_1. As evaluation standards, we only have to solve BDM for a particular x and α_1, materialized in a particular period after the fact. If we want to obtain a multifactor flexible budget before the fact as a motivational devise, then we would problably have to solve BDM for certain representative values of x and α_1, and do some kind of approximation for other values of x and α_1. In this context, certain interesting behaviors of objective values, θ°, might help. Denoting by $c\ (\alpha_1)$ the α_1 part of c and so on, the following remarks apply.

> θ°, seen as a function of x and $b\ (\alpha_1)$, is an increasing, piece-wise linear, convex function of x and a decreasing, piece-wise linear, convex function of $b\ (\alpha_1)$. θ° is also a concave function of $c\ (\alpha_1)$.[7]

Once we get the optimal solution, z°, for BDM for a particular value of x and α_1, budgeting itself becomes rather trivial. Let us denote this optimal solution by

$$z^\circ=z^\circ\ (x,\ \alpha_1)$$

recognizing the fact that z° is a function of x and α_1. This determines the optimal combination of activities.

Then, the corresponding input mix is,

[7] Proofs are in the appendix.

(18) $y^{\circ}=y^{\circ}(x, \alpha_1)=A(\alpha_1, \bar{\alpha}_2, \bar{\beta})\, z^{\circ}(x, \alpha_1)+d(\alpha_1, \bar{\alpha}_2, \bar{\beta})$

Each budget component is now given by (1),

(19) $u^{\circ}=u^{\circ}(x, \alpha_1)=f(c(\alpha_1, \bar{\alpha}_2, \bar{\beta}), y^{\circ}(x, \alpha_1))$.

This is the standard budgets to be used in performance evaluation in a multi-factor flexible budgeting system.

Although budgeting by PPM would be most powerful when the manager have much discretion in his technology selection under the given circumstance, it is still meaningful even if the production process is rigid enough to give the manager no more than one feasible solution or technology in BDM. In that case, BDM will serve as a kind of budget simulation model in which we can get budgets for all kinds of changes in x and α_1.

Having obtained the standard budgets, our next task is to examine how to compare observed costs with these budgets meaningfully (variance analysis) and how to assess the significance of the magnitude of the observed variances (control limit). These are the topics that we now turn to in the next section.

VI. *New Variance Analysis in a Multi-factor Flexible Budgeting System*

Generally speaking, analysis and investigation of variances would be done in the following manner. First, the manager reports the actual costs of the period with the values of whatever flexible factors are designated by the flexible budgeting system in use. Variances would be computed for each budget component and also, of course, for the total cost using the standards from the flexible budgeting system. If these variances are judged 'within the limits' by higher management, then no more action is taken. If the variances (or parts of them) are judged 'out of the limits', the first stage of investigation begins. It is usually to collect more detailed data to help assess the significance of variances more closely and accurately. At this stage, the disaggregation of variances into more meaningful component variances might be attempted. Depending on the outcomes of this more detailed analysis of variances, further actions (either corrective actions or more investigative actions) may or may not be taken.

What we attempt to do in this section is to make the above general procedures more specific and operational in the framework of the multi-factor flexible budgeting system developed in the earlier sections of this paper.

In our framework, the first step is for the manager to report actual costs (let us denote this by $\tilde{\mu}$) and its total (denoted by $\tilde{\theta}$) together with actual levels of x and α_1 (denoted by $\tilde{x}$ and $\tilde{\alpha}_1$). No report of the actual values of α_2 and β (denoted by $\tilde{\alpha}_2$ and $\tilde{\beta}$) is made at this stage.

Then, variances are computed as follows quite easily.

$$V(\theta)=\tilde{\theta}-\theta^{\circ}(\tilde{x}, \tilde{\alpha}_1)$$
$$V(u)=\tilde{u}-u^{\circ}(\tilde{x}, \tilde{\alpha}_1)$$

,where $V(\theta)$ and $V(u)$ stand for 'variance of total cost' and 'variances of cost components', respectively. $V(\theta)$ is a scalar, and $V(u)$ is a $l\times 1$ vector.

In assessing the significance of $V(\theta)$ and $V(u)$, we have to separate them into controllable variances and uncontrollable variances, because even in this multi-factor flexible budgeting system $u^{\circ}(\tilde{x}, \tilde{\alpha}_1)$ does not account for the uncontrollable cost increase due to the deviations of $\tilde{\alpha}_2$ from $\bar{\alpha}_2$,[8] and, in principle, we want to know controllable variances.

Without knowing $\tilde{\alpha}_2$, the assumption which is made at this stage, we have to determine somehow whether there exist significant controllable variances in $V(\theta)$ and $V(u)$. Here controllable variances mean variances due to planning inefficiencies or / and supervisory inefficiencies (deviations of $\tilde{\beta}$ from $\bar{\beta}$).

Although any way of disaggregating variances into the controllable part and the uncontrollable part would have some degree of arbitrariness, we think the following disaggregation is reasonable.

Define $\mu(x, \alpha_1, \tilde{\alpha}_2)$ and $\tau(x, \alpha_1, \tilde{\alpha}_2)$ as component costs and total cost (counterparts of u and θ respectively) which would be incurred if the initial production plan is made as $z^{\circ}(x, \alpha_1)$ (which is determined at $\alpha_2=\bar{\alpha}_2$, $\beta=\bar{\beta}$), but adjustments to activity levels, z°, are made during the period to cope with $\tilde{\alpha}_2$ as they become known, that is, to satisfy the given output goal, x, and the constraints under $\alpha_2=\tilde{\alpha}_2$, but keeping β at $\bar{\beta}$.

After having obtained the adjusted z, z', costs are calculated as follows.

$$y'=A(\alpha_1, \tilde{\alpha}_2, \bar{\beta})z'+d(\alpha_1, \tilde{\alpha}_2, \bar{\beta})$$
$$u=\mu(x, \alpha_1, \tilde{\alpha}_2)=f(c(\alpha_1, \tilde{\alpha}_2, \bar{\beta}), y')$$
$$\theta=\tau(x, \alpha_1, \tilde{\alpha}_2)=c(\alpha_1, \tilde{\alpha}_2, \bar{\beta})y'.$$

Therefore,

$$\tau(x, \alpha_1, \bar{\alpha}_2)=\theta^{\circ}(x, \alpha_1)$$
$$\mu(x, \alpha_1, \bar{\alpha}_2)=u^{\circ}(x, \alpha_1),$$

because no adjustment are necessary if $\tilde{\alpha}_2=\bar{\alpha}_2$.

Functions τ and μ are clearly dependent on how intraperiod adjustments are made.[9] However these adjustments are made, we can conceptually define the uncontrollable variances, denoted by $UCV(\theta)$ and $UCV(u)$, and controllable variances, denoted by $CV(\theta)$ and $CV(u)$, as follows.

$$(20)\quad UCV(\theta)=\tau(\tilde{x}, \tilde{\alpha}_1, \tilde{\alpha}_2)-\theta^{\circ}(\tilde{x}, \tilde{\alpha}_1)$$
$$(21)\quad CV(\theta)=\tilde{\theta}-\tau(\tilde{x}, \tilde{\alpha}_1, \tilde{\alpha}_2)$$
$$(22)\quad UCV(u)=\mu(\tilde{x}, \tilde{\alpha}_1, \tilde{\alpha}_2)-u^{\circ}(\tilde{x}, \tilde{\alpha}_1)$$
$$(23)\quad CV(u)=\tilde{u}-\mu(\tilde{x}, \tilde{\alpha}_1, \tilde{\alpha}_2).$$

Of course,

$$V(\theta)=UCV(\theta)+CV(\theta)$$
$$V(u)=UCV(u)+CV(u).$$

Notice, again, that by assumption we do not know (or have not done any investigation to know) the values of $\tilde{\alpha}_2$ at this stage. Therefore, we do not know the values of $\tau(x, \tilde{\alpha}_1, \tilde{\alpha}_2)$ and $\mu(x, \tilde{\alpha}_1, \tilde{\alpha}_2)$. Only things we know are $\tilde{\theta}$, $\tilde{u}$, $\theta^{\circ}(\tilde{x}, \tilde{\alpha}_1)$, and $u^{\circ}(\tilde{x}, \tilde{\alpha}_1)$. Suppose,

[8] Although it is possible to get u° and θ° from BDM using $\tilde{\alpha}_2$ instead of $\bar{\alpha}_2$, resulting budget standards would be too much or too little to expect from the manager because, by definition, he cannot respond to any changes in α_2, but getting budgets with $\tilde{\alpha}_2$, implies that he should respond optimaly to realized values of α_2, thus making α_2, the same flexible factors as α_1 in their characteristics.

[9] One example of intraperiod adjustments is the following. Use only those activities which are employed in $z^{\circ}(x, \alpha_1)$, (in the linear programming terminology, keep the same basis), and find an adjusted z to satisfy the output goals and the constraints. If there are none of those z's we have to specify some way of changing to or / and adding another set of activities. If there are more than one such z, then select the z, for example, as close as possible to $z^{\circ}(x, \alpha_1)$, that is, with the minimum distance from z°. This reflects the often-advocated principle of stay-close-to-the-original-plan. Instead of the minimum distance criterion, we might impose the minimum 'adjustment cost' criterion, which may complicate the problem more.

however, we know the probability distribution of $\tilde{\alpha}_2$, denoted by $p\ (\tilde{\alpha}_2)$. Then, τ and μ become random variables whose joint probability distribution function can be obtained using $p\ (\tilde{\alpha}_2)$ and $\tau\ (\tilde{x},\ \tilde{\alpha}_1,\ \tilde{\alpha}_2)$ and $\mu\ (\tilde{x},\ \tilde{\alpha}_1,\ \tilde{\alpha}_2)$.

Let us denote this joint probability function by $q\ (\tau,\ \mu)$.[10] It is possible to get $q\ (\tau,\ \mu)$ operationaly, if tedious, because we know functions τ and μ, although we do not know values of τ and μ for yet unknown values of $\tilde{\alpha}_2$.

From here, it is just one step to get the joint probability distribution of $CV\ (\theta)$ and $CV\ (u)$. Now we can get information on important probabilities in the following manner.

$$Pr(CV(\theta)=0,\ CV(u)=0)=q(\tilde{\theta},\ u),$$

$$Pr(CV(\theta)=0)=\int_\mu q(\tilde{\theta},\ \mu)d\mu,$$

$$Pr(CV(u)=0)=\int_\tau q(\gamma,\ \tilde{u})d\tau.$$

If these values are sufficiently large, say .80 or over, then we probably do not have to investigate any further. If these probabilities are not large enough, say .80 or less, then we would go ahead for further investigation.[11]

We can, of course, base our decision on other kinds of probabilities, like

$$Pr(|CV(\theta)|\leq r)$$

or $Pr\ (|CV\ (\theta)|\leq r_1,$ and$|$some component of $CV(u)|\leq r_2)$, where r, r_1, and r_2 are small positive numbers indicating higher management's judgement. In the latter example, higher management is concerned on the controllable variances of the total cost and some component cost they think important. Thus, having a joint probability distribution for $CV\ (\theta)$ and $CV\ (u)$ is really helpful, and we can do a variety of statistical tests before we decide to go ahead for further investigation.

As the second major stage of cost variance analysis, let us suppose higher management have decided to investigate further and got information on $\tilde{\alpha}_2$, $\tilde{\beta}$. Now we know the value of $\tau\ (\tilde{x},\ \tilde{\alpha}_1,\ \tilde{\alpha}_2)$ and $\mu\ (\tilde{x},\ \tilde{\alpha}_1,\ \tilde{\alpha}_2)$. Therefore values of $CV\ (\theta)$ and $CV\ (u)$ are known, too.

Now having the values of controllable variances, $CV\ (\theta)$ and $CV\ (u)$, it would be useful if we can break $CV\ (\theta)$ and $CV\ (u)$ down so that we can see the causes of variances. In particular, we are interested in disaggregating the controllable variances by two major causes, planning inefficiency and supervisory inefficiency.[12] Each of these two is a very important item in performance evaluation of the manager.

Following the basic idea behind the derivation of τ and μ, let us define $\pi\,(x,\ \alpha_1,\ \tilde{\alpha}_2,\ \tilde{\beta})$ and $\lambda\ (x,\ \alpha_1,\ \tilde{\alpha}_2,\ \tilde{\beta})$ as the total cost and component costs which would be incurred if the initial production plan is made as $z^\circ\ (x,\ \alpha_1)$, but adjustments are made to activity levels, z°, during the period to cope with $\tilde{\alpha}_2$, *and* $\tilde{\beta}$ as they become known, that is, to satisfy the

[10] Since functions τ and μ are hardly analytic functions, we may have to use the Monte Carlo simulation technique to get $q\ (\tau,\ \mu)$.

[11] Here, instead of these classical hypotheses-testing analysis (hypotheses being $CV\ (\theta)=0$ or $CV\ (u)=0$), we might do Bayesian analysis. Or we can obtain the optimal investigation policy, assuming investigation cost and benefit. See Dyckman (2).

[12] Here we are assuming perfect implementation of the plan, perfect in the sense that actual activity levels does not deviate at all from what the manager (or the planner) wants them to be. If there is any degree of imperfection in implementation, the terms planning inefficiency and planning variances should be interpreted as meaning planning-implementation inefficiency and variance in the rest of this paper.

given output goal, x, and the constraints under $\alpha_2=\tilde{\alpha}_2$, and $\beta=\tilde{\beta}$.[13]

Therefore,

$$\pi\ (x,\ \alpha_1,\ \tilde{\alpha}_2,\ \tilde{\beta})=\tau\ (x,\ \alpha_1,\ \tilde{\alpha}_2)$$
$$\lambda\ (x,\ \alpha_1,\ \tilde{\alpha}_2,\ \tilde{\beta})=\mu\ (x,\ \alpha_1,\ \tilde{\alpha}_2).$$

The same remarks about the intraperiod adjustment mechanisms apply to π and λ as to τ and μ.

From the definitions of τ, μ, λ and π functions, it seems reasonable to define supervisory variances (meaning cost variances due to supervisory inefficiency or efficiency) as follows. They are denoted by $SV\ (\theta)$ and $SV\ (u)$, for the total cost and component costs respectively.

(24) $SV(\theta)=\pi\ (\tilde{x},\ \tilde{\alpha}_1,\ \tilde{\alpha}_2,\ \tilde{\beta})-\tau\ (\tilde{x},\ \tilde{\alpha}_1,\ \tilde{\alpha}_2)$

(25) $SV(u)=\lambda\ (\tilde{x},\ \tilde{\alpha}_1,\ \tilde{\alpha}_2,\ \tilde{\beta})-\mu\ (\tilde{x},\ \tilde{\alpha}_1,\ \tilde{\alpha}_2)$

We may likewise define planning variances[14] (meaning cost variances due to planning inefficiency), denoted by $PV\ (\theta)$ and $PV\ (u)$ respectively for the total cost and component costs, as follows:

(26) $PV(\theta)=\tilde{\theta}-\pi\ (\tilde{x},\ \tilde{\alpha}_1,\ \tilde{\alpha}_2,\ \tilde{\beta})$

(27) $PV(u)=\tilde{u}-\lambda\ (\tilde{x},\ \tilde{\alpha}_1,\ \tilde{\alpha}_2,\ \tilde{\beta})$.

These definitions seem reasonable considering the meanings of π and λ.

From (20)-(27) it is clear that

(28) $CV(\theta)=SV(\theta)+PV(\theta)$

(29) $CV(u)=SV(u)+PV(u)$.

Thus we have arrived at the desired disaggregation of controllable varinaces both for the total cost and for component costs. Especially, (29) means quite something. There, the complicated influence of supervisory and planning inefficiency is carefully incorporated into each component variances. Furthermore, such vague terms as planning ineffciency or supervisory inefficiency are now cast in cost terms having very concrete meanings. They say, for example, if the manager supervises his operation (which affects parameter values β) successfully and keeps them at their standards, the cost reduction would be $SV\ (\theta)$ dollars in total. In this sense, planning and supervisory variances presented here may be considered as valuations of the manager's planning and supervisory in-efficiency (in gross terms) by opportunity costs. Being opportunity costs, they change their values as surrounding conditions change, that is, as x and α_1 change.

The importance of planning and supervisory variances in evaluating the manager's over-all performance, not each specific phase of his operation (like evaluation of each β), is self-evident. Having obtained these two variances, it is now up to higher management to decide how to assess their significance and how to act upon their assessment.

[13] In Demski (1), a seemingly similar but rather different concept of the ex post optimal program is introduced. His ex post optimal program is the optimal solution of the linear programming model with all the parameters having their actual values, not standard values. In our framework, it corresponds to z° (x, α_1) when all the parameters are treated as α_1, or a special intraperiod adjustment mechanism which yields ex post optimal solution. See also the footnote in p. 23.

[14] See the second footnote of the previous page.

VII. *Summary and Concluding Remarks*

Summary and concluding remarks are now in order.

In this paper we examined both the basic philosophy of responsibility accounting and the current state of a single-factor flexible budgeting, and decided that in some situations a single-factor flexible budgeting did not serve the purpose that it should, thereby creating some need for a multi-factor flexible budgeting system.

After presenting a special type of linear programming models, called the activity analysis model, as a basic model-type suitable for modeling manifacturing operations, we noted that the activity analysis model (PPM) was particularly suitable as a basic model in a multi-factor budgeting system.

Using PPM in budgeting, we could obtain a feasible system for multi-factor flexible budgeting and showed how to get budgets operationally.

New variance analysis using PPM framework and distinction among decision parameters, uncontrollable and respondable parameters, α_1, uncontrollable and unrespondable parameters, α_2, and controllable parameters, β, resulted in a very interesting way of disaggregating variances. First we indicated an operational way of separating uncontrollable variances from controllable ones. Further disaggregation of controllable variances lead to the notions of planning variances and supervisory variances, which would be important pieces of information in evaluation of the manager's over-all performance.

Thus, we have laid down the conceptual as well operational framework for a multi-factor flexible budgeting system.

And now the following discussions of merits and demerits of the proposed multi-factor flexible budgeting system are in order.

In this system, the manager is held responsible when he does not take the opportunity of profitting from the favorable environment (favorable α_1) and he is excused when the environment turns hostile to him (unfavorable α_1). In this sense he will be given fair performance evaluation in line with the idea of responsibility accounting. Whatever good motivational effects the fair evaluation may have on the attitude of the manager should certainly be credited to the merits of the system. In this regard, careful selection of α_1 by higher management becomes very important. Another obvious merit of this proposed system is that it will save unnecessary cost variance investigation which would be done in the traditional system. That is the investigation of those variances due to α_1 which will be costly as well as unfruitful and even detrimental (probably) to the manager's motivation.

There is also an obvious demerit in this new system. That is, the difficulty of interium reports and evaluation during the period. As can be seen easily from the mechanism of budget determination in this new system, we do not have any evaluation standards until the end of the period when we start calculating standard budgets using BDM.

True and net merits of a multi-factor flexible budgeting system are yet to be seen in practical terms. Their empirical evaluation as well as the elaboration of the proposed system in much greater detail would be topics for further research.

REFERENCES

1. Demski, J., "An Accounting System Structured on a Linear Programming Model," *The Accounting Review*, 42 (4), October 1967, pp. 701-712.
2. Dyckman, T., "The Analysis of Cost Variances," *Journal of Accounting Research*, 7 (2), Fall 1969, pp. 215-244.
3. Heckert, J. and J. Wilson, *Business Budgeting and Control*, 3rd ed., New York: The Ronald Press, 1967.
4. Horngren, C., *Cost Accounting: A Managerial Emphasis*, Englewood Cliffs, N.J.: Prentice-Hall, 1962.
5. Ijiri, Y., *Management Goals and Accounting for Control*, Amsterdam: North-Holland, 1965.
6. Ijiri, Y., F. Levy and R. Lyon, "A Linear Programming Model for Budgeting and Financial Planning," *Journal of Accounting Research*, 1 (2), Autumn 1963, pp. 198-212.
7. Koopmans, T., ed., *Activity Analysis of Production and Allocation*, New York: John Wiley, 1951.
8. Shillinglaw, G., *Cost Accounting: Analysis and Control*, 2nd ed., Homewood, Ill.: Richard Irwin, 1967.
9. Stedry, A., *Budget Control and Cost Behavior*, Englewood Cliffs, N.J.: Prentice-Hall, 1960.