THE GROWTH OF FIRMS IN THE JAPANESE MANUFACTURING INDUSTRIES*

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SUMMARY

The purpose of this article is to make analysis of the size and growth of firms in the manufacturing industries in Japan, from a long-term point of view. At first, we will present in rather discriptive manner changes in the size distribution and business concentration, and then, proceed to analyse the relationships between the size distribution and the growth of firms, by use of the method of Markovian stochastic process. At this stage of investigation, attention will be paid only to the steel industry due to the availability of statistical data. Lastly, paying due regard to the results of the above analysis, an attempt will be mabe for a Monte-Carlo simulation of size distributions, thus showing experimental patterns of growth of firms for the simulated industries.

I. *Introduction*

In recent years, many efforts have been presented on the subject of the size and growth of firms;[1] it is noteworthy that in these efforts, special consideration is given to stochastic approaches to the problem. Viewed from the author's point of view, the approaches employed in this field can be classified roughly into four categories, as below.

The approach of the first category is such that, in light of the fact that the size distribution of firms is highly skewed like the distribution of income, a statistical examination is made on the shapes of the distribution, on the basis of the result of which an attempt is undertaken to interprete a variety of significant phenomena, such as business concentration, from new standpoints. A study, published earlier, by P. E. Hart and S. J. Prais[2] regarding business concentration is a representative example of the works carried out by use of this type of approach. Estimations, which are presented in II in this paper, follow the same methodology as employed in that study. In addition, it can be pointed out that a recent effort by H. A. Simon and C. P. Bonini[3] is a sophisticated extension of the forerunner in respect of statistical procedures.

An approach falling into a second category is the one to predict the size distributions of firms by means of making analysis of the growth of firms in terms of a stochastic process,

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1 See the references listed at the end of this paper.

2 [10].

3 [28].

on the basis of the study of the generating mechanism of the size distribution of firms. Needless to say, this approach will too often be undertaken, together with such a study on the distribution as mentioned above. As efforts directly meant for the prediction, it is possible to point out studies by I. Adelman on the steel industry in the United States,[4] and by L.E. Preston and E. J. Bell on food industries in the same country.[5]

The effort pertaining to third group is to give economic interpretations to the results of calculation, meant for supplementing statistical analysis as by the former approaches with analysis into market structure and technological innovations, etc. Such study is, as yet, not numerous, and it is noteworthy that the effort by E. Mansfield[6] in this field points to considerably interesting conclusions.

A fourth type of approach is an attempt to present some new interpretation concerning the problem of long-term market equilibrium as pursued by Alfred Marshall, by means of studying an equilibrium size distribution of firms in terms of a stochastic process. In this field, an attempt has been undertaken by P. Newman and J. Wolf,[7] which is very ambitious as well as interesting. As D. H. Robertson says,[8] however, it must be left to those qualified to judge whether they have succeeded in furnishing Marshall's theory of long-term value.

In the following, we will consider the dynamics of growth of firms in manufacturing industries, especially the steel industry in Japan, with resort to the first and second types of approach as described above. At the same time, a comparison will be made between the U. S. steel industry and the Japanese counterpart, on the basis of the results arrived at by the author, with a result of clarifying some characteristic features of firm growth in Japan.

II. *The Size Distribution and the Growth of Firms*

In the process of dynamic growth of firms, a wide variety of factors, such as market conditions, technological innovations, managerial abilities, effects of economic policy, etc., are in full play, acting on each other. If such factors influence each other not in an aditive manner but rather in a multiplicative manner, then it is possible to derive that the size distribution of firm is lognormal under certain assumptions.[9] (Orelse, it is possible to obtain a general pattern of distribution defined as Yule distribution, by changing some of the assumptions.[10])

Then, admitting that the size distribution of firms approximates a lognormal distribution, the pattern of that distribution will manifest itself in terms of means and variances of the logarithmical values of firm sizes. By means of pursuit of a statistical measure for them, therefore, we will be able to represent the dynamism of growth of firms in terms of a simple but meaningful index. For instance, the log variances of firm sizes can be made use of as a

[4] [1].
[5] [25].
[6] [21].
[7] P. Newman, "The Erosion of Marshall's Theory of Value", *the Quarterly Journal of Economics*, November 1960, pp. 587–99. Their monograph titled *An Essay on the Theory of Value* should have already published, it has not yet come to the hand of the author.
[8] D. H. Robertson, "Comment", *The Quarterly Journal of Economics*, November 1960, pp. 600–1.
[9] See Aitchison, J. and J. A. C. Brown [2], pp. 20–27.
[10] See Simon H. A. [29], pp. 145–64.

much more effective measure than the ratio of concentration in traditional use.

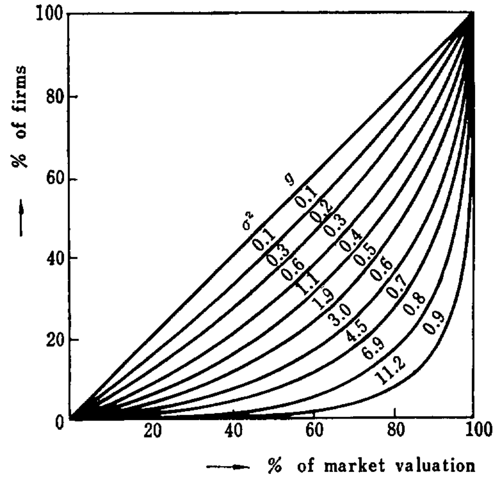
As a measure for business concentration, the proportion of production activity occupied by several firms from the top (called as an absolute concentration ratio) and the proportion of production activity by a certain percentage of firms (called as a relative concentration ratio) have long been utilized. And, analysis has been made by use of the Lorenz curve, and the Gini's coefficient. As compared with these analytical apparatus, the measure represented by log variances can be considered to have a few advantages to follow.[11]

i) The log variance will be one of characteristic values describing its totality, if the distribution of firms is considered as lognormal;
ii) A statistical test has direct applicability by the assumption of lognormality;
iii) The log variances are subject to a change in any part of the size distribution of firms, since they are dependent upon the whole of information on it;
iv) The value of a variance can be decomposed into some constituent elements, i. e. 1. the entry of new firms; 2. the exit of old firms; 3. the growth of survival firms.

[11] The method of expressing the degree of business concentration in terms of a log variance is, however, not generally familiar, and therefore, it may not be intuitionally clear what extent of concentration they will correspond with. Here, a figure prepared by Hart and Prais is presented to show the relation between the variance of lognormal distribution, theoretical Lorenz curve and Gini coefficient. (Fig. A)

A curve in this figure represents Lorenz curve with which Gini coefficient corresponds by every 0.1, and in each case, a log variance with 2 as base is described, which will present a considerably clear numerical image. For example, when the sizes of firms registered in the stock exchange market are measured in terms of assets, a log variance (log 2 as a unit) becomes 5.5, which means about 0.75 in terms of Gini coefficient. Thus, this figure shows that about 12 percent of a totality of firms occupies approximately 75 percent of the total assets.

Fig. A. The Relation between variance of lognormal distribution, theoretical Lorenz curve and Gini's coefficient



Gini coefficient g is obtained by deviding an area under a diagnal in Lorez curve by a total area of a triangle and the diagnal.

$$g=\frac{1}{2}\Delta/\bar{x}$$

$$\Delta=\sum_i\sum_j|x_i-x_j|/N^2$$

where x_i stands for the size of the i the firm, $\bar{x}$ the average of firm sizes, and N the number of firms.

TABLE 1. SUMMARY OF CHANGES IN THE SIZE DISTRIBUTION OF THE STEEL INDUSTRY IN JAPAN
——pre-war and post-war years

unit: $\log_2$

	1918~1929			1953~1963		
	N	$\bar{x}$	σ^2	N	$\bar{x}$	σ^2
1 Total at first year	21	3.17	5.76	43	1.77	3.34
2 Deaths in period	1	4.10	0	3	1.92	0.68
3 Survivors at first year	20	3.12	6.00	40	1.76	3.54
4 Survivors at second year	20	4.41	6.83	40	4.16	4.14
5 Births in period	18	0.44	6.07	15	3.05	0.67
6 Total at second year	38	2.53	10.41	55	3.85	3.43
7 β		0.88			0.98	
8 ρ		0.82			0.90	
9 β^2/ρ^2		1.14			1.17	

Notes: N=Number of Firms; $\bar{x}$=Means of the Firm Sizes; σ^2=Variances of the Firm Sizes. Firm Sizes are measured by $\log_2$ of Million Ton of Steel in pre-war years and of Million Yen of Production in post-war years.

Data: *Seitetsu-Gyo Sanko-Shiryo* (Reference Statistics for the Steel Industry) for pre-war years and *Jyojyo-Gaisha Soran* (Statistical Abstracts of the Quoted Companies in Japan) for post-war years.

Reference Table

Table A. Summary of Changes in the Size Distribution of the selected Japanese Manufacturing Industries in pre-war and post-war years

unit: $\log_2$

	Pre-war Years (1914~1930)						Post-war Years (1953~1963)					
	Manufacturing Industry —major firms			Cotton and Spinning —major firms only			Cotton and Spinning Industry			Chemical Industry		
	N	$\bar{x}$	σ^2	N	$\bar{x}$	σ^2	N	$\bar{x}$	σ^2	N	$\bar{x}$	σ^2
1 Total at first year	53	2.84	1.99	10	5.01	.94	49	.97	3.76	70	.83	1.93
2 Deaths in period	1	3.78	—	—	—	—	9	.14	2.92	8	−.09	1.51
3 Survivors at first year	52	2.83	2.02	10	5.01	.94	40	2.39	3.03	62	1.35	1.71
4 Survivors at second year	52	5.70	1.89	10	5.99	.80	40	3.92	2.73	62	3.87	1.54
5 Births in period	23	5.10	2.31	—	—	—	6	2.05	.58	23	2.18	3.34
6 Total at second year	75	5.52	2.10	10	5.79	.80	46	3.67	2.84	85	5.05	2.29
7 β		.80			.84			.84			.78	
8 ρ		.83			.92			.89			.83	
9 β^2/ρ^2		.94			.86			.90			.90	

Note: see the footnote of Table 1.

Data: Honpo Jigyo-Gaisha Keiei-Kōritsu (Analysis of the Financial Statements of the Japanese Manufacturing Industries) for pre-war years and Jyojyo Gaisha Soran (Statistical Abstracts of the Quoted Companies in Japan) for post-war years.

Now, we will consider the steel industry both in the pre-war and post-war periods, and estimate log variances corresponding with these three factors so that an influence due to each element can be distinctively known.

Reference to Table 1. For the sake of reference, the results of estimation with respect to other major manufacturing industries are demonstrated in Table A in footnotes.

In order to understand the effects brought about by disappeared firms and new firms in these tables, it is sufficient to keep in mind the following equation. Now, assuming, with respect to two groups of firms in the same population, that the respective means are $\bar{x}_1$ and $\bar{x}_2$, and the variances σ_1^2 and σ_2^2, a variance for a sum of the two groups will obtain by

(1) $\sigma^2=\omega_1\sigma_1^2+\omega_2\sigma_2^2+\omega_1\omega_2(\bar{x}_1-\bar{x}_2)^2$.

Here, ω_1 and ω_2 stand for the proportions of the samples of the respective groups to the toaal. For example, a look at a column for the post-war period in Table 1 indicates that a variance for 1953 is 3.34, which changes to 3.54 due to the influence of disappeared firms. ($3.34=\omega_1\ 3.54+\omega_2\ 0.68+\omega_1\omega_2\ (1.76-1.92)^2$; $\omega_1=40/43$, $\omega_2=3/43$). Similarly, the entry of new firms causes a change of the variance from 4.14 in the fourth column to 3.43 in the sixth column. ($3.43=\omega_1\ 4.14+\omega_2\ 0.67+\omega_1\omega_2\ (4.16-3.05)^2$; $\omega_1=40/55$, $\omega_2=15/55$).

On the other hand, for understanding the change of a variance over time for a group of survivors, we can establish a regression equation, such as

(2) $x_{t+1}-\bar{x}_{t+1}=\beta(x_t-\bar{x}_t)+\varepsilon$

(which can be considered as a regression towards an optimal size of firm.) Therefore, it is sufficient to consider a relation:

(3) $\sigma_{t+1}^2/\sigma_t^2=\beta^2/\rho^2$

where ρ implies a correlation coefficient.[12] In the bottom of Table 1, estimations are made for β and ρ and β^2/ρ^2. From this table, we can know that a value of β^2/ρ^2 for the steel industry is larger than unity both for the pre- and post-war periods, and, hence, that there is in existence the mechanism by which concentration is always progressing between the surviving firms.

Now, let us turn to interpretation of the results of the analysis. As exhibited by Table A in footnotes, the pattern of changes in the size distribution of firms varies in considerable degree with each type of manufacturing industry in Japan. At this stage, therefore, it may be impossible to derive a general conclusion as to that. In addition, statistical data available for our purpose are tentative.

For these reasons, we will focuss our main concern mainly on the steel industry, and, at the same time, make a comparison between the Japanese steel industry and the U. S. counterpart, based upon the results thus arrived at. (It is possible for us to make a similar estimation for the U. S. Steel Industry. The result of this calculation is shown in Table 2).

The conclusions derived from Table 1 and Table 2 can be summarized as follows. That is, in the case of Japan, the influence of disappeared firms is negligibly small, and the entry of new firms has different effects on the size distribution from the pre-war period to the post-war

[12] $x_{t+1}-\bar{x}_{t+1}=\beta(x_t-\bar{x}_t)+\varepsilon$

$\sigma_{t+1}^2=\beta^2\sigma_t^2+\sigma_\varepsilon^2$

And then ρ is defined as

$\rho^2=1-\sigma_\varepsilon^2/\sigma_{t+1}^2$

therefore

$\sigma_{t+1}^2/\sigma_t^2=\beta^2/\rho^2$.

TABLE 2. SUMMARY OF CHANGES IN THE SIZE DISTRIBUTION OF THE STEEL INDUSTRY IN U.S.A.

unit: $\log_2$

	1916～1925			1926～1935			1935～1945			1945～1954		
	N	$\bar{x}$	σ^2	N	$\bar{x}$	σ^2	N	$\bar{x}$	σ^2	N	$\bar{x}$	σ^2
1 Total at first year	90	.42	2.04	122	.37	1.95	76	.88	2.11	81	.05	1.48
2 Deaths in period	18	.39	2.35	56	−.20	1.51	2	.50	18.25	12	−.17	.97
3 Survivors at first year	72	.43	1.97	66	.85	1.83	74	.89	1.67	69	.09	1.55
4 Survivors at second year	72	.72	1.98	66	.99	2.13	74	1.07	0.76	69	.30	1.55
5 Births in period	51	−.20	1.49	10	.20	1.36	15	.47	0.91	14	−.31	1.37
6 Total at second year	123	.34	1.98	76	.88	2.11	89	.97	1.40	83	.19	1.57

Note: Computed from the transition matrices cited in E. Mansfield, "Entry, Gibrat's Law, Innovation, and the Growth of Firms," *The American Economic Review*, December 1962, pp. 1023～51., esp., 1045～5.

period. That is to say, in the case of the period before the war, the size of new entrants is extremely small, and therefore, new entries had an effect of increasing the variance of the distribution. In contrast, on the other hand, in the post-war days, the size of new firms entering the industry is relatively large, and therefore, they have brought on a diminution of the variance.

The main factor which has brought about a change in the size distribution of firms in the industry is the progress of business concentration between a group of survivors over a period extending over the pre- and post-war days. This is the greatest characteristic feature of the Japanese industry, as compared with the U. S. industry, in which no such a phenomenon is observed.

With these fact-findings in mind, we will next examine a change in the size distribution of firms in the Japanese steel industry in a more detailed fashion.

III. *Stochastic Process and the Dynamics of Firm Growth*

A change in the size distribution of firms can be understood in a more detailed setting by means of making a transition matrix of sizes of the firms.

The transition matrix is formally expressed in a form such as

$$(4)\qquad (N)=\begin{pmatrix} N_{11}\cdots\cdots N_{1n} \\ \vdots \qquad \vdots \\ N_{n1}\cdots\cdots N_{nn} \end{pmatrix}$$

where element N_{ij} signifies the number of firms that had belonged to size i at time t and transited to size j at time $t+1$. Then, a transition probability or the stochastic matrix is provided by an equation

$$(5)\qquad P_{ij}=N_{ij}/\sum_{j=1}^{n}N_{ij}.$$

And, from the nature of this matrix,

$$(6)\qquad \sum_{j=1}^{n}P_{ij}=1.$$

By the way, such a transition matrix can be made for each year of the period under consideration. $\hat{P}_{ij}$ can then be obtained by summation of the matrix for each year; thus

TABLE 3. STOCHASTIC MATRIX OF THE JAPANESE STEEL INDUSTRY, 1917–1930

	0	1	2	3	4	5	6	7	8	9	10	11	12	Σ
0		.344	.219	.094	.094	.156	.062	.031						1.
1	.059	.706	.118	.059	.029		.029							1.
2	.083	.139	.500	.250	.028									1.
3		.027	.243	.541	.162	.027								1.
4			.027	.108	.649	.189	.027							1.
5	.029			.029	.171	.486	.114	.142	.029					1.
6	.065				.032	.161	.516	.161	.032	.032				1.
7							.149	.660	.191					1.
8							.026	.132	.605	.237				1.
9									.315	.579	.105			1.
10											.833	.167		1.
11												.750	.250	1.
12												.111	.889	1.

Notes: Row "0" stand for "birth" and column "0" for "death." The classification of the size classes is as follow.

unit: ton

1	2	3	4	5	6	7	8	9	10	11	12
~ 500	501 ~ 1,000	1,001 ~ 2,000	2,001 ~ 4,000	4,001 ~ 8,000	8,001 ~ 16,000	16,001 ~ 32,000	32,001 ~ 64,000	64,001 ~ 128,000	128,001 ~ 256,000	256,001 ~ 512,000	512,001 ~

TABLE 4. STOCHASTIC MATRIX OF THE JAPANESE STEEL INDUSTRY, 1953–1963

	0	1	2	3	4	5	9	7	8	9	Σ
0			.059	.059	.352	.235	.176	.118			1.
1		.428	.571								1.
2		.083	.291	.625							1.
3	.015		.121	.439	.394	.015		.015			1.
4	.011			.138	.543	.309					1.
5	.018				.091	.691	.200				1.
6	.020					.184	.612	.184			1.
7							.057	.743	.200		1.
8								.040	.560	.360	1.
9									.100	.900	1.

Notes: Row "0" stand for "birth", and column "0" for "death." The classification of the size classes is as follows.

unit: million yen of sales

1	2	3	4	5	6	7	8	9
~ 300	301 ~ 600	601 ~ 1,200	1,201 ~ 2,400	2,401 ~ 4,800	4,801 ~ 9,600	9,601 ~ 19,200	19,201 ~ 38,400	38,401 ~

(7) $$\hat{P}_{ij}=\frac{\sum_{t=1}^{T} N_{ij}(t)}{\sum_{t=1}^{T}\sum_{j=1}^{n} N_{ij}(t)}$$

which is known to be a maximum likelihood estimates of P_{ij}.[13] The estimates of $\hat{P}_{ij}$ thus derived are exhibited in Tables 3 and 4. Here it is noted that row 0 stands for new firms entering the industry, and column 0 for disappeared ones in the same industry.

Now, let us make analysis into the dynamics of the size distribution of firms by use of the stochastic matrix as obtained above.

Signifying the distribution at time t as dt, from the nature of the stochastic matrix, we obtain

(8) $\{d_t\}(P_{ij})=\{d_{t+1}\}$.

If this stochastic matrix is regular, it is known that there is in existence a stationary distribution such that $\{d_t\}(P_{ij})=\{d_t\}$.

Therefore, such a stationary distribution can be attained only by obtaining an eigen vector in the case where an eigen value is unity.[14] The results derived by iterative calculations for such values are demonstrated in Tables 5 and 6. At the same time, for reference's sake, the values for ten years after both before and after the war are also presented in the tables.

TABLE 5. PREDICTION OF SIZE DISTRIBUTION OF FIRMS BY STOCHASTIC MATRIX IN PRE-WAR YEARS

Size classes	1930	1940	equilibrium
0	.022	.019	.010
1	.112	.074	.037
2	.107	.080	.040
3	.107	.092	.047
4	.115	.110	.059
5	.096	.078	.044
6	.088	.078	.051
7	.129	.124	.086
8	.110	.134	.103
9	.058	.080	.046
10	.019	.042	.043
11	.014	.040	.143
12	.025	.050	.275

Note: For the classification of size classes, see the footnote of Table 3.

TABLE 6. PREDICTION OF SIZE DISTRIBUTION OF FIRMS BY STOCHASTIC MATRIX IN POST-WAR YEARS

Size classes	1963	1973	equilibrium
0	.011	.009	.004
1	.011	.002	.001
2	.044	.011	.004
3	.127	.044	.018
4	.206	.109	.052
5	.260	.230	.098
6	.125	.151	.065
7	.085	.130	.076
8	.053	.100	.154
9	.079	.215	.529

Note: For the classification of size classes, see the footnote of Table 4.

First, by looking at an estimated size distribution of firms on the basis of the stochastic matrix for the pre-war period, it is clearly known that at equilibrium there exists a mechanism which brings about a distribution signifying a so-called industrial dual structure. That is, it is estimated that on the one hand, a group of small-sized firms, ranging from size 1 to size 6,

[13] See Anderson T. W. [3].

[14] For the mathematical theory of stochastic process, see Feller W. [7], or Kemeny, J. G. and J. L. Snell [18].

occupy approximately 28 percent of the total, and, on the other hand, a group of large-sized firms belonging to size 12 have almost the same weight.

A size distribution estimated on the basis of the stochastic matrix for the post-war period is completely skewed towards very large-sized firms: in particular, a tendency is observed for excessive business concentration on the uppermost class of firms with sales in excess of 380 billion yen per year (this group is at present represented by the big six firms in the steel industry.) The results of the estimations as such may well be justified in the light of the economic structure of Japan in the pre-war and the post-war eras. When compared with the results obtained for the U. S. steel industry by application of the same analysis, the conclusions drawn as above are very suggestive.

Here, let us touch in brief upon the case of the U. S. industry. Table 7 represents the outcomes of estimation of a stationary distribution by use of the stochastic matrix based upon the data utilized by E. Mansfield. According to that, it is estimated that business concentration is skewed towards medium-sized firms in the U. S. steel industry. This estimation is in agreement with the conclusion of I. G. Adelman's effort, based upon her stochastic matrix utilizing total assets of the industry for the periods 1929–39 and 1945–56 as statistical data, pointing to the possibility of growth of medium-sized firms.[15]

It follows, therefore, that there has already been in existence an optimal size of the firm in the U. S. steel industry, towards which medium-sized firms are continuously growing, but with no tendency observed for them to grow beyond that size into a group of large-sized firms.

On the other hand, as far as the steel industry in Japan is concerned, there existed a group of small-sized firms in parallel with a group of large-sized ones, respectively with unique growth paths, in the pre-war period, while the tendency towards business concentration on large-sized firms is distinctively recognized, due to their strong orientation towards an increased scale of operation, in the post-war days.

TABLE 7. PREDICTION OF SIZE DISTRIBUTION OF FIRMS OF THE U. S. STEEL INDUSTRY

Size classes	1955	eguilibrium
0	.087	.088
1	.000	.000
2	.032	.032
3	.100	.099
4	.348	.337
5	.281	.282
6	.154	.162

Note: Computed from the transition matrices compiled by E. Mansfield. (*op. cit.*)
The classification of size classes is as follows.

unit: million ton of ingot capacity

0	2	3	4	5	6
~ 4	4 ~ 15.9	16 ~ 63.9	64 ~ 255.9	256 ~ 1023.9	1024 ~

[15] See Adelman I. G. [1].

IV. *Monte-Carlo Simulation of Growth Patterns*

The preceding analysis is an example of the application of a homogeneous Markov chain, where a transition probability is assumed to be constant. In reality, however, there exists no positive reason for the pobability being constant. As a matter of fact, the transition probability will undergo considerable changes in correspondence with the phases of a business cycle. From this arises a strong argument in favor of the necessity of analysis of the movements of the transition probability themselves. The approach proposed for the purpose of this analysis is to seek for the variables which will explain a change of P_{ij}, on the assumption that P_{ij} is deterministic, not a set of constant probabilities. Such an approach is, as might be expected, of considerable interest, but, for the time being, it is deemed to yield an *ad hoc* explanation. It should be noted, at this juncture, that the idea behind an effort to illustrate the size distribution of firms through the stochastic approach is really based upon the consideration that a generalized explanation on the size distribution is extremely difficult to obtain within the frame-work of the traditional theory of the firm; therefore, this theory fails to provide any substantial illumination with regard to such a considerably regular size distribution as observed in reality. For the deterministic approach to be possible, therefore, a new development must be seen of the theory of firm; and such a purpose is beyond the scope of this article.

In the following, then, within the frame-work of the stochastic approach, we will attempt a Monte-Carlo simulation, based upon rather realistic assumptions as compared with the case of constant probability, and, furthermore, we will try to test whether such a simulation experiment can obtain patterns of growth of firms such as to characterize the steel industry in the pre- and post-war periods.

Simulation analysis in this paper follows fundamentally the method attempted by H. A. Simon and Y. Ijiri.[16] However, the present article differs from their work in a) that an initial condition has been adopted for the pre- and post-war eras respectively and a test has been carried out in an attempt to ascertain the effects of the intial conditions, and b) that an analytical effort has been made to ascertain, by changing parameters, meaningful differences in the pattern of growth between the two periods under review.

The fundamental assumptions are two as follows;

1) The entry of new firms into the industry has constant probability α;
2) The current size of a firm and the growth potential of the firm are considered as the size of the firm, and the latter, as being governed by a stochastic process, depends both upon the size to which the firm has grown and the times when its growth has taken place.

In order to introduce the second assumption, we define the growth potential of firm as

$$\omega_j = \sum_{t=1}^{T} x(t)\beta^t, \qquad \beta \leqq 1. \tag{9}$$

where $x(t)$ represents an increment in size of a firm, and β a diminishing ratio of the growth potential.

No mention is made here of the details of simulation procedures. (See Simon H. A. and Y. Ijiri [10] and the computer program by FORTRAN in Appendix of this paper.) Instead, the premises and brief outline of calculations will be expounded.

Considering that a new firm makes entry with a minimum scale permitted in the industry,

[16] [15].

this minimum scale is taken as unit, thus being given a numerical value 1, for the sake of calculational convenience. In addition, an existing firm grows at the rate of this unit, or by 1, per unit of time. In this case, the unit of time is operational time, taken as $T=1000$.

For entry probability α, values 0.30, 0.20, 0.10 or combination of them are assumed. At first, a rectangular random number is drawn between 0 and 1, and if the value of it is smaller than α, the entry of a firm is made. If it is larger α, an existing firm is designated to grow, and the probability of which firm to grow is proportional to the growth potential of each firm.[17] In calculation of a growth potential for each firm, values 0.95, 0.98 or 0.99 are assumed for β.

Simulation, with a hypothetical initial condition that there exist 3 firms of minimum scale 1, based on the above-mentioned procedures, first has confirmed that a size distribution obtained approximates closely the Pareto distribution, as illustrated in Fig. B.[18] (In this case, $\alpha=0.10$, $\beta=0.98$)

Next, providing the following two cases as the initial conditions corresponding with the pre-war and post-war steel industry respectively, we have attempted simulation for each of the cases.

Initial Condition (pre-war) 8 firms
1. 2. 2. 2. 2. 7. 5. 50.
Initial Condition (post-war) 24 firms
1. 1. 1. 1. 1. 1. 1. 1. 1. 1.
2. 2. 2. 5. 5. 10. 10. 20. 20.
30. 50. 80. 90.

The size distributions obtained under these conditions are exhibited in Fig. 1 and 2. From these results it is disclosed that, after departure from the pre-war initial condition, and in case $\alpha=0.30$ in the first, and then, $\alpha=0.10$ after 500 run, that is entry is progressively

[17] That is, drawing again a rectangular random number ν, we let the k-th firm grow such that will have a maximum value of k, enough to satisfy

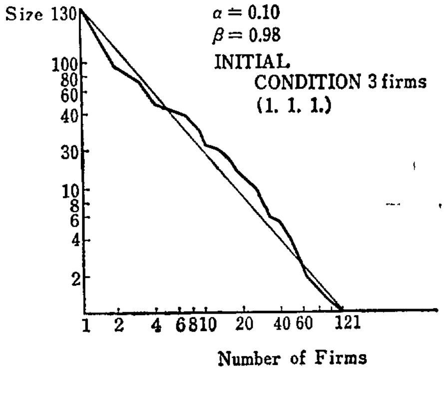
$$\sum_{j=1}^{k} w_j(t-1)/w(t-1) \leqq \nu$$

$$w(t-1)=\sum_{j=1}^{N} w_j(t-1),$$

where N is a total number of firms at time $t-1$.

[18]

Fig B. Size Distribution of Firms in a Simulated Industry



prevented as in the pre-war years, a firm with size as large as 50 initially (Yawata Iron & Steel Works) has continued to maintain a dominant position, being followed by a group of firms less than half in size, and a larger number of firms of increasingly smaller sizes. Therefore, such a size distribution can be concluded to be consistent with the dual size distribution estimated by the method of a stochastic process.

FIG. 1. SIZE DISTRIBUTION OF FIRMS SIGNIFING PRE-WAR PATTERN

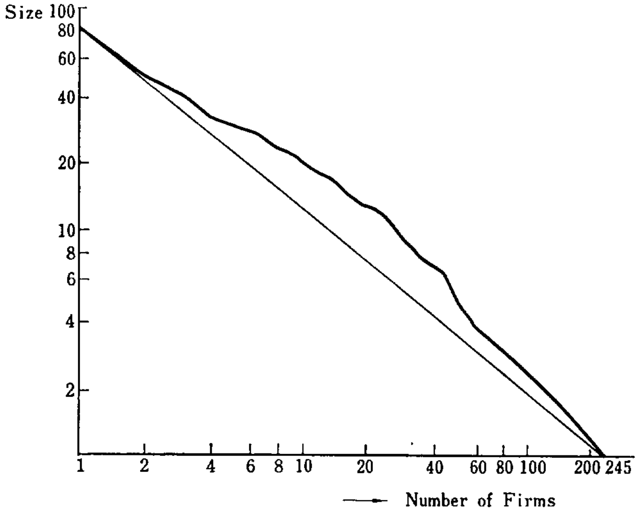
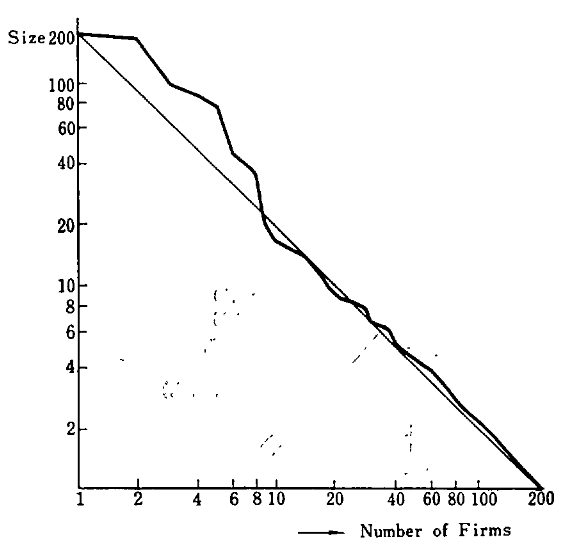


FIG. 2. SIZE DISTRIBUTION OF FIRMS SIGNIFING POST-WAR PATTERN



While, after departure from the post-war initial condition, and if $\beta=0.99$, i.e. the growth potential diminished at a relatively smaller rate than in the pre-war period, a group of firms whose size was large at the initial condition have produced extreme concentration.

FIG. 3. GROWTH PATTERN OF LEADING FIRMS IN A SIMULATED INDUSTRY—PRE-WAR PATTERN

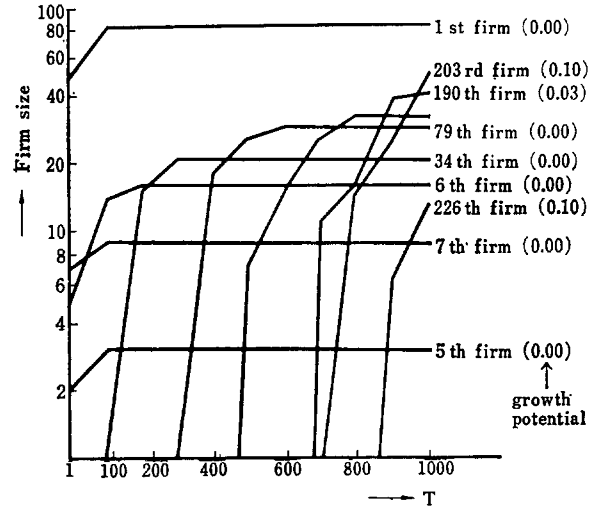
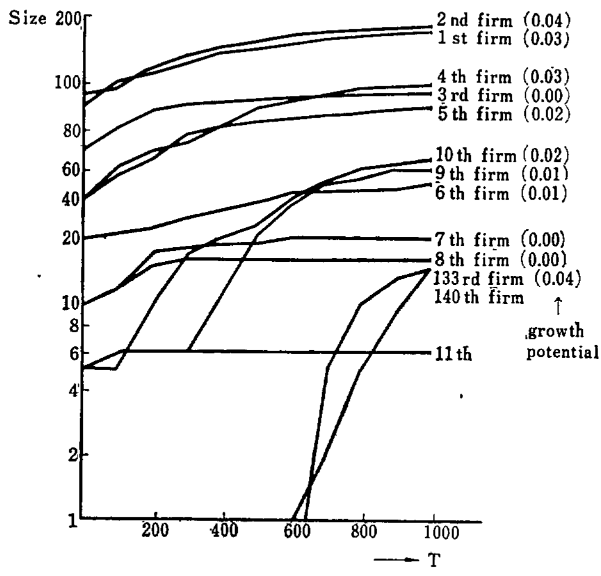


FIG. 4. GROWTH PATTERN OF LEADING FIRMS IN A SIMULATED INDUSTRY—POST-WAR PATTERN



The patterns of growth of firms demonstrated in Fig. 3 and 4 can be considered as symbolic of differences between the two periods concerned in the growth pattern in the steel industry.

In conclusion, the difference from the pre-war to the post-war period is due to the influence of the initial condition for each era, and a factor characterizing the patterns of growth of firms in the pre-war period is changes in entry probability α, while that for the post-war period is the fact that the diminishing ratio of the growth potential of firms is relatively small.

V. *Concluding Remarks*

Throughout this paper, we have emphasized a tentative character of the result of our effort, and now we should like to point out some of the limitations of this study, simultaneously suggesting some direction in which future research efforts may loom promising.

First of all, we must point out the incompleteness of statistical data now available for such purposes. With a view to tracing changes in the size distribution of firms as well as the process of growth of firms in a time series, it is of primary necessity to collect statistical information on every firm appearing in the market, including small-sized ones. So far as this study is concerned, a field of industry which has satisfied such a requirement is only the steel industry. Hence our purposeful limitation of the study to the steel industry. Since the approach to the problem of business concentration based on knowledge of the size distribution of firms is of considerable promise, however, it is necessary to extend the scope of analysis to other industries of major proportions.

In the second place, this study has not made out an effort for a statistical test as regards the pattern of the size distribution of firms in a satisfactory degree. The statistical test will have to be carried out in an over-all manner, after the completion of gathering data.

Thirdly, there is the need for a further examination on the classification of firms in terms of size, on the occasion of formulating a transition matrix. Moreover, there remain a lot of efforts such as to manufacture mobility indexes within a particular industry, on the basis of data supplied by the transition matrix.

In the last place, there arises more experiments on the Monte-Carlo simulation, provided that the ability of calculation permits them.

Despite these limitations, however, it is considered that this effort of analysis has shown a direction of the stochastic approach, with the steel industry in Japan as the objective, and that has thrown light on the problem of business concentration or of industrial organization.

In the last place, it should be mentiond that as pointed out by M. Kalecki, it may run to extremes if all the processes of growth of firms are considered at random. Needless to say, the process of firm growth is governed partially by laws governing economic activity, and in part by random factors, and therefore, a mixture of relevant approaches are deemed necessary. For this purpose, an effort must be undertaken for a reformulation of the traditional static theory of the firm in terms of the theory of growth, thus presenting a challenge of great significance to be responded in the future.

APPENDIX A. SIZE DISTRIBUTION OF FIRMS BY SIZE IN SIMULATED INDUSTRIES

Pre-war Patterns (Initial Firms 8)

α=0.20 ($T\leqq$300) α=0.10 (T>300)		β=0.95	α=0.20 ($T\leqq$500) α=0.10 (T>500)		β=0.95	α=0.30 ($T\leqq$500) α=0.10 (T>500)		β=0.95
Size of Firms	Number of Firms	Growth Potential	Size of Firms	Number of Firms	Growth Potential	Size of Firms	Number of Firms	Growth Potential
178	1	.019	163	1	*	83	1	*
128	1	*	104	1	.378	50	1	.097
103	1	*	83	1	*	41	1	.034
53	1	*	40	1	.001	33	1	*
39	1	.165	38	1	*	29	1	.097
37	1	*	37	1	*	26	1	*
35	2	*	24	1	*	23	2	.001
31	1	*	22	1	*	21	1	*
30	1	*	21	1	*	20	1	*
28	1	.003	18	1	.272	19	1	*
27	1	*	17	1	.035	18	1	.002
25	1	.224	16	2	*	17	1	*
15	1	*	15	2	*	16	1	*
11	1	*	13	3	.052	15	1	*
9	2	.090	12	1	*	14	2	*
8	4	.118	11	2	.002	13	3	.110
7	3	.025	10	5	.030	12	2	*
6	5	.127	9	1	*	11	1	*
5	3	*	8	2	*	10	3	.175
4	9	.059	7	2	*	9	3	.045
3	13	.097	6	6	.011	8	4	*
2	22	.035	5	2	*	7	7	*
1	61	.037	4	9	.116	6	4	.005
			3	11	*	5	4	*
			2	30	.025	4	10	.124
			1	70	.076	3	24	.003
						2	49	.059
						1	113	.250
Σ	137	1.000	Σ	160	1.000	Σ	245	1.000

* less than 0.001

Notation: α=entry probability

β=diminishing ratio of the growth potential

Note: For the unit of the size of firms, a minimum scale permitted in the industry is taken as unit.

Post-war Patterns (Initial Firms 24)

$\alpha=0.20$		$\beta=0.98$	$\alpha=0.20$		$\beta=0.99$	$\alpha=0.30$ ($T\leqq500$) $\alpha=0.20$ ($T>500$)		$\beta=0.98$
Size of Firms	Number of Firms	Growth Potential	Size of Firms	Number of Firms	Growth Potential	Size of Firms	Number of Firms	Growth Potential
154	1	*	186	1	.045	155	1	*
144	1	*	173	1	.032	137	1	*
99	1	*	100	1	.032	64	1	*
61	1	*	90	1	.001	54	1	.129
45	1	.062	78	1	.018	50	1	*
44	1	*	45	1	.025	49	1	.001
40	1	*	40	1	.011	44	1	.143
36	1	.076	35	1	.010	37	1	*
30	1	*	20	1	*	36	1	*
29	1	.140	17	1	.043	32	1	.124
21	1	.029	16	1	*	26	1	.018
20	1	.001	15	2	.052	21	1	*
18	1	.001	14	1	.047	19	1	*
17	2	.051	13	1	.031	18	2	.071
16	3	.003	12	1	.010	16	2	.004
14	1	*	11	1	.016	15	1	.015
13	2	*	10	1	.032	14	1	*
12	1	.042	9	3	.052	12	4	.059
11	2	.022	8	7	.035	11	2	.087
10	2	*	7	1	*	9	4	.023
9	5	.123	6	8	.065	7	3	.008
8	3	.006	5	5	.010	6	4	*
7	5	.001	4	17	.084	5	13	.050
6	6	.053	3	16	.069	4	13	.071
5	7	.029	2	41	.091	3	17	.035
4	11	.062	1	100	.196	2	43	.093
3	16	.109				1	116	.069
2	31	.074						
1	105	.116						
Σ	215	1.000	Σ	216	1.000	Σ	238	1.000

Note: See the footnote of the last Table.

APPENDIX B. THE COMPUTER PPOGRAM OF THE MONTE-CARLO SIMULATION OF SIZE DISTRIBUTIONS

```
C         SIMULATION OF SIZE DISTRIBUTION
C
C         SUBROUTINE RANSUI (UTILITY ROUTINE)
C
          DIMENSION  X (1000),  Y (1000),  D (500),  W (500)
     101  FORMAT (2 I 10)
     102  FORMAT (3 F 10.2,  2 I 10)
     103  FORMAT (7 F 10.0)
     104  FORMAT (I 10)
     201  FORMAT (1 H+,  6 HENTRY=I 5,  3 X,  E 12.5)
     202  FORMAT (1 H ,  I 5,  F 10.0,  3 X,  E 12.5)
     203  FORMAT (1 H ,  I 5,  F 10.0,  3 X,  F 10.5)
     204  FORMAT (18 H 1 SIZE DISTRIBUTION,  2 I 10,  E 12.5)
     205  FORMAT (11 H 1 PARAMETER, 3 F 10.2,  2 I 10)
          READ INPUT TAPE 5, 101,  NITA,  NRUN
          DO 999 NZ=1,  NITA
C
          READ INPUT TAPE 5,  102,  AA,  BC,  AA 2,  II,  NHN
          WRITE OUTPUT TAPE 6,  205,  AA,  BC,  AA 2,  II NHN
          DO 2 I=1, 1000
          X (I)=0.
       2  Y (I)=0.
          READ INPUT TAPE 5,  104,  NHD
          IF (NHD) 4,  4,  3
       3  READ INPUT TAPE 5,  103,  (D(I),  I=1,  II)
       4  DO 10 I=1,  II
          X (I)=D (I)
      10  Y (I)=D (I)
C
          KK=1
          K 1=101
          L=II
          BB=BC
    1000  CALL RANSU 1 (RANDOM)
          IF (RANDOM-AA) 11,  11,  12
      11  L=L+1
          X (L)=1.0
          Y (L)=Y (L)+1.0/BB
          WRITE OUTPUT TAPE 6,  201,  L,  (Y (L))
          GO TO 88
      12  SUM=0.
          DO 13 I=1,  L
      13  SUM=SUM+Y (I)
          CALL RANSU 1 (RANDOM)
          LL=1
          SS=0.
      19  SS=SS+Y (LL)/SUM
          IF (SS-RANDOM) 15,  14,  14
      15  LL=LL+1
          IF (LL-L) 19,  14,  14
      14  X (LL)=X (LL)+1.0
```

```
          Y (LL)=Y (LL)+1.0/BB
          WRITE OUTPUT TAPE 6, 202, LL, (X (LL), Y (LL))
C
      88  KK=KK+1
          BB=BB*BC
          IF (KK-NHN) 21, 21, 23
      23  AA=AA 2
      21  IF (KK-K 1) 22, 71, 22
      71  SY=0.
          DO 90 I=1, L
      90  SY=SY+Y (I)
          DO 91 I=1, L
      91  W (I)=Y (I)/SY
          WRITE OUTPUT TAPE 6, 204, KK, L, SY
          DO 92 I=1, L
      92  WRITE OUTPUT TAPE 6, 203, (I, X (I), W (I))
          K1=K1+100
      22  IF (KK-NRUN) 99, 999, 999
      99  GO TO 1000
     999  CONTINUE
          CALL EXIT
          END
```

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