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THE FORMAL STRUCTURE OF METAMORPHOSIS OF CAPITAL

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Abstract

Metaeconomics as a seperated economic discipline gives formal expression to a transcendental observation of economic structure in human society. Such expression is mainly constructed through logistics and abstract mathematics.

In this article, Marx's theory on metamorphosis and circulation of capital is analyzed by means of the mathematical theory of group. However our interest does not lie in explaining and interpreting Marx's economic thinking embodied in his classical work *Capital*, but in formalizing and structuralizing his terminology and his own metaphysical notions as our starting point of metaeconomic investigation.

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I

Is it possible to construct a conceptual image of metaeconomics, that is, a 'theory' to analyze the structures of various economics and political economy, and to restructure their characteristics as historically situated phenomena? The aim of this article is to present an affirmative answer to this question. The prefix 'meta' in metaeconomics is interpreted to have the same meaning as the 'meta' in metaphysics (τὰ μετά τὰ φυσικὰ) as constituted by Aristotle, or as the 'meta' in metamathematics (*Metamathematik*), which can be defined as a universal theory that makes up the logical structure of mathematical reasoning and proofs, as originally defined by David Hilbert.

First of all, let us introduce a fundamental concept —*alienatedness*— necessary for our study of 'metamorphosis of capital,' which Marx analyzed from the viewpoint of classical political economy.

II

Alienatedness is defined as a situation in which the organic body (*Leib*) and life or genus

(*Gattung*) are separated from each other in social context. Marx, observing Hegel's definition of *Gattung* as life in general, claimed:

> A worker is always laying his life in outer objects. However, they have belonged not to him, but to objects themselves. The more these activities extend, the more the worker is losing his own objects. What he produces with his work is not his.[1]

As Marx suggests, such a situation makes it possible to apply the method of physical science to economic analysis. But our aim is to clarify the image of alienatedness as a structure, which means an ensemble of factors including relationship of factors. Now we try to construct a structure of alienatedness with mathematical terminology. The following chart expresses a simple form of the structure, in which ϕ and Φ denote 'morphism' and 'functor' respectively, and the 'category' Γ consists of the 'objects'—the response of *Gattung* to things—and the 'morphism,' which shows relinquishment or alienation of *Gattung*.[2] If Φ, a function of correlating 'category' Γ with Γ' , is defined as 'human labour', $\Phi(X)$, $\Phi(Y)$ and$\Phi(\phi)$ are to express the labour power, the products of labour (commodities) and the relinquishment of labour respectively. The condition that ϕ or $\Phi(\phi)$ is irreversible in a mathematical meaning, is indispensable to the formal expression of alienatedness.

$$
\begin{array}{lcl}
\Gamma & \qquad\qquad\qquad & \Gamma' \\
X & & \Phi(X) \\
\downarrow \phi & & \downarrow \Phi(\phi) \\
Y & & \Phi(Y)
\end{array}
$$

The alienatedness, whose structure is graphically shown in the chart, has social reality resulting from rational grounds. But every action of persons who have already been alienated may be regarded as irrational, because their repeated action has nothing to do with their own will or orientation. A profit-making activity that can be taken as an extraordinary act from the angle of the satisfaction of economic wants is an ordinary business in a commercial society where people are not aware of it to do so. They cannot act rationally, for it is not a person, but an alienated image that behaves rationally. The irrationality of alienatedness was analyzed by Marx, but he did not reduce it to the personal level of social behavior. As profit-making activity is inevitably squeezed in an endless social process, it is the same type of social behavior that Max Weber called 'traditional'. The secularly reiterating movement of alienatedness has no bearing on individual human life. When a social action is repeated unconsciously as a result of the situation that the consciousness of the rationality has been lost with the emergence of autonomous alienated images, it may be called 'metempsychotic'. Among the four types of social action that Weber defined in his comprehensive work 'Economy and Society',[3] the 'instrumentally rational' and 'value-rational' actions can also become metempsychotic through the repetition which makes the alienated situation rational and autonomous, and deprive themselves of their consciously rational character they originally assume. The social composi-

[1] Karl Marx (1968), *Ökonomisch-philosophische Manuskripte*, S.152.

[2] See, MacLane, S. (1998), *Categories for the Working Mathematician*, 2nd ed., Chapter I.

[3] Max Weber (1968), *Economy and Society: An Outline of Interpretive Sociology*, edited by Guenther Roth and Claus Wittich, Vol.1-2, Berkeley, University of California Press.

tion of these metempsychotic actions, i.e. the structure of alienatedness is the main object of our metaeconomic analysis in later sections.

III

As mentioned above, some sets of social actions often show a marked tendency to become factors of an alienated situation. There is a school of political economy which investigates these social processes through applying several methods of natural science. It may be called social economics or socioeconomics, where David Ricardo and J. M. Keynes are both important English figures.

There are various branches and sub-branches in socioeconomics, of which three are relevant. The first is represented by the economic doctrines of Ricardo and Marx, who were mainly concerned about profit-making activity as an important factor in determining the conditions of reproduction. The second is the theory of effective demand, which also lays stress on these conditions. It includes the theories of James Steuart, Robert Malthus and J. M. Keynes. The third type of socioeconomics mainly consists of theories of reproduction structure. François Quesnay and Marx constructed this type of theory, which provides the analytical framework of repetition and cycles of economic relationship. Our present object is to point out several characteristics of their theoretical works.

Quesnay's famous works *Tableau Économique* and *Formule du Tableau Économique*[4] —hereafter *Tableau* and *Formule* respectively— visualize economic life governed by natural law, and illustrate the economic structure of absolute monarchy from the viewpoint of the reproduction process. He considered the social complex of economic interests as consisting of three classes: landowner (*propriétaire*), productive class (*classe des dépenses productives*) and unproductive class (*classe des dépenses stériles*). According to his diagrams depicted in *Tableau* and *Formule*, we will reconstruct the reproduction structure below as a cyclical process.

First, we consider the essence of development of production theory from W. Petty and R. Cantillon to Quesnay. In particular, Cantillon considers of the economic structure of the real world as a cyclical system, placing particular emphasis on structural economic change. He exerted significant influence upon Quesnay's economic doctrine. Petty and Cantillon accept the general relationship based on 'value', and assert that every product (Q) or 'wealth' can be obtained from a combination of land (l) and labour (w) as factors of production according to a certain rule (F). In other words, factors of production vector $V=(l, w)$ can be changed into Q through a production mapping F (production function), that is, a linear form F: $V \to Q$ can always be assumed. In Quesnay's *Tableau* there appears a production process (F) and a product (Q) with two factors of production vectors $X=(x_1, x_2, x_3)$ and $V=(v_1, v_2, v_3)$. The numbers of suffix (1, 2, 3) indicate in order the factors of the production of peasantry, landlords and handicraftsmen. The fundamental structure of *Tableau* can be reduced to a bilinear form

$$\mathrm{F}(X, V) = \sum^{i,j=1,2,3} T_{ij} x_i \cdot v_j \ ,$$

in which T represents the element of this second order tensor, and $x_i \cdot v_j$ a product as the result

[4] See, F. Quesnay (1968), *Oeuvres économiques et philosophiques de F. Quesnay*, publiées par Auguste Oncken.

of 'association' of the two factors of production. Quesnay assumes that there cannot be any products without peasantry factors, therefore $T_{22}=T_{23}=T_{32}=T_{33}=0$. As a result, there are five classes of product obtained from one cycle process of production. If these classes are regarded as *units* (Y) of product, the total sum equals to five units.

The *Formule* consists of three sorts of expenditure, i.e. those of the 'unproductive class' (U), 'productive class' (P) and landowners. The time-pass of these expenditures can be determined by the same calculation as in the so-called multiplier theory. Suppose that the initial income is Y, the multipliers of U's and P's expenditures are 2 and 4 respectively. Then the incomes of U and P in terms of products become 2Y and 4Y. Among them half of 2Y is transformed into 'primitive advances', and the other half becomes the 'annual advances' of U. On the other hand, half of 4Y is expropriated by landowners and the other 2Y are divided into two equal sums. Half of them is delivered to U and the other half remains to be 'annual advances' of P. Naturally, total products consist of 5 units except 1 unit consumed by U. These processes may be illustrated as follows:

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Unproductive Class (U)        Y---------2Y--------------------------→Y (annual advances)
                                         ↓
                                              Y (P→U) consumption
Landowners              2Y--------Y    exchange→
                                              Y(U→P)→Y(primitive consumption)
                                         ↑
Productive Class              Y---------4Y--------------------→Y (annual advances)
                                                               →2Y (to landowners)
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Thus the *Formule* describes a process in which *produits net* can be reproduced. This reproduction process is expressed synchronically by the *formule arithmétique*. The exchange transactions depicted there are supposed to pass through the following five stages:

1st; the landowners buy from the 'productive class' 1 billion *livres* worth of products,
2nd; the 'unproductive class' buys from the 'productive class' their materials and foodstuffs, which are processed to make final industrial products,
3rd; the landowners buy from the 'unproductive class' these industrial products, and the latter receives payments of 1 billion *livres*,
4th; the 'unproductive class' buys again from the 'productive class' their materials and foodstuffs to make further industrial products,
5th; the industrial products made in the 4th stage are purchased by the 'productive class' and served as an annual supplement of 'primitive advances'.

Here, we introduce a new terminology to express the above exchange transactions. Suppose the 'productive class', the landowners and the 'unproductive class' correspond to numbers 1, 2 and 3. We can denote an exchange transaction between i and j $(i, j,=1,2,3,\ i\neq j)$ as a permutation (i, j, k), or (j, i) in a mathematical term. Similarly, (i, k, j) expresses another permutation, and (i, j, k) means the identity permutation.

Now let $T_1=(1,2,3)$, $T_2=(2,3,1)$, $T_3=(3,1,2)$, $T_4=(3,1)$, $T_5=(3,2)$, $T_6=(2,1)$ be all the permutation considered here, and an association of two kinds of permutation be denoted as T_i & T_j $(i, j=1,2,3,4,5,6)$. Then, the following correspondences are established:

Stages of exchange transactions		Permutation
1st	$\longleftrightarrow$	T_6 & T_6
2nd	$\longleftrightarrow$	T_4
3rd	$\longleftrightarrow$	T_5 & T_6
4th	$\longleftrightarrow$	T_4
5th	$\longleftrightarrow$	T_4

If we associate all these transactions, we can get the equality

$$T_4 \,\&\, T_4 \,\&\, T_5 \,\&\, T_6 \,\&\, T_4 \,\&\, T_6 \,\&\, T_6 = T_6.$$

Therefore, a sequence of exchange transactions beginning from the landowners as a starting point is to finish at the same transaction, that is, it performs a cyclical movement.

Since T_4 & T_4 & T_5 & T_6 & T_4 & $T_6 = T_1$ & T_2 & T_3, it is clear that this movement expresses a synchronic mutual exchange system through three poles of exchange (in Quesnay's case, landowners and two classes), or, in other words, it depicts an exchange structure that contains both the mutual exchange between three poles and the non-exchange situation (T_1). In order to analyze the structure, it is necessary for us to decide theoretically the interrelationship of three poles, which Quesnay tried to construct from the angle of his contemporary economic life. We can obtain a stricter theoretical expression that is consistent with the reality of economic life by way of regarding exchange transactions as a process of metamorphosis as Marx did in his famous works on capital.

As profit-making activities gradually spread and create a situation of alienatedness in the modern world, movements of capital as a form of alienatedness undergo a permanent cyclical process. Human behavior in economic life is to repeat itself almost mechanically in spite of difference from the satisfaction of needs as a primary economic activity. That is to say, our material life in capitalist economy circulates as if it might follow the route of metempsychosis. Marx explained and formulated such a process of economic circulation.

There are several forms of the circuit of capital. Our starting point is the metamorphosis of commodities, which should be related to the circuit of capital. According to Marx,

> As C—M means M—C for the buyers, and M—C means C—M for the seller, the circulation of capital presents only the ordinary metamorphosis of commodities.[5]

Therefore, the metamorphosis must include all the relations between commodities (C) as concrete wealth and money (M) as abstract wealth. Now only three cases may occur: ① M—C (or C—M), ② C—C, ③ M—M. The meaning of ① is obvious. In order that the expression ② can bear an economic meaning the first C must differ from the second C in economic qualities, namely, C—C has to be transformed into C_1—C_2 ($C_1 \neq C_2$). On the other hand, the meaning of ③ must be interpreted in another way. As M is always constant in quality, there must be a difference between the first M and the second M, that is, the latter M has to contain a certain increment δM. Thus M—M may be transformed into M—M$'$($=$M$+\delta$M), which represents movements of 'interest bearing capital'. In mathematical formulation, the above forms of metamorphosis can be represented in terms of permutation as follows:

[5] Karl Marx (1967), *Capital: A Critique of Political Economy*, Vol.II (Moscow), p.103.

$$\begin{pmatrix} M & C \\ M & C \end{pmatrix}, \quad \begin{pmatrix} C & M \\ M & C \end{pmatrix}.$$

If we replace these permutations with a mapping Φ, which performs two functions $\Phi(M)=C$ and $\Phi(C)=M$, and define the identity and reverse mappings as Φ & $\Phi=i$, and $\Phi^{-1}=\Phi$ respectively, we can obtain a symmetric group $\{i, \Phi\}$ of degree 2.

Now, through entering an element of circulating production in the process of metamorphosis we have a new relationship, that is, 'industrial capital', which cannot be represented by the above symmetric group. Marx claims:

> The two forms assumed by capital-value at the various stages of its circulation are those of money-capital and commodity capital. The form pertaining to the stage of production is that of productive capital. The capital which assumes these forms in the course of total circuit and then discards them and in each of them performs the function corresponding to the particular form, is industrial capital....[6]

The following three categories are useful, that is, commercialization of human beings (abbreviated as CH), forced labour (FL) and exploitable factors of production (EFP). Two types of CH can be distinguished in historical perspective. The first type is slavery in general, which consists of commodity-slaves or slaves for obligations as in the ancient Mediterranean world or in America of the 18th and 19th centuries. The second is commercialization of labour force, which is pre-requisite for the circuit of 'industrial capital'. Moreover, there are also two distinct types of FL.First, there appeared several forms of directly 'outer-economic' forced labour in slavery and other various compulsory services for autocracy. The second type of FL is a system of indirectly forced labour of the 'proletariat' based on commercialization of labour force in a capitalist society.

Thirdly and lastly, there is EFP, defined as a factor of production that is able to create a positive difference between its total working hours and its reproduction costs per day or per week. Since it contains two kinds of labour, i.e. 'free' labour of the 'proletariat', and labour of slaves and livestock, it can duplicate diachronically a necessary condition for the existence of 'industrial capital'. By way of these interpretations we can modify Marx's 'circuit' diagram as follows:

$$M—C < \begin{matrix} \text{Pm (=means of production)} \\ \\ \text{EFP} \end{matrix} \cdots P \cdots C' \cdots M'$$

In such an abstract process of the circuit of 'industrial capital', a certain 'essence' of capital can be preserved continually throughout all phases of metamorphosis, as indicated in the following diagrams:

$$M—C \cdots P \cdots C'—M=M—C \cdots P \cdots C' \cdots \rightarrow \text{infinity}$$

· ———————— · the circuit of money capital

· ———————— · the circuit of commodity capital

· ———————— · the circuit of productive capital

[6] *Ibid*., p.50.

Further formalization and abstraction of the circuit makes the unit process M—C—P into the process of infinite circulation. Such metamorphosis of capital can be represented in mathematical terms. First, three forms of metamorphosis, M—M, C—C and P—P, can be defined as identity permutation. Among them C—C and M—M imply barter in general and the 'interest bearing capital' respectively. This sets up a rigid circuit, as Marx argued:

> In P · · · P′, P′ does not indicate that surplus-value has been produced but that the produced surplus-value has been capitalized, hence that capital has been accumulated and that therefore P′, in contrast to P, consists of the original capital-value plus the value of capital accumulated because of the capital-value's movement.[7]

What is then the meaning of P—P? It means a change of inner conditions for production, particularly an alteration of the technical conditions for it. The P—P circuit reflects the structural transformation of the production technology as a consequence of 'depreciation value' or relative outdatedness ('a moral depreciation'). In other words, P—P represents the choice of a capitalist between different systems of production technology in relation to modes of capital accumulation and competition. Second and third forms of metamorphosis are those of continuous and alternate repetition. They correspond with normal or reverse permutation respectively.

Now, three forms of metamorphosis can be represented by the following three types of permutation:

$$\begin{pmatrix} \mathrm{M} & \mathrm{C} & \mathrm{P} \\ \mathrm{M} & \mathrm{C} & \mathrm{P} \end{pmatrix}, \quad \begin{pmatrix} \mathrm{M} & \mathrm{C} & \mathrm{P} \\ \mathrm{C} & \mathrm{P} & \mathrm{M} \end{pmatrix}, \quad \begin{pmatrix} \mathrm{M} & \mathrm{C} & \mathrm{P} \\ \mathrm{P} & \mathrm{M} & \mathrm{C} \end{pmatrix}.$$

Suppose the first permutation be denoted as ι, the second one as ω, and the association law as &. Then the third permutation can be denoted as $\omega \,\&\, \omega = \omega^2 = \omega^{-1}$. Since $\omega \,\&\, \omega^{-1} = \omega \,\&\, \omega^2 = \iota$, the set $\{\iota, \omega, \omega^2\}$ become a subgroup of the symmetric group of degree 3. This is an abstract formulation of the metamorphosis in the system of 'industrial capital'. A sort of group by which metamorphosis is represented may be called 'metamorphosis group' (abbreviated as M-group).

Before discussing the logical extension of M-group to the Marxian scheme of reproduction, we will compare Quesnay's *Tableau* with Marx's theory of metamorphosis concerning to several categories they employed in their own theoretical explanations. Approximate correspondences shown below may be recognized on the basis of our above-mentioned arguments:

Quesnay		*Marx*
landowners	$\longleftrightarrow$	money capital
unproductive class	$\longleftrightarrow$	commodity capital
productive class	$\longleftrightarrow$	productive capital

From this chart we can also assign T_1, T_2 and T_3 to ι, ω and ω^2 respectively.

[7] *Ibid*., p.82.

IV

An abstract space in which commodity production prevails or the world of commodities may be regarded as a 'category' in a mathematical term. Within this space, the metamorphosis of capital can be formulated mathematically as a certain type of group, i.e. M-group. It is an 'object' of the 'category' which can be transformed into another mathematical structure. In order to explain the logical development from metamorphosis to reproduction scheme we use mathematical concepts such as those of morphism or homomorphism or the representation of groups.

Let M-group be identified with a three-order cyclic group $\{\iota, \omega, \omega^2\}$ which has an operation denoted by (&). Then we can define the rule of group operation such as ω^2 & $\omega = \iota$ and $\omega^{-1} = \omega^2$ concerning to the permutation $\omega = (2, 3, 1)$. If we take column vectors ε_1, ε_2 and ε_3 denoted as

$$\varepsilon_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \varepsilon_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \varepsilon_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

then the permutation $(\varepsilon_i, \varepsilon_j, \varepsilon_k)$ $(i, j, k = 1, 2, 3)$ can be represented by three order square matrices. There is a one-to-one correspondence between the elements of M-group and these three matrices as follows:

$$\iota \longleftrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \omega \longleftrightarrow \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \qquad \omega^2 \longleftrightarrow \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

These representation matrices may be denoted I, ρ and ρ^2 in the above order.

We introduce here several new notions relating to the Marxian reproduction scheme. First, V is the sum of total 'variable capital' and 'surplus value' in a closed system of reproduction. Next, total 'constant capital' in the system is denoted as C. Moreover, let our system be consisted of only two departments of production denoted 1 as that of articles of consumption, and 2 as that of means of production. Then W_i shows the volume ('value') of total production of i department $(i = 1,2)$.

Now we will consider operations of representation matrices I, ρ, ρ^2 upon the column vector

$$\begin{bmatrix} V \\ C \\ W_i \end{bmatrix}.$$

We can derive two equalities from working the operators ρ and ρ^2 on the left side of the above vector (The operation of I, as it is self-evident, may be omitted.):

$$\rho\begin{bmatrix} V \\ C \\ W_i \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V \\ C \\ W_i \end{bmatrix} = \begin{bmatrix} W_i \\ V \\ C \end{bmatrix},$$

$$\rho^2\begin{bmatrix} V \\ C \\ W_i \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V \\ C \\ W_i \end{bmatrix} = \begin{bmatrix} C \\ W_i \\ V \end{bmatrix}.$$

From these results three equalities can be deduced, i.e.
(1) $C=V$, that is, the quantity of 'constant capital' equals to that of 'variable capital',
(2) $V=W_i=W_1$, that is, the quantity of 'variable capital' plus that of 'surplus value' equals to the quantity of articles of consumption,
(3) $V=W_i=W_2$, that is, the quantity of 'constant capital' equals to that of means of production.

These equalities (1)—(3) are necessary conditions for the continuance of total reproduction. Let us compare these conditions with Marx's notions on the reproduction scheme. In his terminology c_i, v_i and m_i indicate the 'quantity of value' of the 'constant capital', the 'variable capital' and the 'surplus value' respectively, which are produced in the i department ($i=1,2$). $C=V$ is a stronger than that of Marx for 'simple reproduction', which has not, however, an essential meaning for our problem concerned. From conditions (2) and (3) we draw expressions,

$$v_1+m_1+v_2+m_2=c_2 \ +v_2+\mathrm{m}_2 \qquad ①$$

and

$$c_1+c_2=c_1+v_1+m_1 \qquad ②$$

respectively. Putting together ① and ②, we can obtain a simple expression

$$c_2=v_1+m_1, \qquad ③$$

which stands for the condition of 'simple reproduction' formulated by Marx, but which has a greater meaning.

As explained above, a permanently repeated structure of reproduction appears in consequence of the operation of M-group's representation matrices on the column vector of C, V and W_i. This reflects the 'laws' of motion of the capitalist economic system, which Marx analyzed by means of his reproduction scheme. But our method of explanation may enrich the theoretical content of Marx's discussions on the reproduction process, because it clarifies the logical structure of reproduction, that is, the formal or mathematical structure of reproduction that is deduced continuously from structural analysis of alienatedness. The representation group operating on the column vector in three-dimensional space of the reproduction process is a subgroup of three-order general linear group or an orthogonal group of three orders. The law formalizing reproduction scheme through such a representation group constructed from the M-group can be regarded as a mapping or isomorphism. Therefore, we have identified a 'law' indicating a one-to-one correspondence between the 'category' of group and that of a general linear group. Its transformation rule may be called 'functor'. In other words, the structure of alienatedness as expressed in 'capital' can be described as that of 'category' in mathematical or formal terminology.