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# LABOR MARKET SEARCH, NOMINAL RIGIDITIES AND MONETARY PROPAGATION*

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## *Abstract*

Business cycle models with nominal rigidities do not readily generate persistent and hump shaped aggregate output dynamics to monetary shocks. In this paper, we consider labor market search in models with nomial rigidities to obtain realistic monetary propagation. While existing research combined labor market search with nominal price stickiness, greater persistence and hump shaped output dynamics as well as plausible labor market movements are obtained when labor market search is combined with nominal wage stickiness rather than nominal price stickiness.

*Keywords:* Labor market search, Nominal rigidities
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## I. *Introduction*

Labor market seach models like Mortensen and Pissarides (1994) have become widely used in the modern explanation of unemployment. These models have also been used in business cycle research. Andolfatto (1996) and Merz (1995) first combined labor market search with a real business cycle model to explain business cycle regularities. den-Haan et al.

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(2000) added endogenous job destruction in a real business cycle model with labor market search and made the effects of productivity shocks more pronouced and persistent. Labor market search has also been incorporated into monetary business cycle models. Cooley and Quadrini (1999) combined labor market search with a limited participation model of money and found that many important qualitative features of labor markets and the Phillips curve relation are captured by doing so.

Recently labor market search is introduced into the so-called New Keynesian type models with nominal price stickiness to improve on the models' weak propagation of monetary shocks as noted by Chari et al. (2000) and Dotsey and King (2001). But it cannot be determined yet whether labor market search significantly enhances the performance of the models with nominal price stickiness. Walsh (2002) shows that labor market search can induce a delayed and hump-shaped output response to a monetary shock. The model includes a 'cost channel' of monetary shocks, however, which is not tied closely with labor market search or nominal price stickiness. Trigari (2004) also builds a business cycle model with labor market search and nominal price stickiness. The model generates plausible dynamics of output, but it incorporates habit formation, which is controversial.[1] Krause and Lubik (2003) find that a business cycle model with labor market search and Rotemberg-type quadratic price adjustment costs does not generate any substantial propagation of monetary shocks.

In this paper, we consider nominal wage stickiness in stead of or in additon to nominal price stickiness in monetary business cycle models with labor market search to obtain realistic monetary propagation and plausible labor market dynamics. Our model can be thought of a variant of New Keynesian models and we are basically trying to improve on the performance of the models with nominal rigidities as Christiano et al. (2005) by incorporating labor market search. The main findings from this attempt can be summarized as follows. First, nominal wage stickiness combined with labor market search can be an important mechanism in generating delayed and hump shaped responses of aggregate variables to monetary shocks. Second, monetary shocks can also generate plausible labor market dynamics such as a negative correlation between job creation and destruction rate when we combine nominal wage stickiness and labor market search. Our attempt will be interesting in the following respects. First, Walsh (2002) and Trigari (2004) incorporated nominal price stickiness into monetary business cycle models with labor market search, but no attempt has been made to incorporate nominal wage stickiness in those models. Second, Krause and Lubik (2003) combined Hall (2004) type real wage rigidity and Rotemberg-type quadratic price adjustment costs in a model with labor market search. While their model can be broadly interpreted to have incorporated both nominal price and wage stickiness, the effects of nominal wage stickiness alone cannot be analyzed in their model. Also, Krause and Lubik (2003) find no substantial propagation of monetary shocks in their model, but we observe delayed and hump shaped responses of economic variables to monetary shocks in our model with nominal wage stickiness. Finally, nominal wage stickiness is naturally introduced in our model following Hall (2004) without relying on monopolistic wage setting unions, which may not play a significant role in the US economy but are typically assumed in models with nominal wage stickiness.

The rest of the paper is organized as follows. In section 2, we construct the model. We first write down a flexible wage version of the model so that we can refer to it afterward and then

[1] See Karen (2000) and Otrok et al. (2002) for some evidences against habit formation.

incorporate nominal wage stickiness into it. In section 3, we characterize the equilibrium of the model and calibrate parameters. In section 4, we summarize findings from our benchmark model and its variants. And in section 5, we conclude.

## II. *The Model*

Our model is a monetary business cycle model with labor market search based on Cooley and Quadrini (1999). We will introduce nominal wage stickiness *a la* Fischer (1977) into it in our benchmark case.

In each period $t$, the model economy experiences an event $s_t$ in $S_t$. We denote by $s^t=(s_0, \ldots, s_t)$ the history of states of the economy up through and including period $t$. The probability as of period 0, of any particular history $s^t$ is $\pi(s^t)$. The initial realization $s_0$ is given. The economy is populated with workers (or households) and firms distributed on the interval [0,1) respectively.

### 1. Households

Workers as households purchase consumption goods and acquire real balances. They also supply labor to the firms when they engage in production; otherwise they search for a job. We consider household $i$ without loss of generality. Household $i$ maximizes

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)U(c_i(s^t), h_i(s^t), M_i(s^t)/P(s^t), b),\ 0<\beta<1 \tag{1}$$

where $c_i(s^t)$, $M_i(s^t)/P(s^t)$, $h_i(s^t)$ and $b$ are consumption, real balances, labor hours and exogenous unemployment benefits respectively. Momentary utility function is[2]

$$U(c, h, M/P, b)=c-\chi(\varpi\frac{h^\gamma}{\gamma})+(1-\chi)b+\omega\frac{(M/P)^{1-\zeta}}{1-\zeta},\ \varpi,\ \gamma,\ \omega, \text{ and } \zeta>0 \tag{2}$$

where $\chi$ is an indicator function which equals 1 if the worker (or household) is employed and 0 otherwise. The budget constraints are

$$\begin{aligned} & c_i(s^t)+\frac{M_i(s^t)}{P(s^t)}+\frac{\Sigma s_{t+1}Q(s^{t+1}|s^t)B_i(s^{t+1})}{P(s^t)} \\ & =y_i^l(s^t)+\frac{M_i(s^{t-1})}{P(s^t)}+\frac{B_i(s^t)}{P(s^t)}+\frac{\Pi_i(s^t)}{P(s^t)}+\frac{T_i(s^t)}{P(s^t)} \end{aligned} \tag{3}$$

where $y_i^l(s^t)$ is real labor income, $\Pi_i(s^t)$ is the nominal profit transfer from firms, and $T_i(s^t)$ is the nominal transfer from the government. $B_i(s^{t+1})$ is state dependent nominal bond claim (amount) and a bond which pays one dollar in state $s^{t+1}$ costs $Q(s^{t+1}|s^t)$ dollars in state $s^t$. Households' first order conditions with respect to consumption, money and bond holdings can be obtained by maximizing (1) subject to (3).

[2] We assume a utility function linear in the consumption to avoid complications with heterogeneity.

## 2. The Labor Market Search

### Flexible wage

To produce the good used for final consumption and investment, a firm and a worker need to be matched. We assume, without loss of generality, that there is a single firm for each worker so that each firm hires only one worker. Firms searching for a worker incur a fixed flow cost of $k$ due to vacancy posting. Workers searching for a job (or firm) do not face any search cost, however. Firms and workers in the economy are matched according to the following Cobb-Douglas matching technology

$$\phi(v(s^t),\ u(s^t))=\varphi v(s^t)^{\phi_1}u(s^t)^{\phi_2},\ \varphi>0 \text{ and } \phi_1,\ \phi_2>0 \tag{4}$$

where $v(s^t)$ is the number of vacancies posted by firms, $u(s^t)$ is the number of workers searching for a job, and $\phi(v(s^t), u(s^t))$ is the number of newly formed matches in the economy. The probability that a firm finds a worker, denoted by $q^f(s^t)$, is given as

$$q^f(s^t)=\frac{\phi(v(s^t),\ u(s^t))}{v(s^t)} \tag{5}$$

The probability that a worker finds a job, denoted by $q^w(s^t)$, is given as

$$q^w(s^t)=\frac{\phi(v(s^t),\ u(s^t))}{u(s^t)} \tag{6}$$

If a firm and a worker are successfully matched, the matched pair may operate through the following Cobb-Douglas production technology

$$y(s^t)=h(s^t)^{1-\alpha}k(s^t)^{\alpha},\ 0<\alpha<1 \tag{7}$$

where $h(s^t)$ is labor hours and $k(s^t)$ is capital input. Firms act in a perfectly competitive market and sell their output at the price $P(s^t)$. The period $t$ profit of a representative firm in real terms (or in terms of period $t$ output) is given as

$$z(s^t,\ \epsilon)=y(s^t)-\epsilon-w(s^t,\ \epsilon)-r(s^t)k(s^t) \tag{8}$$

where $r$ is real rental rate of capital and $w(s^t, \epsilon)$ is real wage. $\epsilon$ is an idiosyncratic cost factor which is independently and identically distributed across firms and the states of the economy with a distribution function $F$: $[0, \infty] \rightarrow [0, 1]$. Matches may not produce if the realization of $\epsilon$ is too high. We denote the value of $\epsilon$ above which matches decide not to produce as $\bar{\epsilon}(s^t)$ (endogenous separation margin, henceforth) and we note that it is a function of the state of the economy $s^t$. The probability that matches will not produce due to high enough realization of $\epsilon$ is, then, $1-F(\bar{\epsilon}(s^t))$. We also assume that matches separate exogenously with a probability of $\xi^x$, apart from the endogenous separation.[3] A matched firm and worker pair sets real wage $w(s^t, \epsilon)$ so that the worker gets a constant share $\eta$ of the surplus generated by the match following standard literature.

Let $J(s^t, \epsilon)$ be the value of a match for a matched firm expressed in real terms. Then

[3] Exogenous job separation may occur in the real world due to retirement or some other non-economic reasons.

$$J(s^t,\epsilon)=z(s^t,\epsilon)+\beta(1-\xi^x)\sum_{s_{t+1}}\pi(s^{t+1}|s^t)\int^{\bar{\epsilon}(s^{t+1})}J(s^{t+1},\epsilon)dF(\epsilon) \tag{9}$$

That is, $J(s^t,\epsilon)$ is the sum of current period (or period $t$) real profit and the next period discounted expected value of a match for the firm provided that the firm remains matched.[4] Let $Q(s^t)$ be the value of vacancy posting for a firm. Then

$$\begin{aligned}Q(s^t)=&-\kappa+q^f(s^t)\beta(1-\xi^x)\sum_{s_{t+1}}\pi(s^{t+1}|s^t)\int^{\bar{\epsilon}(s^{t+1})}J(s^{t+1},\epsilon)dF(\epsilon)\\&+(1-q^f(s^t))\beta\sum_{s_{t+1}}\pi(s^{t+1}|s^t)Q(s^{t+1})\end{aligned} \tag{10}$$

$Q(s^t)$ consists of three parts. First, vacancy posting costs $\kappa$. Second, we assume matches formed in period $t$ start to produce only at period $t+1$, and thus a successful match (which occurs with probability of $q^f(s^t)$) contributes to $Q(s^t)$ as a form of next period discounted expected value of a match.[5] Third, a vacancy posting firm cannot find a worker with probability $(1-q^f(s^t))$ and gets another chance of posting a vacancy in the next period. In equilibrium, the values of vacancy posting become zero, that is $Q(s^t)=Q(s^{t+1})=0$. Then (10) becomes in equilibrium

$$\kappa=q^f(s^t)\beta(1-\xi^x)\sum_{s_{t+1}}\pi(s^{t+1}|s^t)\int^{\bar{\epsilon}(s^{t+1})}J(s^{t+1},\epsilon)dF(\epsilon) \tag{11}$$

We can interpret (11) as an arbitrage condition for vacancy posting. It says that, in equilibrium, the vacancy posting cost ($\kappa$) equals the discounted expected value of a successful match (which occurs with probability of $q^f(s^t)$). It is also helpful in computation to express the arbitrage condition (11) more explicitly in a recursive form using (9) as

$$\begin{aligned}\frac{\kappa}{q^f(s^t)}=&\beta(1-\xi^x)\sum_{s_{t+1}}\pi(s^{t+1}|s^t)\int^{\bar{\epsilon}(s^{t+1})}z(s^{t+1},\epsilon)dF(\epsilon)\\&+\beta(1-\xi^x)\sum_{s_{t+1}}\pi(s^{t+1}|s^t)\int^{\bar{\epsilon}(s^{t+1})}\frac{\kappa}{q^f(s^{t+1})}dF(\epsilon)\end{aligned} \tag{12}$$

Let $M(s^t,\epsilon)$ be the value of a match for a matched worker and $U(s^t)$ be the value of being unemployed. Then $M(s^t,\epsilon)$ is given as

$$\begin{aligned}M(s^t,\epsilon)=&w(s^t,\epsilon)-\varpi\frac{h(s^t)^\gamma}{\gamma}+\beta\sum_{s_{t+1}}\pi(s^{t+1}|s^t)U(s^{t+1})\\&+\beta(1-\xi^x)\sum_{s_{t+1}}\pi(s^{t+1}|s^t)\int^{\bar{\epsilon}(s^{t+1})}[M(s^{t+1},\epsilon)-U(s^{t+1})]dF(\epsilon)\end{aligned} \tag{13}$$

$M(s^t,\epsilon)$ consists of three parts. First, a matched worker gets a real wage net of the disutility of work in the current period.[6] Second, the worker can get at least the value of being

[4] Here, we utilize the fact that the value of a discontinued match for a firm is zero.

[5] Because labor market matching is a time-consuming process, we follow the literature in assuming that new matches may start to produce with a lag of one period.

[6] Given the utility function linear in consumption, we can express the disutility of labor in real terms as in (13).

unemployed for all cases in the next period. Third, if the worker remains matched and produces in the next period, he or she gets the extra value of the match over that of being unemployed. The value of being unemployed, $U(s^t)$, is given as

$$U(s^t)=b+\beta\sum_{s_{t+1}}\pi(s^{t+1}|s^t)U(s^{t+1})+q^w(s^t)\beta(1-\xi^x)\times$$
$$\sum_{s_{t+1}}\pi(s^{t+1}|s^t)\int^{\bar{\epsilon}(s^{t+1})}[M(s^{t+1},\epsilon)-U(s^{t+1})]dF(\epsilon) \tag{14}$$

$U(s^t)$ consists of three parts. First, an unemployed worker gets some exogenous benefits in the current period, denoted $b$. Second, the unemployed worker can get at least the value of being unemployed in the next period. Third, the unemployed worker can be matched with a firm with the probability $q^w(s^t)$ and if the match does not separate and does engage in production, the worker gets the extra value of the match over that of being unemployed in the next period.

Using (9) - (14) and the constant match surplus sharing rule, real wage rate, $w(s^t,\epsilon)$, can be expressed respectively as

$$w(s^t,\epsilon)=\eta\left[h(s^t)^{1-\alpha}k(s^t)^{\alpha}-\epsilon-r(s^t)k(s^t)+\kappa\frac{q^w(s^t)}{q^f(s^t)}\right]$$
$$+(1-\eta)\left[\varpi\frac{h(s^t)^{\gamma}}{\gamma}+b\right] \tag{15}$$

A matched firm and worker pair jointly maximizes the total surplus, subject to the production function (7). The first order conditions with respect to capital and labor hours are

$$r(s^t)=\alpha\left(\frac{h(s^t)}{k(s^t)}\right)^{1-\alpha} \tag{16}$$

$$\varpi h(s^t)^{\gamma-1}=(1-\alpha)\left(\frac{h(s^t)}{k(s^t)}\right)^{-\alpha} \tag{17}$$

Thus, the marginal product of capital is equated to its rental rate and the marginal product of labor (labor hours) is equated to the marginal disutility of labor. A matched pair breaks up endogenously whenever $\epsilon$ is higher than the endogenous separation margin, $\bar{\epsilon}(s^t)$. $\bar{\epsilon}(s^t)$ can be characterized with the condition that total surplus is zero. And we note that if total surplus is zero then $J(s^t,\bar{\epsilon}(s^t))=0$ thanks to the constant match surplus sharing rule. Then using (9), (11) and $J(s^t,\bar{\epsilon}(s^t))=0$, we obtain the following condition to determine $\bar{\epsilon}(s^t)$ in equilibrium.

$$z(s^t,\bar{\epsilon}(s^t))+\frac{\kappa}{q^f(s^t)}=0 \tag{18}$$

**Predetermined wage**

In this section, we introduce nominal wage stickiness *a la* Fischer (1977) instead of assuming flexible wages.[7] We suppose that it is costly to negotiate wages every period, leading firms and workers negotiate wages at intervals of $N$ periods for the next $N$ periods. The matches are indexed by $i$ as follows: those indexed $i\in[0, 1/N)$ determine new wages in 0, $N$,

[7] Other features of the model remain unchanged, if not mentioned otherwise.

$2N$ and so on; those indexed $i\in[1/N,\ 2/N)$ determine new wages in 1, $N+1$, $2N+1$; and so on, for the $N$ cohorts of matches. But unlike commonly adopted Calvo or Taylor type nominal wage stickiness, we do not restrict wages to be the same throughout the $N$ subsequent periods after an adjustment. Corresponding to each cohort of matches, firms and workers are indexed so that matches in the cohort $[0,\ 1/N)$ are formed by firms indexed $i\in[0,\ 1/N)$ and workers indexed $j\in[0,\ 1/N)$; matches in the cohort $[1/N,\ 2/N)$ are formed by firms indexed $i\in[1/N,\ 2/N)$ and workers indexed $j\in[1/N,\ 2/N)$; and so on. Given the restriction on match formation, it is natural that the labor market matching process is shared only in the same cohort or between pair cohorts. Namely firms indexed $i\in[0,\ 1/N)$ share the processes with other firms indexed $i'\in[0,\ 1/N)$, workers indexed $j\in[0,\ 1/N)$ share the processes with other workers indexed $j'\in[0,\ 1/N)$, and firms indexed $i\in[0,\ 1/N)$ share the processes with workers indexed $j\in[0,\ 1/N)$; and so on. Then there will be $N$ different matching processes comprised of matching function, job-finding probability and worker-finding probability corresponding to (4) - (6) in the flexible wage setting.

Without loss of generality, we consider a match $i$, which predetermines wages in period $t-m$. The labor contract signed between the firm and the worker in the match specifies rules for the wage determination and the labor hours.[8] The rule for the wage determination will be detailed below. The rule for labor hours is assumed to be the same as the labor hours optimality condition in the flexible wage setup, namely,

$$\varpi\, h_i(s^t)^{\gamma-1}=(1-\alpha)\left(\frac{h_i(s^t)}{k_i(s^t)}\right)^{-\alpha} \tag{19}$$

Thus while wages are set in advance, labor hours are adjusted to equate the marginal disutility of labor to the marginal productivity of labor period by period depending on the state of the economy. We denote the nominal wage predetermined for the period $t-m+\gamma$ in period $t-m$ as $W_{t-m+\gamma|t-m}$. Given the predetermined wage (which will be detailed below), the period $t-m+\gamma$ profit of the firm in terms of period $t-m+\gamma$ output (or in real terms) is given as

$$z_i(s^{t-m+\gamma},\ \epsilon_i)=y_i(s^{t-m+\gamma})-\epsilon_i-\frac{W_{t-m+\gamma|t-m}}{P(s^{t-m+\gamma})}-r(s^{t-m+\gamma})k_i(s^{t-m+\gamma}) \tag{20}$$

The first order condition with respect to capital input implies

$$r(s^t)=\alpha\left(\frac{h_i(s^t)}{k_i(s^t)}\right)^{1-\alpha} \tag{21}$$

which is the same as that in the flexible wage setup.

We can now specify the rule for the wage determination. Nominal wages are predetermined every $N$ periods for the next $N$ periods in advance so that the corresponding real wages for the next $N$ periods are set to be the expected values of the real wages if the wage setting were flexible. Then the predetermined wages are given as:

$$W_{t-m+\gamma|t-m}=\left(\sum_{s^{t-m+\gamma}}\pi(s^{t-m+\gamma}|s^{t-m})(P(s^{t-m+\gamma}))^{-1}\right)^{-1}\times$$

[8] Given predetermined wages, rules for labor hours are also required since firms will demand as many labor hours as possible and workers will supply as few hours as possible without such rules.

$$\left(\sum_{s^{t-m+\gamma}} \pi(s^{t-m+\gamma}|s^{t-m}) \int^{\bar{\epsilon}_i(s^{t-m+\gamma})} w_i^*(s^{t-m+\gamma}, \epsilon_i) \frac{dF(\epsilon_i)}{F(\bar{\epsilon}_i(s^{t-m+\gamma}))}\right)$$

$$\text{for } \gamma = 1, \cdots, N \tag{22}$$

where $w_i^*(s^{t-m+\gamma}, \epsilon_i)$ is real wage for the period $t-m+\gamma$ if wage setting were flexible:

$$\begin{aligned} w_i^*(s^{t-m+\gamma}, \epsilon_i) = \eta \Big[ & h_i(s^{t-m+\gamma})^{1-\alpha} k_i(s^{t-m+\gamma})^{\alpha} - \epsilon_i \\ & - r(s^{t-m+\gamma}) k_i(s^{t-m+\gamma}) + \kappa (q_i^w(s^{t-m+\gamma}) / q_i^f(s^{t-m+\gamma})) \Big] \\ & + (1-\eta) \left[ \varpi \frac{h_i(s^{t-m+\gamma})^{\gamma}}{\gamma} + b \right] \end{aligned} \tag{23}$$

Since the wages are set in advance conditional on the match not being separated endogenously, we need to integrate out the idiosyncratic factor conditional on its being smaller than the separation margin in the period $t-m+\gamma$ and thus we have $F(\bar{\epsilon}_i(s^{t-m+\gamma}))$ as an integrating factor.

Given the predetermined wages and input rules, we can also characterize the endogenous job destruction condition as

$$z_i(s^t, \bar{\epsilon}_i(s^t)) + \frac{\kappa}{q_i^f(s^t)} = 0 \tag{24}$$

which is analogous to the endogenous job destruction condition (18) in the flexible wage setup and can be derived by the same procedures used in the flexible wage setup.[9] And we can characterize the job posting condition as

$$\begin{aligned} \frac{\kappa}{q_i^f(s^t)} = & \beta(1-\xi^x) \sum_{s_{t+1}} \pi(s^{t+1}|s^t) \int^{\bar{\epsilon}_i(s^{t+1})} z_i(s^{t+1}, \epsilon) \, dF(\epsilon) \\ & + \beta(1-\xi^x) \sum_{s_{t+1}} \pi(s^{t+1}|s^t) \int^{\bar{\epsilon}_i(s^{t+1})} \frac{\kappa}{q_i^f(s^{t+1})} dF(\epsilon) \end{aligned} \tag{25}$$

Since nominal wages are predetermined, these can cause inefficiencies and there can be mutual benefits to the firm and worker from re-adjusting the wage ex post as noted by Barro (1977), Mankiw and Reis (2002) and Hall (2004). That is, the predetermined wage cannot be enforced so it cannot exist. We would like to rule out this problem based on the following reasons. First, as pointed out by Hall (2004), rigid wage (nominal wage stickiness in our model) does not cause inefficiencies in labor market matching models as long as the rigid wage remains within the bargaining range of wage (the difference between the reservation wage of the firm and the worker). Thus, when the bargaining range is wide enough[10] and shocks are small enough, regular wage negotiations in every $N$ period will suffice to keep the rigid wage

[9] It is assumed that workers always want to be employed given predetermined wages and thus there is no separating condition reflecting workers' willingness to separate.

[10] Empirically, Blanchflower et. al (1996) find range of pay is, for rent-sharing reasons alone, approximately 24% the mean wage, suggesting wide bargaining range. In our benchmark model, worker's share of match surplus is 52% real wage at the steady state.

stay within the bargaining range. Then there will be no inefficiency in the first place. This argument can apply easily in the special case of our model when we do not allow endogenous separation of matches as in Hall (2004). When we allow endogenous separation of matches, this argument does not apply exactly since bargaining range can be as small as zero due to the idiosyncratic cost factor. So there will be some matches which will experience inefficiencies.

Second, for those matches which experience inefficiencies due to predetermined wages, we assume that they also do not readjust wages due to some non-economic concerns. That is, given other majority of matches do not readjust wages, these matches also find it better not to adjust wage and rather break up due to some social and psychological concerns such as fairness at work place.

**The matching processes**

Since we have $N$ different cohorts of matches, there will be $N$ different matching processes. We consider the $i$th cohort matching processes without loss of generality.

The number of workers employed at the beginning of period $t$, denoted $n_i(s^t)$, evolves according to

$$n_i(s^t) = (1 - \xi_i(s^{t-1}))\, n_i(s^{t-1}) + v_i(s^{t-1})\, q_i^f(s^{t-1}) \tag{26}$$

where $\xi_i(s^{t-1})$ is the sum of the endogenous and exogenous separation rate given as

$$\xi_i(s^{t-1}) = \xi^x + (1 - \xi^x)(1 - F(\bar{\epsilon}_i(s^{t-1}))) \tag{27}$$

Thus, workers employed at the beginning of period $t-1$ separate with the rate $\xi_i(s^{t-1})$ during the period $t-1$ and the remaining workers stay employed at the beginning of period $t$. In addition, workers who found a job in period $t-1$ are also employed at the beginning of period $t$. The number of unemployed workers looking for a job in period $t$, denoted $u_i(s^t)$, satisfies the following relation

$$u_i(s^t) = 1 - n_i(s^t) + \xi_i(s^t)\, n_i(s^t) \tag{28}$$

That is, $1 - n_i(s^t)$ unemployed workers search for a job from the beginning of period $t$ and $\xi_i(s^t)$ $n_i(s^t)$ separated workers in period t also search for a job possibly to produce in the next period.

Job creation and destruction rates are of independent interest. We define them following den-Haan et al. (2000). Job creation rate, denoted $jc_i(s^t)$, is defined as

$$jc_i(s^t) \equiv \frac{\phi(v_i(s^t),\, u_i(s^t))}{n_i(s^t)} - q_i^f(s^t)\xi^x \tag{29}$$

Thus, job creation rate is the proportion of newly-created matches out of total employed workers (or matches), net of the proportion of matches serving to refill the vacancies resulting from exogenous separation.[11] Job destruction rate, denoted $jd_i(s^t)$, is defined as

$$jd_i(s^t) = \xi_i(s^t) - q_i^f(s^t)\xi^x \tag{30}$$

That is, job destruction rate is the separation rate, net of the proportion of matches serving to refill the vacancies resulting from exogenous separation.

[11] den-Haan et. al. (2000) interpret matches serving to refill the vacancies resulting from exogenous separation are not new job creation.

We will be eventually interested in the aggregate (or economy-wide) numbers of employed workers, workers looking for a job and so on instead of the cohort-wide numbers. These aggregate numbers are calculated as the weighted averages of the corresponding cohort-wide numbers, using the masses of the cohorts or employment shares as the weights.

### 3. Capital Supply

We introduce capital leasing firms for ease of analysis. Capital is provided by competitive capital leasing firms which maximize

$$\sum_{\gamma=t}^{\infty}\beta^{\gamma-t}\sum_{s^{\gamma}}\pi(s^{\gamma}|s^{t})\{r(s^{\gamma})k^{s}(s^{\gamma-1})-i(s^{\gamma})\} \tag{31}$$

subject to the following law of motion for capital accumulation[12]

$$k^{s}(s^{t})=(1-\delta)k^{s}(s^{t-1})-\frac{a}{2}\left(\frac{i(s^{t})}{k^{s}(s^{t-1})}-\delta\right)^{2}k^{s}(s^{t-1})+i(s^{t}) \tag{32}$$

where $k^{s}(s^{t})$ is capital supply and $i(s^{t})$ is investment. $a$ is capital adjustment cost parameter and $\delta$ is the depreciation rate of capital. First order conditions with respect to investment and capital can be obtained by maximizing (31) subject to (32).

### 4. Monetary Shocks

The nominal money supply process is given by

$$M(s^{t})=\mu(s^{t})M(s^{t-1}) \tag{33}$$

The growth rate of money, $\mu(s^{t})$, follows a first order autoregressive process

$$\log\mu(s^{t})=\rho_{\mu}\log\mu(s^{t-1})+\epsilon_{\mu}(s^{t}) \tag{34}$$

where $\epsilon_{\mu}(s^{t})$ is independently, identically and normally distributed with mean zero and standard deviation $\sigma_{\varepsilon_{\mu}}$. Newly injected money is distributed to the households in a lump sum fashion, satisfying $T(s^{t})=M(s^{t})-M(s^{t-1})$.

## III. *Computation of Equilibrium and Parametrization*

### 1. Computing the Equilibrium

We impose market-clearing conditions and resource constraints in addition to conditions obtained from agents' optimization problems and law of motions to define equilibrium. We then use a standard log-linearization method such as Sims (2002) to compute the solution.

The parameters are taken mainly from Cooley and Quadrini (1999). Time period is a quarter. Subjective discount factor, $\beta$, is set to be 0.98. The labor hours disutility parameter, $\gamma$, is set to be 2. The weight of labor disutility in the momentary utility function, $\varpi$, is

[12] This law of motion comes from Chari et al. (2000).

calibrated so that the share of time allocated to the labor hours is around 1/3 of the available time. Exogenous unemployment benefits, $b$, is assumed to be zero. We set the interest elasticity parameter, $\zeta$, to be 2.56 and the weight of real balance in the momentary utility function, $\omega$, to be 0.66 on the basis of the estimate of the money demand function in Chari et al. (2000).[13] Exogenous job separation rate parameter, $\xi^x$, is set to be 0.068 following den Haan et al. (2000).[14] For the Cobb-Douglas matching function parameters, we set $\phi_1=0.6$ and $\phi_2=0.4$. We fix the steady state value of the probability that a firm finds a worker, $q^{f*}$, to be 0.7 and steady state value of the probability that a worker finds a job, $q^{\omega*}$, to be 0.6. We set steady state value of the number of workers employed, $n^*$, to be 0.94 implying a steady state unemployment rate of 0.06. Given these restrictions, the parameter $\varphi$ in the matching function and steady state value of the endogenous separation margin, $\bar{\epsilon}^*$, can be determined. We assume that the idiosyncratic cost factor, $\epsilon$, follows an exponential distribution with parameter $\rho$. Then using the steady state relationship of (24) and (25), we can set the value for the parameter $\rho$ together with the vacancy posting cost parameter, $\kappa$. We set the Cobb-Douglas production function parameter, $\alpha$, to be 0.36 and the capital depreciation rate parameter, $\delta$, to be 0.025. Also we set the monetary shock parameters $\rho_\mu$ and $\sigma_{\varepsilon_\mu}$ to be 0.48 and 0.00623 respectively. The capital adjustment cost parameter, $a$, is adjusted to match the relative volatility of investment to output with the corresponding data statistic as in Chari et al. (2000). Surplus sharing parameter $\eta$ is set to be 0.5 following den-Haan et al. (2000). Finally we set $N$ to be 4 so that wages are predetermined for one year.

## IV. *Findings*

### 1. Benchmark Case

Figure 1 plots the responses of variables to a standard deviation monetary shock in the benchmark model. Total output, labor, investment and consumption all increase due to the monetary shock. Job destruction rate falls due to the monetary shock while job creation rate rises initially and then falls below the steady state before returning to it.

When the monetary shock hits the economy, demand for (final) good increases. But the real wage goes down due to the nominal wage stickiness. Wage costs then fall, raising firm profits. And due to the increased firm's profits, the endogenous job destruction margin goes up as can be seen from the condition (24). And thus fewer firm and worker matches break up endogenously. Further, vacancy posting increases because of the expectation of the falling real wage costs and rising profits in the future as can be seen from the job posting arbitrage condition (25). This pulls up employment and pushes down unemployment. Also the job destruction rate goes down and job creation rate goes up. Finally total output, consumption, total investment all increase due to the rise of employment.

We observe somewhat delayed effects of the monetary shock. That is, total output, consumption and labor peak two periods after the shock, instead of reaching their maxima in

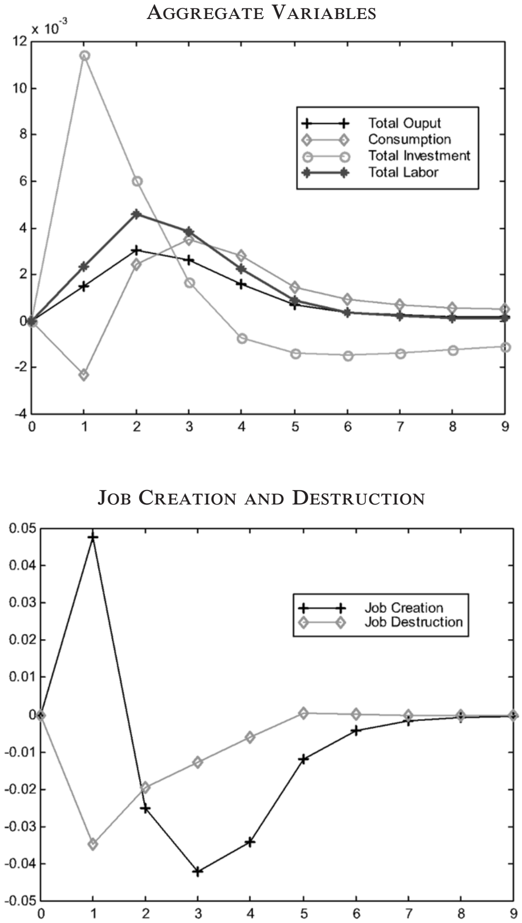
[13] We do not follow Cooley and Quadrini (1999) strictly in inroducing demand for money via cash in advance constraint. Instead, we induce demand for money via money in utility function as in Chari et al. (2000).

[14] Cooley and Quadrini (1999) do not assume exogenous separation.

FIG. 1.[15] BENCHMARK

AGGREGATE VARIABLES

JOB CREATION AND DESTRUCTION




the first period and declining thereafter. This occurs for the following reasons: When the shock hits the economy, endogenous separation decreases and this increases total output, consumption and labor initially. But the maximum effects on those variables are realized in the second period when newly formed matches are beginning to be productive after the time-consuming process of labor market matching. Finally we observe job creation decreases below the steady state after the first period. This is due to the (gradual) rise of the real wage per job and the fall of the worker finding rate. It becomes unprofitable to post vacancies after the first period due to the rising wage costs (to the steady state) and the low rate of finding a worker.

Table 1 shows autocorrelations of output growth in the model. The model economy exhibits persistence in output growth as shown by the positive first-order autocorrelation. Table 2 shows some correlations among job creation, destruction rate and employment. The

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[15] All variables are in log-deviation form. The shock hits the economy at 1st period.

TABLE 1.[16] AUTOCORRELATION OF OUTPUT GROWTH

| Autocorrelation at lags | | | |
|---|---|---|---|
| t-4 | t-3 | t-2 | t-1 |
| Benchmark | | | |
| −0.27 | −0.40 | −0.23 | 0.48 |
| Sticky Price | | | |
| −0.01 | −0.01 | 0.00 | −0.49 |
| U.S. Economy | | | |
| −0.04 | −0.07 | 0.22 | 0.20 |

TABLE 2.[17] CROSS-CORRELATION OF EMPLOYMENT, JOB CREATION AND JOB DESTRUCTION

| | Correlation at lags and leads | | | | | | |
|---|---|---|---|---|---|---|---|
| | t-3 | t-2 | t-1 | t | t+1 | t+2 | t+3 |
| Benchmark Model | | | | | | | |
| corr($Cre_{t+k}$, $Emp_t$) | 0.30 | 0.30 | 0.05 | −0.75 | −0.68 | −0.34 | −0.02 |
| corr($Des_{t+k}$, $Emp_t$) | −0.24 | −0.66 | −0.96 | −0.41 | −0.08 | 0.15 | 0.25 |
| corr($Cre_{t+k}$, $Des_t$) | −0.19 | −0.27 | −0.24 | −0.28 | 0.65 | 0.68 | 0.39 |
| Sticky Price | | | | | | | |
| corr($Cre_{t+k}$, $Emp_t$) | −0.05 | −0.32 | −0.99 | −0.32 | −0.02 | 0.09 | 0.13 |
| corr($Des_{t+k}$, $Emp_t$) | −0.09 | −0.32 | −0.90 | 0.12 | 0.11 | 0.11 | 0.10 |
| corr($Cre_{t+k}$, $Des_t$) | −0.11 | −0.10 | −0.12 | 0.90 | 0.33 | 0.07 | −0.03 |
| U.S. Economy | | | | | | | |
| corr($Cre_{t+k}$, $Emp_t$) | 0.27 | 0.15 | 0.04 | −0.19 | −0.58 | −0.68 | −0.60 |
| corr($Des_{t+k}$, $Emp_t$) | −0.63 | −0.65 | −0.59 | −0.35 | −0.01 | 0.29 | 0.45 |
| corr($Cre_{t+k}$, $Des_t$) | −0.39 | −0.44 | −0.47 | −0.43 | −0.14 | 0.18 | 0.34 |

model generates correlations among these variables that are in reasonable accord with the data.

## 2. Comparison with Nominal Price Stickiness

In this section, we compare our benchmark model with nominal wage stickiness with models with nominal price stickiness as in Walsh (2002) and Trigari (2004).[18]

Figure 2 shows the response of variables to one standard deviation monetary shock when nominal price stickiness (not nominal wage stickiness) is introduced into the flexible wage search model. Now the response of total output, labor, consumption and investment do not display delayed and hump shaped dynamics. Also job creation and destruction rate do not show the negative correlation observed in the data, and instead they move together very closely.

When the monetary shock hits the economy, demand for final goods increases and

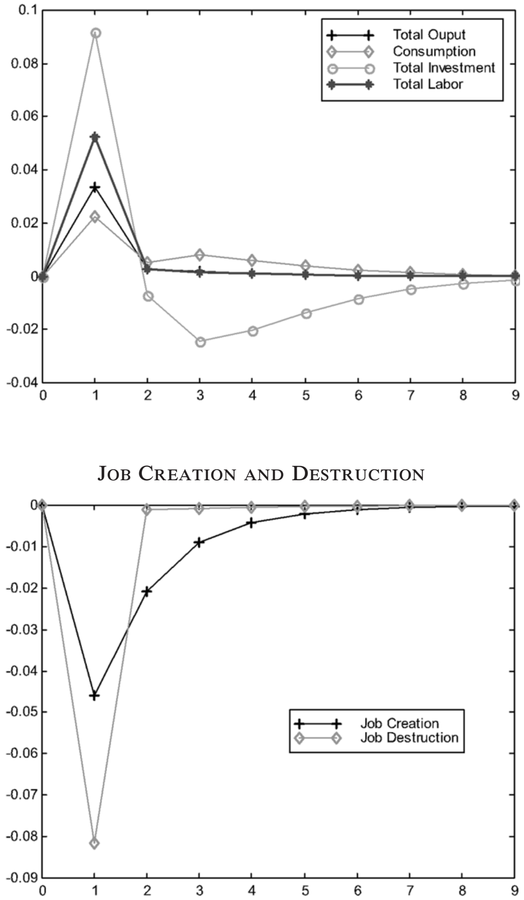
[16] Statistics for the model model economy are computed on HP-detrended data generated by simulating the model for 200 periods and repeating the simulation 100 times. The statistics are average over these 100 simulations. Statistics for the US economy are computed using HP-detrended data from 1959.1 through 1996.4.

[17] See the footnote for table1.

[18] See Walsh (2002) and Trigari (2004) for detailed setup.

FIG. 2.[19] STICKY PRICE

AGGREGATE VARIABLES




production rises. But more production is mainly done by breaking up less with existing workers rather than hiring new workers initiated by posting vacancies. That is fewer vacancies are posted expecting rapid (real) wage increase when wages are flexible. And increased demands in the first period due to nominal price stickiness are met by separating less with existing workers. Accordingly, we do not observe any delayed response of economic variables initiated by more vacancy posting. Since vacancy posting drops from the beginning due to rapid wage adjustments, total vacancies and unemployment show almost a perfect positive correlation. Accordingly job creation and destruction rates decrease together showing strong positive correlation.

[19] All variables are in log-deviation form. The shock hits the economy at 1st period.

## V. *Conclusion*

Monetary business cycle models with nominal rigidities have their weakness in producing persistent and hump shaped aggregate output responses to monetary shocks. Several ways to improve on this weakness have been suggested in the literature, but they are sometimes controversial and often ad hoc. In this paper, we investigate the possibility of obtaining realistic propagation of monetary shocks by combining labor market matching and nominal wage stickiness. We find that nominal wage stickiness and labor market matching can be important propagation mechanism in inducing realistic dynamics of aggregate output to monetary shocks as well as plausible labor market movements.

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