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PRICE COMPETITION BETWEEN ONLINE AND OFFLINE FIRMS IN AN ELECTRONIC COMMERCE MARKET AND DISCRIMINATORY TAXATION*

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Abstract

This paper formulates an equilibrium model to examine price competition in an electronic commerce market where the offline firms compete against an online firm when commodity taxes are imposed on transactions. We compare the price differential between offline and online markets at the symmetric and coexistence equilibrium. We then analyze government tax revenues, investigating the optimal tax difference between online and offline taxes. This model demonstrates that there is an optimal difference between the two taxes and thus, the equilibrium prices depend not only on the ratio of online access cost and offline transportation cost, but on tax distortion cost

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I. *Introduction*

The explosive growth in information technology (IT) has created an online purchasing environment where goods are easily purchased, causing the dramatic rise of web-based businesses or "e-commerce" (electronic commerce). According to Weisman (2000), the Internet economy has already created more than 1.2 million jobs generating more than $300 billion in revenue. In terms of user base, 100 million Americans and 20 million South Koreans had accessed to the web in 2000. In terms of an e-commerce base, there were more than 17.4 million commercial websites in 2000.

The drastic change in consumption manner and new market opportunities for business model is becoming commonplace. With the lowering of commercial barriers new debates on policy considerations have come to the forefront regarding business profitability, government

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taxation, and consumer welfare. In particular, the rapid rise in Internet sales (where Internet buyers pay no sales taxes in the US) has ignited a considerable debate over Internet taxation. Thus, one important task for the new information economy is to examine e-commerce policy issues, especially in relation to the pricing strategies of goods via online purchasing and Internet taxation.

Although e-commerce studies are still an up-and-coming research field, recent work into e-commerce has been conducted mainly through conceptual discussions and empirical approaches. Weisman (2000) and Kauffman and Walden (2001) effectively organize and review some business and economic issues to e-commerce, such as price competition between online and offline firms, content differentiation between online firms, and Internet taxation. As far as being related to price levels of online firms, Bakos (1997), Bailey (1998), and Harrington (2001) analyze the relationship between search costs and product price in online marketplaces, compare prices for products between online and offline purchases, and point out the possibilities of higher price of online products. Zettelmeyer (2000) examines firms which compete on multiple channels such as retailing and e-commerce, and indicates that how the pricing and communications strategies of the firms are affected by the size of the Internet. Dumans (2002) and Chun and Kim (2005) investigate the relative efficiency between transaction cost and delivery cost, and examine the equilibrium of the strategic game model between online and offline firms. Lee (2003) and Cho and Lee (2006) consider two competing channels between online and offline markets, analyzing the pricing strategies of the firms and their social costs in an equilibrium model determined by online business maturity.

Some empirical works have been done on the possible effects of imposing an Internet sales taxes and the relevant compliance costs on the Internet. Goolsbee (2000) examines the effect of local sales taxes on e-commerce, indicating that the local sales tax rate significantly affects an individual's choice to purchase online goods. He also demonstrates that applying existing tax rates to the Internet will reduce the number of buyers online from 20 percent to 25 percent, thereby reducing total sales by 25 percent to 30 percent. Shy (2001) indicates that people in high sales tax locations are more likely to make purchases online as the Internet matures. Thus, high sales tax states will lose greater tax revenues than low sales tax states. However, these works do not take into consideration the strategic aspects of the firms that firms can charge different prices for identical products in different channels, depending on the competitive situations they face.

As a theoretical step to examine taxation issues in the e-commerce market using the equilibrium analysis, we raise an economic question of how much government revenue will be adversely affected by e-commerce. This includes examining which level of online tax is optimal for Internet transaction at equilibrium. Using the conventional product differentiation model by Hotelling (1929), this paper considers an e-commerce market where offline firms compete against an online firm and investigates the equilibrium prices between the two.[1] We then compare the prices difference between the two markets at the symmetric and coexistence equilibrium where the commodity taxes in transactions are imposed. We show how the price equilibrium depends on offline transportation cost, online access cost, and commodity taxes. In

[1] As far as concerned the equilibrium model of e-commerce, Hotelling's linear city model is very conventional. For the reference, see Dumans (2002), Lee (2003), Kim and Chun (2005), and Cho and Lee (2006) among others.

succession, we also analyze the government tax revenues and investigate the optimal tax difference between online and offline taxes. It is shown that the optimal difference between the two taxes depends not only on the ratio of online access and offline transportation costs, but on tax distortion cost.

The organization of this paper is as follows: In the next section, policy debates are discussed regarding Internet taxation. In Section 3, using the Hotelling's linear city model, we formulate the basic e-commerce market model between offline and online firms, explore equilibrium prices, and provide some comparative statics. In Section 4, we discuss tax revenues and discriminatory Internet taxation, deriving the optimal tax difference between online and offline taxes. A conclusion is provided in the final section.

II. *Policy Debates on Internet Taxation*

The subject of Internet taxation is at the forefront of recent public policy debates over e-commerce after introducing the Internet Tax Freedom Act (ITFA) in the United States.[2] The original sponsors of the ITFA have also introduced legislation requesting the World Trade Organization (WTO) to enact a permanent global moratorium on taxation of Internet commerce.

However, with international policy developments such as the European Union's 2000 announcement to extend a value added tax to electronically transmitted goods from non-EU companies, the issues of taxing e-commerce are still being debated in the US Congress. The main controversy over taxing e-commerce is that the moratorium policy will have made all online purchases exempt from sales taxes, thus putting offline firms at a disadvantage and depriving local governments of potential revenue.

In fact, the U.S. government and its states are concerned that the movement of consumers to the web will drastically reduce their available tax base. This could amount to more than $10 billion a year, thus beginning to pressure law makers to introduce an Internet tax base.[3]

Their assertion is also strengthened by the interest of small 'Main Street' businesses that are beginning to feel the competitive pressure of online stores. Online stores are able to offer lower prices, partly because of the tax advantage. For example, *Wal-Mart* has joined with local politicians to address this inequality in the playing field. As a result, local politicians try to persuade large retailers who are apt to raise taxes on small retailers to provide services and tax breaks for Wal-Mart.

At the same time, the current debate over taxing e-commerce has placed the onus on scholars to establish why e-commerce should, or should not be taxed. The conventional 'infant industry' arguments support a moratorium on Internet taxation until the e-commerce channels

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[2] In the U.S., the ITFA (formerly known as S.442, now Title IX of P.L. 105-277, Moratorium on Internet taxes)" was enacted in 1998 and extended to 2006. The current moratorium on Internet tax includes taxes on Internet access, multiple-state taxation of a single item bought online, and discriminatory taxes that threaten Internet purchases differently than other types of sales.

[3] Goolsbee and Zittrain (1999) investigated where tax revenues currently come from and how the Internet is likely to affect them. They claimed that the tax revenue loss was $430 million in 1998, which was less than two percent of potential sales tax revenues. For a discussion on international taxation and tax administration, see Watanabe (2000).

are mature. They argue that states and governments have plenty of time to develop an efficient and useful taxation scheme for e-commerce. In addition, Goolsbee and Zittrain (1999) claim that taxation is needed to push for a moratorium on Internet taxes so that usage will proliferate across all demographic groups. This might allow the maximum benefits of the network to be realized.[4]

However, a few academics have addressed the efficiency and equity issues associated with taxing e-commerce using arguments from public finance economics. McLure (1999), for example, compares e-commerce events to the history of the mail order catalog stating that the moratorium policy on e-commerce taxation gives an unfair advantage to Internet companies. This inevitably favors industries from ever ‘growing up’. He also argues that exemptions for e-commerce, combined with the current taxation system, will lead to significant distortions that will put conventional retailers at a great disadvantage. Therefore, economic decisions will suffer gross inequalities and distortions. Local merchants will face unfair competition from out-of-state vendors who pay no sales tax.

Lukas (1999) posits whether e-commerce must be taxed to level the playing field between online and offline firms. He claims that the tax differential will merely encourage conventional ‘bricks and mortar’ retailers to move to the Internet and thus, the government should investigate harmonizing tax rates downward for local retailers, rather than imposing new taxes on the Internet to eliminate the tax differential.

Another factor to consider in taxing e-commerce is the possible compliance costs associated with different taxation policy. Goolsbee (1999) examines the costs and benefits associated with enforcing taxes on Internet commerce, including revenue loss, retail trade competition, income distribution, and external factors. For example, the average sales tax rate in the US is about 6.3%, but the sales taxes are currently imposed on approximately 30,000 different locals of local governments and are not uniform across all goods. If merchants were required to collect taxes from states where they do not have a presence and intimate knowledge of the tax code, compliance costs would be higher.[5] Thus, legislation on restructuring tax system and technology for compatible programs should help keep compliance costs from becoming unwieldy.

Some research suggests that e-commerce has already accelerated the existing downward trend in state and local taxes. It is a serious threat for the U.S. government since sales tax revenues account for almost 50 percent of all state tax revenues. This increased burden might tempt local governments to either change their fiscal structure to derive revenues from alternative sources such as property or income taxes, reducing their overall expenditures. It would be a deliberate move away from the ever-dwindling tax base in order to be less dependent on the traditional tax sources. Varian (2000), for instance, suggests eliminating

[4] For a detailed discussion about network effects in information goods, see Shapiro and Varian (1999) and Shy (2001).

[5] The taxation situation in the U.S. is complex since local taxes are utilized for local services such as paving local roads, lighting local streets, and providing police and fire station services. Extending the tax to Internet companies means that Internet companies would pay for services for which they received no benefits. It would create a reverse injustice, a situation where Internet companies pay for services consumed by their competitors. In addition, consumers who already pay taxes on their cable or ADSL/DSL phone lines that carry their Internet service would encounter double taxation. This would also further raise the costs for other consumers who are not connected. On this point, see Delaney (1999) and Mark (2003).

state and local sales taxes altogether in favor of a revenue-equivalent increase in income tax or the establishment of a consumption tax. Both scenarios would be less distortional and less awkward to implement than the existing sales taxes.

III. *An Equilibrium Model*

Following the model of Hotelling (1929), we consider a liner city of unit length where consumers are uniformly distributed on this interval. Each consumer is indexed by $x \in [0, 1]$, so x is a location from the origin.

Suppose that there are two conventional offline firms at either ends of the city. They sell the same product and compete against each other with zero marginal cost. We denote the price of the offline firm A, p_f^A, which is located at point 0, while p_f^B is for the offline firm B located at point 1. Each consumer buys one unit of the product from the offline firms by paying the price and transportation cost of τ per unit of distance. For example, a consumer located at a point x has to pay transportation cost of τx for shopping at firm A, or $\tau(1-x)$ for shopping at firm B. In addition, the consumer has to pay the commodity tax of t_f per unit product when they purchase the product from the offline firm.

In the analysis, we will focus on the market-covered case where offline market consumers should buy one good from one of the two offline firms.[6] Then, we can define the total payment of a consumer located at point x by $p_f^A + t_f + \tau x$ if a consumer buys from the offline firm A, while $p_f^B + t_f + \tau(1-x)$ if they buy from offline firm B.

On the other hand, there is a pure online firm that has no physical location and sells the same good with offline firms. The price of the online firm is denoted by p_n and the unit cost is also zero.[7]

Assume that every consumer at each location point has access to the Internet. Thus, consumers may buy goods from offline firms or from an online firm. If the consumer buys the good from the online firm, irrespective of the location point of the consumer, they incur online access cost of a, which include search cost, uncertainty cost, security cost, order tracking cost, and delivery cost.[8] Then, if a consumer buys one unit of the product from the online firms, they have to pay the total payment of $p_n + t_f + a$, where t_n is commodity tax for an online transaction.

Let x_A(or x_B) denote the consumer who is indifferent to whether they purchase from an online firm or offline firm A (or firm B) when they access the Internet. Then, from the equality that $p_f^A + t_f + \tau x = p_n + t_f + a$, we have

$$x_A = \frac{p_n - p_f^A + t_n - t_f + a}{\tau}. \tag{1}$$

Similarly, we have

[6] For full equilibrium characterization including the market-uncovered case, see Wang (2004).

[7] In order to focus on the relative magnitude between transportation cost and delivery cost, we assume that the cost of online channel is the same as that of offline channel. For a discussion on production cost of online channel in an e-commerce market, see, for example, Lynch and Ariely (2000) and Chun and Kim (2005) among others.

[8] On the discussions about the cost incurred by online shopping, see Strader and Shaw (1997), and Chun and Kim (2005).

$$x_B = 1 - \frac{p_n - p_f^B + t_n - t_f + a}{\tau}. \tag{2}$$

We will restrict our analysis into the *symmetric and coexistence* equilibrium where two offline firms set the same price at the equilibrium and both online and offline firms sell the product in the equilibrium, i.e., $0 < x_A < x_B < 1$. Let $\Delta \equiv t_n - t_f$. Then, we need the following assumption in the analysis:

$$\text{Assumption: } -\tau/2 - a < \Delta < \tau - a. \tag{3}$$

In the coexistence equilibrium, the demand functions for each firm are given by:

$$D_f^A = x_A = \frac{p_n - p_f^A + \Delta + a}{\tau},$$

$$D_f^B = 1 - x_B = \frac{p_n - p_f^B + \Delta + a}{\tau},$$

$$D_n = x_B - x_A = \frac{\tau - (2p_n - p_f^A - p_f^B + 2\Delta + 2a)}{\tau},$$

where $D_f^A + D_f^B + D_n = 1$.
Then, from the first-order conditions of the profit functions of offline firms, $\pi_f^i = p_f^i D_f^i$, $i = A, B$ and the profit function of the online firm, $\pi_n = p_n D_n$, we have the following equilibrium prices:

$$p_f^* = \frac{\tau + 2\Delta + 2a}{6} \text{ and } p_n^* = \frac{\tau - \Delta - a}{3}, \tag{4}$$

where $p_f^A = p_f^B = p_f^* > 0$ and $p_n^* > 0$.

We now compare the equilibrium price differences and examine the properties of equilibrium prices. We then investigate the changes of market demands of online and offline firms in a symmetric and coexistence equilibrium.

First, comparing the properties of equilibrium prices in (4), we show that the difference of the equilibrium prices between offline and online firms depends on the parameters in the model as follows:[9]

$$p_f^* - p_n^* = \frac{4\Delta - \tau + 4a}{6}. \tag{5}$$

A few remarks are in order. First, we have $p_f^* > p_n^*$ if $\Delta > \tau/4 - a$ while $p_f^* < p_n^*$ otherwise. Therefore, p_n^* might be greater than p_f^* when t_n is sufficiently smaller than t_f. For instance, if $\Delta = 0$, then $p_n^* > p_f^*$ when $a < \tau/4$.

Second, we have $\partial p_n^*/\partial a < 0 < \partial p_f^*/\partial a$ and thus $\partial(p_f^* - p_n^*)/\partial a > 0$. But, $\partial p_n^*/\partial \tau > \partial p_f^*/\partial \tau > 0$ and thus $\partial(p_f^* - p_n^*)/\partial \tau < 0$. It indicates that the differences of equilibrium prices are dependent on the relative cost between online access and offline transportation. In particular, as the access

[9] A lot of empirical research has analyzed the competition between offline and online markets and yielded conflicting results about the relative prices between two markets according to the product types, place an order, payment system for the product. On this point, see Brynjolfsson and Smith (1999) and Chun and Kim (2005).

cost increases, consumers prefer to buy products from offline firms and thus, the equilibrium price of offline firms increases while the equilibrium price of online firm decreases. However, the equilibrium prices of both online and offline firms increase as transportation cost increases, and the upward pressure is strong on the online firm. This is so because, as the transportation cost increases, the offline firm comes under pressure to decrease its price because of the competition effect between online firm, while it can take product differentiation effect between the other offline firm. Hence, the equilibrium price is affected by both product differentiation effect within offline market and competition effect between online market.

Finally, we have $\partial p_f^*/\partial t_f < 0 < \partial p_n^*/\partial t_f$ and $\partial p_n^*/\partial t_n < 0 < \partial p_f^*/\partial t_n$. It demonstrates that taxation negatively affects its own price yet positively affects the price of the counterpart firm. For example, the online tax will reduce the price of the online firm, but will raise the price of offline firms.

Next, we investigate the changes of market demands in e-commerce market competition. From the equilibrium prices in (4), we have

$$x_A^* = 1 - x_B^* = \frac{2\Delta + \tau + 2a}{6\tau}, \tag{6}$$

where $0 < x_A^* < 1/2$ from the assumption in (3). Thus, in a symmetric and coexistence equilibrium, we have the equilibrium market demand of offline firm, $D_f^* = x_A^*$, and that of online firm, $D_n^* = 1 - 2x_A^*$.

A few remarks are needed for clarification. First, $D_f^* > D_n^*$ if $x_A^* > 1/3$ or, equivalently, $\Delta > \tau/2 - a$ while $D_f^* < D_n^*$ otherwise. Therefore, in a symmetric and coexistence equilibrium, D_f^* might be greater than D_n^* when t_n is sufficiently smaller than t_f .

Proposition 1. The equilibrium prices and the corresponding demands depend on access costs, transportation costs, and tax rates.
(i) If $\tau/2 - a < \Delta < \tau - a$, then $D_f^* > D_n^*$ and $p_f^* > p_n^*$.
(ii) If $\tau/4 - a < \Delta < \tau/2 - a$, then $D_f^* < D_n^*$ and $p_f^* > p_n^*$.
(iii) If $-\tau/4 - a < \Delta < \tau/4 - a$, then $D_f^* < D_n^*$ and $p_f^* < p_n^*$.

Second, we have $\partial x_A^*/\partial a > 0 > \partial x_A^*/\partial \tau$ and thus, $\partial D_f^*/\partial a > 0 > \partial D_f^*/\partial \tau$ or $\partial D_n^*/\partial a < 0 < \partial D_n^*/\partial \tau$. This implies that as the access cost increases or as transportation cost decreases, the market size of offline firms increases while that of online firm decreases.

Finally, we have $\partial x_A^*/\partial t_n > 0 > \partial x_A^*/\partial t_f$ and thus, $\partial D_f^*/\partial t_n > 0 > \partial D_f^*/\partial t_f$ or $\partial D_n^*/\partial t_n < 0 < \partial D_n^*/\partial t_f$. This implies that as the online commodity tax decreases or as the offline commodity tax increases, the market size of the offline firms decreases while that of online firm increases.[10]

IV. *Optimal Discriminatory Taxation*

We now examine the tax revenues which are dependent on the difference between offline and online taxes. From there, the optimal tax difference, specifically optimal online tax rate

[10] Goolsbee (2000) indicates that local sales tax rates affect an individual's choice to purchase online goods in a positive manner. Controlling for observable characteristics, people living in high sales tax locations are significantly more likely to buy online.

when the offline tax rate is given exogenously, is derived.

Let me first examine the tax revenues which depend on the tax rate difference between online and offline taxes in the model. Then, the tax revenues for a single product at equilibrium are as follows:

$$G=G_f+G_n \tag{7}$$

where

$$G_f=2t_f D_f^*=2t_f x_A^*=t_f(2\Delta+\tau+2a)/3\tau$$

and

$$G_n=t_n D_n^*=t_n(1-2x_A^*)=2t_n(\tau-\Delta-a)/3\tau.$$

A few remarks on comparative statics are in order. First, $\partial G_f/\partial\tau>0$ and $\partial G_n/\partial\tau<0$ if $\Delta>-a$, while $\partial G_f/\partial\tau<0$ and $\partial G_n/\partial\tau>0$ otherwise. Thus, if $\Delta\geq 0$, tax revenues from offline firms increase while tax revenues from online firms decrease as transportation cost increases. However, the change of total tax revenues, G, with respect to transportation costs depends on the sign of $\Delta(\Delta+a)$. Therefore, $\partial G/\partial\tau>0$ if $\Delta>0$, while $\partial G/\partial\tau<0$ if $\Delta<0$.

Second, $\partial G_f/\partial a>0$ and $\partial G_n/\partial a<0$. This implies that tax revenues from offline firms increase while tax revenues from online firms decrease as access cost increases. However, the change of total tax revenues depends on the sign of $-\Delta$. Therefore, $\partial G/\partial a<0$ if $\Delta>0$, while $\partial G/\partial a>0$ if $\Delta<0$.

Third, $\partial G_f/\partial t_f\geq 0$ if $\Delta\geq t_f-\tau/2-a$ and $\partial G_f/\partial t_f\leq 0$ if $\Delta\leq t_f-\tau/2-a$, while $\partial G_n/\partial t_f>0$. Thus, if the offline tax is sufficiently low, under the assumption in (3), then total tax revenues increase as the offline tax rate increases. However, if offline tax is sufficiently large, then total tax revenues decrease as offline tax increases. In particular, there is a threshold for the change of tax revenues, $-(\tau+2a)/4$, below which tax revenues decrease as Δ decreases (or t_f increases), while tax revenues increase as Δ decreases above that threshold.

Finally, $\partial G_n/\partial t_n\geq 0$ if $\Delta<\tau-a-t_n$ and $\partial G_n/\partial t_n\leq 0$ if $\Delta\geq\tau-a-t_n$, while $\partial G_f/\partial t_n>0$. Thus, if the online tax is sufficiently low, under the assumption in (3), then total tax revenues increase as the online tax rate increases. However, if the online tax is sufficiently large, then the total tax revenues decrease as the online tax increases. In particular, there is a threshold for the change of tax revenues, $(\tau-a)/2$, below which tax revenues increase as Δ increases (or t_n increases), while tax revenues decrease as Δ increases above that threshold.

The optimal online tax rate which should be imposed on the online commodity to minimize economic inefficiency is next examined. In particular, based on the partial equilibrium analysis, we will find an optimal online tax rate which maximizes the social welfare and compare it to the tax rate on offline commodity. In so doing, we will measure the appropriate tax difference between online and offline taxes, providing the rationale why the government should keep the differentiated tax system between online and offline transactions.[11]

Define the social welfare in the online and offline markets as the sum of consumer surplus, profits, and tax revenues, which is raised from the questioned product, G, as follows:

[11] Note that the optimal tax theory does not necessarily call for the rates to be equal within the two types of commerce.

$$W = CS + \Pi + (1-\alpha)G, \tag{8}$$

where α is the tax distortion rate as an opportunity cost for the public funds, where $0<\alpha<1$,[12] $\Pi = \pi_n + \pi_f^A + \pi_f^B$ is total sum of firms profits, and CS is consumers surplus incurring from online and offline shopping, which is defined as

$$CS = \int_0^{x_A} (V - p_f^A - t_f - \tau s) ds + \int_{x_B}^{1} (V - p_f^B - t_f - \tau(1-s)) ds$$
$$+ (V - p_n - t_n - a)(x_B - x_A).$$

Then, the optimal online tax, t_n^*, which maximizes the above social welfare function, is derived from the following first-order condition:

$$\frac{\partial CS}{\partial t_n} + \frac{\partial \Pi}{\partial t_n} + (1-\alpha)\frac{\partial G}{\partial t_n} = 0,$$

where

$$\frac{\partial CS}{\partial t_n} = -\int_0^{x_A} \frac{\partial p_f^A}{\partial t_n} ds + \frac{\partial x_A}{\partial t_n}(V - p_f^A - t_f - \tau x_A)$$
$$-\int_{x_B}^{1} \frac{\partial p_f^B}{\partial t_n} ds + \frac{\partial x_B}{\partial t_n}(V - p_f^B - t_f - \tau(1-x_B))$$
$$-\left(\frac{\partial p_n}{\partial t_n} + 1\right)(x_B - x_A) + (V - p_n - t_n - a)\left(\frac{\partial x_B}{\partial t_n} - \frac{\partial x_A}{\partial t_n}\right)$$

$$\frac{\partial \Pi}{\partial t_n} = p_n \frac{\partial D_n}{\partial t_n} + D_n \frac{\partial p_n}{\partial t_n} + \sum_{i=A,B}\left(p_f^i \frac{\partial D_f^i}{\partial t_n} + D_f^i \frac{\partial p_f^i}{\partial t_n}\right)$$

$$\frac{\partial G}{\partial t_n} = (t_n - t_f)\left(\frac{\partial x_B}{\partial t_n} - \frac{\partial x_A}{\partial t_n}\right) + (x_B - x_A).$$

Using the results of equilibrium in (4) and (6), where $\frac{\partial x_A^*}{\partial t_n} = -\frac{\partial x_B^*}{\partial t_n}$, and rearranging this yield[13]

$$0 = -2\frac{\partial x_A^*}{\partial t_n}(t_n - t_f + a - \tau x_A^*) + (1 - 2x_A^*) - (1-\alpha)\left(1 - 2x_A^* - 2(t_n - t_f)\frac{\partial x_A^*}{\partial t_n}\right). \tag{9}$$

Then, substituting the result of (6) into (9), we can derive the following optimal online tax:

$$t_n^* = t_f + k_1\tau - k_2 a, \tag{10}$$

where

[12] Notice that tax revenues give less negative effect on the social welfare when α is small. In particular, if $\alpha=0$, $W=CS+\Pi+G$ and thus, tax does not affect the social welfare. For the other extreme case of $\alpha=1$, where $W=CS+\Pi$, tax reduces the social welfare with the same amount of tax revenues.

[13] Notice that the second-order condition is satisfied, i.e., $\partial^2 W/\partial t_n^2 < 0$

$$k_1 = \frac{6\alpha+1}{2(6\alpha-1)} \text{ and } k_2 = \frac{3\alpha+2}{6\alpha-1}.$$

Thus we know that (i) if $0<\alpha<1/6$, then $k_2<k_1<0$ and (ii) $1/6<\alpha<1$, then $k_2>k_1>0$.

In sum, from (10), the optimal online tax might be greater or less than offline tax depending not only on the relative size between transportation cost and access cost, but also on the sizes of k_1 and k_2. Specifically, the optimal difference of the two taxes is as follows:

$$\Delta^* \equiv t_n^* - t_f \gtreqless 0 \text{ if } \frac{\tau(6\alpha+1)}{2(6\alpha-1)} \gtrless \frac{(3\alpha+2)a}{6\alpha-1}.$$

Proposition 2. The optimal difference between online and offline taxes depends not only on the ratio of access cost and transportation cost, a/τ, but also on tax distortion cost, α.

(i) If $0<\alpha<1/6$, then $\Delta^* \gtreqless 0$ if $\dfrac{6\alpha+1}{6\alpha+4} \lesseqgtr a/\tau$.

(ii) If $1/6<\alpha<1$, then $\Delta^* \gtreqless 0$ if $\dfrac{6\alpha+1}{6\alpha+4} \gtreqless a/\tau$.

Proposition 2 implies that if $0<\alpha<1/6$, then $\Delta^*>0$ when $2/5<a/\tau$. However, if $0<a/\tau<2/5$, the optimal tax difference depends on the size of α. In particular, $\Delta^*>0$ if $0<\alpha<(4a-\tau)/6(\tau-a)$ while $\Delta^*<0$ if $(4a-\tau)/6(\tau-a)<\alpha<1/6$. Therefore, when tax distortion cost is low, the optimal condition to have a higher online tax rate than offline tax rate at equilibrium is that the ratio of access cost and transportation cost should be high.

The economic intuition of this condition is clear when α is sufficiently low, i.e., tax distortion rate is small and thus, imposing tax contributes to the social welfare positively. Then, higher online tax, not only reducing online transaction but also increasing tax revenues, is beneficial to the society when the ratio of access cost and transportation cost is high. Put differently, when access cost is sufficiently greater than transportation cost, it is welfare-improving to discourage online shopping and to encourage offline shopping by imposing higher online tax. Otherwise, lower online tax rate than offline tax rate is beneficial to the society.

On the other hand, if $1/6<\alpha<1$, then $\Delta^*<0$ when $a/\tau>1$ while $\Delta^*>0$ when $0<a/\tau>2/5$. However, if $2/5<a/\tau<1$, the optimal tax difference depends on the size of α. In particular, $\Delta^*<0$ if $1/6<\alpha<(4a-\tau)/6(\tau-a)$ while $\Delta^*>0$ if $(4a-\tau)/6(\tau-a)<\alpha<1$. Therefore, when tax distortion cost is high, the optimal condition for a higher online tax rate than offline tax rate at equilibrium is that the ratio of access cost and transportation cost is sufficiently low, while the optimal condition for a lower online tax rate than offline tax rate at equilibrium is that the ratio of access cost and transportation cost is sufficiently high.

The economic explanations of the case that α is high should focus on the strong effect of tax revenues on the social welfare. That is, the size of tax revenues matters to the society when $1/6<\alpha<1$, and thus, tax should be placed to the balanced level between the two markets at the equilibrium.[14] For example, when the ratio of access cost and transportation cost is sufficiently low, higher online tax rate than offline tax rate at equilibrium should be imposed to balance tax revenues between the two markets. Otherwise, tax revenues will be drastically reduced since

[14] We can reconsider the social welfare function in (8) as the problem of maximizing W subject to $G \leq \bar{G}$, where $\bar{G}$ is the tax revenues requirement, which is given exogenously. Then, the size of tax revenues should be binding at the optimum, i.e., $G^* = \bar{G}$.

many consumers will purchase online goods. On the contrary, when the ratio of access cost and transportation cost is sufficiently high, lower online tax rate than offline tax rate at equilibrium should be imposed to balance tax revenues between the two markets. Otherwise, tax revenues will be drastically increased since many consumers will purchase offline goods.

V. *Concluding Remarks*

This paper has considered an e-commerce market and examined the equilibrium prices of online and offline firms when commodity taxes on transactions are imposed. We then compared price differences between online and offline markets at the symmetric and coexistence equilibrium.

We demonstrated that the equilibrium prices depend on the online access cost, offline transportation cost, and taxes rates. After accounting for the government tax revenues, we investigated the optimal tax difference between online and offline taxes. It is indicated that the optimal tax difference depends not only on the ratio of online access cost and offline transportation cost, but on tax distortion cost. We found that when tax distortion cost is sufficiently low, the optimal condition to have a higher online tax than offline tax is that the ratio of access cost and transportation cost is sufficiently high. However, when tax distortion cost is sufficiently high, tax revenues effect is strong to the social welfare and thus, tax should be placed to the balanced level between the two markets at the equilibrium. Specifically, the optimal condition (*i*) for a higher online tax than offline tax is that the ratio of access cost and transportation cost is sufficiently low; and (*ii*) for a lower online tax than offline tax is that the ratio of access cost and transportation cost is sufficiently high.

Although the results in this article are based on a model attempting to characterize the price equilibrium and optimal taxation on the Internet, the findings should be applied to other business and policy perspectives. First, we can observe the recent trend in the e-commerce market where offline firms enter online markets as a form of a hybrid firm. This might occur due to offline firms taking advantage of the transportation cost savings from online market through multiple channels. In this sense, it is worthwhile to extend the model where offline firms operate not only in offline markets, but also in online markets. Second, it is an important aspect to investigate some dynamic issues of Internet growth. For future research, it is needed to analyze the dynamic interactions between the maturity or growth of online market and online taxation.

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