Hitotsubashi Journal of Economics 46 (2005), pp.41-63. © Hitotsubashi University

DIRECT SALES OF FINANCIAL INFORMATION BY A MONOPOLISTIC SELLER*

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Received August 2003; Accepted November 2004

Abstract

I refer to as a direct sale of financial information any case in which the end users of the information get to observe the information before they decide to act on it. This paper investigates the conditions under which a possessor of valuable information may prefer to sell her information instead of trading on it. The sale of information has an important commitment effect in that it credibly commits a risk neutral possessor of information to a strategy which promotes more intense competition among informed traders in the market and makes the trading strategies of other informed traders less aggressive. It is this strategic externality that makes the selling of information an optimal strategy. The model in this paper shows that if the security price does not fully reflect the private information of all the traders, diluting the seller's information before selling it is not optimal even if the seller trades on her own account while selling her information. The price of information in equilibrium is such that privately informed traders never find it optimal to purchase additional information from the seller. It is also shown that if the information seller is more risk averse than her clients, then she finds it optimal to commit not to trade on the basis of her information.

Keywords: Information sales,Strategic trading, Strategic substitution, Risk allocation
JEL classification: D82; G104

I. *Introduction*

Participants in financial markets typically have access to a wide variety of financial services whose providers claim to help clients achieve better results from their trading activities. Examples of such services for sale include investment newsletters, private banking services, and financial consultants' and brokers' advice. The gamut of these financial services may be thought of as constituting an information market. This paper focuses on one element

* I thank Professor Sugato Bhattacharyya for his helpful comments from the inception of idea of this paper. I also appreciate Professors Rick Green, Duane Seppi, Bart Lipman, Murugappa Krishnan and an anonymous referee for their constructive suggestions. Financial support from William Larimer Mellon Fund, Margaret and Richard M. Cyert Family Funds, and Sloan Foundation are gratefully acknowledged. The usual disclaimer applies.

of this market-the direct sale of financial information; the sale of information by a mechanism like the subscription to an investment newsletter of limited circulation.

I refer to direct sale of information as any case in which the end user of the information gets to observe the information before he decides to act on it. Thus, delegated portfolio management through a mutual fund manager for a fee is not covered by the definition of a direct sale mechanism.[1] The typical question that confronts an information seller in a financial market is why she has to resort to the sale of information if she could directly trade on it herself and, presumably, make greater profits thereby. This paper investigates the conditions under which a possessor of valuable information may prefer to sell her information instead of trading on her own account. In addition, this paper explores the nature of the optimal sales strategy under different structures and the effect of sales of information on the welfare of other market participants.

First, it is established that for a risk neutral possessor of information who has monopolistic access to information about a financial security, it is more profitable to trade on the information than to sell it. This conclusion is based on the assumption that trading in securities can be achieved in an anonymous fashion. Once the strong assumption of monopolistic access to information is removed, it may no longer be optimal for an information possessor to abjure the direct sale of her information. In fact, the sale of information to clients who will then optimally use the information to decide their trading strategies has important effects on the nature of trading in the financial markets; this gives rise to incentives to sell information rather than trade on it directly. Such a sale of information has important commitment effects in that it credibly commits the information possessor to a strategy that would not be credible if she were to avoid such sales. Intuitively, the sale of information to a number of clients provides for more intense competition in the financial market. While this does reduce the total profits available to informed traders as a group, it also has the effect of making the trading strategies of other information-based traders less aggressive. As a result, although the reduction in overall profits from trading may be substantial, the individual seller of information does not bear the full cost of the reduction in overall profits. This strategic externality may make the selling of information an optimal strategy for the possessor of information.

The issue of selling information in the context of financial markets has been analyzed by Admati & Pfleiderer (1986, 1988) in two papers. In Admati & Pfleiderer (1986), they show that in a competitive rational expectations setup, the optimal way to sell information is to make it coarser by means of adding '*personalized noise*' to the information. This addition of noise prevents full revelation of information by the market price in the rational expectations equilibrium, and thus preserves the value of private information. In Admati & Pfleiderer (1988), they show that it may be optimal for a monopolistic risk averse information possessor to sell her information in order to achieve better risk sharing.

The analysis in this paper yields results that are different from those of Admati and Pfleiderer, due to the use of a strategic model of financial market trading. In such a model, given the specified sequence of moves on the part of various players, the final price never reveals the information in full as in a competitive rational expectations framework, and it may indeed be optimal to sell information even without appealing to risk sharing considerations. In particular, this paper also shows that it is optimal for the seller never to dilute her information

[1] Please refer to Bhattacharya & Pfleiderer(1985) for the analysis of delegated portfolio management.

by the addition of noise, whether '*personalized*' or not, even if she trades on her own account as well. Thus, the results in this paper mitigate the objection that the optimal strategy established may be illegal due to discrimination amongst customers. In fact, given the moral hazard problems typically associated with the sale of financial information for trading, it is likely that the sale of information with added '*personalized*' noise is indistinguishable from the sale of spurious information. While the moral hazard problems are not directly focused, in this model all buyers of information are allowed to check that the information seller treated them equally well.

Fishman & Hagerty (1995) and Sabino (1993) also investigate the incentive for the sales of information. There are two major differences between Fishman & Hagerty (1995) and this paper. Firstly, in Fishman & Hagerty (1995), the sequence of the game is as follows: (i) the market maker chooses the price schedule; (ii) given the price schedule, information seller decides the optimal information selling strategy; (iii) trading of securities commences. In this sequence of game, no matter what sales strategy is taken by the information seller, the price schedule chosen by the market maker does not change and consequently market liquidity is not affected by the sale of information. However, in this paper and Sabino (1993), the first two stages are reversed, and considering the effect of information sales on the market liquidity, the information seller optimally decides the strategy of information sales. As will be shown in the following section, the condition for the information sales is not affected by the sequence of the game, but different implications on the welfare of liquidity traders are derived. Secondly, and more importantly, the model in this paper has fewer restriction than Fishman & Hagerty (1995) and Sabino (1993): the information seller is allowed to dilute her information before sales and she can also trade on her account while selling her information. In addition, other information-based traders can purchase information from the seller and enhance the precision of their information. This paper demonstrates that diluting the seller's information before selling it is not optimal even if the seller trades on her own account while selling her information, and the price of information in equilibrium is such that privately informed traders never find it optimal to purchase additional information from the seller. Therefore, the results derived in this paper are a lot stronger than those from Fishman & Hagerty (1995) and Sabino (1993).

The model is extended to the case where the information seller and her clients are risk averse. In addition to the strategic externality that can be caused by the sales of information, the seller can achieve better risk sharing through the sales of information. Since the value of information is higher to the less risk averse trader, if the information seller is more risk averse than her clients, then she finds it optimal to commit not to trade on her own account. Therefore, the sales of information has the function of allocating information to the traders who value most.

As mentioned above, this paper does not deal with the moral hazard aspect of the sale of financial information. That issue is the focus of a paper by Allen (1990), in which he shows that in order to convince the buyer of the veracity of the information, the seller may have to make her own wealth contingent on the price outcome of the security about which she claims to have private information. The model in this paper, on the other hand, is based on the assumption that the buyers of information can costlessly verify whether the seller has engaged in adequate information gathering or not, although the precise outcome of the investigative process is not directly observable to the buyer. The analysis in this paper is related to that of

Kamien & Tauman (1986), in which the single patent holder of a cost reducing innovation in a product market finds it optimal to license unless he is a monopolist in the product market. Kane & Marks (1990) and Brennan & Chordia (1993) compare direct sales of information to other methods of indirect sales of information. Kane & Marks (1990) shows that in the presence of borrowing constraints, investors prefer direct sales of information. In Brennan & Chordia (1993), different ways to charge customers of information sales are compared.

The rest of this paper is organized in four sections. Section 2 presents the basic model of the financial market which will be used throughout the paper. The model is an adaptation of the model in Kyle (1985), and the condition for the sales of information is derived. The basic model introduced in Section 2 will be generalized in Section 3. The case of risk averse information seller and her clients are analyzed in Section 4. Section 5 discusses directions for future research and conclusions. Most of the proofs are presented in Appendix.

II. *Basic model*

A single risky security is traded in a financial market. The ex post payoff of this security, denoted $\tilde{v}$, is normally distributed with mean $\bar{v}$ and precision (inverse of variance) of h_v. All participants in the financial market are assumed to be risk neutral. A monopolistic *information seller* has costless access to a private observation of $\tilde{v}$ without any noise. The information seller can either trade on her own account or sell her information, but she is not allowed to do both.

There are *n speculators*, who are not allowed to buy information from the information seller. Any trader who trades on the information obtained by studying the market by himself is termed a speculator. Arbitrageurs and fund managers working for brokerage firms and investment banks, and even insiders, are included in this group. The speculators' information is less precise that the information seller's in that each speculator i privately observes the noisy signal of $\tilde{v}$, $\tilde{v}+\tilde{\eta}_i$, $i = 1,2,\ldots, n$. It is assumed that $\{\tilde{\eta}_i\}_{i=1,2,\ldots,n}$ are mutually independent normal random variables independent of $\tilde{v}$ with mean 0 and precision h_η.

Let $\tilde{u}$ denote the net market order of liquidity traders, where $\tilde{u}$ is normally distributed with mean 0 and precision h_u, and independent of $\tilde{v}$ and $\{\tilde{\eta}_i\}_{i=1,2,\ldots,n}$'s. Liquidity traders buy or sell a certain number of shares for exogenous reasons such as immediate consumption or tax purposes. Institutional investors such as insurance companies and pension funds which have stochastic influx and outflow of their assets can be included in this group. Even if they have information about the payoff of the security, this may not affect the size of their trading orders.

Sufficiently many *outside investors* have neither liquidity demand nor any information about $\tilde{v}$, and they are potential clients of the information seller. Finally, a competitive *market maker* takes the net trading orders that clear the market, and determines the price of the security. The number of speculators and the statistical properties of $\tilde{v}$, $\{\tilde{\eta}_i\}_{i=1,2,\ldots,n}$ and $\tilde{u}$ are common knowledge.

FIG. 1 TIMELINE

- Information seller announces the price of the information, and it is sold to her clients.
- Informed traders observe realization of signal on $\tilde{v}$.
- Market makers announce price schedules.
- Liquidity demands are realized, and traders submit market orders.
- Price at time t is set by the market maker.
- Market observes $\tilde{v}$.

The sequence of trading is as follows. In period zero, information about $\tilde{v}$ may be sold to outside investors. In period one, trading takes place in the market. In the last period, $\tilde{v}$ is publicly revealed, and payoff of the security is given to security holders. Sequence of trading is given in Figure (1).

If the information seller decides to sell her information, then in period zero, she announces the price and the precision of the information to be sold. For simplicity, it is assumed that the information seller is restricted to sell her information 'as is' without adding any noise, which will be relaxed in the next section. Outside investors who choose to become clients of the information seller pay for the service, and then they privately observe the information. Subsequently, they base their trading strategies on the purchased information.

Since the information seller is a monopolist, and there are sufficiently many potential clients for each price and precision of the information, the equilibrium number of clients is uniquely determined such that the expected trading profit of each client equals the price that he pays for the information. The clients of the information seller purchase the unrestricted use of the seller's information in trading securities, but resale of the information is assumed to be prohibited.[2] The incentive problems of the information seller will not be discussed in this paper, and it is assumed that if the information is sold, it is communicated truthfully. The price and the precision of information sold by the information seller are all common knowledge, and the equilibrium number of clients are correctly inferred by all the market participants.[3]

[2] While information sellers are likely to be established financial institutions with reputation and long-term relationship with their current and future customers, clients of information seller tend to be general investors who cannot credibly convince other investors of the quality of the information they try to resell. Since the analysis of this paper is based on one-period model in which the information is assumed to be short-lived, in addition to the clients' lack of credibility, it is conceivable that information buyers cannot have enough time to resell their purchased information to other investors before trading begins.

[3] As is analyzed in the remainder of this paper, the price of the information and the total profit earned by the seller crucially depends on the number of information buyers. The assumption that the equilibrium number of clients is correctly inferred by all market participants is critical for the existence of equilibrium in this model. Suppose the seller sets the price of the information claiming that it would be sold to K clients. If the seller is the

In period one, a competitive market maker announces the price schedule on the basis of all the available public information. Then, traders place their market orders to the market maker, who takes the aggregate net trading order to clear the market and sets the price such that he expects to earn zero profits. The market maker is assumed to observe only the aggregate net trading order, denoted $\tilde{y}$, and not the individual trading orders submitted. As in Kyle (1985), the price schedule set by the market maker satisfies the following equation thanks to the zero expected profits condition induced by the competition on the trading floor:

$$P = \bar{v} + \lambda\tilde{y} = E[\tilde{v}\,|\,\tilde{y}]. \tag{1}$$

Both speculators and clients of the information seller are information-based traders who trade on their private information to earn trading profits. λ is a measure of market liquidity and it represents how sensitively price moves as net trading order submitted to the market maker changes. The equilibrium λ is determined by the number of two different types of information-based traders and the precision of their information. The information seller is a leader of this trading game in that she is able to affect the equilibrium λ, and consequently influence the trading strategy of all the information-based traders. The instrument she uses to do this is the price that she announces for access to her information.

The model presented in this section is a modification of Kyle (1985), and it has a couple of important characteristics, which lead to the results of this paper. First, in this model, the seller's information cannot be leaked to non-clients before trading commences by being reflected in the price of the risky security. When traders place their market orders, they only observe the price schedule, not the actual price. Therefore, they decide the size of their trading orders only on the basis of their own information or their liquidity demand, taking into account the effects of their trading orders on the price.

Second, the price cannot fully reflect all traders' private information, and the market can never collapse due to the presence of the traders with perfect information. This is because the market maker is not able to distinguish random liquidity demand separately from trading orders of other traders who trade on the basis of their information.

In the following analysis, the factors that make the monopolistic information seller choose to sell her information, rather than keeping it to herself and trading on it, are going to be investigated. When selling information is desirable, optimal sale strategy is characterized.

Suppose the seller determines the price to charge for access to her information such that m outside investors pay the price for it, and become her clients. Given the number of speculators and the precision of their information, let $\Pi(m\,|\,[n, h_\eta])$ denote the expected trading profit of each individual client. Since the seller is a monopolist in the market for information, and there are sufficiently many outside investors who are potential clients of the information seller, the seller is able to charge the price of information such that she can fully extract the expected trading profit to be earned by her clients. Thus, $\Pi(m\,|\,[n, h_\eta])$ is also the

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only one who correctly knows the number of information buyers, then she has an incentive to sell her information secretly to additional clients on the side after the first round of sales. This obviously debases the value of information to the clients who purchase the information in the first round, and being aware of this possibility, clients would refuse to pay the price set by the seller and the equilibrium cannot be sustained. Therefore, market participants' ability to correctly infer the number of information buyers is a critical condition for the existence of the equilibrium of this model. This type of cheating on the part of the seller is less likely to happen if the seller is established financial institution whose reputation and long-term relationship with its customers are at stake.

price of the information, and the seller's total profit from information sales is $m\Pi(m\,|\,[n, h_\eta])$. Her problem is equivalent to choosing the optimal number of clients, denoted m^*, to maximize her total profit $m\Pi(m\,|\,[n, h_\eta])$.[4] Equilibrium is derived by backward induction, and following lemma presents the equilibrium in trading stage.

Lemma 1 *1. Given the price schedule $P(\tilde{y}) = \bar{v}+\lambda\tilde{y}$, the information seller's clients and speculators place the following market orders respectively.*

$$\frac{h_\eta+2h_v}{\lambda[(m+1)(h_\eta+2h_v)+nh_\eta]}(\tilde{v}-\bar{v}). \tag{2}$$

$$\frac{h_\eta}{\lambda[(m+1)(h_\eta+2h_v)+nh_\eta)]}(\tilde{v}+\tilde{\eta}_i-\bar{v}) \qquad \text{i} = 1,2,\ldots, n \tag{3}$$

2. The equilibrium λ is

$$\lambda = \sqrt{h_u}\sqrt{h_v}\frac{\sqrt{m(h_\eta+2h_v)^2+nh_\eta(h_\eta+h_v)}}{(m+1)(h_\eta+2h_v)+nh_\eta}. \tag{4}$$

3. The total profit that the information seller expects to earn is

$$\Pi^*[m|(n, h_\eta)] = \frac{m(h_\eta+2h_v)^2}{\lambda[(m+1)(h_\eta+2)+nh_\eta]^2}$$

$$= \frac{1}{\sqrt{h_u}\sqrt{h_v}}\frac{m(h_\eta+2h_v)^2}{\sqrt{m(h_\eta+2h_v)^2+nh_\eta(h_\eta+h_v)}[(m+1)(h_\eta+2h_v)+nh_\eta]}. \tag{5}$$

Proof: Suppose information buyers and speculators are believed to take trading strategies of $\alpha(\tilde{v}-\bar{v})$ and $\beta(\tilde{v}+\tilde{\eta}_i-\bar{v})$ respectively. Given $P = \bar{v}+\lambda\tilde{y}$, $\tilde{v}$ and trading strategies of other buyers and speculators, an infomration buyer determines his optimal trading strategy by solving following optimization problem.

$$\max_x E[x(\tilde{v}-\bar{v}-\lambda(x+(m-1)\alpha(\tilde{v}-\bar{v})+\sum_{i=1}^{n}\beta(\tilde{v}+\tilde{\eta}_i-\bar{v})+u))\,|\,\tilde{v}]$$

The first order condition yields

$$x^* = \frac{\tilde{v}-\bar{v}}{2\lambda}[1-\lambda(m-1)\alpha-\lambda n\beta]$$

For the consistent belief to hold in equilibrium, $x^* = \alpha(\tilde{v}-\bar{v})$ should be satisfied, and following equality is obtained from it.

$$1 = \lambda(m+1)\alpha+\lambda n\beta \tag{6}$$

Similarly, a speculator's optimal trading strategy is derived from following maximization problem taking $P = \bar{v}+\lambda\tilde{y}$, $\tilde{v}+\tilde{\eta}_i$ and trading strategies of other buyers and speculators.

$$\max_z E[z(\tilde{v}-\bar{v}-\lambda(z+m\alpha(\tilde{v}-\bar{v})+\sum_{j\neq i}^{n}\beta(\tilde{v}+\tilde{\eta}_j-\bar{v})+u))\,|\,\tilde{v}+\tilde{\eta}_i]$$

[4] In this paper only $m \geq 1$ are considered, and for simplicity, the integer problem is ignored.

The first order condition yields

$$z^* = \frac{\tilde{v}+\tilde{\eta}_i-\bar{v}}{2\lambda}\left[\frac{h_\eta}{h_\eta+h_v} - \lambda m\alpha \frac{h_\eta}{h_\eta+h_v} - \lambda(n-1)\beta\frac{h_\eta}{h_\eta+h_v}\right]$$

In equilibrium, $z^* = \beta(\tilde{v}+\tilde{\eta}_i-\bar{v})$ should be satisfied, and following equality is obtained from it.

$$\frac{h_\eta}{h_\eta+h_v} = \lambda m\alpha\frac{h_\eta}{h_\eta+h_v} + \lambda\beta\frac{(n+1)h_\eta+2h_v}{h_\eta+h_v} \tag{7}$$

By solving simulataneous equations of (6) and (7), equilibrium trading strategies in equations (2) and (3) are obtained. Expected trading profit earned by each informed trader is

$$\Pi[m\,|\,(m, h_\eta)] = E[\alpha(\tilde{v}-\bar{v})(\tilde{v}-\bar{v}-\lambda(m\alpha(\tilde{v}-\bar{v})+\sum_{i=1}^{n}\beta(\tilde{v}+\tilde{\eta}_i-\bar{v})+u))]$$

which is given in equation (5). From equation (1) and $\tilde{y} = m\alpha(\tilde{v}-\bar{v})+\Sigma_{i=1}^{n}\beta\ (\tilde{v}+\tilde{\eta}_i-\bar{v})+u$, the price schedule satisfies

$$\begin{aligned} P = E[\tilde{v}\,|\,\tilde{y}] &= \bar{v}+\lambda\tilde{y} \\ &= \bar{v}+\frac{Cov(\tilde{v},\tilde{y})}{Var(\tilde{y})}\tilde{y} \end{aligned}$$

and from equations (2) and (3), equilibrium λ in equation (4) is derived. ■

Note that either $n = 0$ or $h_\eta = 0$ means that there are no speculators in the market, and that $m = 1$ is equivalent to information seller choosing to keep her information to herself and trade on it rather than sell it to outside investors. As a leader in this game in period zero, given the number of speculators and the precision of their information, the information seller determines the optimal number of her clients to maximize her profit, taking into account the price schedule to be set by the market maker in period one.

Proposition 1 below demonstrates that the seller's decision to sell her information or trade on it depends on whether she has monopolistic access to the information about $\tilde{v}$ or not.

Proposition 1 *1. A monopolistic seller of information obtains the highest profit by trading on her information instead of selling it if she also has monopolistic access to information.*

2. A monopolistic seller of information is able to obtain a higher profit by selling her information instead of keeping it to herself and trading on it when she is not a monopolistic owner of information.

A proof of the first part of Proposition 1 is given in Admati and Pfleiderer (1988), and a similar proof applies in the context of this paper. Any market participants who trade on their private information regarding $\tilde{v}$ are termed '*informed traders*', and they all expect to earn positive trading profits. The total trading profit they expect to make will be called the '*market trading profit*'. Since the seller is the monopolist in the market for information, her share of the market trading profit is her clients' total expected trading profit which is extracted by the price charged for the information.[5]

[5] Since the seller commits to sell her information 'as is' without adding any noise and she fully extracts the expected trading profits to be earned by her clients by charging a price equal to $\Pi(m\,|\,[n, h_\eta])$, the total profit she

When there are no speculators in the market, the seller is also a monopolistic owner of the information about $\tilde{v}$, and either the seller or her clients are the only ones who trade on the information regarding $\tilde{v}$ depending on the seller's decision and, therefore, her total profit $m\Pi(m)$ is exactly equal to the market trading profit. Clearly, the seller's profit $m\Pi(m)$ is decreasing in m, and her profit is maximized if she does not sell her information to anyone in the market. The intuition here leads to the same outcome as in the Cournot oligopoly model where industry profits are decreasing in a number of identical firms. The sale of information creates unnecessary competition among her clients, and the profit from the sale of information always falls short of the seller's profit which could be earned by trading on her information without selling it. Therefore, it is desirable for her to keep the information to herself and trade on it instead of selling it. The following proposition specifies a set of conditions under which the seller finds it optimal to sell her information.

If the seller does not have monopolistic access to information, she has to face competition from speculators. This forces her to share the market trading profits with the speculators. Since the seller is not able to appropriate the entire market trading profit by herself, her objective now is to maximize her share, not the market trading profit itself.[6]

The condition for the information seller to sell her information rather than to keep it to herself and trade on it is similar to the one derived in Fishman & Hagerty (1995) and Sabino (1993) although the sequence of game assumed in Fishman & Hagerty (1995) is different from this model. In Fishman & Hagerty (1995), information sellers decide her sales strategy after market maker announces price schedule, and no matter what sales strategy is taken by the information seller, the price schedule chosen by the market maker does not change and consequently market liquidity is not affected by the sales of information. However, in this paper and Sabino (1993), the sequence of game is such that market maker chooses price schedule given the seller's sales strategy, and consequently market liquidity is clearly affected by how widely information is sold to outside investors. In period zero, the information seller optimally decides the strategy of information sales considering the effect of information sales on the market liquidity. Unlike Fishman & Hagerty (1995), this model demonstrates that the sales of information affects the welfare of every market participant including liquidity traders. Since liquidity traders' expected trading loss is $\frac{\lambda}{h_u}$, sales of information directly affect the welfare of liquidity traders in that as the information is sold more widely, due to more intense competition among information-based traders, liquidity traders are better off with smaller expected trading loss. [7]

The sale of information to clients who will then optimally use the information to decide their trading strategies has an important commitment effect in that it credibly commits the information seller to a strategy that would not be credible if she were to avoid such sales. The

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expects to make by selling her information to m-1 clients and trading on her own account is exactly same as that by selling her information to m clients without being engaged in any trading herself.

[6] The analysis is conducted based on the strategic trading model in which traders allowed to submit only market orders. The condition for the seller to find the sales of her information optimal is that $m\Pi(m)$ does not decrease monotonically, and Proposition 1 shows that it is true in case that traders submit market orders. Although it is conjectured that the same result would be obtained in other trading mechanisms such as limit orders, formal analysis is left for further research.

[7] This can be easily shown from equation (4) since λ is a decreasing function of m.

sale of information to a number of clients provides for more intense competition in the financial market. While this does reduce the market trading profits, it also has the effect of making the trading strategies of speculators less aggressive. This is actually the 'strategic substitutability' in the sense of Bulow, Geanokoplos and Klemperer (1985). Although the reduction in overall market trading profits may be substantial, the seller of information does not bear the full cost of the reduction in the market trading profits. By selling her information to a number of clients, the information seller is able to make the trading strategy of the speculators less aggressive and increase her share of the market trading profits at the expense of the speculators' profit. It is this strategic externality that makes the sales of information an optimal strategy for the information seller.

A natural question is why the information seller herself does not place the same size of trading order as the one collectively submitted by herself and by her clients, instead of selling information. Notice that if the information seller decides not to sell her information, this then becomes a quantity game simultaneously played by n speculators and the information seller. In this game, once trading begins, the information seller does not have any strategic advantage against other informed traders, and cannot credibly commit herself to a trading strategy of placing collective trading orders by her and her clients under the information sales scenario because that cannot be sustained as a Nash equilibrium anymore.[8] However, by selling her information to outside investors, which is observable to speculators and the market maker, the information seller is able to ascertain herself as leader of this trading game in that she can manage to affect the equilibrium λ, and consequently influence the trading strategy of all the information-based traders. The instrument she uses to do this is the price that she announces for access to her information, and she can credibly commit herself to a strategy that makes the speculators' trading strategy less aggressive.

In Admati & Pfleiderer (1988), only a risk averse information seller chooses to sell her information for the purpose of better risk sharing with her clients. As shown in Proposition 1, however, even with a risk neutral information seller, the presence of other information owners in the market justifies her decision to sell her information, and she obtains higher profits by committing herself to a strategy that promotes more intense competition in the market.[9] Although it is assumed that the seller observes $\tilde{v}$ without any noise, the seller's decision to sell

[8] Suppose the seller trades on her own account instead of selling her information. It is a special case of Lemma 1 with $m=1$. Then, it is now a quantity game played by n speculators and the information seller who now become another informed trader. The seller decides her optimal trading strategy *given* other informed traders' trading strategy by solving $\max_{\alpha}\hat{\Pi}_s(\alpha, \beta_1, \beta_2, \cdots, \beta_n)$, where $\hat{\Pi}_s$ is the expected trading profit earned by the seller and β_i is the trading strategy taken by speculator i. The equilibrium trading strategies taken by the seller and speculators are given in equations (2) and (3) respectively with $m=1$, and it is clear that placing collective orders by the seller and her clients is not the seller's optimal strategy anymore. Kane & Marks(1990) analyze the game among speculators in a set-up in which one speculator becomes a Stackelberg leader who can credibly commit to a particular trading strategy at the beginning of game, and force other speculators to adjust their trading strategy accordingly. Analysis in this paper is different from Kane & Marks(1990) in that this paper assumes that once trading stage begins, there is no endogenous mechanism that enables the information seller to become Stackelberg leader of speculators.

[9] This is quite a contrast to Admati & Pfleiderer(1988). They demonstrate that if either information seller or outsider investor is sufficiently risk tolerant (i.e., close to risk neutral), it is optimal to have only one informed trader in the market. However, Proposition 1 implies that if the seller and outside investors are risk neutral, it is always optimal to increase the number of informed traders in the market unless the seller is the monopolistic owner of the information.

her information depends not on the precision of her information, but on the strategic effect of selling her information which makes the trading strategy of the speculators less aggressive. In particular, even if the seller's information is coarser than the speculators' information, she will still choose to sell her information since this promotes the information-based competition in the market, by which she is able to obtain higher profits.

These results were derived under the set of restrictive assumptions on the strategies of the players in the game. In the following section, it is shown that the results follow even when some of these assumptions are relaxed.

III. *Extensions of the Basic Model*

In the last section, the condition for the sales of information was derived under rather restrictive set of assumptions: (i) information seller can either trade on her account or sell her information, but she is not allowed to do both, (ii) if she sells her information, she is restricted to sell her information 'as is' without adding any noise to it, and (iii) speculators are not permitted to purchase information from the seller. In this section, it will be shown that the equilibrium obtained in the previous section does not change even if these three assumptions are relaxed. Put differently, it will be shown that in equilibrium speculators never find it optimal to purchase information from the seller and improve the precision of their information, and it is always optimal for the information seller not to dilute her information before selling it even when she trades on her own account.

In the last section, it is assumed that conditional on the sale of information, the seller is restricted to sell her information 'as is.' Suppose she is allowed to sell signals of lower precision which she generates by adding to her original signal a realization of mutually independent noise terms. Now, in period zero, the information seller is supposed to announce the statistical properties of the added noise along with the price of the information. In this case, although clients purchase the signals of the same precision, they may observe different signals and thus submit different trading orders.[10]

Suppose the information seller trades on her own account and sells the information of precision h_ε by adding to her original signal of $\tilde{v}$ a realization of mutually independent noise terms to generate each of the signals to be sold. The price of information is set such that in equilibrium information is sold to z outside investors, and k speculators purchase information from the seller to improve the precision of their information by h_ε. The total number of clients, $z+k$ is denoted m. Each client privately observes $\tilde{v}+\tilde{\varepsilon}_j$, $j = 1, \ldots, m$ where $\{\tilde{\varepsilon}_j\}_{j=1,\ldots,m}$ are

[10] Another way of generating signals of lower precision is to add the noise before selling her information so that all of her clients observe the same signal and submit the same sized trading orders. In Admati & Pfleiderer(1986), this way of adding noise is called '*photocopied noise*,' and the way assumed in this paper is labeled '*personalized noise*.' Admati & Pfleiderer(1986) show that the information seller obtains higher profit by adding personalized noise than by adding photocopied noise to her information before selling it. Unlike signals with personalized noise, the signals with photocopied noise lead all clients to submit the identical size of trading orders to the market maker. The market maker is then able to extract more information about $\tilde{v}$ by looking at the size of the net market order, and the price schedule is more sensitive to the net market order. That is why signals with photocopied noise generate smaller trading profits for the seller's clients than signals with personalized noise, which results in smaller information sales profit for the seller. The same result can be proved in the context of this model but is omitted for the brevity of the model.

normally distributed with mean 0 and precision h_ε, and they are independent of $\tilde{\upsilon}$, $\{\tilde{\eta}_i\}_{i=1,\ldots,n}$ and $\tilde{u}$. Notice it is assumed that the information seller cannot observe the realizations of $\{\tilde{\varepsilon}_j\}_{j=1,\ldots,m}$ and resale of information is prohibited.[11]

Also, the seller cannot charge discriminatory price against speculators, and therefore every information buyer pays the same price for the information purchased from the seller.

If k speculators out of n purchase information from the seller, then there are four types of information-based traders in the market: (i) information seller with perfect information on $\tilde{\upsilon}$, (ii) z outside investors who purchase the information of precision h_ε, (iii) $n-k$ speculators with the private information of precision h_η, and (iv) k speculators with the information of precision $h_\varepsilon+h_\eta$ by observing two pieces of signals.

Given (h_ε, m) and (h_η, n), the expected trading profits of each trader depend on the number of the speculators who purchase information. Let $\Pi^{ib}(k, z)$, $\Pi^{ub}(k, z)$ and $\Pi^{i}(k, z)$ denote the expected trading profits of a speculator with extra information, that of a seller's client who used to be outside investor, and that of a speculator who does not buy information, respectively. Then, [12]

$$\Pi^{ib}(k,z)=\frac{(h_\varepsilon+h_\eta)(h_\varepsilon+h_\eta+h_\upsilon)(h_\varepsilon+2h_\upsilon)^2(h_\eta+2h_\upsilon)^2h_\upsilon}{\sqrt{Q}\sqrt{h_uh_\upsilon}\,T},$$

$$\Pi^{ub}(k,z)=\frac{h_\varepsilon(h_\varepsilon+h_\eta+2h_\upsilon)^2(h_\varepsilon+h_\upsilon)(h_\eta+2h_\upsilon)^2}{\sqrt{Q}\sqrt{h_uh_\upsilon}\,T},$$

$$\Pi^{i}(k,z)=\frac{h_\eta(h_\varepsilon+h_\eta+2h_\upsilon)^2(h_\varepsilon+2h_\upsilon)^2(h_\eta+h_\upsilon)}{\sqrt{Q}\sqrt{h_uh_\upsilon}\,T}. \qquad (8)$$

In the previous section, it was assumed that speculators cannot have any access to the seller's information. If speculators are also allowed to purchase the seller's information, they are given the option of improving their information with the help of the seller's information, and thereby, of increasing their trading profits. The question is whether such improvement in the speculators' information is worth the price. The next proposition shows that the answer is no.

Proposition 2 *Speculators never find it optimal to purchase the seller's information and improve the precision of their information*.

Suppose that, in equilibrium, the seller's clients include some speculators as well as outside investors. The information seller cannot charge discriminatory price to her clients, and every information buyer pays the same price. Since there are sufficiently many outside investors, in equilibrium, the number of clients who used to be an outside investor is determined such that what they pay for the information equals exactly to their expected trading profit. Due to the decreasing marginal returns of information, speculators cannot increase their expected trading

[11] Admati & Pfleiderer (1986) explains that "Signals may be personalized in other, less direct ways. For example, the seller may provide information that is vague and open to interpretation, so that the buyers themselves make personal, independent, errors of interpretation.". If noise is added in this way, each buyer interprets the information provided by the seller differently and the seller cannot possibly know how each buyer interprets the information.

[12] For the derivations of T and Q, please refer to the proof of Proposition 2 in the Appendix.

with the help of the purchased information as much as outside investors. Thus, no speculator purchases information from the seller regardless of the precision and the price of the seller's information.

Fishman & Hagerty (1995) assume that both speculators and information seller have the perfect information on the payoff of the security, which effectively rules out the possibility of speculators' purchasing information from the seller. If speculators' information is less than perfect, then they could increase their expected trading profit by improving the precision of their information with extra pieces of information purchased from the seller. As Proposition 2 shows, due to the large number of outside investors who earn greater marginal return from the purchased information than the speculators, regardless of precision of the information sold by the seller, there is no speculator purchasing information from the seller.

If the seller herself is also a trader in the market, as information of higher precision is provided, there might exist a tradeoff between her own trading profit and the profit from information sales. By creating more intense competition in the market, the seller's own trading profit might suffer. In order to maximize her total profit from trading and information sales, the seller may have an incentive to sell information of lower precision by adding noise to her information before selling it.

Suppose the seller provides information of precision h_ε to m clients by adding personalized noise, and she trades on her own account as well. The seller's total profit from trading and the sale of information is given by the next equation. [13]

$$\Pi^s[(m, h_\varepsilon) \mid (n, h_\eta)] = \frac{1}{\lambda^s} \frac{mh_\varepsilon(h_\varepsilon + h_\upsilon)(h_\eta + 2h_\upsilon)^2 + (h_\varepsilon + 2h_\upsilon)^2(h_\eta + 2h_\upsilon)^2}{[mh_\varepsilon(h_\eta + 2h_\upsilon) + nh_\eta(h_\varepsilon + 2h_\upsilon) + 2(h_\varepsilon + 2h_\upsilon)(h_\eta + 2h_\upsilon)]^2}. \tag{9}$$

As the next proposition shows, in spite of this tradeoff, the seller's profit is maximized by selling her information 'as is' without adding any noise to it.

Proposition 3 *The information seller never finds it optimal to add any noise to her information before selling it.*

Proposition 3 contrasts with Admati & Pfleiderer (1986) in which the seller with very precise information prefers to add personalized noise to her information before selling it. In their paper, based on a rational expectations model, the information purchased by clients is leaked to non-clients by being reflected in the price before trading commences. As more precise information is sold by the seller, the price carries more of her information due to her clients' more aggressive use of it, causing faster deterioration of its value and even market breakdown. To prevent market collapse, the precision of the information sold to the clients needs to be lower than a critical level. Since more noise needs to be added as the seller's information gets better, the seller cannot fully exploit her improved information for her profit. Since the added noise terms are independent random variables, they observe different signals and submit different sized trading orders to the market maker although information sold to the seller's clients has the same precision. Therefore, it is even possible that some clients make *ex post* trading profits while others suffer *ex post* losses.

[13] Equilibrium λ^s is derived in the proof of Proposition 3 in the Appendix.

In this model, however there is no leakage of the seller's information to non-clients or to speculators before trading commences, and price cannot fully carry traders' private information thanks to the random liquidity demand which is exogenous noise in the market. As more precise information is sold to the seller's clients, the value of information increases without ever causing market collapse. Therefore, the seller is able to sell the best information she possibly can, and obtains the highest profit possible by selling her information 'as is' without adding any noise. In addition, her clients make exactly the same *ex post* as well as *ex ante* trading profits because they observe identical information and submit exactly the same sized trading orders.

Proposition 3 shows that once the seller decides to sell her information, it is sold to more than one client, and the information sales profit always makes up a larger portion of her total profits than her own trading profit. Therefore, maximizing her information sales profit helps maximize her total profits as well.

Proposition 3 is a lot stronger than a related result in Admati & Pfleiderer (1988) and Fishman & Hagerty (1995), where the seller is restricted not to dilute her information. But Proposition 3 demonstrates that even if there is no such restriction imposed on the seller, and she is allowed to dilute her information before selling it while she trades on her own information, the seller still never finds it optimal to dilute her information before selling it.

There are a couple of important implications derived from Proposition 3. First, the seller trades on exactly the same information as that sold to her clients. This implies that her expected trading profit is equal to that of each of her clients which is the price she charges for the information. Therefore, the total profit she expects to make by selling her information to $m-1$ clients and trading on her own account is exactly same as that by selling her information to m clients without being engaged in any trading herself. In equilibrium, as far as the seller communicates honestly there are the same number of traders in the market who trade on the seller's information whether the seller utilizes both options or not. Second, if the statistical properties of the seller's information and her decision on the sale of her information are common knowledge, one of the incentive problems on the part of the information seller can be avoided. Since the seller trades after she is paid for her information, in order to increase her trading profit, she has an incentive to cheat her clients by actually providing the information with lower precision than the one for which they pay. Suppose the seller is restricted to choosing between selling her information and trading on it, and not allowed to do both. Proposition 3 shows that as long as the seller communicates honestly with her clients, this restriction does not change the seller's total profit. Since the seller no longer trades when she sells her information, she has no reason to cheat her clients, and her information is communicated honestly. Thus, this model is able to provide a strong answer to the question of why an information possessor may abjure trading totally and sell information only. This contradicts the central result in Allen (1990). Note, however, that this paper has not dealt with the general moral hazard problem in Allen (1990).

Comparative statics on the equilibrium are collected in the following proposition. [14]

Proposition 4 *1. The information seller sells her information to more clients at a lower price, and*

[14] As part of the proof of Proposition 3, it can be shown that if the seller trades on her own account and dilutes her information before selling it, as h_ε increases, i.e., as the seller provides more accurate information to her clients, the number of optimal clients decreases while her total profit increases.

earns smaller profit as the number of speculators increases or as their information becomes more accurate.

2. *As the precision of liquidity demands improves, the seller's profit decreases as she charges lower price for her information while selling it to the to the same number of clients.*
3. *The increase in the precision of the security's payoff reduces the seller's total profit as she sells her information to a smaller number of clients at a lower price.*

As more speculators trade on their information or as their information becomes more precise, they collectively trade more aggressively, which reduces the seller's information sales profit. She is unable to recover all of the loss incurred by the more aggressive trading of speculators, but she can still retrieve part of the loss by selling her information to more clients at a lower price, diluting the speculators' trading profits, and thereby enhancing her share of the market trading profits. [15]

The precision of liquidity demands does not affect the seller's decision on the number of her clients, but since the market maker gets more accurate information about the payoff of the security as the precision of liquidity demands improves, the seller's profit decreases. As the speculators' information becomes more accurate or the number of speculators increases, then the speculators collectively trade more aggressively, which causes information seller's share of market profit to shrink as far as the seller still sells her information to the same number of information buyers. Therefore, the seller's optimal response is to sell her information to more clients to prevent her share of market trading profit from decreasing further.

The increases in the precision of security's payoff, i.e., the decreases in the variance of security's payoff cause the seller's information to be less valuable to potential clients, and consequently a smaller number of outside investors are willing to purchase information from the seller, which forces the seller to charge a lower price.

IV. *Risk Averse Information Seller and Buyers*

In previous sections it is assumed that all the market participants are risk neutral, and the sale of information enables the risk neutral seller to credibly commit to a strategy that promotes more intense competition among informed traders and thereby earns her a greater profit. In this section, the information seller and outside investors are assumed to be risk averse and the seller's optimal use of her private information will be analyzed. For the simplicity of analysis, speculators are still assumed to be risk neutral, and the seller does not add any noise to the information sold to her clients.

Suppose the information seller and outside investors have quadratic utility functions with risk aversion coefficient of γ and μ respectively. The reservation utility of the outside investors without purchasing information from the seller is assumed to be zero. If the seller trades on her own account while selling her information to k clients, expected utility of each information buyer is [16]

[15] If the number of speculators is low, then the equilibrium number of information buyers would be also low. In this case, information buyers have less difficulty in verifying the actual number of clients to whom the seller's information is sold.

[16] Exact derivations of expected utilities of speculators and information seller are given in the Appendix.

$$EU_B(\gamma, \mu, k) \equiv E[E[U_B|\tilde{v}]]$$

$$= \frac{\sigma_v^2}{2}(1+\gamma\lambda\sigma_u^2)\frac{\gamma\mu\sigma_u^4\lambda^2+\sigma_u^2\lambda(2\gamma+\mu)+2}{\lambda(\lambda^2(n+1)\gamma\mu\sigma_u^4+\lambda(\mu(n+2)+\gamma(k+n+1))\sigma_u^2+k+n+2)^2} \quad (10)$$

$E\Pi_S^T(\gamma, \mu, k)$ denotes the seller's total certainty equivalent of her expected utility of trading and the proceeds from the sales of her information. Since there are sufficiently many outside investors, the price paid by each information buyer is equal to $EU_B(\gamma, \mu, k)$, and the information seller's expected utility is

$$E\Pi_S^T(\gamma, \mu, k) \equiv EU_S(\gamma, \mu, k)+kEU_B(\gamma, \mu, k) \quad (11)$$

where

$$EU_S(\gamma, \mu, k)$$

$$= \frac{\sigma_v^2}{2}(1+\mu\lambda\sigma_u^2)\frac{\gamma\mu\sigma_u^4\lambda^2+\sigma_u^2\lambda(2\gamma+\mu)+2}{\lambda(\lambda^2(n+1)\gamma\mu\sigma_u^4+\lambda(\mu(n+2)+\gamma(k+n+1))\sigma_u^2+k+n+2)^2} \quad (12)$$

If the seller decides not to trade on her own account, $E\Pi_S(\mu, k)$ denotes the total proceeds from selling her information to k clients. Since there are sufficiently large number of outside investors who are potential clients of the information seller, $E\Pi_S(\mu, k) = kEP_B(\mu, k)$, where $EP_B(\mu, k)$ is the certainty equivalent of the information buyer's expected utility of trading on the basis on the information purchased from the seller. [17] As the next proposition demonstrates, the seller's decision on whether or not to trade on her own account depends on the degree of her risk aversion relative to outside investors' risk aversion.

Proposition 5 *If the information seller is more risk averse than her clients, then it is optimal for her to commit not to trade on her own account. Otherwise, the seller is better off by selling her information while trading on the basis of her information as well.*

Proposition 5 can be illustrated by the following equation.

$$E\Pi_S^T(\gamma, \mu, k) \geq (\leq)\ (k+1)\ EP_B(\mu, k+1) \text{ if } \gamma \leq (\geq)\ \mu$$

If all market participants including the information seller and her clients are risk neutral, the sale of information makes the trading strategy of the speculators less aggressive and increase her share of the market trading profits at the expense of the speculators' profit. It is this strategic externality that makes the sales of information an optimal strategy for the risk neutral information seller. If the seller and her clients are risk averse, then better risk sharing among the seller and buyers of information can be achieved through the sale of information, which is another incentive for the sales of private information owned by the seller. Contrary to the case of risk neutral seller derived in Proposition 1, a risk averse information seller finds it optimal to

[17] If the seller decides not to trade on her own account, then she collects the proceeds from the sales of her information in period 0, and does not face risk. Therefore, the total profits from the sales of her information does not depend on her risk aversion.

sell her information to achieve better risk sharing even if there are no speculators.[18]

The result derived in Proposition 5 is straightforward. The less risk averse a trader is, the more aggressively he trades based on his private information, and the higher the value of information is to him. The sales of information has the function of allocating information to the traders who value the most. If the seller is less risk averse than her clients, the expected utility from trading on the basis of her information is greater than the price that her clients are willing to pay for, and therefore it is optimal for her to trade on her own account while selling her information to her clients. But the opposite is true if the seller is more risk averse. Since the value of information is higher to her client than to her, the seller can be better off by selling her information while committing not to trade at all.

TABLE 1 OPTIMAL SALE OF INFORMATION BY RISK AVERSE SELLER TO RISK AVERSE BUYERS

	$\gamma = 0$	$\gamma = 0.3$	$\gamma = 0.6$	$\gamma = 0.9$	$\gamma = 1.2$	$\gamma = 1.5$	$\gamma = 1.8$	$\gamma = 2.1$
$\mu = 0$	20 (0.117866)							
$\mu = 0.3$	22 (0.115101)	22 (0.11497)						
$\mu = 0.6$	23 (0.112604)	23 (0.112464)	24 (0.112351)					
$\mu = 0.9$	25 (0.110335)	25 (0.110199)	25 (0.110056)	25 (0.10999)				
$\mu = 1.2$	26 (0.108269)	26 (0.108126)	26 (0.108007)	27 (0.10791)	27 (0.10783)			
$\mu = 1.5$	27 (0.106367)	28 (0.106226)	28 (0.106109)	28 (0.106011)	28 (0.105927)	28 (0.105856)		
$\mu = 1.8$	29 (0.104624)	29 (0.10448)	29 (0.104358)	29 (0.104255)	30 (0.104168)	30 (0.104097)	30 (0.104037)	
$\mu = 2.1$	30 (0.10301)	30 (0.102861)	31 (0.102737)	31 (0.102637)	31 (0.102551)	31 (0.102477)	31 (0.102414)	31 (0.102359)

The first number is $k^*(\gamma, \mu)$, optimal number of information buyers for the seller with risk aversion coefficient γ to the clients with risk aversion coefficient of μ, and the number in the bracket is the seller's total profits. For $\gamma \leq \mu$, the seller trades on her own account and sell her information to is $k^*(\gamma, \mu)$ clients. But for $\gamma > \mu$, the seller commits not to trade on her own account but sells her information to $k^*(\mu, \mu)+1$ clients

Numerical example is given in Table 1 for $\sigma_u^2 = \sigma_v^2 = 1$ and $n=10$. As the seller and her clients become risk averse, the seller finds it optimal to sell her information to the greater number of clients but her expected utility decreases as the value of information to her clients and herself diminishes. [19]

[18] This result can be obtained in the context of the model in this paper but the derivation is similar to the one in Admati & Pfleiderer(1988) and omitted.

[19] For $\gamma=\mu$, if the seller decides to trade on her own account while selling her information to k clients, then there are $k+1$ traders with identical utility functions. It is equivalent to the case that the seller commits not to trade at all but sells her information to $k+1$ clients. Therefore $E\Pi_S^t(\mu, \mu, k)=E\Pi_S(\mu, k+1)=(k+1)EP_B(\mu, k+1)$ is obtained, and the seller with $\gamma \geq \mu$ is better off by selling her information without trading on her own account and thereby earning $E\Pi_S(\mu, k+1)=(k+1)EP_B(\mu, k+1)$ instead of being engaged in both sales of information and trading.

V. *Conclusion*

This paper analyzes the direct sales of information in both monopolistic and oligopolistic markets for financial information. For agents with private information, competition based on information makes the selling of their information more profitable than trading on their own accounts. Since price cannot reflect all the private information held by traders in the market, information sellers find it optimal to sell their information 'as is' without adding any noise before selling it to clients. This is still true even when the seller herself also trades on her own account.

If a trader who already has private information about a security is allowed to buy information from the seller, he will never find it optimal to do so due to decreasing marginal returns of information. As more intense competition among traders occurs due to the increase in the number of traders or the improvement in their information, a seller sells her information to more clients at a lower price. The model is extended to the case that information seller and her clients are risk averse. A risk averse seller can achieve better risk sharing through the sales of information, and the sales of information has the function of allocating information to the traders who value most.

This paper explores only the issues raised by the direct sales of information. There are many other ways in which a possessor of valuable information may offer it for use in trading. For instance, mutual fund managers sometimes claim to invest their shareholders' money based on private information and research, but shareholders of a mutual fund never directly observe this information. A broader comparison between such different selling methods is much needed, and this paper is best viewed as a first step.

Another important issue in this context concerns the incentive problems of sellers. It is assumed both that the statistical properties of a seller's information are common knowledge, and that truthful communication can be guaranteed. This paper demonstrates, however, that a risk neutral information seller need not trade to maximize her profit if her information is truthfully communicated. This, clearly, is not a complete solution to the general incentive problems. A more detailed appraisal of these issues in an integrated framework remains a topic for further research.

Appendix

Proof of Proposition 1 The seller's problem is to decide the optimal m^* to maximize $m\Pi[m \mid (n, h_\eta)]$ given (n, h_η). If there is no speculator, *i.e.*, $n=0$ or $h_\eta=0$, then from equation (5), $m^*=1$ is obtained, which means that the seller trades on her information instead of selling it. If there are speculators in the market i.e., $n>0$ and $h_\eta>0$, then, $\left.\frac{\partial m\Pi[m \mid (n, h_\eta)]}{\partial m}\right|_{m=1}>0$, therefore the seller obtains higher profits by selling her information.

Proof of Proposition 2 Suppose the seller provides information of precision h_ε at the price of P_i^* by adding personalized noise to her information, and in equilibrium k clients are speculators, and z are outside investors. These speculators observe two noisy signals of $\tilde{v}$, and if $\tilde{\varepsilon}_i$'s are independent of $\tilde{\eta}_l$'s, then the precision of their information is improved by h_ε. Given the price schedule $P(\tilde{y})=\bar{\upsilon}+\lambda\tilde{y}$, the speculators who purchase information from the seller submit the following trading orders to the market maker:

$$\frac{1}{\lambda}\frac{(h_\varepsilon+h_\eta)(h_\varepsilon+2)(h_\eta+2)}{T}(\tilde{\upsilon}+\tilde{\delta}_l-\bar{\upsilon}) \qquad l=1,2,\ldots,k$$

in which

$$\tilde{\delta}_l=\frac{h_\varepsilon\tilde{\varepsilon}_l+h_\eta\tilde{\eta}_l}{h_\varepsilon+h_\eta} \qquad l=1,2,\ldots,k$$

and

$$\begin{aligned}T &= k(h_\varepsilon+h_\eta)(h_\varepsilon+2h_\upsilon)(h_\eta+2h_\upsilon)+zh_\varepsilon(h_\varepsilon+h_\eta+2h_\upsilon)(h_\eta+2h_\upsilon)\\ &\quad+(n-k)h_\eta(h_\varepsilon+h_\eta+2h_\upsilon)(h_\varepsilon+2h_\upsilon)+2(h_\varepsilon+h_\upsilon)(h_\varepsilon+h_\eta+2h_\upsilon)(h_\eta+2h_\upsilon)\end{aligned}$$

The equilibrium is $\lambda=\sqrt{h_u h_\upsilon}\dfrac{\sqrt{Q}}{T}$, in which

$$\begin{aligned}Q &= k(h_\varepsilon+h_\eta)(h_\varepsilon+h_\eta+h_\upsilon)(h_\varepsilon+2h_\upsilon)^2(h_\eta+2h_\upsilon)^2\\ &\quad+zh_\varepsilon(h_\varepsilon+h_\upsilon)(h_\varepsilon+h_\eta+2h_\upsilon)^2(h_\eta+2h_\upsilon)^2\\ &\quad+(n-k)h_\eta(h_\eta+h_\upsilon)(h_\varepsilon+h_\eta+2h_\upsilon)^2(h_\varepsilon+2h_\upsilon)^2\\ &\quad+(h_\eta+2h_\upsilon)^2(h_\varepsilon+2h_\upsilon)^2(h_\varepsilon+h_\eta+2h_\upsilon)^2(h_\varepsilon+2h_\upsilon)^2\end{aligned}$$

Since there are sufficiently many outside investors as potential clients of the seller, given the price of information and conjectured number of speculators who purchase information from the seller, the equilibrium number of outside investors who become the clients of the seller is determined in such a way that the price paid for the seller's information is $\Pi^{ub}(k,z)$, i.e., $P_i^*=\Pi^{ub}(k,z)$. However, from equation (8), it is shown that the following inequality always holds for any z and k.

$$\Pi^{ib}(k,z)-\mathrm{P}_i^*=\Pi^{ib}(k,z)-\Pi^{ub}(k,z)\le\Pi^{i}(k,z)$$

Therefore, no speculator buys extra information from the seller regardless of the precision of the information.

Proof of Proposition 3 Suppose the seller provides information of precision h_ε to m clients by adding personalized noise, and she trades on her own account as well. Given the price schedule $P(\tilde{y})=\bar{\upsilon}+\lambda^s\tilde{y}$, the seller submits the following order to the market maker:

$$\frac{(h_\varepsilon+2h_\upsilon)(h_\eta+2h_\upsilon)}{\lambda^s[mh_\varepsilon(h_\eta+2h_\upsilon)+nh_\eta(h_\varepsilon+2h_\upsilon)+2(h_\varepsilon+2h_\upsilon)(h_\eta+2h_\upsilon)]}(\tilde{\upsilon}-\bar{\upsilon}), \tag{A.1}$$

where equilibrium λ_s is

$$\lambda^s=\sqrt{h_u h_\upsilon}\frac{\sqrt{mh_\varepsilon(h_\varepsilon+h_\upsilon)(h_\eta+2h_\upsilon)^2+nh_\eta(h_\eta+h_\upsilon)(h_\varepsilon+2h_\upsilon)^2+(h_\varepsilon+2h_\upsilon)^2(h_\eta+2h_\upsilon)^2}}{mh_\varepsilon(h_\varepsilon+2h_\upsilon)+nh_\eta(h_\varepsilon+2h_\upsilon)+2(h_\varepsilon+2h_\upsilon)(h_\eta+2h_\upsilon)}. \tag{A.2}$$

It can be shown that $\Pi^s\,[(m, h_\varepsilon)\,|\,(n, h_\eta)]$ increases in h_ε, and selling her information 'as is' maximizes her total profits.

Proof of Proposition 4 From the proof of Proposition 1, it is known that optimal m^* is derived from the following equation:

$$\left.\frac{\partial m\Pi[m\,|\,(n,h_\eta)]}{\partial m}\right|_{m=m^*}=0, \tag{A.3}$$

which can be rearranged to yield the following quadratic equation

$$m^{*2}\,(h_\eta+2h_\upsilon)^3-m^*((n+1)h_\eta+2h_\upsilon)(h_\eta+2h_\upsilon)^2-2nh_\eta(h_\eta+h_\upsilon)((n+1)h_\eta+2h_\upsilon)=0. \tag{A.4}$$

From this equation, $m^* = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$ is obtained, where

$$a = (h_\eta+2h_\upsilon)^3,\ b=-((n+1)h_\eta+2h_\upsilon)(h_\eta+2h_\upsilon)^2,\ c=-2nh_\eta(h_\eta+h_\upsilon)((n+1)h_\eta+2h_\upsilon).$$

By using the solution of the quadratic equation, m^* is derived, which increases in both n and h_η but decreases in h_υ. Thanks to the envelope theorem, it can be shown that the price charged by the seller and her total profit decrease in n, h_η and h_υ. Since equation (A.4) does not contain h_u, m^* is not affected by the precision of liquidity demands, but equation(5) shows that the seller's profit is monotonically decreasing in it.

Proof of Proposition 5 Utility function of information seller is denoted U_S while that of information buyer is denoted U_B. Suppose information seller trades on her own account while selling her information to k clients. Given the price schedule of $P = \bar{\upsilon}+\lambda\tilde{y}$ and $\tilde{\upsilon}$, information seller, her clients and speculators are believed to be taking the trading strategies of $\alpha\ (\tilde{\upsilon}-\bar{\upsilon})$, $\beta(\tilde{\upsilon}-\bar{\upsilon})$ and $\delta(\tilde{\upsilon}-\bar{\upsilon})$ respectively. Taking the price schedule of $P = \bar{\upsilon}+\lambda\tilde{y}$, $\tilde{\upsilon}$ and trading strategies of information seller and speculators, the information seller's optimal trading order is derived from the solution of the following maximization problem.

$$\max_x E[U_S\,|\,\tilde{\upsilon}]$$

$$\max_x E[x(\tilde{\upsilon}-\bar{\upsilon}-\lambda(x+k\beta(\tilde{\upsilon}-\bar{\upsilon})+n\delta(\tilde{\upsilon}-\bar{\upsilon})+\tilde{u}))\,|\,\tilde{\upsilon}]$$

$$-\frac{\gamma}{2}\,Var[x(\tilde{\upsilon}-\bar{\upsilon}-\lambda(x+k\beta(\tilde{\upsilon}-\bar{\upsilon})+n\delta(\tilde{\upsilon}-\bar{\upsilon})+\tilde{u}))\,|\,\tilde{\upsilon}]$$

Solving the first order condition yields

$$x = \frac{1}{\lambda(2+\gamma\sigma_u^2\lambda)}\,(1-k\lambda\beta-n\lambda\delta)(\tilde{\upsilon}-\bar{\upsilon})$$

where $\sigma_u^2 = 1/h_u$. For the market's belief on the seller's trading strategy to be consistent, following equality should hold.

$$\alpha = \frac{1}{\lambda(2+\gamma\sigma_u^2\lambda)}(1-k\lambda\beta-n\lambda\delta) \tag{A.5}$$

Each of information buyers solves following maximization problem given the price schedule, $\tilde{v}$ and trading strategies of information seller, speculators and other information buyers.

$$\max_z E[U_B|\tilde{v}]$$

$$\max_z E[z(\tilde{v}-\bar{v}-\lambda(z+\alpha(\tilde{v}-\bar{v})+(k-1)\beta(\tilde{v}-\bar{v})+n\delta(\tilde{v}-\bar{v})+\tilde{u}))|\tilde{v}]$$

$$-\frac{\mu}{2}Var[z(\tilde{v}-\bar{v}-\lambda(z+\alpha(\tilde{v}-\bar{v})+(k-1)\beta(\tilde{v}-\bar{v})+n\delta(\tilde{v}-\bar{v})+\tilde{u}))|\tilde{v}]$$

From the first order condition, following optimal trading order is obtained.

$$\begin{aligned} z &= \frac{1}{\lambda(2+\mu\sigma_u^2\lambda)}(1-\lambda\alpha-(k-1)\lambda\beta-n\lambda\delta)(\tilde{v}-\bar{v}) \\ &= \beta(\tilde{v}-\bar{v}) \\ &\Rightarrow \beta = \frac{1}{\lambda(k+1+\mu\sigma_u^2\lambda)}(1-\lambda\alpha-n\lambda\delta) \end{aligned} \tag{A.6}$$

The trading order submitted by each of speculators is the solution of following maximization problem taking the price schedule, $\tilde{v}$ and trading strategies of information seller, information buyers, and other speculators.

$$\max_w E[w(\tilde{v}-\bar{v}-\lambda(\alpha(\tilde{v}-\bar{v})+k\beta(\tilde{v}-\bar{v})+(n-1)\delta(\tilde{v}-\bar{v})+\tilde{u}))|\tilde{v}]$$

Optimal trading order of speculators is obtained from the first order condition.

$$\begin{aligned} w &= \frac{1}{2\lambda}(1-\lambda\alpha-k\lambda\beta-(n-1)\lambda\delta)\ \tilde{v} \\ &= \delta(\tilde{v}-\bar{v}) \\ &\Rightarrow \delta = \frac{1}{\lambda(n+1)}(1-\lambda\alpha-k\lambda\beta) \end{aligned} \tag{A.7}$$

Solutions of simultaneous equations of (A.5), (A.6) and (A.7) produce the equilibrium trading orders submitted by information seller, information buyers and speculators.

$$\begin{aligned} \alpha &= \frac{1+\lambda\mu\sigma_u^2}{\lambda(\lambda^2(n+1)\gamma\mu\sigma_u^4+\lambda(\mu(n+2)+\gamma(k+n+1))\sigma_u^2+k+n+2)} \\ \beta &= \frac{1+\lambda\gamma\sigma_u^2}{\lambda(\lambda^2(n+1)\gamma\mu\sigma_u^4+\lambda(\mu(n+2)+\gamma(k+n+1))\ \sigma_u^2+k+n+2)} \\ \delta &= \frac{(1+\lambda\mu\sigma_u^2)(1+\lambda\gamma\sigma_u^2)}{\lambda(\lambda^2\ (n+1)\gamma\mu\sigma_u^4+\lambda(\mu(n+2)+\gamma(k+n+1))\sigma_u^2+k+n+2)} \end{aligned} \tag{A.8}$$

Due to the zero expected profit condition of market maker, the equilibrium $\lambda^*(\gamma, \mu, k)$ is determined from following equation.

$$E[\tilde{v}|\tilde{y} = \alpha\ (\tilde{v}-\bar{v})+k\beta(\tilde{v}-\bar{v})+n(\tilde{v}-\bar{v})+\tilde{u}] = \bar{v}+\lambda\tilde{y}$$

$$\Rightarrow \frac{(1+\lambda\mu\sigma_u^2)(1+\lambda\gamma\sigma_u^2)(k+n+1+\lambda\sigma_u^2\ (\mu(n+1)+\gamma(n+k))+n\gamma\mu\lambda^2\sigma_u^4)}{(\lambda^2\ (n+1)\ \gamma\mu\sigma_u^4+\lambda(\mu(n+2)+\gamma(k+n+1))\sigma_u^2+k+n+2)^2}$$

$$= \lambda^2\ \sigma_u{}^2 \qquad \text{(A.9)}$$

where $\sigma_v^2 = 1/h_v$. From equation (A.9), it can be shown that $\lambda^*(\gamma, \mu, k)$ increases in γ. Expected utility of each information buyer is

$$EU_B(\gamma, \mu, k) \equiv E[E[U_B|\tilde{v}]]$$

$$= \beta(1-\lambda(\alpha+k\beta+n\delta))\sigma_v^2 - \frac{\mu}{2}\lambda^2\beta^2\sigma_v^2\sigma_u^2$$

$$= \frac{\sigma_v^2}{2}(1+\gamma\lambda\sigma_u^2)\frac{\gamma\mu\sigma_u^4\lambda^2+\sigma_u^2\lambda(2\gamma+\mu)+2}{\lambda(\lambda^2(n+1)\gamma\mu\sigma_u^4+\lambda(\mu(n+2)+\gamma(k+n+1))\sigma_u^2+k+n+2)^2}$$

Since there are sufficiently many outside investors, the price paid by each information buyer is equal to $EU_B(\gamma, \mu, k)$, and the information seller's expected utility is

$$E\Pi_S^T(\gamma, \mu, k) \equiv EU_S(\gamma, \mu, k)+kEU_B(\gamma, \mu, k)$$

where

$$EU_S(\gamma, \mu, k) = \alpha(1-\lambda(\alpha+k\beta+n\delta))\sigma_v^2 - \frac{\gamma}{2}\lambda^2\alpha^2\sigma_v^2\sigma_u^2$$

$$= \frac{\sigma_v^2}{2}(1+\mu\lambda\sigma_u^2)\frac{\gamma\mu\sigma_u^4\lambda^2+\sigma_u^2\lambda(2\gamma+\mu)+2}{\lambda(\lambda^2(n+1)\gamma\mu\sigma_u^4+\lambda(\mu(n+2)+\gamma(k+n+1))\sigma_u^2+k+n+2)^2}$$

From equations (10) and (12), it can be shown that $E\Pi_S^T(\gamma, \mu, k)$ decreases in λ and γ. Since λ^* is an increasing function of γ, $E\Pi_S^T(\gamma, \mu, k)$ decreases in γ. $EP_B(\mu, h)$ denotes the price charged to each information buyer if the seller's information is sold to h clients while the seller commits not to trade on her own account. In this case, the seller gains the total profit of $E\Pi_S(\mu, h) = hEP_B(\mu, h)$ regardless of γ. For $\gamma = \mu$, if the seller trades on her own account while selling her information to k clients, then there are $k+1$ informed traders with identical risk aversion, which is equivalent to the seller's information sold to $k+1$ clients while the seller committing not to trade. Therefore, $E\Pi_S^T(\gamma, \mu, k) = (k+1)\ EU_B(\gamma, \mu, k) = (k+1)EP_B(\mu, k+1)$ holds for $\gamma = \mu$. Since $E\Pi_S^T(\gamma, \mu, k)$ is a decreasing function of γ, following inequality is obtained.

$$E\Pi_S^T(\gamma, \mu, k) \geq (\leq)\ (k+1)EP_B(\mu, k+1) \text{ if } \gamma \leq (\geq)\ \mu$$

and the result follows.

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