

FORECASTING THE VELOCITY OF CIRCULATION IN THE JAPANESE ECONOMY*

IMAD A. MOOSA**

*Department of Economics and Finance, La Trobe University
Victoria 3086, Australia
I.Moosa@latrobe.edu.au*

JAE H. KIM

*Department of Econometrics and Business Statistics, Monash University
Caulfield East, VIC 3145, Australia
Jae.Kim@BusEco.monash.edu.au*

Received October 2002, Accepted January 2004

Abstract

This paper compares the direct and indirect methods of predicting the velocity of circulation in the Japanese economy. Forecasts are generated using the autoregressive (AR) model and Harvey's structural time series model. In addition to point forecasts, prediction intervals (calculated by using the recently proposed bootstrap-after-bootstrap) are used as a criterion for evaluating forecasting accuracy. The results indicate the superiority of the direct method. While this result is not consistent with the theoretical appeal of the indirect method, it can be explained on the grounds that the pooling of time series reduces the noise associated with individual time series.

Key Words: Monetary Velocity, Forecasting, Bootstrapping, Harvey's Structural Time Series Modelling

JEL classifications: E47, E52, C15, C22

I. Introduction

The velocity of circulation is a concept that monetary economists have been preoccupied with for a long time. The variable is important from a theoretical as well as a policy perspective as it plays a crucial role in the economy (see, for example, Bordo, 1989).

Accurate forecasting of the velocity of circulation (defined as the ratio of nominal output to the money supply) is useful for policy purposes. One important issue involved in the choice of a monetary aggregate for policy purposes is the predicability of the relationship between the

* We are grateful to an anonymous referee for some useful comments.

** The corresponding author. Fax: (03) 9479 1654. E-mail: i.moosa@latrobe.edu.au.

aggregate and nominal GDP, where the velocity is the link. If we take the ability to achieve a desired level of the money supply as given, the success in achieving a nominal GDP target is based on the precision with which velocity can be forecast. Moreover, the stability of the velocity is crucial for the monetarist model of inflation as portrayed by the quantity theory of money. For a stable relationship to exist between the general price level and the money supply per unit of real output, velocity must be stable. Hence, accurate forecasting of velocity is required to predict (with precision) the inflationary effect of a monetary expansion.

Some attempts have been made to devise accurate forecasting models of the velocity of circulation, invariably using U.S. data. For example, Hein and Veugelers (1983) examined the predicability of velocity using several time series methods. The results show that velocity fluctuates randomly about a fixed mean. Hence, they concluded that forecasting velocity using average values over some extended period of time is as effective as any other more sophisticated forecasting technique. They also found that the accuracy of average velocity forecasts improved with the time horizon over which the forecast is made.

Other studies have been concerned with the issue of forecasting the velocity as a defined variable, which is a variable that is defined in terms of, or constructed from, other variables. The velocity is a defined variable because it can be defined in terms of, or calculated from, the general price level, real output and the money supply. Defined variables can be forecast either directly, by estimating a model from historical data on the whole series, or indirectly by estimating separate models for the components. Specifically, indirect forecasting of the velocity as a defined variable involves the following steps: (i) estimation of models for the price level, real output and the money supply (the components); (ii) using the estimated model to generate forecasts for these variables; and (iii) using its definition to generate forecasts for the velocity, given the forecasts of the components.¹ Kang (1986) made such an attempt and found mixed results for the velocity of circulation as the direct method turned out to be superior according to some criteria and inferior according to others. He attributed the inconclusiveness of the results to the U.S. monetary policy shift to monetary targeting in October 1979. Still, he maintained that the indirect method makes more sense, and should be more powerful.

One may suggest an analytical comparison of the accuracy of direct and indirect methods, but it should be extremely difficult or even impossible. This is mainly because the velocity is a complicated non-linear function of other variables. In the simpler case of aggregation, there have been theoretical studies that provided the analytical conditions under which one method performs better than the other. They include Grunfeld and Griliches (1960), Orcutt et al. (1968), Edwards and Orcutt (1969), Rose (1977), Tiao and Guttman (1980), Wei and Abraham (1981), Kohn (1982), Lütkepohl (1984) and Clark (2000). However, these conditions are so restrictive and unrealistic that the choice should be made entirely on an empirical basis.

¹ It is arguable that the terms “direct” and “indirect” may be inadequate here. Alternatively, this issue can be considered in terms of the error term of the predicted values, specifically, whether they are constrained or unconstrained (or whether they are dependent or independent). In this case, the constraint arises from the need to satisfy the identity that the logarithm of the velocity is equal to the logarithm of the price level plus the logarithm of income minus the logarithm of the money supply. Since this identity has to be satisfied, the error term of the predicted value cannot vary freely. For example, if velocity is stationary, then the three right-hand-side variables of the identity must be cointegrated in a specific way. Having said that, however, we will still use the terms “direct” and “indirect”, as is normally done in the literature.

Whereas the superiority of either method over the other is a purely empirical matter, indirect forecasting may sound more appealing because definitions can embrace variables of diverse characters. The components of a defined variable are heterogenous with respect to the time series properties and the units of measurement. It is the objective of this paper to compare the ability of the direct and indirect methods to forecast the velocity of circulation in the Japanese economy.

II. *An Informal Examination of the Data*

The velocity of circulation is defined as nominal GDP (or real GDP multiplied by the GDP deflator) divided by the money supply. The data sample, obtained from Datastream, consists of quarterly observations covering the period 1970:1-1999:1 and four variables: money supply (M1 and M2), real GDP (Y) and GDP deflator (P). The observations up to 1995:4 are used for model identification and estimation, and the remaining 13 observations are held out for forecast accuracy evaluation. We consider M1 and M2 velocities, which are defined as $V_1 = PY/M1$ and $V_2 = PY/M2$ respectively. Note that M1, M2, Y and P are transformed into natural logarithms for testing, estimation and forecasting. However, their forecasts are transformed back into the original scales to generate indirect forecasts. Figure 1 shows time plots of the components (M1, M2, Y and P) and the defined variables (V1 and V2). It can be seen that all component variables show strong upward trends. Furthermore, it appears that the defined variable V1 shows random fluctuations around the fixed mean, whereas V2 shows a downward trend.

In the following two sections we present an outline of the methodology employed for forecasting the two velocities using the direct and indirect methods. Forecasts are generated based on univariate autoregressive modelling and Harvey's structural time series modelling. In the former case, prediction interval (calculated by using the bootstrap-after-bootstrap) is employed to compare the direct and indirect forecasting methods. Note that we fit the two models to all time series in levels without considering the presence of unit roots. This modelling strategy, which is widely used for forecasting time series with a possible unit root, has been found to provide reasonably accurate forecasts (see, for example, Diebold and Kilian, 2000). For this reason, we do not attempt to conduct unit root testing in this paper. This may be justified on the grounds that unit root testing as applied to small samples is subject to widespread scepticism (see, for example, Rudebusch, 1993).

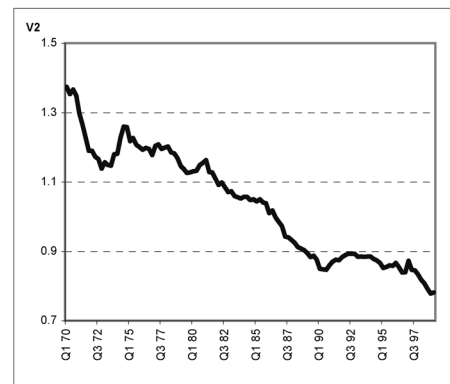
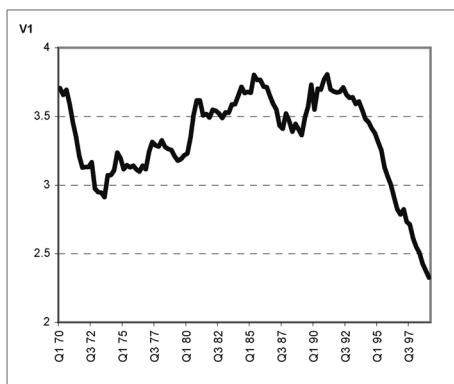
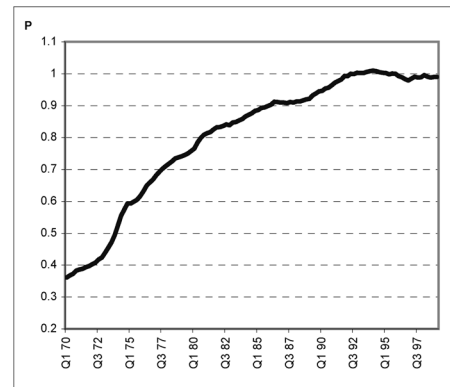
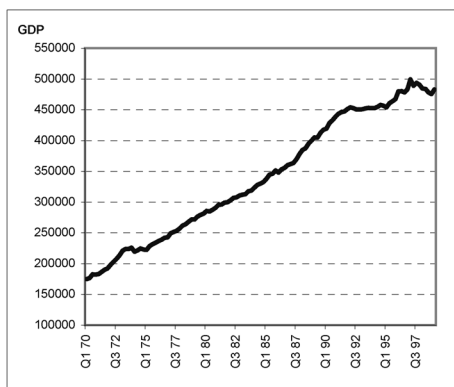
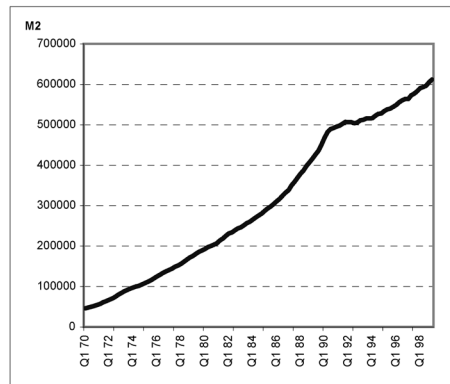
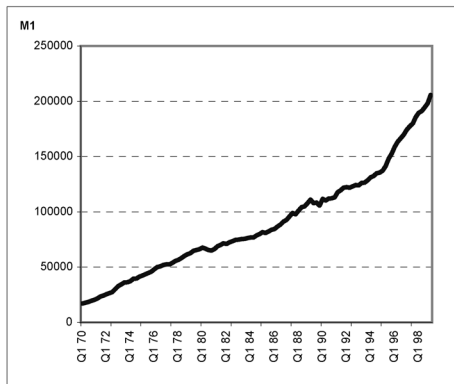
III. *Methodology: Autoregression and the Bootstrap*

Consider an autoregressive (AR) model with a linear time trend of the form

$$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \dots + \alpha_p Y_{t-p} + \alpha_{p+1} t + u_t \quad (1)$$

where u_t is a white noise process and $t = 1, \dots, n$. In this paper, we attempt to fit equation (1) to all time series in levels. This modelling strategy is popularly adopted in practice for forecasting, although it can yield severely biased parameter estimates in small samples, especially when the model is a unit root or a near-unit root process. To generate bias-corrected

FIG. 1. TIME SERIES PLOTS OF THE VARIABLES



point and interval forecasts, we resort to the bootstrap method of Efron and Tibshirani (1993).

The unknown parameters are estimated using the least-squares (LS) method, such that the LS estimators are denoted $(\hat{\alpha}_0, \hat{\alpha}_1, \dots, \hat{\alpha}_p, \hat{\alpha}_{p+1})$. The AR order, p , is determined using Akaike's (1974) information criterion, combined with the residual diagnostics and over-fitting tests. The latter include the Ljung-Box test for white noise errors and the over-fitting test on AR coefficients using the asymptotic-t test. The time trend coefficient, α_{p+1} , is restricted to zero when there is no global trend or when the coefficient is found to be statistically no different from zero. The forecasts for Y_{n+h} at time n can be generated using the estimated coefficients as

$$Y_n(h) = \hat{\alpha}_0 + \hat{\alpha}_1 Y_n(h-1) + \dots + \hat{\alpha}_p Y_n(h-p) + \hat{\alpha}_{p+1}(n+h), \quad (2)$$

where $Y_n(j) = Y_{n+j}$ for $j \leq 0$. Suppose that Y_t is a defined variable, say $Y_t = X_{1t}X_{2t}/X_{3t}$, which is the definition of the velocity if X_1 , X_2 , and X_3 are the price level, GDP and the money supply respectively. Obviously, $Y_n(h)$ is the direct forecast for Y_{n+h} . Equation (1) can be fitted to the X_{it} 's, and forecasts can be generated as in (2) to yield $X_{1,n}(h)$, $X_{2,n}(h)$ and $X_{3,n}(h)$. The indirect forecast for Y_{n+h} is generated as

$$Z_n(h) = X_{1,n}(h)X_{2,n}(h) / X_{3,n}(h) \quad (3)$$

We also use interval forecasts to compare the forecasting performance of the direct and indirect methods. Past studies compared forecast accuracy between the two methods based only on point forecasts. This can be restrictive, as point forecasts carry no information as to how much uncertainty is associated with forecasting (Chatfield, 1993). Comparison based on interval forecasts is more useful because of richer information content. Prediction intervals based on the direct and indirect methods can be constructed by using the bootstrap method. Past studies that proposed the use of bootstrap prediction intervals for AR models include, *inter alia*, Thombs and Schucany (1990) and Kim (1999). The bootstrap adopted by these authors generates pseudo-data sets based on the backward AR model associated with the forward model (1). This procedure is used to incorporate the conditionality of AR forecasts on past observations into bootstrap replicates.

To construct bias-corrected bootstrap prediction intervals for AR models, we use the bootstrap-after-bootstrap recently proposed by Kilian (1998a, 1998b). It involves two successive applications of the standard (non-parametric) bootstrap detailed in Efron and Tibshirani (1993, p 96). In a recent study, Kim (2001) extended the work of Thombs and Schucany (1990) and Kim (1999) by applying the bootstrap-after-bootstrap to prediction intervals for AR models. Kim (2001) simulated a wide range of univariate and bivariate AR models with or without linear time trend, under normal and non-normal innovations including Student t and ARCH. It was found that bootstrap-after-bootstrap prediction intervals perform much better than other asymptotic and standard bootstrap alternatives in small samples, particularly when the AR model has characteristic roots that are equal or close to 1. The asymptotic validity of the bootstrap for autoregressive models with a possible unit root can be found in Inoue and Kilian (2001).

The bootstrap-after-bootstrap prediction intervals for AR forecasts (2) can be constructed by following a three-stage procedure (further details can be found in Kim, 2001). In stage 1, equation (1) is fitted to observed data realisations (Y_1, \dots, Y_n) to obtain $(\hat{\alpha}_0, \hat{\alpha}_1, \dots,$

$\hat{\alpha}_p, \hat{\alpha}_{p+1}$) and residuals $\{\hat{u}_t\}_{t=p+1}^n$. The LS residuals are centred and scaled as in Thombs and Schucany (1990). In stage 2, the standard bootstrap for AR models is conducted, using $\hat{\alpha}_i$'s and $\{\hat{u}_t\}$ obtained in stage 1, to estimate bias of parameter estimates denoted as $Bias(\hat{\alpha}_i)$. For this purpose, the pseudo-data sets are generated as

$$Y_t^* = \hat{\alpha}_0 + \hat{\alpha}_1 Y_{t-1}^* + \dots + \hat{\alpha}_p Y_{t-p}^* + \hat{\alpha}_{p+1} t + u_t^* \quad (4)$$

where u_t^* is a random draw with replacement from $\{\hat{u}_t\}$. Adapting the procedure proposed by Kilian (1998a), the bias-corrected estimators $\hat{\alpha}_i^c$'s are calculated using $Bias(\hat{\alpha}_i)$.

Stage 3 is concerned with the generation of the pseudo-data sets using $\hat{\alpha}_i^c$'s based on the backward model as

$$Y_t^* = \hat{\alpha}_0^c + \hat{\alpha}_1^c Y_{t+1}^* + \dots + \hat{\alpha}_p^c Y_{t+p}^* + \hat{\alpha}_{p+1}^c t + u_t^* \quad (5)$$

where u_t^* is a random draw with replacement from $\{\hat{u}_t\}$. Using this bias-corrected pseudo-data set, the unknown parameters are estimated using the LS method, which are denoted as $(\hat{\alpha}_0, \hat{\alpha}_1, \dots, \hat{\alpha}_p, \hat{\alpha}_{p+1})$. By adapting the bias-correction procedure of Kilian (1998a), biases of $\hat{\alpha}_i$'s are corrected using the $Bias(\hat{\alpha}_i^c)$'s as a proxy for $Bias(\hat{\alpha}_i)$'s. These bias-corrected estimates are denoted as $\tilde{\alpha}_i^c$'s. The bootstrap replicates of forecasts are generated recursively as

$$Y_n^*(h) = \tilde{\alpha}_0^c + \tilde{\alpha}_1^c Y_n^*(h-1) + \dots + \tilde{\alpha}_p^c Y_n^*(h-p) + \tilde{\alpha}_{p+1}^c (n+h) + u_n^* \quad (6)$$

where $Y_n^*(j) = Y_{n+j}$ for $j \leq 0$ and u_n^* is a random draw with replacement from $\{\hat{u}_t\}$.

By repeating (6) sufficiently many times, say B times, the bootstrap distribution of $Y_n(h)$ can be obtained and denoted as $\{Y_n^*(h; i)\}_{i=1}^B$. From this distribution, the $100(1-\alpha)$ per cent prediction interval for Y_{n+h} based on the direct method can be obtained by resorting to the percentile method of Efron and Tibshirani (1993), which gives $[Y_n^*(h, \tau), Y_n^*(h, 1-\tau)]$, where $Y_n^*(h, \tau)$ is the 100τ th percentile of the bootstrap distribution $\{Y_n^*(h; i)\}_{i=1}^B$ and $\tau = 0.5\alpha$. For example, for 95% prediction interval with $\alpha = 0.05$ and $\tau = 0.025$, and the end points of the prediction interval are the 2.5th percentile and 97.5th percentile of the bootstrap distribution.

In (4), (5) and (6), the same set of residuals are resampled for different purposes. The idea is that the empirical distribution function of the residuals is used as an approximation to the true distribution of error term. Since the resampling is done with replacement, there is no loss of efficiency in approximating the true distribution, even when it is done a number of times for different purposes.

Recalling that $Y_t = X_{1t}(X_{2t}/X_{3t})$, the bootstrap prediction interval based on the indirect method can be obtained by applying the above bootstrap procedures to individual X_{it} 's and generating bootstrap replicates of indirect forecasts as

$$\{Z_n^*(h, i)\}_{i=1}^B = \left\{ X_{1,n}^*(h, i) \frac{X_{2,n}^*(h, i)}{X_{3,n}^*(h, i)} \right\}_{i=1}^B \quad (7)$$

The $100(1-\alpha)$ per cent bootstrap prediction interval for Y_{n+h} based on the indirect method is $[Z_n^*(h, \tau), Z_n^*(h, 1-\tau)]$, where $Z_n^*(h, \tau)$ is the 100τ th percentile of the bootstrap distribution $\{Z_n^*(h, i)\}_{i=1}^B$.

For bias-corrected point forecasting, the bias-corrected parameter estimators $\hat{\alpha}_i^c$'s obtained in stage 2 can be used instead of $\hat{\alpha}_i$'s in equation (2). This provides direct point forecasts, generated from parameter estimates adjusted for small sample biases. This idea can

also be applied to $Z_n(h)$ in (3) to obtain bias-corrected indirect point forecasts.

IV. Methodology: Harvey's Structural Time Series Modelling

The structural time series model of Harvey (1989) is used to decompose an observed time series into unobserved components. These components can be forecast individually and combined to produce a forecast for the total series. This model arguably represents the main features of a time series by considering its various constituent components. The univariate version of the model may be written as

$$Y_t = \mu_t + \phi_t + \gamma_t + \varepsilon_t \quad (8)$$

where Y_t is the observed time series, μ_t is the trend, ϕ_t is the cyclical component, γ_t is the seasonal component and ε_t is the random component. This model postulates general specifications for the processes generating the components, thus allowing for any possibility such as the presence of stochastic (rather than deterministic) trend and seasonality. The trend, cyclical and seasonal components are assumed to be uncorrelated while ε_t is assumed to be white noise.

The trend, which represents the long-term movement in a series, can be represented by

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \quad (9)$$

$$\beta_t = \beta_{t-1} + \zeta_t \quad (10)$$

where $\eta_t \sim NID(0, \sigma_\eta^2)$, and $\zeta_t \sim NID(0, \sigma_\zeta^2)$. μ_t is a random walk with a drift factor, β_t , which follows a first order autoregressive process as represented by equation (10). This process collapses to a simple random walk with drift if $\sigma_\zeta^2 = 0$, and to a deterministic linear trend if $\sigma_\eta^2 = 0$ as well. If, on the other hand, $\sigma_\eta^2 = 0$ while $\sigma_\zeta^2 \neq 0$, the process will have a trend that changes relatively smoothly.

The cyclical component, which is assumed to be a stationary linear process, may be represented by

$$\phi_t = a \cos(\theta t) + b \sin(\theta t) \quad (11)$$

where t is time and the amplitude of the cycle is given by $(a^2 + b^2)^{0.5}$. In order to make the cycle stochastic, the parameters a and b are allowed to evolve over time, while preserving continuity is achieved by writing down a recursion for constructing ϕ before introducing the stochastic elements. By introducing disturbances and a damping factor we obtain

$$\phi_t = \rho(\phi_{t-1} \cos \theta + \phi_{t-1}^* \sin \theta) + \omega_t \quad (12)$$

$$\phi_t^* = \rho(-\phi_{t-1} \sin \theta + \phi_{t-1}^* \cos \theta) + \omega_t^* \quad (13)$$

where ϕ_t^* appears by construction such that ω_t and ω_t^* are uncorrelated white noise disturbances with variances σ_ω^2 and $\sigma_{\omega^*}^2$ respectively. The parameters $0 \leq \theta \leq \pi$ and $0 \leq \rho \leq 1$ are the frequency of the cycle and the damping factor on the amplitude respectively. The period of the cycle, which is the time taken by the cycle to go through its complete sequence of values, is $2\pi/\theta$ (Harvey, 1989, p 38). The stochastic cycle in equations (12) and (13) collapses to AR(1) process when $\theta = 0$ or π . In order to make numerical optimisation easier, the constraint $\sigma_\omega^2 = \sigma_{\omega^*}^2$ is imposed.

While there are a number of different specifications for seasonality (see Harvey, 1989, Chapter 2), the trigonometric specification is the most preferred. For an even s , where s is the number of seasons per year (four for quarterly data), the seasonal component is written as

$$\gamma_t = \sum_{j=1}^{s/2} \gamma_{j,t} \quad (14)$$

$\gamma_{j,t}$ is given by

$$\gamma_{j,t} = \gamma_{j,t-1} \cos \lambda_j + \gamma_{j,t-1}^* \sin \lambda_j + \kappa_{j,t} \quad (15)$$

$$\gamma_{j,t}^* = -\gamma_{j,t-1} \sin \lambda_j + \gamma_{j,t-1}^* \cos \lambda_j + \kappa_{j,t}^* \quad (16)$$

with $j = 1, \dots, (s/2) - 1$, $\lambda_j = 2\pi j/s$ and

$$\gamma_{j,t} = -\gamma_{j,t} + \kappa_{j,t}, \quad j = s/2 \quad (17)$$

where $\kappa_{j,t} \sim NID(0, \sigma_{\kappa}^2)$ and $\kappa_{j,t}^* \sim NID(0, \sigma_{\kappa}^{*2})$. Again, the assumption $\sigma_{\kappa}^2 = \sigma_{\kappa}^{*2}$ is imposed. One advantage of this specification is that it allows for smoother changes in the seasonals.

The extent to which the trend, seasonal and cyclical components evolve over time depends on the values of σ_{η}^2 , σ_{ϵ}^2 , σ_{κ}^2 , σ_{ω}^2 , θ and ρ which are known as the hyperparameters. These parameters can be estimated by maximum likelihood in the time or frequency domain once the model has been written in a state space form (Harvey, 1989, Chapter 4). The frequency domain estimation is much faster but it provides slightly different results because the procedure is based on an approximation to the frequency domain likelihood function. When these parameters have been estimated via the Kalman filter, it is possible to obtain estimates of the unobserved components. For details, see Harvey (1989) and Koopman et al. (1995).

In order to induce more dynamics in the model, which may be necessary to remove serial correlation, the model is modified by including a lagged dependent variable. Hence, the model that we estimate takes the form

$$Y_t = \mu_t + \phi_t + \gamma_t + \delta Y_{t-1} + \varepsilon_t \quad (18)$$

In equation (18) the components should turn out to be insignificant if the explanatory variable is capable of fully explaining the variation in the dependent variable, which would otherwise be explained by the components. This would be the case if the dependent variable is generated by a pure autoregressive process of order 1.

V. Empirical Results

Table 1 reports selected diagnostics for the estimated AR models of all time series along with selected diagnostics. The linear time trend is included for M1, Y and V2. AR models of orders in the range 1-4 are fitted. For all fitted models, the residuals are found to mimic a white noise process. It can also be seen that the estimated AR coefficients yield characteristic roots fairly close to the unit circle for all time series, suggesting the possibility of unit or near-unit root AR models. The Bera-Jarque (BJ) test for non-normality and the LM test for ARCH innovations are also reported. It is evident that some time series show strong evidence for non-normality and conditional heteroskedasticity in their residuals. Simulations conducted by

TABLE 1. DIAGNOSTICS OF THE AR MODELS

	p	Trend	$Q(12)$	BJ	LM	Root
M1	1	Yes	12.33 (.42)	8.82	8.34	1.06
M2	3	No	11.70 (.47)	1.08	7.08	1.02
Y	4	Yes	19.53 (.08)	50.07	0.11	1.17
P	4	No	9.87 (.62)	1.41	7.17	1.02
V1	3	No	9.13 (.69)	1.77	5.01	1.11
V2	3	Yes	13.93 (.31)	4.65	13.23	1.28

p : the order of the autoregressive process.

Q : the Ljung-Box test statistic for the joint significance of the residual SACF's up to lag 12. The p-values are given in parentheses.

LM: test statistic for ARCH(4) errors, which asymptotically follows chi-squared distribution with 4 degrees of freedom.

BJ: Bera-Jarque test statistic for normality, asymptotically following chi-squared distribution with 2 degrees of freedom.

"Root" indicates the modulus of the smallest characteristic root from AR coefficients.

TABLE 2. GOODNESS OF FIT MEASURES AND DIAGNOSTICS OF HARVEY'S MODELS

	$\tilde{\sigma}$	H	DW	Q	R_s^2	$CUSUM$	PF
M1	0.02	0.99	1.99	7.79	0.29	0.067	3.25
M2	0.01	0.74	2.00	13.03	0.77	0.27	9.64
Y	0.01	0.28	2.12	11.69	0.05	-0.98	20.36
P	0.01	0.42	1.90	12.72	0.50	0.03	21.47
V1	0.07	1.09	1.94	4.48	0.01	-0.24	3.67
V2	0.01	0.14	2.00	8.41	0.08	-0.22	14.95

H is distributed as $F(33,33)$, Q as $X^2(7)$, $CUSUM$ as $t(101)$ and PF as $X^2(13)$.

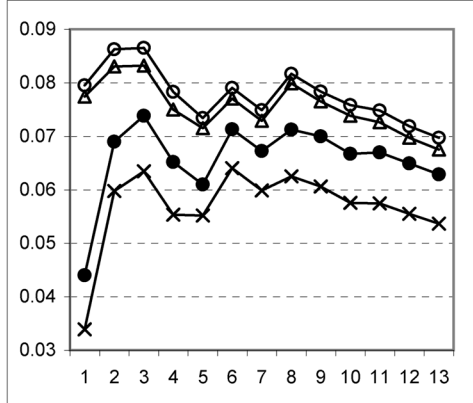
Kilian (1998a, 1998b) and Kim (2001) provide evidence suggesting that the bootstrap-after-bootstrap is a highly effective tool for obtaining bias-corrected bootstrap replicates in small samples, especially for the AR models whose characteristic roots are equal to or close to 1. Moreover, Kim (2001) found that bootstrap-after-bootstrap prediction intervals perform reasonably well in the presence of non-normal innovations including ARCH processes.

Now, we turn to the estimation results of the structural time series models. The following goodness-of-fit and statistical adequacy measures are presented: $\tilde{\sigma}$ is the standard error of the estimate; H is a heteroskedasticity test statistic calculated as the ratio of the sum of squares of the last h residuals to that of the first h residuals where h is the closest integer to one third of the sample size; DW is the Durbin-Watson statistic; Q is the Ljung-Box test statistic for serial correlation; and R_s^2 is the modified coefficient of determination. Finally, two structural stability test statistics are reported: the $CUSUM$ test (which has a t distribution) and the PF test (which has a X^2 distribution). Further details of these diagnostic tests and goodness-of-fit measures are given in Harvey (1989) and in Koopman et al. (1995). The results reported in Table 2 show that the estimated models are well determined in terms of goodness of fit measures and that they pass the diagnostics tests.

We consider both updated and multi-step forecasts. The updated forecasts are one-step-ahead forecasts generated by updating the data sample and re-estimating the unknown coefficients over the forecast period. To compare the accuracy of point forecasts, we use cumulative root mean squared error (CRMSE) over the forecast period.

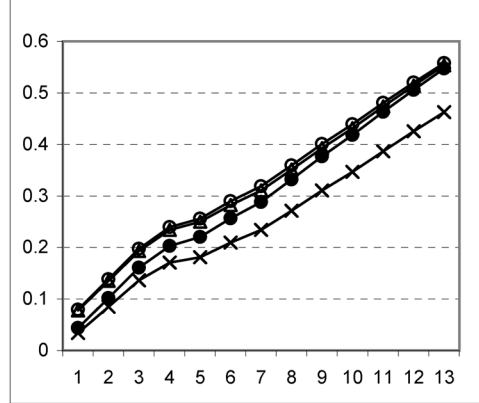
FIG. 2. DIRECT AND INDIRECT FORECASTS OF THE M1 VELOCITY(V1): AR MODELS
Cumulative Root Mean Squared Error

Updated forecasts



● : direct forecasts
○ : indirect forecasts

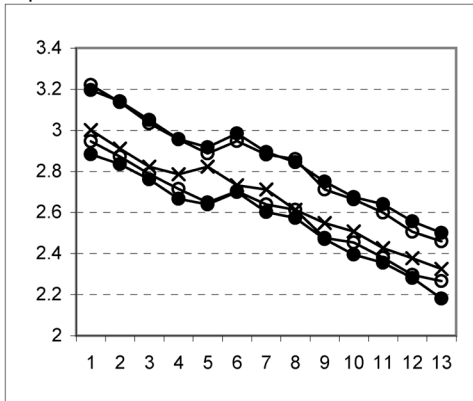
Multi-step forecasts



× : direct forecasts with bias-correction
△ : indirect forecasts with bias-correction

95% Bootstrap Prediction Intervals and Actual Values

Updated forecasts



× : actual values
● : prediction intervals for direct forecasts
○ : prediction intervals for indirect forecasts

Multi-step forecasts

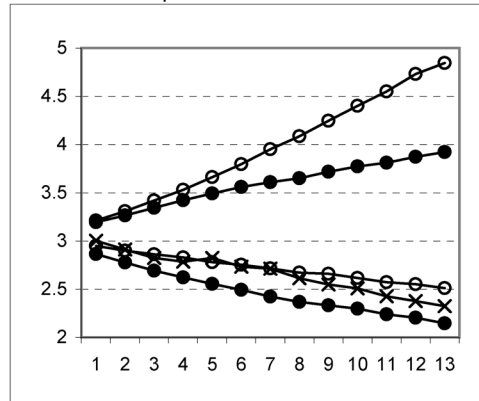
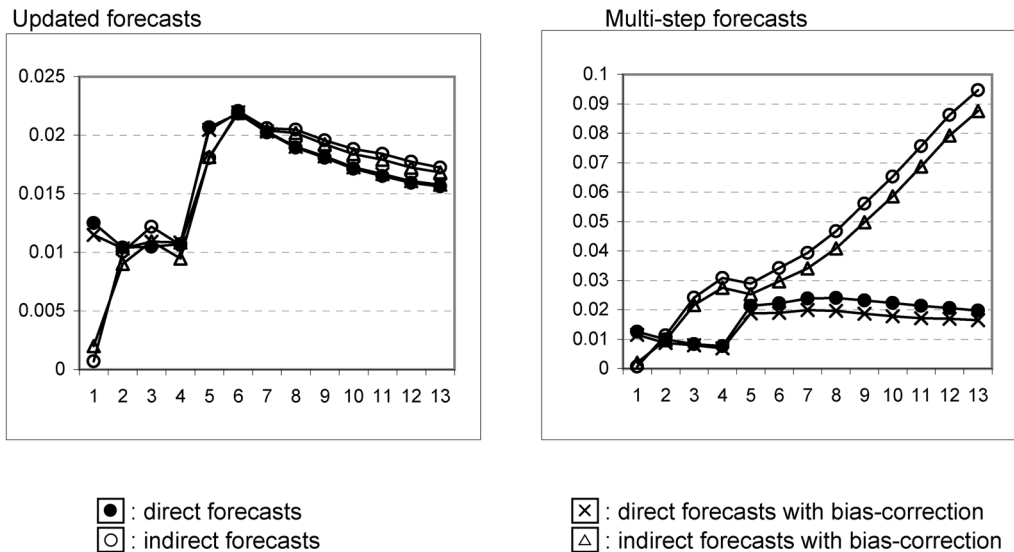


FIG. 3. DIRECT AND INDIRECT FORECASTS OF THE M2 VELOCITY(V2): AR MODELS
Cumulative Root Mean Squarred Error



95% Bootstrap Prediction Intervals and Actual Values

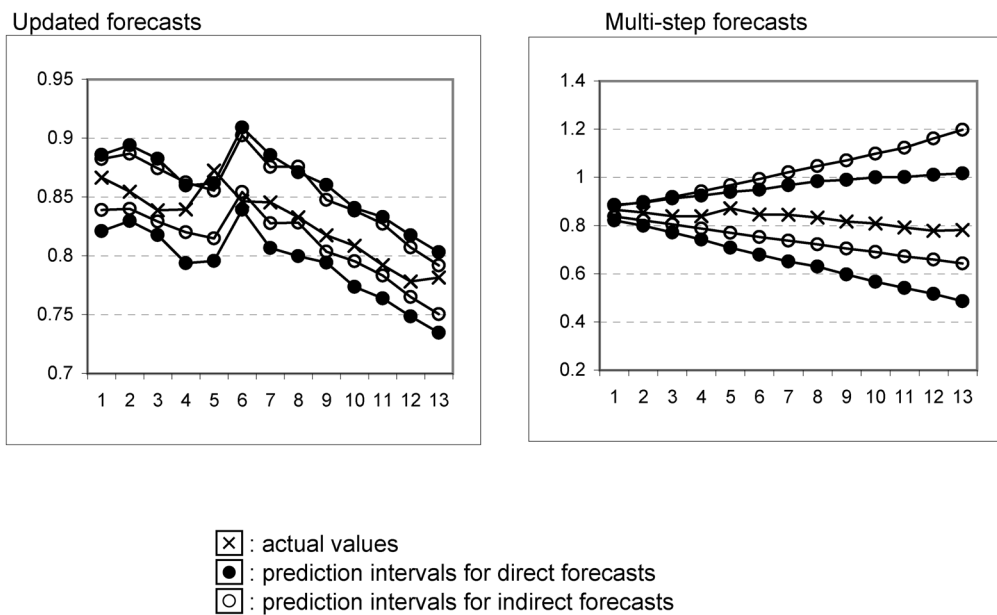
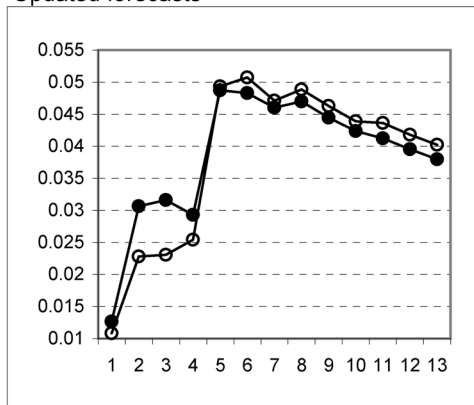


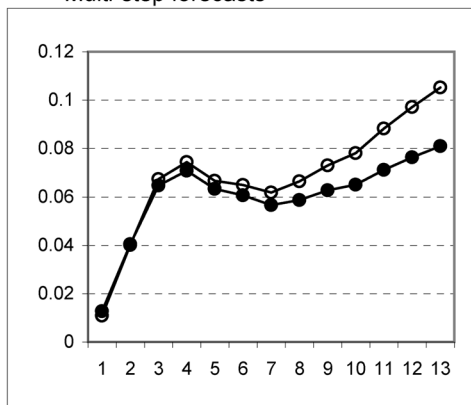
FIG. 4. DIRECT AND INDIRECT FORECASTS: HARVEY'S MODEL

V1: Cumulative Root Mean Squared Error

Updated forecasts

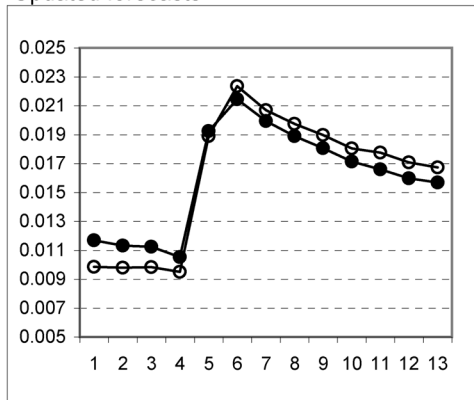


Multi-step forecasts

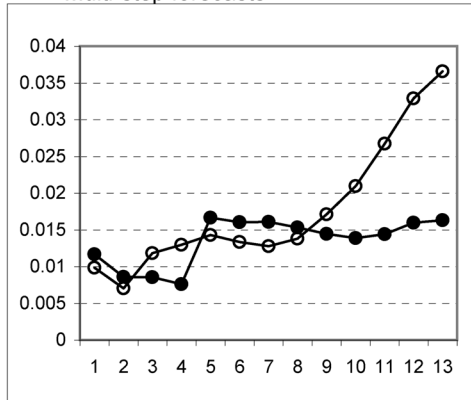


V2: Cumulative Root Mean Squared Error

Updated forecasts



Multi-step forecasts



● : direct forecasts

○ : indirect forecasts

Figure 2 reports the case of V1 when forecasts are generated from AR models. It is evident that V1 can be forecast more accurately by the direct method, for both updated and multi-step forecasts. In particular, the direct AR forecasts with bias-corrected coefficients show the smallest CRMSE values. In the second panel of Figure 2, the 95 per cent bootstrap-after-bootstrap prediction intervals generated from the direct and indirect methods are reported. In the case of updated forecasts, prediction intervals from the direct and indirect methods are quite similar, indicating that the degrees of uncertainty associated with the two methods are nearly the same. In the case of multi-step forecasts, the prediction intervals from

the direct method are much narrower than those from the indirect method, indicating that less uncertainty is associated with the direct method. The evidence from point and interval forecasting suggests that the direct method provides more accurate forecasts for V1.

Figure 3 reports the case of V2. For updated forecasts, the two methods perform similarly, although there is tendency for the direct method to perform better. For multi-step forecasts, it is evident that the direct method performs much better than the indirect method. As before, the gain in accuracy due to bias-correction is evident. In the case of prediction intervals, those based on the direct method are slightly wider than those based on the indirect method. For updated forecasts, the mean width of prediction intervals from the direct method is 0.07, and that from the indirect method is 0.04. For multi-step forecasts, the mean widths of prediction intervals from the direct and indirect methods are 0.31 and 0.29 respectively.

Finally, Figure 4 compares the CRMSE values of the forecasts for V1 and V2, when forecasts are generated from Harvey's model based on the direct and indirect methods. For all cases, it is evident that the direct method generates more accurate forecasts than the indirect method. Thus, the evidence points to the superiority of the direct forecasting method.

VI. Conclusion

In this paper we examined the forecastability of the velocity of circulation in the Japanese economy by employing two models and two forecasting methods: the direct method and the indirect method. This choice is available because the velocity is a defined variable, which means that it can be measured residually from its constituent components.

We compared the accuracy of direct and indirect forecasting of the velocity of circulation of two monetary aggregates: M1 and M2. For forecasts derived from AR models, we used two criteria: the cumulative mean square error and the bootstrap prediction intervals. Only the first criterion was used in conjunction with the forecasts derived from Harvey's structural time series model.

In general, the results supported the superiority of the direct method. While these results are not consistent with the theoretical appeal of the indirect method, they can be justified on the basis of the following proposition. It is arguable that direct forecasting can be more accurate if it leads to a reduction in the noise associated with the individual components. After all, the direct method is based on a definition whereby differing time series are pooled, and this may lead to a smaller random component (to use Harvey's terminology) than those of the individual time series.

REFERENCES

- Akaike, H. (1974) A New Look at the Statistical Model Identification, *IEEE Transactions on Automatic Control*, AC-19, pp.716-723.
- Bordo, M. (1989) Equation of Exchange, in: Eatwell, J., Milgate, M. and Newman, P. (eds) *The New Palgrave: Money*, Macmillan: London, vol 2, pp.151-156.
- Clark, T. E. (2000) Forecasting an aggregate of cointegrated disaggregates, *Journal of Forecasting*, 19, pp.1-21.

- Chatfield, C. (1993) Calculating Interval Forecasts, *Journal of Business and Economic Statistics* 11, pp.121-135.
- Diebold F.X. and Kilian, L. (2000), Unit Root Tests are Useful for Selecting Forecasting Models, *Journal of Business and Economic Statistics* 18, pp.265-273.
- Edwards, J. B. & Orcutt, G. H. (1969) Should aggregation to estimation be the rule? *Review of Economics and Statistics* 51, pp.409-420.
- Efron, B. and Tibshirani, R.J. (1993) *An Introduction to the Bootstrap*, New York: Chapman & Hall.
- Grunfeld, Y. & Griliches, Z. (1960) Is aggregation bad? *Review of Economics and Statistics* 42, pp.1-13.
- Harvey, A.C. (1989) *Forecasting: Structural Time Series Models and the Kalman Filter*, Cambridge, Cambridge University Press.
- Hein, S.E. and Veugelers, T.W.M. (1983) Predicting Velocity Growth: A Time Series Perspective, *Federal Reserve Bank of St. Louis Review*, October, pp.34-43.
- Inoue, A. and Kilian, L. (2001) Bootstrapping Autoregressive Processes with Possible Unit Roots, *Econometrica* (forthcoming).
- Kang, H. (1986) Univariate ARIMA Forecasts of Defined Variables, *Journal of Business and Economic Statistics* 4, pp.81-86.
- Kilian, L. (1998a) Small Sample Confidence Intervals for Impulse Response Functions, *Review of Economics and Statistics* 80, pp.218-230.
- Kilian, L. (1998b) Confidence Intervals for Impulse Responses Under Departure from Normality, *Econometric Reviews* 17, pp.1-29.
- Kim, J.H. (1999) Asymptotic and Bootstrap Prediction Regions for Vector Autoregression, *International Journal of Forecasting* 15, pp.393-403.
- Kim, J.H. (2001) Bootstrap-after-Bootstrap Prediction Intervals for Autoregressive Models, *Journal of Business and Economic Statistics* 19, pp.117-128.
- Kohn, R. (1982) When is an aggregate of a time series forecast by its past? *Journal of Econometrics* 18, pp.337-349.
- Koopman, S.J., Harvey, A.C., Doornik, J.A. and Shephard, N. (1995) *Stamp 5.0: Structural Time Series Analyser, Modeller and Predictor*, London: Chapman and Hall.
- Lütkepohl, H. (1984) Forecasting contemporaneously aggregated vector ARMA processes, *Journal of Business and Economic Statistics* 2, pp.201-214.
- Orcutt, G. H., Watts, H. & Edwards, J. B. (1968) Aggregation and information loss, *American Economic Review* 58, pp.773-787.
- Rose, D. (1977) Forecasting aggregates of independent ARIMA processes, *Journal of Econometrics* 5, pp.323-345.
- Rudenbusch, G.D. (1993) The Uncertain Unit Root in Real GNP, *American Economic Review* 83, pp.264-272.
- Tiao, G. C. and Guttman, I. (1980) Forecasting contemporaneous aggregates of multiple time series, *Journal of Econometrics* 12, pp.219-230.
- Thombs, L.A. and Schucany, W.R. (1990) Bootstrap Prediction Intervals for Autoregression, *Journal of the American Statistical Association* 85, pp.486-492.
- Wei, W.W.S. and Abraham, B. (1981) Forecasting temporal time series aggregates, *Communications in Statistical Theory and Methods* 10, pp.1335-1344.