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TAX POLICIES AND UNEMPLOYMENT IN A DYNAMIC EFFICIENCY WAGE MODEL*

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Abstract

In this paper we make a dynamic analysis of the effects of various tax policies on the unemployment of an economy with a labour-efficiency function which shifts over time. Stock of knowledge, which is produced as a durable public good, accumulates over time; and the efficiency of the worker varies positively with the stock of knowledge in addition to wage and unemployment. We analyze the accumulation of physical capital and human capital stock (stock of knowledge) and the properties of long-run equilibrium of the system. The comparative steady-state effects on unemployment with respect to change in various tax rates are analyzed assuming that the production of the public good (educational output) is financed by the tax revenue. In many cases, these results are different from the corresponding comparative static results.

Key Words: Tax; Unemployment; Efficiency
JEL classification: H20; J64; O41

I. *Introduction*

There exists a substantial literature on the properties of the various trade and fiscal policies in the two-sector efficiency wage model of unemployment. This model is a special case of the closed economy version of the two-sector Heckscher-Ohlin-Samuelson (HOS) variety with an important departure. Labour is measured in efficiency unit here; and the efficiency of the worker varies positively with wage and unemployment. This literature includes the works

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of Agell and Lundborg (1992, 1995), Brecher (1992), Akerloff and Yellen (1990), Solow (1979), Katz (1986), Copeland (1989), Davidson, Martin and Matuz (1988), Shapiro and Stiglitz (1984), Pisauro (1991) etc. However, all the existing works in this area are essentially static in nature. The efficiency function of the worker, which is the most important feature of all these models, does not include any argument which accumulates over time. So the efficiency function of the worker in a static model does not shift over time.

In the present paper, we consider a dynamic extension of the static efficiency wage model. The efficiency function now includes an additional argument which accumulates over time; and this causes the efficiency of the worker to shift upwards. This variable is the stock of knowledge. Gross addition of this variable is produced as a public good and the production is financed by tax revenue net of unemployment subsidy. Govemment expenditure in education is substantially higher in a less developed economy like India in comparison to private expenditure. Why the efficiency of the worker varies positively with his level of education (skill) is formally explained in the appendix. The efficiency function[1] in the existing literature is derived as the optimum effort function of the utility maximizing worker. So the optimum effort varies postively with the level of education when the marginal disutility of labour is lower (higher) for a more (less) educated worker.

In the standard efficiency wage model, unemployment is determined by the efficiency function. So if the efficiency function of the worker shifts over time, the unemployment level should also change over time. Hence we need a dynamic intertemporal framework to analyze the effects of various policies on the equilibrium level of unemployment. Using such a dynamic framework, where physical capital and human capital (stock of knowledge) accumulate over time we analyze the comparative steady-state effects of changes in the rates of various taxes imposed to finance the production of public good (educational output). The importance of this dynamic exercise becomes clear when we look at the various comparative steady state effects on unemployment. The corresponding comparative static effects on unemployment which are available in the static literature are not necessarily identical to the comparative steady-state results obtained in this paper. The parametric change in the policy variable affects the long-run equilibrium value of the stock of human capital in this model. This produced an additional effect on the efficiency function and hence on unemployment. This may weaken or even strengthen the comparative static effects.

The plan of the paper is as follows. In section 2 the static model has been described. The comparative static effects of various taxes are discussed in this section. The dynamic version of the model has been presented in section 3. The comparative steady-state effects of changes in the tax rate (and the subsidy rate) are described in section 4. The concluding remarks are made in section 5.

II. *The Static Model*

We consider a closed economy consisting of two sectors, sectors1 and 2. Both these

[1] Actually it is the worker who in the context of his labour supply function finds an endogenous probability of being fired (if caught shirking). See Shapiro and Stiglitz (1984) for its details. However, in the general equilibrium literature, it is called the efficiency function. See Brecher (1992), Agell and Lundborg (1992, 1995), Pisauro (1991) etc.

sectors produce private goods with the help of physical capital and labour. There is perfect intersectoral mobility of physical capital and labour leading to equalization of the wage rates and the interest rates. Production function in each sector satisfies all the standard neoclassical properties including CRS. Physical capital is fully utilized. A part of the labour force may, however, remain unemployed. Markets for both the private goods are perfectly competitive. Stock of physical capital is exogenously given at some point of time though it accumulates over time. Number of workers available in the economy is also given.[2] Apart from the private goods there exists a public good in the economy. We consider proportional tax on domestic factor income, advalorem tax on labour and capital as alternative forms of taxation. We also consider that total tax revenue net of unemployment subsidy is used to finance the public good. This public good is the educational output which contributes to the expansion of the stock of knowledge. We assume this stock of knowledge to be identical to the accumulated skill of the worker. Labour is measured in efficiency unit, and the efficiency of the representative worker varies positively with wage rate, unemployment and the stock of knowledge. The efficiency of the representative worker, however, varies negatively with the interest rate (or the rate of return on capital) and unemployment subsidy rate. The stock of knowledge (human capital) is given at a particular point of time though it accumulates over time. This causes the intertemporal accumulation of labour force measured in efficiency unit, i.e. the efficiency function of the worker, to shift over time. This particular property of the efficiency function has not been considered in the existing literature; and this justifies the importance of the dynamic analysis made in the later sections of the paper.

The notations used in this model are sated in the following manner. Let X_j, K_j, f_j, L_j, E_j, k_j, C_{Ej}, C_{Kj}, C_j, P_j, θ_{Ej}, θ_{Kj}, λ_{Ej} and λ_{Kj} denote respectively the level of output, the capital stock employed, intensive production function, the level of employment, the level of employment in efficiency unit, capital-labour ratio, employment (in efficiency unit) -output ratio, capital-output ratio, unit cost of production, price of the product, share of labour cost (in efficiency unit), share of capital cost, share of efficient labour used and share of capital used in sector j (for $j=1,2$). Let e, p, w, r, Υ_Y, Υ_E, Υ_K, N, U, K, Q, R, b and Y denote respectively efficiency per worker, relative price of product 1 in terms of product 2, wage rate received by the workers, interest rate or rate of retum on capital received by the capitalists, rate of proportional income tax, rate of advalorem tax on employment, rate of advalorem tax on capital, the labour endowment, number of unemployed workers, stock of physical capital, stock of knowledge, the level of output of the educational sector (public good), rate of unemployment subsidy and domestic factor income. It is to be noted that $E_j=eL_j$, $k_j=(K_j/E_j)$, $C_{Ej}=(E_j/X_j)$, $C_{Kj}=(K_j/X_j)$. It is also to be noted that $wT_E=$wage rate paid to the workers, where $T_E=(1/(1-\Upsilon_E))\geqq 1$ for $0\leqq \Upsilon_E<1$, and $rT_K=$rate of retum on capital paid to the capitalists, where $T_K=(1/(1-\Upsilon_K))\geqq 1$ for $0\leqq \Upsilon_K<1$. Thus $\theta_{Ej}=(wT_E C_{Ej}/ep)$ where for $j=2$ we have $p=1$, and $\theta_{Kj}=(rT_K C_{Kj}/ep)$ where for $j=2$ we have $p=1$. Finally, $\lambda_{Ej}=(C_{Ej}X_j/e(N-U))$ and $\lambda_{Kj}=(C_{Kj}X_j/K)$.

The equational structure of the model can be stated as follows.

$$X_1=f_1(k_1)E_1 \tag{1}$$

[2] Major results of the paper will remain unchanged even if we assume that the number of workers grows over time at a constant rate—a standard assumption.

$$X_2=f_2(k_2)E_2 \tag{2}$$
$$p=C_1(wT_E/e),\ rT_K) \tag{3}$$
$$1=C_2((wT_E/e),\ rT_K) \tag{4}$$
$$p=\Psi(X_1/X_2) \tag{5}$$

with $\Psi'<0$

$$C_{E1}X_1+C_{E2}X_2=e(N-U) \tag{6}$$
$$C_{K1}X_1+C_{K2}X_2=K \tag{7}$$
$$Y=w(N-U)+rK \tag{8}$$
$$\Upsilon_Y Y+\Upsilon_E wT_E(N-U)+Y_K rT_K K-bU=R \tag{9}$$
$$e=e(w,\ U,\ Q,\ r,\ b) \tag{10}$$

with $(\partial e/\partial w)>0$, $(\partial e/\partial U)>0$, $(\partial e/\partial Q)>0$, $(\partial e/\partial r)<0$ and $(\partial e/\partial b)<0$

$$\text{Finally, } (\partial e/\partial w)(w/e)=1 \tag{11}$$

Equations (1) and (2) imply the production function of the two private goods. Due to the assumption of CRS $k_j=k_j(wT_E/e)/rT_K)$ for $j=1,2$, where (wT_E/e) implies the wage paid to the workers (measured in efficiency units) and rT_K is the rate of return on capital paid to the capitalists. In the absence of taxes on labour and capital $\Upsilon_E=0$ and $\Upsilon_K=0$ so that $T_E=1$ and $T_K=1$. The assumption of CRS also implies that the unit cost functions (as shown by equations (3) and (4)) are linear homogeneous. Here the demand functions for good 1 and good 2 are assumed to be homothetic so that the price ratio of good 1 with respect to good 2 can be expressed as a negative function of output ratio as shown by equation (5). The assumption of CRS again implies that in equations (6) and (7) $C_{Ej}=C_{Ej}((wT_E/e)/rT_K)$ and $C_{Kj}=C_{Kj}((wT_E/e)/rT_K)$ for $j=1,2$. Domestic factor income is given by equation (8). Equations (1) to (8) implies a simple two-sector closed economy general equilibrium model. After equation (8) we introduce a public good (the level of output of the education sector) in our model. Equation (9) implies that the tax revenue resulting from tax on income and also on factors is used to finance the public good and also unemployment subsidy. We assume that each unemployed worker receives an allowance at the rate b and a part of the tax revenue is spent to finance this.[3]

The readers not familiar with the efficiency wage models may ask for some explanations of equations (10) and (11). Equation (10) is the endogenous efficiency function of the representative worker. It shows how the optimal effort of the representative utility maximizing worker is sensitive to the changes in the wage rate, unemployment level, his level of education (skill), the rate of return on capital and unemployment subsidy rate (treated as a parameter). A micro foundation of this efficiency function is analyzed in the appendix. Existing static literature on the efficiency wage theory assumes that efficiency varies positively with wage and unemployment and negatively with rate of return on capital. The derivation of this property clearly follows from the works of Shapiro and Stiglitz (1984), Pisauro (1991), Agell and Lundborg (1992, 1995) etc.[4] In this model, stock of knowledge, Q, has been considered as an additional efficiency raising factor. This can be justified if the marginal disutility of labour is

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[3] Tax revenue here is used to finance both educational expenditure (expressed in terms of commodity 2) and unemployment subsidy. Educational expenditure (expressed in terms of commodity 2) is actually referred to as educational output in this paper.

[4] In Agell and Lundborg (1992), for example, the efficiency function is actually a positive function of wage-rental ratio implying as the rate of return on capital (or rental on capital) increases the efficiency level falls.

lower for a more educated worker. The stock of knowledge accumulates over time and hence the efficiency function shifts over time. This is the most important dynamic character of this model and we shall focus on the dynamic analysis in the next section. In the efficiency function we have also introduced a parameter reflecting unemployment subsidy with which efficiency varies negatively.

Equation (11) is the traditional Solow (1979) condition i.e. the first order condition of minimizing (w/e) with respect to w. This implies that the wage elasticity of efficiency (ϵ_1) is equal to unity. We assume this elasticity to be independent of U, Q, r and b. This is valid if the efficiency function (10) takes the following special form.

$$e=e_1(w)\ e_2(U, Q, r, b) \tag{10.1}$$

In this static model we have eleven equations (equation (1) to equation (11)) with eleven endogenous variables: X_1, X_2, E_1, E_2, w, e, r, Y, R, U and p. The LHS of equation (9) shows various types of tax revenue (net of subsidies) accruing to government. The first component, $\Upsilon_Y Y$, implies revenue from proportional income tax. The second component, $\Upsilon_E w T_E\ (N-U)$, shows tax revenue resulting from advalorem tax on labour employed. Finally, the term bU shows the total unemployment subsidy. In equation (9) total tax revenue net of subsidies is used to finance the public good, R. Equation (11) implies that we can solve uniquely for w. From equations (3), (4), (5), (6), (7) and (10) we can solve for six unknowns e, r, U, p, X_1 and X_2. Once r and U are known, then given the already determined w, we can determine Y from equation (8). It implies that total tax revenue (net of subsidies) is known and hence R is known. The capital intensities are functions of (wT_E/e) and rT_K. So, when the factor prices, efficiency level and the output levels of the private good producing sectors are known, E_1 and E_2 can easily be determined from equations (1) and (2).

In order to examine the comparative static effects we assume, on the basis of the works of Agell and Lundborg (1992), that

$$(\partial e/\partial r)\ (r/e)=-1$$

In other words, the elasticity of e with respect to r, ϵ_4, is equal to minus unity. From equation (10) we find that

$$\hat{e}=\hat{w}+\epsilon_2\hat{U}+\epsilon_3\hat{Q}-\hat{r}+\epsilon_5\hat{b} \tag{12}$$

where $\hat{z}=(dz/z)$, $\epsilon_1=(\partial\log e/\partial\log w)=1$, $\epsilon_2=(\partial\log e/\partial\log U)>0$, $\partial_3=(\partial\log e/\partial\log Q)>0$, $\epsilon_4=(\partial\log e/\partial\log r)=-1$ and $\epsilon_5=(\partial\log e/\partial\log b)<0$.

From equations (3) and (4) and substituting equation (12) we find that

$$(\hat{P}_1-\hat{P}_2)=-(\theta_{E1}-\theta_{E2})\ (\epsilon_2\hat{U}+\epsilon_3\hat{Q}-\hat{r}+\epsilon_5\hat{b}-\hat{T}_E+\hat{T}_K) \tag{13}$$

where $(\theta_{K1}-\theta_{K2})=-(\theta_{E1}-\theta_{E2})$, given $\theta_{Ej}+\theta_{Kj}=1$ for $j=1,2$.

Again equation (5) implies

$$\hat{X}_1-\hat{X}_2=-\sigma_D\ (\hat{P}_1-\hat{P}_2) \tag{14}$$

where $\sigma_D>0$ is the aggregate elasticity of substitution in demands.[5]

[5] This follows from homothetic demand functions. In fact $\sigma_D=-(\epsilon_{11}+\epsilon_{22})$ where ϵ_{ii} is the compensated price elasticity of demand for $i=1,2$; $\epsilon_{ii}<0$. See Atkinson and Stiglitz (1987) in this connection. In fact in our framework it can be shown that $(P_1/P_2)\ (\hat{P}_1-\hat{P}_2)=\Psi'(X_1/X_2)\ (\hat{X}_1-\hat{X}_2)$. Comparing it with equation (14) we find that $\sigma_D=-[(1/\Psi')(P_1/P_2)/(X_1/X_2)]>0$.

From equations (6) and (7) (and using equation (12)) we find that

$\hat{C}_{Ej}=\theta_{Kj}\sigma_j(\epsilon_2\hat{U}+\epsilon_3\hat{Q}+\epsilon_5\hat{b}-\hat{T}_E+\hat{T}_K)$ ···(15) and (16) for $j=1$ and 2 respectively
$\hat{C}_{Kj}=\theta_{Ej}\sigma_j(\epsilon_2\hat{U}+\epsilon_3\hat{Q}+\epsilon_5\hat{b}-\hat{T}_E+\hat{T}_K)$ ···(17) and (18) for $j=1$ and 2 respectively

where σ_j is the elasticity of factor substitution for the production of jth commodity ($j=1,2$). From equation (6) we get

$$C_{E1}X_1(\hat{C}_{E1}+\hat{X}_1)+C_{E2}X_2(\hat{C}_{E2}+\hat{X}_2)=-edU+(N-U)de.$$

Using equations (15) and (16) and after some manipulation we get

$$\lambda_{E1}\hat{X}_1+\lambda_{E2}\hat{X}_2=(\epsilon_2\hat{U}+\epsilon_3\hat{Q}+\epsilon_5\hat{b}-\hat{T}_E+\hat{T}_K)\ (1-\lambda_{E1}\theta_{K1}\sigma_1-\lambda_{E2}\theta_{K2}\sigma_2)+(\hat{w}-\hat{r}) - (U/(N-U))\hat{U}+(\hat{T}_E-\hat{T}_K) \quad (19)$$

where $\lambda_{E1}+\lambda_{E2}=1$ (λ_{Ej} is the share of efficient labour used in sector $j=1,2$). We can interpret $E^d=(N-U)$ as the effective demand for labour. It is to be noted that $E^d=E^d(wT_E/rT_K)$, where $(dE^d/d(wT_E)/rT_K))<0$.

The intuition behind the idea can be explained as follows. As the relative price of labour (paid to the workers) increases demand for labour falls. Given the total labour endowment, N, it implies that unemployment, U, increases. So $(N-U)$ falls. We assume that the effective demand for labour curve as a rectangular hyperbola,[6] i.e. $(N-U)=\xi/\ (wT_E/rT_K)$, where ξ is some constant. It implies that

$$(\hat{w}+\hat{T}_E-\hat{r}+\hat{T}_K)=(U/(N-U))\hat{U} \quad (20)$$

Again from equation (7) we find that

$$\lambda_{K1}\hat{X}_1+\lambda_{K2}\hat{X}_2=-(\epsilon_2\hat{U}+\epsilon_3\hat{Q}+\epsilon_5\hat{b}-\hat{T}_E+\hat{T}_K)\ [\lambda_{K1}\theta_{E1}\sigma_1+\lambda_{K2}\theta_{E2}\sigma_2]+K\hat{K} \quad (21)$$

Using equation (20) and also using the fact that $\lambda_{E2}=1-\lambda_{E1}$ and $\lambda_{K2}=1-\lambda_{K1}$ we can deduct equation (21) from equation (19) to get (after some manipulation)

$$(\lambda_{E1}-\lambda_{K1})\ (\hat{X}_1-\hat{X}_2)=(\epsilon_2\hat{U}+\epsilon_3\hat{Q}+\epsilon_5\hat{b}-\hat{T}_E+\hat{T}_K)A-K\hat{K} \quad (22)$$

where $A=[1-\lambda_{E1}\theta_{K1}\sigma_1-\lambda_{E2}\theta_{K2}\sigma_2+\lambda_{K1}\theta_{E1}\sigma_1+\lambda_{K2}\theta_{E2}\sigma_2]$
$=[(1-\sigma)+\theta_{E1}\sigma_1(\lambda_{E1}+\lambda_{K1})+\theta_{E2}\sigma_2(\lambda_{E2}+\lambda_{K2})]$
and $\sigma=\lambda_{E1}\sigma_1+\lambda_{E2}\sigma_2$. Thus, σ is the weighted average of σ_1 and σ_2. If $\sigma\leqq 1$, $A>0$.[7]
From equations (13), (14) and (22), after putting $\hat{T}_E=\hat{T}_K=\hat{b}=\hat{K}=0$, we get

$$(\hat{P}_1-\hat{P}_2)/\hat{Q}=0 \quad (23)$$
$$(\hat{X}_1-\hat{X}_2)/\hat{Q}=0 \quad (24)$$
$$\text{and } \hat{U}/\hat{Q}=(\epsilon_3/\epsilon_2)<0 \quad (25)$$

Thus an increase in the stock of knowledge changes only the level of unemployment without any change in the relative price or relative output.[8]

[6] This is just a simplifying assumption.

[7] In case of Cobb-Douglas production function for sectors 1 and 2 we find that $\sigma_1=\sigma_2=\sigma=1$ and $A>0$.

[8] Suppose $\hat{b}=\hat{T}_E=\hat{T}_K=0$ and $Q>0$ and we treat (13), (14) and (22) as equations to solve for $(\hat{P}_1-\hat{P}_2)$, $(\hat{X}_1-\hat{X}_2)$ and $(\epsilon_2\hat{U}+\epsilon_3\hat{Q})$. It is to be noted that in this case $(\epsilon_2\hat{U}+\epsilon_3\hat{Q})=C$, where C is some constant. It implies that an increase in Q reduces only U but it leads to no change in the relative price and relative output as $(\hat{P}_1-\hat{P}_2)$ and $(\hat{X}_1-\hat{X}_2)$ are unique.

Again from equations (13), (14) and (22), after putting $\hat{T}_E=\hat{T}_K=\hat{b}=\hat{Q}=0$, we get

$$(\hat{P}_1-\hat{P}_2)/\hat{K}=(1/\Delta)\ (-K) \tag{26}$$

$$(\hat{X}_1-\hat{X}_2)/\hat{K}=(1/\Delta)\ [\epsilon_2(\theta_{E1}-\theta_{E2})\sigma_D K] \tag{27}$$

$$\text{and } \hat{U}/\hat{K}=(1/\Delta)K \tag{28}$$

where $\Delta=-\epsilon_2[(1-\sigma)+\theta_{E1}\lambda_{E1}(\sigma_1-\sigma_D)+\theta_{E2}\lambda_{E2}(\sigma_2-\sigma_D)+\theta_{E1}(\sigma_1\lambda_{K1}+\sigma_D\lambda_{E2}) + \theta_{E2}(\sigma_2\lambda_{K2}+\sigma_D\lambda_{E1})]$

$\Delta<0$ if $\sigma\leq 1$ and if $\sigma_1>\sigma_D$ and $\sigma_2>\sigma_D$. Under these conditions $((\hat{P}_1-\hat{P}_2)/\hat{K})>0$, $((\hat{X}_1-\hat{X}_2)/\hat{K})<0$ and $(\hat{U}/\hat{K})<0$.[9]

On the basis of the assumption that the elasticity of e with respect to r is -1 we can write equation (10) in the following manner.

$$e=e(w,\ U,\ Q,\ b)/r \tag{10.2}$$

The assumption of CRS implies that the unit cost function is linear homogeneous. Hence using equation (10.2) we can write

$$r=1/C_2((wT_E/e(w,\ U,\ Q,\ b)),\ T_K) \tag{4.1}$$

From equation (4), assuming $\hat{T}_E=\hat{T}_K=\hat{b}=0$, we get $\hat{r}=\theta_{E2}\,\hat{e}$ as $\hat{r}=-\hat{C}_2$ and $\hat{C}_2=-\theta_{E2}\,\hat{e}$. Using equation (12) we find that

$$(\hat{r}/\hat{Q})=0 \tag{29}$$

In other words, from equation (4.1) we find that the effect of an increase in Q on U is such that e remains unchanged. Thus, r remains unchanged.

Again an increase in K reduces e (.). Thus $(wT_E/e\ (.))$ increases or C_2 (.) (in equation (4.1)) increases resulting in a fall in r. It can be checked that

$$(\hat{r}/\hat{K})=[\theta_{E2}\epsilon_2\,K/\Delta(1+\theta_{E2})]<0 \tag{30}$$

We summarise the major results so far derived in the form of the following proposition.

Proposition 1: An increase in the stock of physical capital reduces both the levels of unemployment and the rate of return of capital received by the capitalists. An increase in the stock of knowledge, on the other hand, reduces only the level of unemployment. It has no effect on the rate of return on capital received by the capitalists.

We now consider an increase in the rate of proportional income tax assuming that other tax rate and the subsidy rate do not change. So $\hat{T}_Y>0$ and $\hat{T}_E=\hat{T}_K=\hat{b}=0$. In this static model we find that increase in the value of T_Y lead to no change in the levels (P_1/P_2), U, (X_1/X_2) and Y. From equation (9) we, however, find that, other things remaining same, an increase in T_Y leads to an increase in R.

We next consider an increase in advalorem tax on labour employment, assuming that all other tax rates (together with the subsidy rate) are undisturbed. So $\hat{T}_Y=\hat{T}_K=\hat{b}=0$ and $\hat{T}_E>0$. Putting $\hat{T}_K=\hat{b}=\hat{Q}=\hat{K}=0$ and using the fact that $\hat{T}_E=(T_E/(1-T_E))\ \hat{T}_E$ we can derive from equations (13), (14) and (22) that an increase in T_E causes no change in the levels of relative price and relative output. We also find that

[9] In case of Cobb-Douglas production function for sectors 1 and 2 we find that the sufficient condition for $\Delta<0$ is $\sigma_D<1$

$$(\hat{U}/\hat{T}_E)=(T_E/(1-T_E))\ (1/\epsilon_2)>0 \tag{31}$$
$$\text{and } (\hat{r}/\hat{T}_E)=(T_E/(1-T_E))\ [\theta_{E2}/(1+\theta_{E2})\ ]>0 \tag{32}$$

The effect of an increase in T_E on domestic factor income cannot be easily predicted. Using equations (31) and (32) we get from equation (8) that

$$(dY/dT_E)=-(1/(1-T_E))\ [1/\epsilon_2(1+\theta_{E2})]\ [w\mu(N-U)+\theta_{E2}(w\mu(N-U)-\epsilon_2 rK)] \tag{33}$$

where $\mu=U/(N-U)$. From equation (33) we get $(dY/dT_E)<0$ if $\epsilon_2<[\mu\tilde{\theta}_{(N-U)}/\tilde{\theta}_K]$ where $\tilde{\theta}_{(N-U)}=w(N-U)/Y$ is the share of employment in domestic factor income and $\tilde{\theta}_K=rK/Y$ is the share of capital in domestic factor income.

The impact of an increase in T_E on the level of output of the educational sector can be easily predicted if we assume that unemployment subsidy rate is sufficiently small. We can rewrite equation (9) of the model as

$$T_Y[wT_E(N-U)+rK]+(1-T_Y)T_E wT_E(N-U)+T_K rT_K K-bU=R \tag{9.1}$$

On the basis of the assumption that the effective demand for labour curve with respect to relative wage rate is a rectangular hyperbola (as made in the context of equation (19)) we find that $wT_E(N-U)=\xi rT_K$. We also know that r increases when T_E increases. Hence the first three terms on the LHS of equation (9.1), i.e. the total tax revenue, increases. Under the assumption that b is sufficiently small (close to zero) we find that R increases.

We next consider an increase in advalorem tax on capital input, assuming that all other taxes/subsidies do not change, i.e. $\hat{T}_Y=\hat{T}_E=\hat{b}=0$ and $\hat{T}_K>0$. Hence putting $\hat{T}_E=\hat{b}=\hat{Q}=\hat{K}=0$ and using the fact that $\hat{T}_K=(T_K/\ (1-T_K))\ \hat{Y}_K$ we get from equations (13), (14) and (22) that again there is no change in the levels of (P_1/P_2) and (X_1/X_2) due to an increase in T_K. We also find that

$$(\hat{U}/\hat{T}_K)=(T_K/(1-T_K))\ (-1/\epsilon_2)<0 \tag{34}$$
$$\text{and } (\hat{r}/\hat{T}_K)=(T_K/(1-T_K))\ [-\theta_{E2}/(1+\theta_{E2})]<0 \tag{35}$$

Using equations (34) and (35) we get from equation (8) that

$$(dY/dT_K)=-(1/(1-T_K))[1/\epsilon_2(1+\theta_{E2})][w\mu(N-U)+\theta_{E2}(w\mu(N-U)-\epsilon_2 rK)] \tag{36}$$

where $\mu=U/(N-U)$. From equation (36) we get $(dY/dT_K)<0$ if $\epsilon_2<[\mu\tilde{\theta}_{(N-U)}/\tilde{\theta}_K]$ where $\tilde{\theta}_{(N-U)}=w(N-U)/Y$ and $\tilde{\theta}_K=rK/Y$.

From equation (35) we find that

$$(\hat{r}/\hat{T}_K)=[-\theta_{E2}/(1+\theta_{E2})\ ] \tag{35.1}$$

It implies that $|(\hat{r}/\hat{T}_K)|<1$. In other words, the absolute value of the elasticity of r with respect to T_K is less than one. Hence, as T_K increases we find that r falls and T_K increases, and as the absolute value of the elasticity of r with respect to T_K is less than one we can conclude that rT_K increases. It implies that an increase in T_K leads to an increase in revenue from tax on capital. An increase in T_K also implies from equation (9.1) that there is an increase in revenue from tax on employment. Hence, there is an increase in $(1-T_Y)T_E\ wT_E(N-U)$. This follows from the fact that an increase in T_K reduces the level of unemployment. Again, from equation (36) we find that, under some reasonable assumptions, an increase in T_K leads to an increase in domestic factor income. Hence, equation (9.1) implies that an increase in T_K raises total revenue. Under the assumption that the unemployment subsidy rate, b, is sufficiently small (close to zero) an increase in T_K raises the level of educational output, R.

Finally, we examine the effects of an increase in unemployment subsidy rate, b, in our model. When $\hat{b}>0$ (and $\hat{T}_E=\hat{T}_K=\hat{Q}=\hat{K}=0$) we find from equations (13), (14) and (22) that there is no change in the levels of (P_1/P_2) and (X_1/X_2) due to an increase in b. We also find that

$$(\hat{U}/\hat{b})=-\epsilon_5>0 \tag{37}$$

$$\text{and } (\hat{r}/\hat{b})=-[\theta_{E2}\epsilon_2\epsilon_5/(1+\theta_{E2})]>0 \tag{38}$$

In order to find out the effect on domestic factor income as a result of an increase in b we get from equation (8), after some algebraic manipulation, that

$$(\hat{Y}/\hat{b})=-[\epsilon_5/Y(1+\theta_{E2})][w\mu(N-U)+\theta_{E2}(w\mu(N-U)-\epsilon_2 rK)] \tag{39}$$

$(\hat{Y}/\hat{b})<0$ if $\epsilon_5<[\mu\tilde{\theta}_{(N-U)}/\tilde{\theta}_K]$ where $\mu=U/(N-U)$, $\tilde{\theta}_{(N-U)}=w(N-U)/Y$ and $\tilde{\theta}_K=rK/Y$.

As an increase in b under the above condition, reduces Y we find that there is reduction in the revenue from proportional income tax. Again, an increase in the subsidy rate raises unemployment, so the revenue from tax on employment also falls. In case of the effect of an increase in unemployment subsidy rate on the revenue from tax on capital we find that the sign is positive. This follows from equation (38) as an increase in the subsidy rate reduces the level of output of the educational sector.

We summarise the effects of an increase in all the above mentioned types of tax rates and also an increase in the unemployment subsidy rate on some major variables in the context of our model in the form of the following proposition.

Proposition 2: (i) An increase in the proportional income tax rate causes no change in the level of unemployment, rate of return on capital received by the capitalists and domestic factor income. It, however, raises the level of output of the educational sector. (ii) An increase in the tax rate on employment raises the unemployment level and the rate of return on capital received by the capitalists. It reduces the domestic factor income and raises the equilibrium level of output of the educational sector under some reasonable assumption. (iii) An increase in the tax rate on capital (unemployment subsidy) reduces (raises) both the equilibrium unemployment level and the equilibrium rate of return on capital received by the capitalists. It, however, raises (reduces) both the equilibrium levels of domestic factor income and the output of the educational sector under some meaningful sufficient conditions.

III. *The Dynamic Analysis*

We assume that a fraction, s, of domestic factor income net of tax revenue from proportional income tax is saved and is invested to augment the physical capital stock. We also assume that the total tax revenue, which is used to finance the output of the educational sector, adds to the stock of knowledge, Q. Thus, the output of the public good (educational sector) is the gross addition to the stock of knowledge. Let m and ρ stand for the constant rates of depreciation of physical capital and human capital (stock of knowledge) respectively. We thus introduce the following differential equations.

$$\dot{K}=s(1-\Upsilon_Y)Y-mK \tag{40}$$
$$\text{and } \dot{Q}=R-\rho Q \tag{41}$$

Using equation (8) and equation (9) and using the fact that U can be expressed in terms of K and Q and r can be expressed in terms of K we can rewrite equations (40) and (41) as follows.

$$\dot{K}=s(1-\Upsilon_Y)[w(N-U(K, Q, \Upsilon_E, \Upsilon_K, b))+r(K, \Upsilon_E, \Upsilon_K, b)\, K]-mK=S(K, Q, \Upsilon_Y\Upsilon_E, \Upsilon_K, b) \tag{40.1}$$

$$\dot{Q}=\Upsilon_Y[w(N-U(K, Q, \Upsilon_E, \Upsilon_K, b))+r(K, \Upsilon_E, \Upsilon_K, b)K]+\Upsilon_E wT_E(N-U(K, Q, \Upsilon_E, \Upsilon_K, b))+ \Upsilon_K r(K, \Upsilon_E, \Upsilon_K, b)T_K K-bU(K, Q, \Upsilon_E, \Upsilon_K, b)-\rho Q=H(K, Q, \Upsilon_E, \Upsilon_K, b) \tag{41.1}$$

where $(\partial U/\partial K)<0$, $(\partial U/\partial Q)<0$ and $(\partial r/\partial K)<0$.

In the long-run equilibrium we have $\dot{K}=\dot{Q}=0$. In order to analyze the properties of long-run equilibrium we establish the following lemma.

Lemma: If (i) $m>(\partial Y/\partial K)$, (ii) $\rho>(\partial R/\partial Q)$ and (iii) $/\epsilon_{r,K}/<1$ (where $\epsilon_{r,K}$ is the elasticity of r with respect to K) we find that the long-run eluilibrium is locally stable if and only if the slope of the $\dot{Q}=0$ locus exceeds the slope of the $\dot{K}=0$ locus.[10]

The sufficient conditions stated in the lemma can be explained in the following manner. The condition $m>(\partial Y/\partial K)$ implies that the marginal contribution of an increase in physical capital stock on domestic factor income is less than the rate of depreciation of physical capital stock.[11] As $s(1-\Upsilon_Y)<1$, we find that $m>s(1-\Upsilon_Y)(\partial Y/\partial K)$. The condition $\rho>(\partial R/\partial Q)$ implies that the marginal contribution of an increase in human capital stock on the level of output of the educational sector is less than the rate of depreciation of human capital stock.[12]

From the lemma it follows that $S_1=(\partial\dot{K}/\partial K)<0$, $S_2=(\partial\dot{K}/\partial Q)>0$, $H_1=(\partial\dot{Q}/\partial K)<0$ and $H_2=(\partial\dot{Q}/\partial Q)<0$.

The slope of the $\dot{K}=0$ locus is given by

$$(dK/dQ)|_{\dot{K}=0}=[-s(1-\Upsilon_Y)w(\partial U/\partial Q)]/[s(1-\Upsilon_Y)\{-w(\partial U/\partial K)+r(\epsilon_{r,K}+1)\}-m] \tag{42}$$

Here $(dK/dQ)|_{\dot{K}=0}>0$ as $(\partial U/\partial Q)<0$, $(\partial U/\partial K)<0$, $|\epsilon_{r,K}|<1$ and $0<\Upsilon_Y<1$.

The slope of the $\dot{Q}=0$ locus is given by

$$(dK/dQ)|_{\dot{Q}=0}=[-(\partial U/\partial Q)\{w\Upsilon_Y+w\Upsilon_E T_E+b\}+\rho]/[\Upsilon_Y\{-w\ (\partial U/\partial K)+r(\epsilon_{r,K}+1)\} -\Upsilon_E wT_E(\partial U/\partial K)+\Upsilon_K r(\epsilon_{r,K}+1)T_K-b(\partial U/\partial K)] \tag{43}$$

In expression (43) $(\partial U/\partial Q)<0$, $(\partial U/\partial K)<0$ and $|\epsilon_{r,K}|<1$. From the lemma we find that $\rho>(\partial R/\partial Q)$. It implies that $\rho>-(\partial U/\partial Q)\{w\Upsilon_Y+w\Upsilon_E T_E+b\}$. Hence the numerator of the expression given by relation (43) is positive so that $(dK/dQ)|_{\dot{Q}=0}>0$.

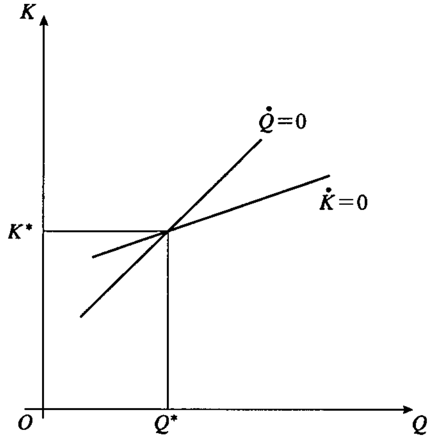
The point of intersection of the $\dot{K}=0$ locus and the $\dot{Q}=0$ locus in figure 1 implies a locally stable long-run equilibrium as the slope of the $\dot{Q}=0$ locus exceeds the slope of the $\dot{K}=0$ locus. The long-run equilibrium values of K and Q are given by K^* and Q^* respectively.

[10] The proof of the lemma is simple and is available on request.

[11] This is true $\forall K\in(0, \bar{K}]$ and $\forall Q\in(0, \bar{Q}]$, where $\bar{K}$ and $\bar{Q}$ are seme given values of K and Q respectively.

[12] This condition is also valid $\forall K\in(0, \bar{K}]$ and $\forall Q\in(0, \bar{Q}]$.

FIG. 1



IV. *The Comparative Steady-state Effects*

We first of all consider the comparative steady-state effects of an increase in proportional income tax rate. An increase in Υ_Y, at the initial long-run equilibrium levels of physical capital stock and human capital stock, reduces the gross (and also the net) addition to physical capital stock. Given human capital stock, Q, at its initial long-run equilibrium level, the physical capital stock, K, must fall to maintain $\dot{K}=0$. Hence, the $\dot{K}=0$ locus (as shown in figure1) shifts to the right. Due to the shift of the $\dot{K}=0$ locus and the $\dot{Q}=0$ locus, the effects on both K^* and Q^* are indeterminate. However, it can be shown that under the sufficient conditions as stated in the lemma K falls when Υ_Y increases.[13] It can also be shown that in the absence of all types of taxes and subsidies, other than proportional income tax, increase in Q^* raises the long-run equilibrium level of output of the educational sector, R^* as $\dot{Q}=0$ locus gives us $R^*=\rho Q^*$. The effect on the long-run equilibrium levels of Y^* and U^* are, however, indeterminate. We summarise our results in the form of the following proposition.

Proposition 3: An increase in the proportional income tax rate, under some reasonable sufficient conditions, reduces the long-run equilibrium level of physical capital stock but lead to an increase in the long-run equilibrium levels of human capital stock and output of the educational sector. As a result of this policy, there is change in the long-run equilibrium levels of domestic factor income and unemployment though their direction of movement is ambiguous. In the short run, however, an increase in the income tax rate produces no change in the levels of domestic factor income and unemployment.

We next consider the effects of an increase in the tax rate on employment, Υ_E, in our model. From the static part of the model we find that an increase in Υ_E reduces domestic factor income, Y, and raises the output of the educational sector, R, under some reasonable conditions. When the shift of the $\dot{Q}=0$ locus dominates over the shift of the $\dot{K}=0$ locus we find

[13] The derivations of the comparative steady-state results are available on request.

increase in the long-run equilibrium levels of both physical capital stock, K^*, and human capital stock, Q^*. As $\dot{K}=0$ implies that $Y^*=[m/s(1-\Upsilon_Y)]K^*$, an increase in K^* raises Y^*. Again, $\dot{Q}=0$ gives us $R^*=\rho Q^*$ so that an increase in Q^* raises R^*. Increase in K^* and Q^* leads to change in the equilibrium value of unemployment, U^*, as $U^*=U^*(K^*, Q^*, \Upsilon_E, \Upsilon_K, b)$. In the short run $(dU/d\Upsilon_E)<0$. Hence, the long-run effect on unemployment is given by

$$(dU^*/d\Upsilon_E)=(\partial U^*/\partial K^*)\ (dK^*/d\Upsilon_E)+(\partial U^*/\partial Q^*)\ (dQ^*/d\Upsilon_E)+(\partial U^*/\partial \Upsilon_E)$$

When K^* and Q^* increase we find that there is reduction in the long-run equilibrium level of unemployment. It may happen that in the long run the equilibrium level of unemployment increases due to an increase in Υ_E. This may happen when there is reduction in the levels of K^* and Q^* due to an increase in Υ_E and the dynamic effect of an increase in Υ_E on U^* (through reduction in K^* and Q^*) dominates over the corresponding static effect. We summarise our results in the form of the following proposition.

Proposition 4: An increase in the tax rate on employment may increase the long-run equilibrium levels of both domestic factor income and output of the educational sector. In the short run, however, an increase in the tax rate on employment reduces the levels of domestic factor income and raises the level of output of the educational sector under some sufficient conditions. The effect on the long-run equilibrium level of unemployment may also be different from its short run effect as a result of an increase in tax rate on employment.

We next consider the comparative steady-state effects of an increase in the tax rate on capital and an increase in the unemployment subsidy rate. An increase in the tax rate on capital, Υ_K, lead to increase in the short-run equilibrium levels of both domestic factor income and output of the educational sector under some sufficient condition (see proposition 2). Hence, in the long-run the $\dot{K}=0$ locus shifts to the left and the $\dot{Q}=0$ locus shifts to the right. The net outcome is increase in the long-run equilibrium levels of both physical capital stock, K^*, and human capital stock, Q^*. As $Y^*=[m/s(1-\Upsilon_Y)]K^*$ and $R^*=\rho Q^*$ it implies increase in the long-run equilibrium levels of both domestic factor income and output of the educational sector.

The long run effect of an increase in Υ_K on U^* consists of both the static effect and the dynamic effect through increase in the levels of K^* and Q^*. In the short run an increase in Υ_K reduces U. In the long-run also we find that an increase in Υ_K leads to unambiguously a reduction in the level of unemployment, U^*. The long-run effect of an increase in the tax rate on capital on the unemployment level, however, is greater than the short run effect.

The effects of an increase in the unemployment subsidy rate, b, on the above variables are *exactly opposite* to that of an increase in the tax rate on capital input. As the explanations behind the results are same we are not explaining them here in detail. We combine the results of these two comparative steady-state effects in the form of the following proposition.

Proposition 5: An increase in the tax rate on capital (unemployment subsidy rate) lead to increase (decrease) in the long-run equilibrium levels of physical capital stock, human capital stock, domestic factor income and output of the educational sector. The increase in the tax rate on capital (unemployment subsidy rate) reduces (raises) the long-run equilibrium level of unemployment. The effect on the long-run equilibrium levels of domestic factor income and unemployment, as a result of an increase in the tax rate on capital (unemployment subsidy rate) are similar to that of the short run ones. However, the long run effects are greater than the short run effects.

V. *Concluding Remarks*

In this paper we have analyzed the effects of some direct and indirect taxes on the long-run equilibrium level of unemployment in a dynamic efficiency wage model where the efficiency of the worker shifts over time. An increase in the rate of proportional income tax in the long-run equilibrium changes the level of unemployment and the level of domestic factor income when the tax revenue finances the production of a public good like the output of the educational sector. Similar results cannot be obtained in the static model where the physical capital stock and the human capital stock are exogenously given. Existing literature on the efficiency wage theory of unemployment is static in nature, and the efficiency function does not include any argument which changes over time. In this model, the stock of knowledge (skill of the worker) accumulates over time and this accumulation is affected by the tax revenue of the government. Also the investment of physical capital depends on the post-tax income, and so the accumulation of physical capital and expansion of the stock of knowledge become a simultaneous phenomenon.

Taxes on factors affect the factor prices and this affects the equilibrium level of unemployment even in a static model where the stock of physical capital and human capital are given. However, the comparative static effect on unemployment is not necessarily identical to the comparative steady-state effect on unemployment and domestic factor income. This is because the long-run equilibrium levels of physical capital stock and human capital stock are altered and this produces additional effect on unemployment and domestic factor income. More importantly, this additional effect may be exactly opposite to the comparative static effect and even may dominate that in some special cases. In the case of an increase in tax on capital or an increase in unemployment subsidy we find that the short run and the long run effects on domestic factor income and also on unemployment are identical. It is, however, to be noted that the long run change in the levels of domestic factor income and unemployment as a result of any one of the above two policies is greater than the corresponding short run change.

APPENDIX

Let Y^E be the expected income of the worker, $\Omega(Y^E)$ be his utility function and $V(e, Q, r)$ be the disutility function where e is his effort and Q is his stock of knowledge. We assume the following (i) $\Omega(.)$ is linear, i.e., the worker is risk neutral; and (ii) $(\partial V/\partial e)>0$; $(\partial^2 V/\partial e^2)>0$; $(\partial V/\partial Q)<0$; $(\partial^2 V/\partial e\partial Q)<0$; $(\partial V/\partial r)>0$; and $(\partial V/\partial e\partial r)>0$.

The objective of the worker is to maximize

$$Z=\Omega(Y^E)-V(e, Q, r) \tag{A.1}$$

through the choice of e. Here

$$Y^E=pw+(1-p)w^a \tag{A.2}$$

where $(1-p)$ is the probability that the worker will be monitored and fired if caught shirking. We assume that this probability is lower (higher) when his effort is higher (lower). Mathematically,

$$p=p(e) \text{ with } p'(.)>0 \text{ and } p''(.)<0 \tag{A.3}$$

Here w^a is the alternative income; and it is given by

$$w^a=(1-u)w^*+ub \tag{A.4}$$

Here u is the unemployment rate, i.e. $u=(U/N)$ where U is the level of unemployment and N is the labour endowment. The wage rate in the alternative job is given by w^* and b is his unemployment allowances.

Using equations (A.1) to (A.4) and the linearity assumption of the utility function, we have

$$Z=p(e).w+(1-p(e))[(1-u)w^*+ub]-V(e,Q,r)$$
$$\text{or, } Z=p(e)[w-w^*+u.(w^*-b)]+(1-u)w^*+u.b-V(e,Q,r)$$

This is to be maximized with respect to e, and the appropriate first-order condition of maximization is

$$p'(e)[(w-w^*)+u(w^*-b)]=(\partial V/\partial e) \tag{A.5}$$

The second order condition is always satisfied because

$$D=\{p''(e)[.]-(\partial^2 V/\partial e^2)\}<0;$$

This is always valid because $w \geqq w^*>b$ (by assumption).

Taking the total differential of the equation (A.5) we have

$$D.de=-p'(e).[dw-udb+(w^*-b)du]+(\partial^2 V/\partial e\partial Q)dQ+(\partial^2 V/\partial e\partial r)dr$$

Hence

$$\left.\begin{aligned} (de/dw)&=-(p'(e)/D)>0;\\ (de/db)&=(p'(e)u/D)<0;\\ (de/du)&=-(p'(e)(w^*-b)/D)>0;\\ (de/dQ)&=((\partial^2 V/\partial e\partial Q)\ /D)>0;\\ \text{and } (de/dr)&=((\partial^2 V/\partial e\partial r)\ /D)<0. \end{aligned}\right\} \tag{A.6}$$

So if e^* is the equilibrium effort of the worker, then

$$e^*=e^*(w,b,u,Q,r) \tag{A.7}$$

is the optimum effort function satisfying the restrictions given by (A.6); and this is called the labour efficiency function in the literature. As $u=U/N$ and as N is given we can rewrite equation (A.7) as

$$e^*=e^*(w,b,U,Q,r) \tag{A.8}$$

Equation (A.8) is actually equation (10) of the text.

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