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REGIME SHIFTS AND FORECASTING ERRORS IN EXCHANGE RATES

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Abstract

In this paper, we empirically investigate the well-documented forward premium anomaly in terms of the forecasting errors of the regime shifts in excess returns on the exchange rates. We use a simple regime-switching model to quantify the forecasted part and the forecasting errors of the shifts. The frequent shifts lead to the systematic forecasting errors of the regime shifts. We find the forecasting errors play a dominant role to explain the anomaly, using quarterly data for the U.S. dollar relative to the British pound, the German mark, and the Japanese yen during 1980's and 1990's.

Key Words: Forward anomaly; Excess returns; Forecasting errors; Regime shifts
JEL classification: F31

I. *Introduction*

The uncovered interest parity states that the expected change in the nominal exchange rate should be equal to the difference in the nominal interest rate across countries. Similarly, the covered interest parity states that the forward premium should be equal to the interest rates difference. These parities hold under no arbitragy condition together with assumptions of risk neutral agents with rational expectations and market efficiency. While the covered interest parity is supported empirically [see Frenkel and Levich (1977); Taylor (1989)], the uncovered interest parity is strongly rejected in general [see MacDonald and Taylor (1992); Taylor (1995)]. If the covered interest parity holds, the uncovered interest parity implies that the expected change in the nominal exchange rate is equal to the forward premium when agents are risk-neutral with rational expectations.

Fama (1984) examines a regression of the rate of change of the nominal exchange rate on the forward premium and finds that the coefficient of the forward premium is significantly negative. This is the so-called forward premium anomaly. In terms of the regression analysis, the negative coefficient means that the forward premium is a biased predictor of the future change in the spot exchange rate. Other researchers have investigated these implications and have confirmed this anomaly for various exchange rates and sample periods [see, for example, Froot and Frankel (1989); Backus, Gregory and Telmer (1993); Bekaert and Hodrick (1993); McCallum (1994)].

The literature extensively seeks possible explanations to this anomaly in terms of time-varying risk premium, irrational expectations and expectations error. A voluminous literature uses the foreign-exchange risk-premium argument to explain a large variation of the excess returns. Hodrick (1987) and Engel (1996) provide an exhaustive survey of the related literature. Recent attempts in this line include affine models of currency pricing by Backus, Foresi and Telmer (1996) and Zhou (1998). A general conclusion is that these models cannot explain the forward premium anomaly unless risk aversion is extremely high. As for the irrational expectations, Frankel and Rose (1995) survey evidence, based on survey data, that the formation of expectations at variance with the rational expectations hypothesis. However, no formal testable model of the irrational expectations has yet been proposed.

The forward anomaly can also be attributed to the expectations error of the rational agents. Frankel and Froot (1987) and Frankel and Meese (1987) examine survey data and claim that expectations error explains the forward anomaly better than time-varying risk premium. This claim is supported by other survey-data-based studies such as Froot and Frankel (1989), Frankel and Froot (1990), and Takagi (1991). Lewis (1995) and Evans (1996) provide excellent surveys on theoretical and empirical implications of the rational systematic forecasting errors and the "peso problem", respectively. Among the three possible explanations, the systematic forecasting errors gain empirical support in the literature.

In this paper, we empirically investigate the forward anomaly in terms of forecasting errors of shifts in the excess returns. We assume that the agent's expectations are rational. However, agents in the market make the forecasting errors of the excess returns earned by holding assets in one currency over another. Such forecasting errors can be caused by the peso effects or the learning effects, discussed by Krasker (1980) and Lewis (1989a, 1989b), respectively.

We use a regime-switching model proposed by Hamilton (1989) to quantify the forecasting errors of shifts in the excess returns. The estimated parameters of the model give the prior and the smoothed probability of regimes. The former is the expected probability based on the currently available information, while the latter is based on the full samples and indicates the probability with which a regime is supposed to have happened at each time. Their difference would capture market's forecasting errors of regimes. We use the spot rates and the three-month forward rates for the U.S. dollar against the British pound, the German mark and the Japanese yen. The data are sampled quarterly. We find that the regime shifts of the excess returns are more frequent than those of the exchange rates documented in the literature. The expected duration of a regime is roughly three quarters. The frequent shifts lead to the market's forecasting errors of the regime in the market. The forecasting errors play a dominant role in explanation of the forward anomaly. In the related literature, regime-switching models are used to capture the long swings in the exchange rates. Bekaert and Hoddrick (1993) and Evans and Lewis (1993) give suggestive evidence that regime shifts in the exchange rate could explain the forward anomaly. Although Engel and Hamilton (1990) find no evidence that the peso effects can explain the forward anomaly, Kaminsky (1993) and Evans and Lewis (1995) show the explanatory power of the peso effects is as high as 75 percent and roughly 20 percent, respectively. In contrast with these previous studies, we apply a regime-switching model to the excess returns, not to the rate of change in the exchange rates.

The remainder of the paper is organized as follows. In the next section, we present an empirical framework to study the forward anomaly. In section III., we quantify the forecasted shifts in the excess returns and the forecasting errors based on a regime-switching model. Then, we investigate the importance of the forecasting error on regime shifts in the excess returns. The final section is allocated to discussion.

II. *Empirical Framework*

1. Interest Parity and Excess Returns

To present an empirical framework, we start with two parity conditions: the convered and the uncovered interest parity. Let e_t the log of the spot foreign exchange rate (the U.S. dollar per a local currency) at time t, f_t the log of the one-period forward exchange rate at time t, r_t the U.S. one-period nominal interest rate at time t, r_t^* a one-period local nominal interest rate at time t. If the covered interest parity holds, we have the following relation between the forward premium and the interest rate difference.

$$f_t - e_t = r_t - r_t^* \tag{1}$$

The left-hand side of equation (1) is called the forward premium. Under no arbitrage condition, the U.S. dollar asset should give rise to the same returns as an equivalent asset in different currencies when the returns are evaluated in a common currency. Suppose the interest rate on a financial asset denominated in the U.S. dollar is higher than that on a local equivalent asset. Then, investors will hold a local asset only if the local currency appreciates next period to offset the interest rate difference. Then, the forward premium should also be positive. If the forward premium is larger than the interest rate difference, the forward market guarantees that the local currency appreciates more than enough to offset the interest rate difference. Therefore, investment on the local assets would produce excess returns due to the excess appreciation. No arbitrage condition requires that the forward premium equals the interest rate difference. Frenkel and Levich (1977) and Taylor (1989) document that the covered interest parity holds for Eurodeposit rates free from capital controls. If we replace the forward rate in equation (1) with the expected appreciation or depreciation, we obtain the following uncovered interest parity.

$$E_t(e_{t+1}) - e_t = r_t - r_t^* \tag{2}$$

where $E_t(e_{t+1})$ is the expected appreciaton for the next period based on information at time t. If the market expectations are rational and homogeneous, no arbitrage condition ensures that the expected appreciation equals the interest rate difference. Suppose the U.S. dollar is expected to depreciate next period. In equation (2), it means $E_t(e_{t+1}) - e_t$ is positive. Then, holding financial assets in the U.S. dollar is risky. To compensate this risk, the returns on the assets denominated in the U.S. dollar should be larger than those on the local assets. Therefore, the U.S. interest rate should be higher than the local interest rate under no arbitrage condition. If the equality in equation (2) does not hold, there is an arbitrage opportunity. Equations (1) and (2) give the following relation.

$$E_t(e_{t+1}) - e_t = f_t - e_t \tag{3}$$

It states that the expected appreciation equals the forward premium. This relation holds if agents in the market are risk neutral and rationally form homogenous expectations. Any deviation from this relation implies an arbitrage opportunity. The difference between the expected appreciation and the forward premium is the expected excess returns. Then, the expected excess returns one period ahead (er_{t+1}) is written as

$$E_t(er_{t+1})=E_t(e_{t+1})-e_t-(f_t-e_t) \tag{4}$$

Here, $E_t(er_{t+1})$ denotes the expectation of er_{t+1} based on information available at time t. Similarly, the realized excess returns are expressed as

$$er_{t+1}=e_{t+1}-e_t-(f_t-e_t) \tag{5}$$

The equations (4) and (5) give rise to the following relation.

$$e_{t+1}-e_t=(f_t-e_t)+E_t(er_{t+1})+(er_{t+1}-E_t(er_{t+1})) \tag{6}$$

If the expectations on the future spot rate are rational and accurate on average, equation (6) is equivalent to equation (3). The empirical literature extensively examines the following regression, postulating the rational expectations.

$$e_{t+1}-e_t=\alpha+\beta(f_t-e_t)+u_{t+1} \tag{7}$$

where u_{t+1} is the regression error. If the parity conditions hold, $\alpha=0$ and $\beta=1$. Note that non-zero α alone will not be evidence of the forward anomaly because such a constant deviation will be explained by transaction costs. The essential anomaly is in the estimate of β different from one. In another word, the forward premium is a conditionally biased predictor of the future spot rate. Further, the rational expectations imply that u_{t+1} has a conditional mean of zero, and that it has no serial correlation as long as the maturity date of the forward contract is same as sampling interval. The empirical regularity is that the estimate of β is negative, indicating the forward premium gives a biased estimate. According to Froot and Thaler (1990), the average value of across some 75 published estimates is -0.88. Only a few are greater than zero, and none is equal to or greater than one. More recent research gives estimates of β ranging from -0.2 to -5.4, using monthly data from the mid 1970's to the late 1980's [see, Backus et.al. (1993); Bekaret and Hodrick (1993); McCallum (1994); Lewis (1995); Engel (1996)].

The biased estimate is obtained when relevant variables in a regression are omitted. Suppose that the forecasting errors persist in spite of the rational expectations and/or that the expected excess returns are time-varying with non-zero conditional mean. Then, an appropriate relation to be examined is in equation (6) instead of equation (7). If the forward premium is negatively correlated with either or both of the forecasted excess returns and the forecasting errors and we estimate equation (7), then we will have negative estimates of β. In this paper, we focus on the forecasting errors related to regime shifts in the excess returns. In the next subsection, we discuss how to quantify the forecasted shifts in the excess returns and the forecasting errors.

2. Quantified Forecasting Errors

Our goal is to examine if the forcasting errors of shifts in the excess returns explain the forward anomaly. Let s_{t+1} the part of the excess returns governed by the shifts, and $E_t(s_{t+1})$ its forecasted part based on information at time t. Then, we consider the following regression.

$$e_{t+1}-e_t = \alpha+\beta(f_t-e_t)+\gamma E_t(s_{t+1})+\delta se_{t+1}+\epsilon_{t+1} \tag{8}$$

where se_{t+1} indicate the forecasting errors, defined as $s_{t+1}-E_t(s_{t+1})$. This equation corresponds to equation (6). The error term (ϵ_{t+1}) captures all other factors that affect the excess returns. It is assumed to have zero mean and follow an identical and independent normal distribution. Here, s_{t+1} and $E_t(s_{t+1})$ are unobservable. To quantify these variables, we use a simple regime-switching model developed by Hamilton (1989). We assume two states in the excess returns: a positive state and a negative state.

$$er_{t+1} = \mu_t+\eta_{it+1}, \quad \eta_{it+1}\sim N(0, \sigma_i^2), \quad (i=1, 2) \tag{9}$$

Here, the disturbance, η_{it+1}, is identically and independently distributed across times and regimes. It is also assumed independent of ϵ_{t+1} in equation (8). μ_t is a constant parameter to indicate excess returns in regime i on average. Let $\mu_1<0$ and $\mu_2>0$. Then, regime 1 gives positive excess returns by holding assets denominated in the U.S. dollar.

We assume that the expectations of regimes formed at time t−1 are updated by Bayes rule after the excess returns at time t are observed. After an investor observes the actual returns, she evaluates her forecasting errors by an error function that takes the largest value when there are no forecasting errors and takes smaller values the larger the deviation of the expected returns from the actual returns. Then, she corrects her forecasting errors, based on Bayes rule. The Bayes rule is supposed to well approximate the rational learning with slow revelation of information [see Lewis (1989a)]. Specifically,

$$\pi_t(i|\Phi_t)=\frac{f_t(\eta_{it})\cdot\pi_t(i|\Phi_{t-1})}{\sum_{i=1}^{2}f_t(\eta_{it})\cdot\pi_t(i|\Phi_{t-1})} \tag{10}$$

where $\pi_t\,(i|\Phi_t)$ is the probability that the regime i occurs at time t, based on the information at time t, and $\pi_t\,(i|\Phi_{t-1})$ is based on the information at time t−1. The error function of the regime i is denoted by $f_t(\eta_{it})$, where η_{it} is the deviation of the expected returns in the regime i from the actual returns at time t. Here, the error function is assumed a normal distribution. Further, we postulate a first-order Markov chain for the evolution of the unobserved regimes. Let p_1 a transition probability from regime 1 at current period to regime 1 next period. A transition probability from regime 1 to regime 2 is given as $1-p_1$. Similarly, p_2 is a transition probability of staying in regime 2 from the current period to the next period. A transition probability from regime 2 to regime 1 is given as $1-p_2$. Then, investors are assumed to forecast the probability of each regime as follows.

$$\begin{pmatrix}\pi_{t+1}(1|\Phi_t)\\ \pi_{t+1}(2|\Phi_t)\end{pmatrix}=\begin{pmatrix}p_1 & 1-p_2\\ 1-p_1 & p_2\end{pmatrix}\begin{pmatrix}\pi_t(1|\Phi_t)\\ \pi_t(2|\Phi_t)\end{pmatrix} \tag{11}$$

$\pi_{t+1}\,(1|\Phi_t)$ is called the prior probability, which indicates a forecasted probability of regime i at time t+1 based on information available at time t. Thus, the forecasted excess returns $(E_t(s_{t+1}))$ due to regime shifts are given by

$$E_t(s_{t+1})=\sum_{i=1}^{2}\pi_{t+1}(i|\Phi_t)\cdot\mu_t \tag{12}$$

To estimate the forecasting errors due to the shifts, we need to infer in which regime the excess returns are historically. The inference can be made with the smoothed probabilities of each regime based on all the samples. The smoothed probabilities indicate the likelihood with which a regime actually happened at each time. Therefore, the difference bewtween the prior and the smoothed probabilities indicates the market's forecasting errors of regimes. Detailed computational methods of the smoothed probability are found in Hamilton (1989) and Kim (1994). Suppose that we have T observations in all, where the 'T' indicates the last period of the sample. Then, a smoothed probability for regime i at time t is denoted by $\pi_t\,(i|\Phi_T)$. Then, the forecating errors on excess returns (se_{t+1}) is given by

$$\begin{aligned} se_{t+1} &= \sum_{i=1}^{2} \pi_{t+1}(i|\Phi_T)\cdot\mu_i - \sum_{i=1}^{2} \pi_{t+1}(i|\Phi_t)\cdot\mu_i \\ &= (\pi_{t+1}(1|\Phi_T) - \pi_{t+1}(1|\Phi_t))\cdot(\mu_1-\mu_2) \end{aligned} \tag{13}$$

We can interpret that equation (13) captures the systematic forecasting errors of the regime shifts in the excess returns due to the rational learning. For example, rational economic agents may only gradually learn a change in the distribution of the underlying economic variables. Such a learning process is captured by the Bayes rule in the model. When the economy enters into a new state or a regime due to a new economic policy and internal or external economic shocks, it would take a long time to learn how such a regime change leads to a shift in the excess returns through changes of the economic fundamentals. If such a learning process persists, we would observe ex post that agents systematically make the forecasting errors, even though they use information efficiently. We estimate the regime-switching model of equations (9) - (11) via the maximum likelihood method proposed by Hamilton (1989). Then, we use the estimates to quantify the expected excess returns and the forecasting errors by equations (12) and (13), respectively. These estimated variables are used to estimate equation (8). We present empirical results in the next section.

III. *Empirical Results*

1. Regime-Switching Model

We use quarterly spot rates and forward rates of the U.S. dollar against the British pound, the German mark, and Japanese yen from the International Monetary Fund's (IMF) *International Financial Statistics* (IFS).[1] The sample covers the periods after exchange controls are lifted, so that the covered parity is likely to hold [see Rivera-Batiz and Rivera-Batiz (1994, pp. 119-125) for discussion]. As a result of excluding the periods with missing values, the samples of the raw data range from the fourth quarter of 1979 to the fourth quarter of 1997 for the dollar-pound rate, from the fourth quarter of 1980 to the first quarter of 1997 for the dollar-mark rate, and from the fourth quarter of 1980 to the second quarter of 2000 for the dollar-yen rate.

Table 1 shows the results of estimation of the regime-switching model. All the estimates of the parameters are statistically significant at 5 percent, except the estimate of the volatility

[1] We compute the U.K. forward rates by $(1/4\times(\text{annual rate of forward premium})-1)\times(\text{spot rate})$ because of the missing values of the forward rates after the 4th quarter of 1984.

TABLE 1. ESTIMATION OF REGIME-SWITCHING MODEL

Model:eq.(9)-(11)	$/Pound:1979Q4-1997Q4 regime 1	regime 2	$/Mark:1980Q4-1997Q1 regime 1	regime 2	$/Yen:1980Q4-2000Q2 regime 1	regime 2
$\hat{\mu}_i$	−0.0709	0.0228	−0.0410	0.0598	−0.0415	0.0632
(standard error)	(0.0143)	(0.0103)	(0.0129)	(0.0158)	(0.0171)	(0.0352)
[p-value]	[0.0000]	[0.0130]	[0.0007]	[0.0001]	[0.0077]	[0.0364]
$\hat{\sigma}_i$	0.0015	0.0022	0.0020	0.0016	0.0016	0.0022
(standard error)	(0.0008)	(0.0006)	(0.0007)	(0.0009)	(0.0009)	(0.0015)
[p-value]	[0.0360]	[0.0001]	[0.0009]	[0.0318]	[0.0407]	[0.0740]
$\hat{p}_i$	0.4899	0.8471	0.7252	0.5703	0.6634	0.4657
(standard error)	(0.1752)	(0.0682)	(0.1341)	(0.1871)	(0.1781)	(0.2672)
[p-value]	[0.0026]	[0.0000]	[0.0000]	[0.0012]	[0.0001]	[0.0407]
White's (1987) testing statistics for residuals						
AR(1)	0.0462	0.0244	0.2066	0.0086	0.0078	5.8594
[p-value]	[0.8297]	[0.8760]	[0.6495]	[0.9262]	[0.9297]	[0.0155]
ARCH (1)	0.4036	0.7377	0.0462	1.5407	0.0002	1.9824
[p-value]	[0.5252]	[0.3904]	[0.8297]	[0.2145]	[0.9902]	[0.1591]
AR(1) & ARCH(1)	10.2732		3.1230		7.0775	
[p-value]	[0.1136]		[0.7933]		[0.3137]	
Nyblom's test*	1.3260		0.8063		0.7084	
[p-value]	[0.1694]		[0.6622]		[0.7816]	
Log-likelihood	101.7247		87.3673		103.6282	
Number of obs.	72		65		78	
Ergodic probs.	0.2307	0.7693	0.6099	0.3901	0.6135	0.3865

Data: International Financial Statistics (IMF). Quarterly excess returns are computed by log of one-period-ahead spot rates over current-period forward rates.

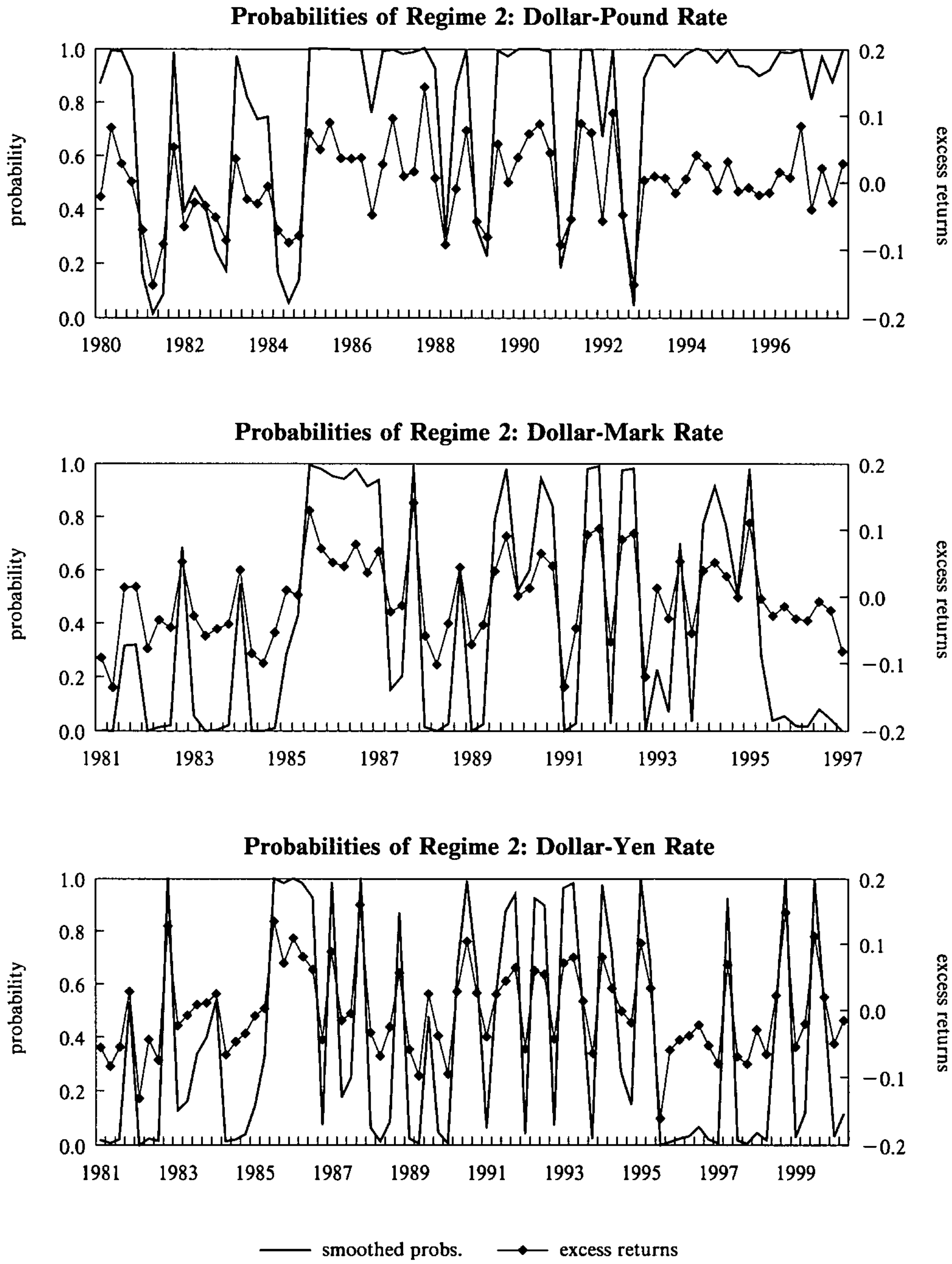
* Test of parameter constancy by Nyblom(1989): upper-tail p-value in the parentheses.

parameter in regime 2 ($\hat{\sigma}_2$) for the dollar-yen rate. We also conduct statistical tests by White (1987) for the first-order autocorrelation(AR(1)) and the first-order autoregressive-conditional-heteroscedasticity (ARCH(1)) effects in residuals to check misspecification of the model. Although the residuals of regime 2 for the dollar-yen rate marginally show a possibility of the first-order autocorrelation, a large p-value of the joint test indicates no serious misspecification of the model. For the dollar-pound and the dollar-mark rates, the statistics indicate no possibility of misspecification at conventional significance levels. Concerning the parameter stability, Nyblom's (1989) joint test shows no evidence against the parameter constancy. Therefore, these estimates deserve consideration of their implications.

For each currency, the expected excess returns are negative in regime 1 and positive in regime 2. That is, investors will have positive excess returns in regime 1 if they hold assets denominated in the U.S. dollar. In regime 2, holding assets in a local currency such as the pound, the mark, and the yen will produce excess returns. The excess losses in regime 1 are 7 percent per quarter for the pound assets, and around 4 percent for the mark and the yen assets. The excess returns of holding the mark and the yen assets in regime 2 are around 6 percent, while those of holding the pound assets are about 2 percent.

We find two characteristics of evolution of the excess returns, distinguished from those of the realized rate of change of the exchange rates that the previous empirical studies find with the same model here [e.g., Engel and Hamilton (1990); Engel (1994)]. First, the volatility parameters have small magnitudes. The ratios, $\hat{\mu}_i/\hat{\sigma}_i$, are substantially different from those

FIG. 1. FORECASTING ERROR IN PROBABILITIES



obtained in the analysis of the log-differenced exchange rate. The ratios are interpreted as the reward per risk to hold a specific currency. The estimates obtained in Engel and Hamilton (1990) and Engel (1994) indicate that the magnitude of the ratio ranges from 0.75 to 0.95, averaged over regimes and the U.S.-dollar rates against different currencies. In contrast, our estimates show that the ratio is 28.36 on average. This implies that once we know the regime shifts, we can infer the mean of the excess returns more accurately than that of the exchange-rate movement and expect large returns per risk.

Secondly, we observe more frequent regime shifts in the excess returns than in the realized rate of change of the exchange rates. The long swings in the dollar are well documented in the literature [e.g., Engel and Hamilton (1990); Evans and Lewis (1995)]. The long swing is generated by a large value of the transition probability p_i because it indicates the probability of staying in the state i next period. The literature finds that the estimates range from 0.8 to 0.9 for the U.S. dollar, which implies that the expected duration of a regime $(1/(1-\hat{p}_1))$ is five to ten quarters. In our results, we only find the value more than 0.8 for the dollar-pound rate in the regime 2 (0.85). The expected duration of regime 1 is 1.96 quarters for the pound, 3.64 for the mark and 2.97 for the yen, while that of regime 2 is 6.54 for the pound, 2.33 for the mark, and 1.87 for the yen. Thus, a regime will continue for only three quarters on average. Comparing these values, we conjecture that the dollar-yen rate will show most frequent shifts in the excess returns, followed by the mark and the pound rate in order.

Figure 1 shows the actual excess returns and the smoothed probabilities from the estimates of our model. The smoothed probabilities are useful to infer ex post in which regime economies are at each time. We observe frequent regime shifts in the excess returns, as expected from the estimates of the transition probabilities. Engel and Hamilton (1990) find that there are four regime shifts for the dollar-mark rate and three shifts for the dollar-pound rate during the period of the fourth quarter of 1973 through the first quarter of 1988. Obviously, more shifts are observed for the British pound data and for the German mark data. Thus we confirm our conjectures from the estimates of the transition probabilities.

The smoothed probabilities well capture the regime shifts in the excess returns. The excess returns of the pound show frequent shifts than that of the mark or the yen before 1985. A large spike observed for the yen in 1983 would be due to the temporary economic boom in the U.S. Otherwise, the excess returns on the mark and the yen tend to stay negative. When the U.S. dollar experiences a large depreciation after the Plaza accord in September 1985, all three currencies show positive excess returns. The U.K. pound generally hangs on to the positive excess returns until the Exchange Monetray System's (EMS) crisis in the fall of 1992. In contrast, the excess returns on the German mark and the Japanese yen show frequent switches during 1987-1995. In this period, the world economy experiences economic and political turmoils caused by enforcement of the Single European Act in January 1987, the trade dispute between the U.S. and Japan, and the collapse of the eastern European economies. After 1995, we do not observe shifts for the excess returns on the U.K. pound and the German mark, while we observe a couple of shifts for the Japanese yen when the Japanese economy has been trapped in the long-lasting recession without effective economic measures to recover.

2. Diagnostic Regression

In this subsection, we investigate how the forecasting errors explain the forward anomaly. All

TABLE 2. DIAGNOSTIC OLS REGRESSION OF FORWARD ANOMALY

Models:	$/Pound:1979Q4-1997Q4		$/Mark:1980Q4-1997Q1		$/Yen:1979Q4-1997Q4	
	Eq.(7)	Eq.(8)	Eq.(7)	Eq.(8)	Eq.(7)	Eq.(8)
$\hat{\alpha}$	-1.3961	-0.3048	0.4757	-0.0889	1.8446	-0.1948
(standard error)	(0.8468)	(0.4729)	(0.8751)	(0.3430)	(1.1922)	(0.4466)
[p-value]	[0.0518]	[0.2607]	[0.2943]	[0.3982]	[0.0630]	[0.3319]
$\hat{\beta}$	-1.9199	0.3374	-0.6654	1.0400	-1.0819	1.1782
(standard error)	(0.9852)	(0.5850)	(0.9547)	(0.3965)	(1.0024)	(0.3880)
[p-value]	[0.0277]	[0.2830]	[0.2442]	[0.0055]	[0.1419]	[0.0017]
$\hat{\gamma}$		0.8851		0.6035		0.7254
(standard error)		(0.3977)		(0.2774)		(0.5173)
[p-value]		[0.0147]		[0.0167]		[0.0825]
$\hat{\delta}$		1.7035		1.5503		1.5157
(standard error)		(0.1333)		(0.0827)		(0.0683)
[p-value]		[0.0000]		[0.0000]		[0.0000]
LM-hetero.*	1.9691	1.1294	2.4524	2.6875	0.0168	0.0611
[p-value]	[0.1605]	[0.2879]	[0.1173]	[0.1011]	[0.8970]	[0.8047]
D.W.	1.9039	1.9575	1.7676	1.7480	1.9309	1.7758
R^2	0.0515	0.7223	0.0077	0.8535	0.0151	0.8714
adjusted R^2	0.0379	0.7101	-0.0081	0.8463	0.0021	0.8662
Number of obs.	72	72	65	65	78	78
p-values of forward anomaly tests (t test and F test)						
$\beta=1$	0.0021	0.1307	0.0430	0.4600	0.0206	0.3237
$\alpha=0$ & $\beta=1$	0.0158	0.5289	0.2240	0.9670	0.1219	0.8933

* Heteroscedasticity Test by Koenker (1981) and Koenker and Bassett (1982) with upper-tail p-values.

the variables are multiplied by 100 to be expressed in percent, which would fascilitate comparison of our results with those in the literature. We first replicate the forward anomaly documented in the literature with our data set. The estimation results of equation (7) are shown in Table 2. Since the sampling interval is same as the maturity of the forward contract, we do not expect any serial correlation in the OLS residuals. The Durbin-Watson statistics show no evidence of the first-order serial correlation. Further, heteroscedasticity is not detected by the Lagrangean-Multiplier(LM) test. Thus, statistical inference based on the OLS estimator of the standard errors would be valid.

The slope coefficient, β, has a negative estimate for all three currencies. The slope coefficient is different from zero at 5 percent significance level for the pound, while not different from zero for the mark and the yen. The null hypothesis, $\beta=1$, implied by the uncovered interest parity with rational expectations, is rejected by *t*-test at 5 percent significance level for all the currencies. The jont hypothesis, $\alpha=0$ and $\beta=1$, is also rejected by *F*-test for the pound at 5 percent but not for the mark and the pound due to large standard errors. These findings are generally consistent with those in the literature [see Lewis (1995); Engel (1996)].

Based on the estimates of the regime-switching model, we construct the forecasted excess returns with equation (12) and the forecasting errors with equation (13). Then, we estimate equation (8). In Table 2, we find that the estimates of β come close to one for the dollar-mark and the dollar-yen rate. The null hypothesis of $\beta=1$ and the joint null hypothesis of $\alpha=0$ and $\beta=1$ are accepted with probability of more than 30 percent.

Turning to the dollar-pound rate, the slope coefficient is still small (0.34) and not

FIG. 2. PROBABILITY DIFFERENCE: SMOOTHED AND PRIOR

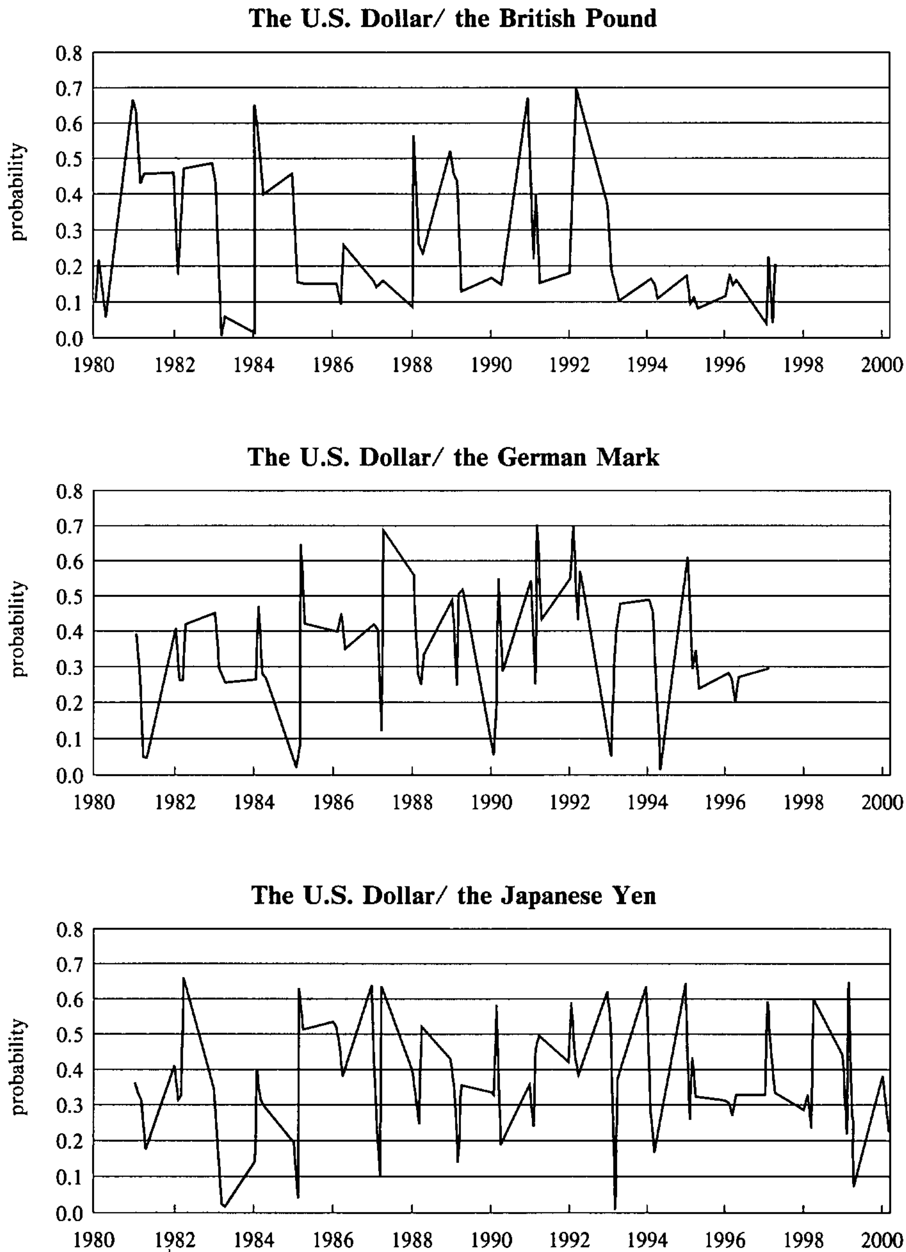


TABLE 3. DIAGNOSTIC OLS REGRESSION WITH RESTRICTIONS

Models:Eq.(8)	\$/Pound:1979Q4-1997Q4		\$/Mark:1980Q4-1997Q1		\$/Yen:1979Q4-1997Q4	
	$\delta=0$	$\gamma=0$	$\delta=0$	$\gamma=0$	$\delta=0$	$\gamma=0$
$\hat{\alpha}$	−1.3703	−0.3789	0.4793	−0.0924	1.8404	−0.1593
(standard error)	(0.8523)	(0.4851)	(0.8810)	(0.3532)	(1.2014)	(0.4487)
[p-value]	[0.0562]	[0.2187]	[0.2942]	[0.3972]	[0.0649]	[0.3618]
$\hat{\beta}$	−1.7680	−0.0301	−0.5569	0.8010	−1.0664	1.0660
(standard error)	(1.0276)	(0.5770)	(0.9984)	(0.3923)	(1.0297)	(0.3821)
[p-value]	[0.0449]	[0.4793]	[0.2895]	[0.0227]	[0.1519]	[0.0033]
$\hat{\gamma}$	0.4006		0.2864		0.1074	
(standard error)	(0.7248)		(0.7138)		(1.4200)	
[p-value]	[0.2911]		[0.3448]		[0.4700]	
$\hat{\delta}$		1.6752		1.5393		1.5105
(standard error)		(0.1365)		(0.0850)		(0.0686)
[p-value]		[0.0000]		[0.0000]		[0.0000]
LM-hetero.*	1.0742	0.4663	3.2423	2.0222	0.0192	0.1469
[p-value]	[0.3000]	[0.4947]	[0.0718]	[0.1550]	[0.8898]	[0.7015]
D.W.	1.9877	1.9718	1.8448	1.8856	1.9424	1.8131
R^2	0.0556	0.7021	0.0102	0.8422	0.0152	0.8680
adjusted R^2	0.0283	0.6935	−0.0217	0.8371	−0.0111	0.8645
Number of obs.	72	72	65	65	78	78
p-values of forward anomaly tests (t test and F test)						
$\beta=1$	0.0044	0.0393	0.0620	0.3069	0.0242	0.4316
$\alpha=0$ & $\beta=1$	0.0317	0.1956	0.3032	0.7748	0.1406	0.9245

* Heteroscedasticity Test by Koenker (1981) and Koenker and Bassett (1982) with upper-tail p-values.

statistically different from zero. This is because the excess returns for the pound less frequently switch regimes and the estimated forecasting errors do not have enough variation to explain the rate of change of the exchange rate. By construction, variation of the forecasting errors only depends on variation of the difference between the smoothed probabilities and the prior probabilities. The standard deiviation of the difference is 0.3 for the pound and 0.4 for the mark and the yen. Figure 2 shows the forecasting errors measured by the absolute value of the difference between the smoothed probabilities and the prior probabilities. The forecasting errors of the excess returns of the dollar-pound rate tend to stay at a low magnitude of 0.2. In contrast, the error of the mark and the yen ranges from 0.2 to 0.4. The full sample average of the magnitude is 0.25 for the pound, 0.35 for the mark and 0.36 for the yen. In the economic and political turbulent periods during 1987-1995, the average is same for the British pound, but increases to 0.38 for the German mark and the Japanese yen. That is, shifts in the excess returns are expected more accurately for the pound than for the mark and the yen. Thus, regime shifts in excess returns will only have a limited explanation power for the anomaly in the dollar-pound rate.

Although the implied forecasting errors are statisitically significant for all three currencies, the forecasted excess returns are also significant at 5 percent. To see which variable plays an important role in explanation of the anomaly, we estimate regression equations with zero restriction on a coefficient of one of the two variables. Table 3 indicates the forecasting errors play a dominant role to explain the anomaly. In case of th German mark and the Japanese yen, most of the bias in the slope coefficients disappears only when the forecasting-error variable is included. For the British pound, inclusion of the forecasting errors does not make the slope coefficient close to one, but brings about substantial reduction of the bias. Further, the coefficients of determination imply that the forecasting errors alone can explain 65-85 percent of the variation of the actual rate of change in the exchange rates, while the forecasted part's contribution is almost negligible.

Finally, the estimates of the coefficients on the forecasting errors are around 1.6. Recall equation (13). Since the estimates of $\hat{\mu}_1-\hat{\mu}_2$ are roughly -0.1, one percentage error of the probability of being in a regime leads to 16-percent forecasting errors of the rate of change of the exchange rates. For example, if we underestimate the probability of the excess returns being in regime 1 by one percentage, we observe 16-percent unexpected appreciation of the U.S. dollar.

IV. *Discussion*

In this paper, we empirically investigate the well-documented forward anomaly in terms of the forecasting errors of the regime shifts in the excess returns on the exchange rates. We use a simple regime-switching model to quantify the forecasted part and the forecasting errors of the shifts. We use quarterly data for the U.S. dollar relative to the British pound, the German mark, and the Japanese yen from the begining of 1980's to the late 1990's. The main findings are as follows. First of all, we find that the regime shifts of the excess returns are more frequent than those of the rate of change of the exchange rates. The expected duration of a regime is roughly three quarters. Secondly, the frequent shifts lead to the forecasting errors of the regime of the excess returns. Then, we observe the forward anomaly. Finally, the estimates of the coefficient on the forecasting errors imply that one percentage error of the regime will lead to 16-percent forecasting errors of the currency appreciation.

A couple of caveats are in order. First, Engel (1996) points out that the forward anomaly is less likely observed in recent sample periods. For example, the slope coefficient becomes positive for the pound and the mark during the sample period of 1987-1995. For the dollar-yen rate, however, it is still negative. As we already discussed, the dollar-yen rate exibits more frequent regime switches than the pound and the mark after 1995. Thus, we can infer that the forecasting errors may become small for the pound and the mark, and the anomaly disappears. To empirically investigate this possibility, we need enough samples for the recent periods. Thus, analysis based on higher-frequency data would be desirable.

Secondly, it is worthwhile to apply more general models to quantifying the forecasting errors. As we found, shifts in the excess returns are expected more accurately for the pound than for the mark and the yen. Thus, the forecasting errors of regime shifts may only have a limited power to explain the anomaly for the dollar-pound rate. We can conjecture that the forecasting errors of the magnitude of the excess returns would be an important factor. One way to capture the forecasting errors of the magnitude directly would be to extend the regime-switching model to allowing the mean parameters to change over time. Then, the forecasting errors depend on the errors of the level of the mean excess returns as well as those of regime shifts.

Finally, in the related literature, Baillie and Bollerslev (2000) argue that the forward anomaly may be viewed as a statistical artifact from small sample sizes and persistent autocorrelation in the forward premium together with FIGARCH process of the exchange rates. However, we still need to know under what economic conditions such a statistical property can be observed from the market data. Our analysis does not answer it, either. We need to identify sources of the forecasting errors and clarify its relation to economic fundamentals. These are left for future research topics.

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