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ON THE COMPETITIVE EQUILIBRIUM IN THE ECONOMY WITH CLUBS

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Abstract

The competitive equilibrium is defined for an economy with a club and many identical consumers. In an example of the economy, the existence of the competitive equilibrium is shown. Also, it is proved that any allocation under the competitive equilibrium in the economy is Pareto optimum.

Key words: Club; Competitive equilibrium; Pareto optimum.
JEL classification: C60, C70, D11, D50, D61, D71, H41.

I. *Introduction*

Groups of people who share and jointly consume goods are called "clubs", or consumption ownership-membership arrangements. Goods consumed in clubs are intermediate goods between purely private goods and purely public goods. In this paper we shall consider a simple model of an economy where there is one club and there are many, but identical consumers. The market of membership of the club is analyzed and the competitive equilibrium for the economy is defined. In an example of the economy, the competitive equilibrium is shown to exist. Our definition of competitive equilibrium is an extension of the usual competitive equilibrium for economies only with private goods. In addition, allocations under the competitive equilibrium are proved to be Pareto optimum.

In his famous paper J. M. Buchanan (1965) presented a model of economy with clubs, and considered Pareto optimality of allocations in the economy. Following his paper, many papers have been published (for detail, confer the survey article by T. Sandler and J. T. Tschirhart (1980)). In most papers such as Y.-K. Ng (1973, 1974, 1978), E. Berglas (1976), and E. Helpman and A. L. Hillman (1977), the optimality of allocations was considered. In a few papers, the competitive equilibrium for economies with clubs is analyzed, for example, by S. Scotchmer and M. H. Wooders (1987). On the other hand, a competitive equilibrium was defined by D. Foley (1967) and D. K. Richter (1974) for economies with public goods, which is a special case of clubs. However, such an equilibrium is quite different from the equilibrium in economies with clubs, because clubs are independent agents and behave for their own purpose.

The definition of competitive equilibrium depends on the behavior of clubs. In this paper we assume that the club maximizes its members' utilities. In our model of economy all individual are assumed to be identical in that their utility functions are the same and they have initially the same amount of wealth. By virture of this assumption, we can easily define a natural concept of competitive equilibrium for the economy. However, in general cases, we expect that many kinds of equilibrium concepts might be defined.

II. *A Model*

We consider an economy in which there are two kinds of commodities, say "commodity 1" and "commodity 2". Commodity 1 is a good shared and consumed in a club. The club is a group of people who share commodity 1 in consumption. We assume that there is only one club in the economy. Commodity 2 is a private good and consumed by each single person. In what follows, we assume that commodity 2 is a numeraire and its price is always unity.

We assume that individuals are "divisible", and the set of all the persons in the economy is denoted by A, which is a unit interval, i.e., $A=[0,1]$. Also, we assume that all individuals are identical, and their utility functions are the same and their incomes are equal.

Let us denote the quantity of commodity 1 consumed in the club by x. Also, let us denote the fraction of people belonging to the club by θ, where $0\leqq\theta\leqq 1$. When the set of the members of the club is a measurable subset M of A, $\theta=\lambda(M)$ where $\lambda(M)$ is the Lebesgue measure of set M. The total number of individuals in the economy is fixed, and fraction θ denotes the number of people participating in the club.

We assume that people do not care about who are members of the club, but only about the number of its members. Therefore, the club is described by pair (x, θ).

The utility function of each person, who becomes a member of club (x, θ), is denoted by

$$u=U((x, \theta), y),$$

where y denotes the quantity of commodity 2.

The variable θ of club (x, θ) indicates degree of congestion. The following assumption means that people prefer a less crowded club.

Assumption 2.1: U is a continuous function and $U((x, \theta), y)$ is increasing in both x and y, and decreasing in θ.

On the other hand, we denote the utility of a person who is not a member of the club by

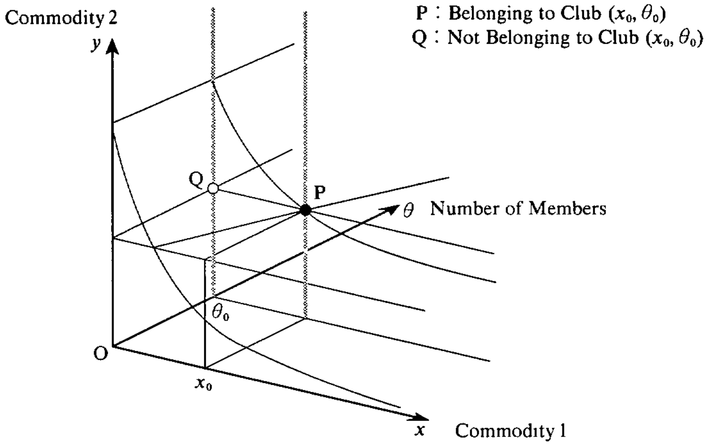
$$u=V(y).$$

Assumption 2.2: $V(y)=U((0, \theta), y)$ for all y and θ.

The above assumption implies that people can get nothing from belonging to the club in which nothing is consumed. Namely, when $x=0$, people in club (x, θ) get as the same level of utility as people out of the club get.

In Fig. 1, an indifference surface for the utility function satisfying the above assumptions

FIG. 1. INDIFFERENT SURFACE FOR UTILITY FUNCTION U



is illustrated.

The production set of commodities 1 and 2 is denoted by Y, which is a subset of the non-negative orthant of a 2-dimensional Euclidean space.

Assumption 2.3: Set Y is non-empty, closed, and convex.

Next, let us denote by m the income of each individual, which arises from production of commodities. The total of incomes is equal to the valued of commodities produced in the economy. When production $(x, y) \in Y$ is chosen and the price of commodity 1 is p, the total value of produced commodities is $px+y$, and the following must hold.

$$m=\int_A mda=px+y$$

Namely, the value of produced commodities is distributed equally among all the individuals in the economy.

Finally, we assume that every individual is initially a member of the club and nothing is consumed in the club, that is, $x=0$ and $\theta=1$ in club (x, θ). Initially, the club is specified by (0, 1), and every individual has the membership of club (0, 1). When an individual wants to leave the club, he sells his membership in the market and the club buys it. If the price of membership of club (0, 1) is r, then the initial income of each individual is $m+r$.

III. *Competitive Equilibrium*

As some individuals leave the club and the club buys some amount of commodity 1, club (0, 1) changes to club (x, θ) Let us denote the price of membership of club (x, θ) by q. Price q is an admission fee that individuals have to pay if they join club (x, θ). Since each individual is negligible, and a single person does not affect variable θ of club (x, θ).

The budget constraint, which each individual must satisfy in joining the club, is denoted by

$$q+y\leqq m+r,$$

where y is the amount of consumption of commodity 2. Thus, each person will continue to join the club if $V(m+r)<U((x,\theta),m+r-q)$, or leave the club if $V(m+r)>U((x,\theta),m+r-q)$.

Given m, r, and x, let us define q_0 and q_1 by

$$q_0=\max\{q \mid V(m+r)\leqq U((x, 0), m+r-q)\}$$

and

$$q_1=\max\{q \mid V(m+r)\leqq U((x, 1), m+r-q)\}.$$

By Assumption 2.1, we have $q_0\geqq q_1$. When $q>q_0$, nobody will join the club, and therefore $\theta=0$. On the other hand, when $q\leqq q_1$, everybody will join the club, and therefore $\theta=1$. When $q_0\geqq q>q_1$, some will join the club, but others will not. The fraction θ of individuals joining the club is determined by

$$V(m+r)=U((x,\theta), m+r-q),$$

and $0\leqq\theta<1$. We write the above relation as $\theta=f(q, x, m, r)$. Thus, the demand for membership of the club is defined by

$$\theta=F(q, x, m, r)\equiv\begin{cases}1 & 0\leqq q<q_1\\ f(q, x, m, r) & q_1\leqq q<q_0.\\ 0 & q_0\leqq q\end{cases}$$

The demand curve of F has a negative slope with respect to q as depicted in Fig. 2.

Now, we assume that the purpose of the club is to maximize its members' utilities. In our simple model of economy, since individuals are all identical, we can assume that the club chooses x, θ, and q so as to maximize $U((x,\theta), m+r-q)$. In addition, there is a budget constraint for the club. Let p be the price of commodity 1. Then, the budget constraint for the club is

$$px+r=q\theta.$$

The behavior of the club can be interpreted as follows. There is a manager in the club,

FIG. 2. DEMAND CURVE OF MEMBERSHIP OF THE CLUB

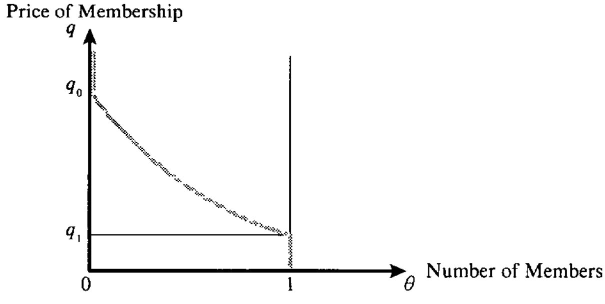
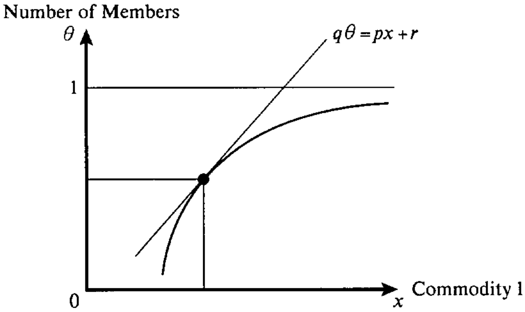


FIG. 3. BEHAVIOR OF THE CLUB



whose job is to maximize the utilities of people joining the club. For that purpose, the manager will determine amount x of commodity 1 consumed in the club, number θ of members of the club, and price q of membership. Thus, given p, m, and r, the club will maximize $U((x, \theta), m+r-q)$ with respect to x, q, and θ under budget constraint $px+r=q\theta$. Therefore, the demand for commodity 1 and the supply of membership by the club are defined by

$$G(p, m, r)\equiv\{(x, \theta, q) \mid px+r=q\theta \text{ and } U((x, \theta), m+r-q)\geqq U((z, n), m+r-s) \text{ for all } (z, n, s) \text{ with } pz+r=sn\}$$

Given q, a situation of the club is illustrated in Fig. 3. Usually, the demand x for commodity 1 by the club will be a decreasing function of p and the supply θ of membership by the club will be an increasing function of q.

Finally, producers maximize the value of commodities, and the supply function of commodity 1 and commodity 2 is defined by

$$H(p)\equiv\{(x, y)\in Y \mid px+y\geqq px'+y' \text{ for all } (x', y')\in Y\}.$$

In equilibrium, the following must hold:

$$\theta=F(q, x, m, r),\ (x, \theta, q)\in G(p, m, r),\ (x, y)\in H(p), \text{ and } m=px+y.$$

Thus, the competitive equilibrium for the economy can be described by $\{p, q, (x, \theta), y, m, r\}$ and defined as follows:

Definition 3.1: $\{p, q, (x, \theta), y, m, r\}$ is said to be a *competitive* equilibrium if the following conditions are satisfied:

(1) If $\theta>0$, then $V(m+r)\leqq U((x, \theta), m+r-q)$, and if $\theta<1$, then $V(m+r)\geqq U((x, \theta), m+r-q)$.
(2) $px+r=q\theta$ and $U((x, \theta), m+r-q)\geqq U((z, n), m+r-s)$ for all (z, n, s) with $pz+r\leqq sn$.
(3) $(x, y)\in Y$ and $m=px+y\geqq px'+y'$ for all $(x', y')\in Y$.

In the above definition, condition (1) means that each person is maximizing utility under a budget constraint. Condition (2) means that in the club members' utilities are maximized. Condition (3) means that producers of commodities are maximizing profits. Conditions (1) and (2) imply that the market of membership is in equilibrium. Also, conditions (2) and (3) imply that the market of commodity 1 is in equilibrium. Therefore, by Walras' law, the market of commodity 2 is in equilibrium.

The competitive equilibrium can be defined in more general cases [see Takekuma (1999)]. In condition (1) of the above definition, each person simply decides whether he (or she) should join the existing club, or not. Therefore, our definition of competitive equilibrium is weaker than, or different from that of S. Scotchmer, S. and M. H. Wooders (1987), in which people choose one club to join among many potentially existing clubs.

IV. *An Example of the Economy*

In this section we are going to show an example of the economy in Section II. Commodity 1, which is consumed in the club, is interpreted as the facilities of the club. Commodity 2, which is a private good, is assumed to be "money".

The set of all the persons in the economy is denoted by $A=[0, 1]$. Let us denote the size of facilities of the club by k and the fraction of people belonging to the club by θ, where $0 \leqq \theta \leqq 1$. Therefore, the club is characterized by a pair (k, θ).

All individuals are identical, and their utility functions and the initial holdings of money are the same. The utility function of each person, when he (or she) is a member of club (k, θ), is assumed to have the following special form.

$$u=18\sqrt{k(1-\theta)}+y,$$

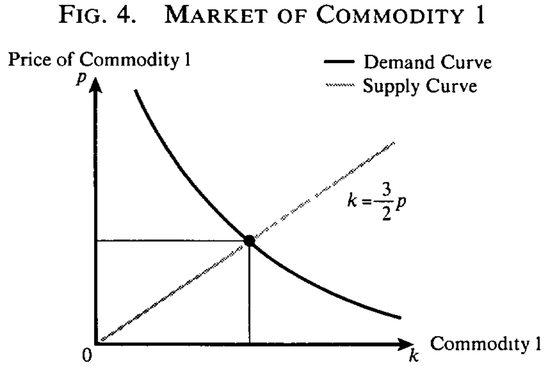
where y denotes the quantity of money. On the other hand, the utility of a person who is not a member of the club is assumed to be

$$u=y.$$

The cost for producing the facilities of the club is denoted by a cost function, which has the following special form.

$$c=\frac{1}{3}k^2.$$

FIG. 4. MARKET OF COMMODITY 1



Let p be the price of commodity 1. Producers of commodity 1 maximize profits,

$$\pi = pk - c = pk - \frac{1}{3}k^2.$$

The condition for profit maximization is

$$\frac{d\pi}{dk} = p - \frac{2}{3}k = 0, \qquad \text{i.e.,} \qquad k = \frac{3}{2}p. \tag{4.1}$$

Therefore, the supply curve of commodity 1 is a straight line with a positive slope illustrated in Fig. 4.

Each individual initially holds the same amount $\bar{y}$ of money, and we assume that $\bar{y}=10$. The profits obtained in production of commodity 1 are equally distributed to all the individuals in the economy. Each individual receives the same amount π of profits from producers. In addition, every individual is initially a member of the club where nothing is consumed. Let r be the price of membership of club (0, 1). The total income of each indvidual is $\bar{y}+\pi+r$, and the budget constraint, which each individual must satisfy in joining the club, is denoted by

$$q + y \leqq \bar{y} + \pi + r,$$

where y is the amount of money and q is the price of membership of club (k, θ). Therefore, each person will join the club if $\bar{y}+\pi+r < 18\sqrt{k(1-\theta)} + \bar{y}+\pi+r-q$, or will not join the club if $\bar{y}+\pi+r > 18\sqrt{k(1-\theta)} + \bar{y}+\pi+r-q$. Hence, the fraction θ of individuals joining the club is determined by

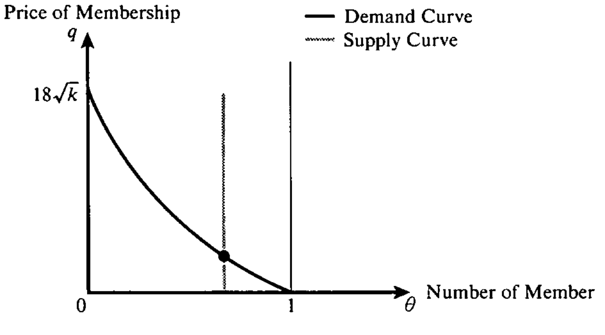
$$\bar{y}+\pi+r = 18\sqrt{k(1-\theta)} + \bar{y}+\pi+r-q, \quad \text{i.e.,} \quad q = 18\sqrt{k(1-\theta)}, \tag{4.2}$$

from which the demand curve of membership in Fig. 5 is derived.

The purpose of the club is to maximize its members' utility. In club (k, θ), members' utility, $18\sqrt{k(1-\theta)} + \bar{y}+\pi+r-q$, is maximized with respect to k, θ, and q under budget constraint $pk + r = q\theta$. The Lagrangian for the maximization problem is defined by

$$L = 18\sqrt{k(1-\theta)} + \bar{y}+\pi+r-q+\alpha(q\theta - r - pk),$$

FIG. 5. MARKET OF MEMBERSHIP



where α is a Lagrangian multiplier. The necessary conditions for maximization are

$$\frac{\partial L}{\partial k}=9\sqrt{\frac{1-\theta}{k}}-\alpha p=0, \tag{4.3}$$

$$\frac{\partial L}{\partial \theta}=-9\sqrt{\frac{k}{1-\theta}}+\alpha q=0, \tag{4.4}$$

$$\frac{\partial L}{\partial q}=-1+\alpha\theta=0, \text{ and} \tag{4.5}$$

$$\frac{\partial L}{\partial \alpha}=q\theta-r-pk=0. \tag{4.6}$$

From (4.3), (4.5), and (4.6), it follows that

$$\frac{1-\theta}{k}=\frac{p}{q} \text{ and } 81\theta^2=pq. \tag{4.7}$$

By (4.6) and (4.7) we have $81\theta^2(1-2\theta)+pr=0$, which implies that θ is determined by p and r. Thus, we have the supply curve of membership, which is a vertical line in Fig. 5.

In equilibrium, by solving six equations from (4.1) to (4.6), we have $\theta=\frac{2}{3}$, $k=3$, $p=2$, $q=18$, $r=6$, and $\alpha=\frac{3}{2}$. Furthermore, $\pi=3$, and the consumption of commodity 2 by each member of the club is $\bar{y}+\pi+r-q=1$, whereas the consumption of commodity 2 by each non-member is $\bar{y}+\pi+r=19$. Thus, a competitive equilibrium is shown to exist for this example of the economy.

Moreover, by (4.7) we have $81\theta^2(1-\theta)=pk$, which implies that

$$81\theta(2-3\theta)\frac{\partial \theta}{\partial p}=k+p\frac{\partial k}{\partial p}.$$

Therefore, since $\theta=\frac{2}{3}$ in equilibrium, $\frac{\partial k}{\partial p}=-\frac{k}{p}<0$ holds in a neighborhood of the equilibrium. Namely, we have the demand curve of commodity 1, which has a negative slope at the equilibrium illustrated in Fig. 4.

V. *Pareto Optimum Allocations*

To describe an allocation in the economy, we have to specify the amount of commodity 1 consumed in the club, its members, and the distribution of commodity 2 among people. Let us denote the amount of commodity 1 consumed in the club by x and the set of its members by a measurable subset M of A. Then, the club is denoted by (x, M).

To denote the distribution of commodity 2 among individuals, we use a real-valued measurable function f on A, where $f(a)$ is the quantity of commodity 2 allocated to person $a \in A$. Thus, an allocation in the economy is indicated by these three elements, $\{(x, M), f\}$. An allocation $\{(x, M), f\}$ in the economy is said to be *feasible* if $(x, \int_A f da) \in Y$.

In allocation $\{(x, M), f\}$, the utility of member $a \in M$ is $U((x, \lambda(M)), f(a))$, whereas the utility of non-member $a \in A \setminus M$ is $V(f(a))$.

Definition 5.1: A feasible allocation $\{(x, M), f\}$ is said to be *Pareto optimum* if there is no other feasible allocation $\{(z, N), g\}$ such that

(1) $U((x, \lambda(M)), f(a)) \leqq U((z, \lambda(N)), g(a))$ for all $a \in M \cap N$,

(2) $U((x, \lambda(M)), f(a)) \leqq V(g(a))$ for all $a \in M \cap (A \setminus N)$,

(3) $V(f(a)) \leqq U((z, \lambda(N)), g(a))$ for all $a \in (A \setminus M) \cap N$,

(4) $V(f(a)) \leqq V(g(a))$ for all $a \in A \setminus (M \cup N)$,

and strict inequalities hold for some $a \in A$ (with positive measure).

Now we can prove the basic theorem of welfare economics for economies with clubs.

Theorem 5.1: Any allocation in the competitive equilibrium is Pareto optimum.

Proof: Let $\{p, q, (x, \theta), y, m, r\}$ be a competitive equilibrium. Define a set M and a function f by

$$M = [0, \theta] \text{ and } f(a) = \begin{cases} m+r-q & \text{for } a \in M \\ m+r & \text{for } a \in A \setminus M \end{cases}$$

By (2) and (3) of Definition 3.1,

$$\int_A f da = \theta(m+r-q) + (1-\theta)(m+r) = m+r-\theta q = px+y+r-\theta q = y,$$

and therefore, $(x, \int_A f da) \in Y$. Namely, allocation $\{(x, M), f\}$ is feasible.

Now, suppose that allocation $\{(x, M), f\}$ were not Pareto optimum. Then, by Definition 5.1, there is a feasible allocation $\{(z, N), g\}$ such that

$$U((x, \theta), m+r-q) \leqq U((z, \lambda(N)), g(a)) \text{ for all } a \in M \cap N, \tag{5.1}$$

$$U((x, \theta), m+r-q) \leqq V(g(a)) \text{ for all } a \in M \cap (A \setminus N), \tag{5.2}$$

$$V(m+r) \leqq U((z, \lambda(N)), g(a)) \text{ for all } a \in (A \setminus M) \cap N, \tag{5.3}$$

$$V(m+r) \leqq V(g(a)) \text{ for all } a \in A \setminus (M \cup N), \tag{5.4}$$

and strict inequalities hold for some $a \in A$ with positive measure.

By (1) of Definition 3.1, $U((x, \theta), m+r-q) \leqq V(m+r)$ holds in (5.3). Therefore, from (5.1) and (5.3), it follows that

$$U((x, \theta), m+r-q) \leqq U((z, \lambda(N)), g(a)) \text{ for all } a \in N,$$

which implies, by (2) of Definition 3.1, that

$$pz+r \geqq (m+r-g(a))\lambda(N) \text{ for all } a \in N. \tag{5.5}$$

By (1) of Definition 3.1, $V(m+r) \leqq U((x, \theta), m+r-q)$ holds in (5.2). Therefore, from (5.2), it follows that $V(m+r) \leqq V(g(a))$ for all $a \in M \cap (A \setminus N)$, which implies, by Assumption 2.2, that

$$m+r \leqq g(a) \text{ for all } a \in M \cap (A \setminus N). \tag{5.6}$$

Moreover, (5.4) and Assumption 2.2 imply that

$$m+r \leqq g(a) \text{ for all } a \in A \setminus (M \cup N). \tag{5.7}$$

Since strict inequalities hold for some $a \in A$ in (5.5), or (5.6), or (5.7), we have, by integration,

$$m < pz + \int_A g da,$$

which contradicts (3) of Definition 3.1. ■

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