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ECONOMIC LANDSCAPES: MULTIPLIER PRODUCT MATRIX ANALYSIS FOR MULTIREGIONAL INPUT-OUTPUT SYSTEMS*

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Abstract

Earlier work (Sonis and Hewings, 1993, 1995; Sonis, Hewings and Miyazawa 1997a) has explored new ways of examining the structure of regional economies using input-output and social accounting systems. In this paper, attention is focused on a new approach to the interpretation of Miyazawa's concepts of left and right multipliers in the decomposition of multiregional input-output systems. Using the technique of the multiplier product matrix (Sonis et al., 1997c), the hierarchical decomposition proposed exploits the insights offered by the fields of influence theory and provides a way of interpreting Miyazawa's left and right multipliers in terms of multiregional feedback loops.

Keywords: Multiplier product matrix; Multiregional imput-output; Block decomposition

JEL classification: C67; D57; R15

I. *Introduction*

In recent years, several new perspectives on economic structure and structural change have been added to and extended from those originally proposed by Miyazawa (1966, 1971).

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In this paper, an attempt is made to link these approaches in a way that provides a clear path from one to the other, thereby revealing the different insights generated by each component more directly comparable or complementary to the others. The paper begins with a presentation of the multiplier product matrix (MPM) and its associated economic landscapes; the hierarchical structure revealed here can be shown to yield a block representation of the field of influence of change, a technique designed to identify analytically important elements in a matrix. From here, the notions of interdependence, especially the identification of internal and external multipliers, originally proposed by Miyazawa, can be generated and reinterpreted with the MPM structure. Finally, the paths of propagation of influence in this integrated system can be revealed through the identification of feedback loops. Hence, the path of decomposition of economic structure can be traced in the way shown in table 1, thus providing a better, more unified sense of the differing yet complementary perspectives offered by the alternative techniques.

TABLE 1: LINKS BETWEEN METHODOLOGIES

Decomposition Approach		*Objectives*
MPM		Comparison of structure
↓		
Field of Influence		Analytically important elements in the structure
↓		
Internal/External Multipliers		Different strength of interdependence
↓		
Feedback Loops		Paths of interdependence

II. *Economic Cross-Structure Landscapes of MPM and the Rank-Size Hierarchies of Backward and Forward Linkages*[1]

This section introduces the notion of artificial economic landscapes and the corresponding multiplier product matrices representing the essence of key sector analysis. The definition of the multiplier product matrix is as follows: let $A = \| a_{ij} \|$ be a matrix of direct inputs in the usual input-output system, and $B=(I-A)^{-1} \| b_{ij} \|$ the associated Leontief inverse matrix and let $B_{\cdot j}$ and $B_{i\cdot}$ be the column and row multipliers of this Leontief inverse. These are defined as:

$$B_{\cdot j}=\sum_{i=1}^{n} b_{ij},\ \ B_{i\cdot}=\sum_{j=1}^{n} b_{ij}\quad j=1,2,\dots,n \tag{1}$$

and represent components of the row and column vectors of column and row multipliers in the following form:

[1] The first part of this section draws on Sonis *et al.*, (1997c)

$$M_c(B)=[B_{\cdot 1}B_{\cdot 2}\dots B_{\cdot p}],\ M_r(B)=\begin{bmatrix} B_{1\cdot} \\ B_{2\cdot} \\ \vdots \\ B_{n\cdot} \end{bmatrix} \tag{2}$$

Let V be the global intensity of the Leontief inverse matrix:

$$V=\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij} \tag{3}$$

Then, the input-output multiplier product matrix (MPM) is defined as:

$$M=\frac{1}{V}\ \| B_{i\cdot}B_{\cdot j} \| = \frac{1}{V}\begin{pmatrix} B_{1\cdot} \\ B_{2\cdot} \\ \vdots \\ B_{n\cdot} \end{pmatrix}(B_{1\cdot}\quad B_{2\cdot}\quad \cdots \quad B_{\cdot n}) = \| m_{ij} \|$$

$$V=\sum_{i=1}^{n} B_{i\cdot}=\sum_{j=1}^{n} B_{\cdot j}=(B_{1\cdot}\quad B_{2\cdot}\quad \cdots \quad B_{\cdot n})\begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}=(11\dots 1)\begin{pmatrix} B_{1\cdot} \\ B_{2\cdot} \\ \vdots \\ B_{n\cdot} \end{pmatrix} \tag{4}$$

or, in vector notation:

$$M=\frac{1}{V}M_r(B)M_c(B);\ V=M_c(B)\times i' = i\times M_r(B) \tag{5}$$

The properties of the MPM that will now be considered will focus on (i) the hierarchy of backward and forward linkages and their economic landscape associated with the cross-structure of the MPM; and (ii) the interpretation of MPM as a matrix of first order intensities of the fields of influence of individual changes in direct inputs.

The concept of key sectors is based on the notion of backward and forward linkages and has been associated with the work of both Rasmussen (1956) and Hirshman (1958). The major thrust of the analytical techniques, and subsequent modifications and extensions, has been towards the identification of sectors whose linkage structures are such that they create an above-average impact on the rest of the economy when they expand or in response to changes elsewhere in the system. Rasmussen (1956) proposed two types of indices drawing on entries in the Leontief inverse:

1. Power of dispersion for the backward linkages, BL_j, as follows:

$$\begin{aligned} BL_j &= \frac{1}{n}\sum_{i=1}^{n} b_{ij}\Big/\frac{1}{n^2}\sum_{i,j=1}^{n} b_{ij} = \\ &= \frac{1}{n}B_{\cdot j}\Big/\frac{1}{n^2}V = B_{\cdot j}\Big/\frac{1}{n}V \end{aligned} \tag{6}$$

and

2. The indices of the sensitivity of dispersion for forward linkages, FL_i, as follows:

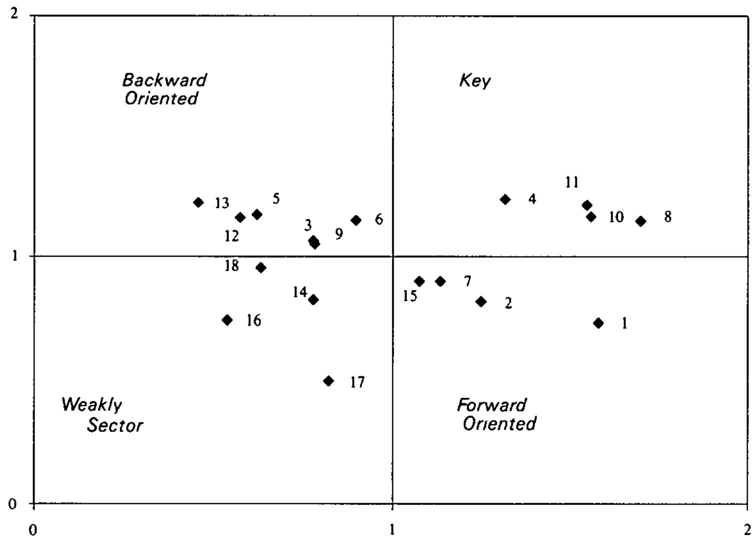
$$FL_i = \frac{1}{n}\sum_{j=1}^{n} b_{ij} \Big/ \frac{1}{n^2}\sum_{i,j=1}^{n} b_{ij} = \tag{7}$$
$$= \frac{1}{n} B_{i\cdot} \Big/ \frac{1}{n^2} V = B_{i\cdot} \Big/ \frac{1}{n} V$$

The usual interpretation is to propose that $BL_j > 1$ indicates that a unit change in final demand in sector j will create an above average increase in activity in the economy; similarly, for $FL_i > 1$, it is asserted that a unit change in all sectors' final demand would create an above average increase in sector i. A key sector is usually defined as one in which both indices are greater than 1; the graphical representation is shown in figure 1 for the Chinese economy.

The definitions of backward and forward linkages provided by (6) and (7) imply that the rank-size hierarchies (rank-size ordering) of these indices coincide with the rank-size hierarchies of the column and row multipliers. It is important to underline, in this connection, that the column and row multipliers for MPM are the same as those for the Leontief inverse matrix. Thus, the structure of the MPM is essentially connected with the properties of sectoral backward and forward linkages.

The structure of the matrix, M, can be ascertained in the following fashion: consider the largest column multiplier, $B_{\cdot j_0}$, and the largest row multiplier, $B_{i_0\cdot}$, of the Leontief inverse, with the element, $m_{i_0 j_0} = \frac{1}{V} B_{i_0\cdot} B_{\cdot j_0}$, located in the place (i_0, j_0) of the matrix, M. Moreover, all rows of the matrix, M, are proportional to the i_0^{th} row, and the elements of this row are larger than the corresponding elements of all other rows. The same property applies to the j_0^{th} column of the same matrix. Hence, the element located in (i_0, j_0) defines the center of the largest *cross* within

FIGURE 1 SECTORS CLASSIFICATION IN CHINESE ECONOMY(1987)



the matrix, M. If this cross is excluded from M, then the second largest cross can be identified and so on. Thus, the matrix, M, contains the rank-size sequence of crosses. One can reorganize the locations of rows and columns of M in such a way that the centers of the corresponding crosses appear on the main diagonal. In this fashion, the matrix will be reorganized so that a descending *economic landscape* will be apparent (see figure 2).

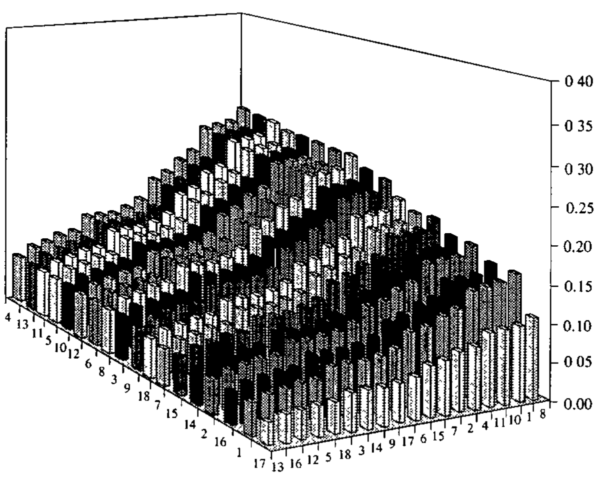
This rearrangement also reveals the descending rank-size hierarchies of the Rasmussen-Hirschman indices for forward and backward linkages. Inspection of that part of the landscape with indices > 1 (the usual criterion for specification of key sectors) will enable the identification of the key sectors. However, it is important to stress that the construction of the economic landscape for different regions or for the same region at different points in time would create the possibility for the establishment of a taxonomy of these economies.

The most important property of the economic landscape is that the components of the M matrix represent the intensities of the first order fields of influence of changes, *i.e.*, the components of the gradients of changes in all direct inputs. This gradient is used as a measure of the inverse importance of direct inputs (see Sonis and Hewings, 1989); inverse important inputs are those whose changes lead to the greatest impact on the economic system.

III. *Hierarchical Inclusion of Economic Landscapes.*

In this section, attention will be directed to a description of multiple shifts in intraregional backward and forward linkages and the associated changes in the positions of key sectors under the influence of interaction between the region and the rest of economy. The approach creates the possibility to evaluate immediately when economic sectors became more important for the regional economy under the influence of synergetic interactions with the rest of economy.

FIGURE 2 CHINESE ECONOMIC LANDSCAPE(1987)



The main analytical tool of the hierarchical inclusion of the economic landscapes will now be revealed. Consider the product, $B=B^{'}B^{''}$, of two matrices, $B^{'}$ and $B^{''}$, of the respective sizes $n\times m$, $m\times p$. Let

$$\begin{aligned} B_{\cdot j} &= \sum_{i=1}^{n} b_{ij};\; B_{i\cdot} = \sum_{j=1}^{n} b_{ij} \\ B^{'}_{\cdot j} &= \sum_{i=1}^{n} b^{'}_{ij};\; B^{'}_{i\cdot} = \sum_{j=1}^{n} b_{ij} \\ B^{''}_{\cdot j} &= \sum_{i=1}^{n} b^{''}_{ij};\; B^{''}_{i\cdot} = \sum_{j=1}^{n} b^{''}_{ij} \end{aligned} \tag{8}$$

be the column and row multipliers of these matrices. Using the definition of V, the global intensity of the matrix B from (3), the following multiplicative connections between the vectors of column and row multipliers of these matrices exist:

$$[B_{\cdot 1}B_{\cdot 2}\ldots B_{\cdot p}] = [B^{'}_{\cdot 1}B^{'}_{\cdot 2}\ldots B^{'}_{\cdot m}] \times B^{''};\; \begin{bmatrix} B_{1\cdot} \\ B_{2\cdot} \\ \vdots \\ B_{n\cdot} \end{bmatrix} = B^{'} \times \begin{bmatrix} B^{''}_{1\cdot} \\ B^{''}_{2\cdot} \\ \vdots \\ B^{''}_{m\cdot} \end{bmatrix};$$

$$V = [B^{'}_{\cdot 1}B^{'}_{\cdot 2}\ldots B^{'}_{\cdot m}] \times \begin{bmatrix} B^{''}_{1\cdot} \\ B^{''}_{2\cdot} \\ \vdots \\ B^{''}_{m\cdot} \end{bmatrix} \tag{9}$$

These expresssions can be checked by direct calculations of the components of the corresponding vectors and matrices.

Further, specify the following vectors:

$$\begin{aligned} M_c(B) &= [B_{\cdot 1}B_{\cdot 2}\ldots B_{\cdot p}] \\ M_c(B^{'}) &= [B^{'}_{\cdot 1}B^{'}_{\cdot 2}\ldots B^{'}_{\cdot m}] \\ M_c(B^{''}) &= [B^{''}_{\cdot 1}B^{''}_{\cdot 2}\ldots B^{''}_{\cdot m}] \end{aligned}$$

$$M_r(B) = \begin{bmatrix} B_{1\cdot} \\ B_{2\cdot} \\ \vdots \\ B_{n\cdot} \end{bmatrix},\; M_r(B^{'}) = \begin{bmatrix} B^{'}_{1\cdot} \\ B^{'}_{2\cdot} \\ \vdots \\ B^{'}_{m\cdot} \end{bmatrix},\; M_r(B^{''}) = \begin{bmatrix} B^{''}_{1\cdot} \\ B^{''}_{2\cdot} \\ \vdots \\ B^{''}_{m\cdot} \end{bmatrix} \tag{10}$$

as the row vectors and column vectors with components that are the column and row multipliers of the matrices, $B, B^{'}, B^{''}$. Using this notation, equation (9) may be presented in the following form:

$$\begin{aligned} M_c(B) &= M_c(B^{'})B^{''}; \\ M_r(B) &= B^{'}M_r(B^{''}); \\ V &= M_c(B^{'})M_r(B^{''}) \end{aligned} \tag{11}$$

Consider the economic system that is comprised of a region r and the rest of economy, R. The corresponding input-output system can be represented by the block matrix

$$A=\begin{pmatrix} A_{rr} & A_{rR} \\ A_{Rr} & A_{RR} \end{pmatrix} \tag{12}$$

Assume that the intra-regional matrix, A_{rr}, of the region r has the following incremental change E_{rr}, and A_{rR}, A_{Rr} are the inter-regional matrices representing direct input connections between region and the rest of the economy, while the matrix A_{RR} represents the intra-regional inputs within the rest of the economy.

The Leontief inverse $B=(I-A)^{-1}$ can be formally presented in the following block form:

$$B=\begin{bmatrix} B_{rr} & B_{rR} \\ B_{Rr} & B_{RR} \end{bmatrix} \tag{13}$$

and this can be further elaborated with the help of the Schur-Banachiewicz formula (Schur, 1917; Banachiewicz, 1937; Miyazawa, 1966; Sonis and Hewings, 1993):

$$B=\begin{bmatrix} B_{rr} & B_{rr}A_{rR}B_{R} \\ B_{RR}A_{Rr}B_{r} & B_{RR} \end{bmatrix}=\begin{bmatrix} B_{rr} & B_{r}A_{rR}B_{RR} \\ B_{R}A_{Rr}B_{rr} & B_{RR} \end{bmatrix} \tag{14}$$

where the matrices $B_r=(I-A_{rr})^{-1}$ and $B_R=(I-A_{RR})^{-1}$ represent the Miyazawa internal matrix multipliers for the region r and the rest of economy (revealing the interindustry propagation effects within the isolated region and isolated rest of economy) while the matrices $A_{Rr}B_r$, B_rA_{rR}, $A_{rR}B_R$, and B_RA_{Rr} show the induced effects on output or input between the two parts of input-output system (Miyazawa, 1966).

Further:

$$\begin{aligned} B_{rr}&=(I-A_{rr}-A_{rR}B_{R}A_{Rr})^{-1} \\ B_{RR}&=(I-A_{RR}-A_{Rr}B_{r}A_{rR})^{-1} \end{aligned} \tag{15}$$

are the extended Leontief multipliers for the region r and the rest of economy. The connections between these extended Leontief multipliers are:

$$\begin{aligned} B_{rr}&=B_r+B_rA_{rR}B_{RR}A_{Rr}B_r \\ B_{RR}&=B_R+B_RA_{Rr}B_{rr}A_{rR}B_R \end{aligned} \tag{16}$$

By using the Miyazawa decomposition, the extended Leontief inverses can be decomposed into the products of internal and external multipliers describing direct and induced self-influences (Miyazawa, 1966, 1976; Sonis and Hewings, 1993):

$$\begin{aligned} B_{rr}&=B_rB^R_{rr}=B^L_{rr}B_r \\ B_{RR}&=B_RB^R_{RR}=B^L_{RR}B_R \end{aligned} \tag{17}$$

where

$$\begin{aligned} B^L_{rr}&=(I-B_rA_{rR}B_RA_{Rr})^{-1};\ B^R_{rr}=(I-A_{rR}B_RA_{Rr}B_r)^{-1} \\ B^L_{RR}&=(I-B_RA_{Rr}B_rA_{rR})^{-1};\ B^R_{RR}=(I-A_{Rr}B_rA_{rR}B_R)^{-1} \end{aligned} \tag{18}$$

are the left and right Miyazawa external multipliers for the region r and the rest of economy.

It is easy to see that for the block Leontief inverse (13), the row vector $M_c(B)$ of the column multipliers has the following block form:

$$M_c(B)=\begin{bmatrix}M_c(B_{rr})+M_c(B_{Rr}) & M_c(B_{rR})+M_c(B_{RR})\end{bmatrix} \tag{19}$$

Using (14), one obtains:

$$\begin{aligned}M_c(B)&=\begin{bmatrix}M_c(B_{rr})+M_c(B_{RR})A_{Rr}B_r & M_c(B_{rr})A_{rR}B_R+M_c(B_{RR})\end{bmatrix}=\\&=M_c(B_{rr})\begin{bmatrix}I & A_{rR}B_R\end{bmatrix}+M_c(B_{RR})\begin{bmatrix}A_{Rr}B_r & I\end{bmatrix}\end{aligned} \tag{20}$$

Analogously, the column block vector of the row multipliers of the Leontief inverse B can be presented in the form:

$$\begin{aligned}M_r(B)&=\begin{bmatrix}M_r(B_{rr})+M_r(B_{rR})\\M_r(B_{Rr})+M_r(B_{RR})\end{bmatrix}=\\&=\begin{bmatrix}M_r(B_{rr})+B_rA_{rR}M_r(B_{RR})\\B_RA_{Rr}M_r(B_{rr})+M_r(B_{RR})\end{bmatrix}=\\&=\begin{bmatrix}I\\B_RA_{Rr}\end{bmatrix}M_r(B_{rr})+\begin{bmatrix}B_rA_{rR}\\I\end{bmatrix}M_r(B_{RR})\end{aligned} \tag{21}$$

Therefore, the expressions (5) and (4) yield the following form of the multiplier product matrix for the block matrix A of the multiregional input-output system and its Leontief inverse:

$$\begin{aligned}M(B)&=\frac{1}{V(B)}M_r(B)M_c(B)=\\&=\frac{1}{V(B)}\left\{\begin{bmatrix}I\\B_RA_{Rr}\end{bmatrix}M_r(B_{rr})+\begin{bmatrix}B_rA_{rR}\\I\end{bmatrix}M_r(B_{RR})\right\}\left\{M_c(B_{rr})\begin{bmatrix}I & A_{rR}B_R\end{bmatrix}+M_c(B_{RR})\begin{bmatrix}A_{Rr}B_r & I\end{bmatrix}\right\}=\\&=\frac{1}{V(B)}\begin{bmatrix}I\\B_RA_{Rr}\end{bmatrix}M_r(B_{rr})M_c(B_{rr})\begin{bmatrix}I & A_{rR}B_R\end{bmatrix}+\frac{1}{V(B)}\begin{bmatrix}I\\B_RA_{Rr}\end{bmatrix}M_r(B_{rr})M_c(B_{RR})\begin{bmatrix}A_{Rr}B_r & I\end{bmatrix}+\\&+\frac{1}{V(B)}\begin{bmatrix}B_rA_{rR}\\I\end{bmatrix}M_r(B_{RR})M_c(B_{rr})\begin{bmatrix}I & A_{rR}B_R\end{bmatrix}+\frac{1}{V(B)}\begin{bmatrix}B_rA_{rR}\\I\end{bmatrix}M_r(B_{RR})M_c(B_{RR})\begin{bmatrix}A_{Rr}B_r & I\end{bmatrix}\end{aligned} \tag{22}$$

It is important to underline that the application of equations (4) and (5) to the extended Leontief inverses, B_{rr}, B_{RR}, will provide the following extended intraregional multiplier product matrices for the region r and the rest of economy:

$$\begin{aligned}M_{rr}&=M(B_{rr})=\frac{1}{V(B_{rr})}M_r(B_{rr})M_c(B_{rr})\\M_{RR}&=M(B_{RR})=\frac{1}{V(B_{RR})}M_r(B_{RR})M_c(B_{RR})\end{aligned} \tag{23}$$

By analogy it is possible to define the interregional extended multiplier product matrices:

$$\begin{aligned} M_{rR} &= \frac{1}{V(B_{rr})} M_r(B_{rr}) M_c(B_{RR}) \\ M_{Rr} &= \frac{1}{V(B_{RR})} M_r(B_{RR}) M_c(B_{rr}) \end{aligned} \tag{24}$$

Therefore, the multiplier product matrix for the block matrix A of the multiregional input-output system reveals the following structure:

$$\begin{aligned} M(B) = {} & \frac{V(B_{rr})}{V(B)} \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_{rr} \begin{bmatrix} I & A_{rR} B_R \end{bmatrix} + \frac{V(B_{rr})}{V(B)} \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_{rR} \begin{bmatrix} A_{Rr} B_r & I \end{bmatrix} + \\ & + \frac{V(B_{RR})}{V(B)} \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} M_{Rr} \begin{bmatrix} I & A_{rR} B_R \end{bmatrix} + \frac{V(B_{RR})}{V(B)} \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} M_{RR} \begin{bmatrix} A_{Rr} B_r & I \end{bmatrix} \end{aligned} \tag{25}$$

Denote the four components of the decomposition (25) as: $M(B)[rr]; M(B)[rR]; M(B)[Rr]; M(B)[RR]$. Then:

$$M(B) = M(B)[rr] + M(B)[rR] + M(B)[Rr] + M(B)[RR] \tag{26}$$

where

$$M(B)[rr] = \frac{V(B_{rr})}{V(B)} \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_{rr} \begin{bmatrix} I & A_{rR} B_R \end{bmatrix} = \frac{V(B_{rr})}{V(B)} \begin{bmatrix} M_{rr} & M_{rr} A_{rR} B_R \\ B_R A_{Rr} M_{rr} & B_R A_{Rr} M_{rr} A_{rR} B_R \end{bmatrix} \tag{27}$$

$$M(B)[rR] = \frac{V(B_{rr})}{V(B)} \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_{rR} \begin{bmatrix} A_{Rr} B_r & I \end{bmatrix} = \frac{V(B_{rr})}{V(B)} \begin{bmatrix} M_{rR} A_{Rr} B_r & M_{rR} \\ B_R A_{Rr} M_{rR} A_{Rr} B_r & B_R A_{Rr} M_{rR} \end{bmatrix} \tag{28}$$

$$M(B)[Rr] = \frac{V(B_{RR})}{V(B)} \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} M_{Rr} \begin{bmatrix} I & A_{rR} B_R \end{bmatrix} = \frac{V(B_{RR})}{V(B)} \begin{bmatrix} B_r A_{rR} M_{Rr} & B_r A_{rR} M_{Rr} A_{rR} B_R \\ M_{Rr} & M_{Rr} A_{rR} B_R \end{bmatrix} \tag{29}$$

$$M(B)[RR] = \frac{V(B_{RR})}{V(B)} \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} M_{RR} \begin{bmatrix} A_{Rr} B_r & I \end{bmatrix} = \frac{V(B_{RR})}{V(B)} \begin{bmatrix} B_r A_{rR} M_{RR} A_{Rr} B_r & B_r A_{rR} M_{RR} \\ M_{RR} A_{Rr} B_r & M_{RR} \end{bmatrix} \tag{30}$$

Using the block structure of the components $M(B)[rr]; M(B)[rR]; M(B)[Rr]; M(B)[RR]$, one can construct the block structure of the multiplier product matrix as:

$$M(B) = \begin{bmatrix} [M(B)]_{rr} & [M(B)]_{rR} \\ [M(B)]_{Rr} & [M(B)]_{RR} \end{bmatrix} \tag{31}$$

by summing the corresponding blocks from (27) - (30);

$$[M(B)]_{rr} = \frac{V(B_{rr})}{V(B)} M_{rr} + \frac{V(B_{rr})}{V(B)} M_{rR} A_{Rr} B_r + \frac{V(B_{RR})}{V(B)} B_r A_{rR} M_{Rr} + \frac{V(B_{RR})}{V(B)} B_r A_{rR} M_{RR} A_{Rr} B_r;$$

$$[M(B)]_{rR} = \frac{V(B_{rr})}{V(B)} M_{rR} + \frac{V(B_{rr})}{V(B)} M_{rr} A_{rR} B_R + \frac{V(B_{RR})}{V(B)} B_r A_{rR} M_{RR} + \frac{V(B_{RR})}{V(B)} B_r A_{rR} M_{Rr} A_{rR} B_R;$$

$$[M(B)]_{Rr} = \frac{V(B_{RR})}{V(B)} M_{Rr} + \frac{V(B_{RR})}{V(B)} M_{RR} A_{Rr} B_r + \frac{V(B_{rr})}{V(B)} B_R A_{Rr} M_{rr} + \frac{V(B_{rr})}{V(B)} B_R A_{Rr} M_{rR} A_{Rr} B_r;$$

$$[M(B)]_{RR}=\frac{V(B_{RR})}{V(B)}M_{RR}+\frac{V(B_{RR})}{V(B)}M_{Rr}A_{rR}B_R+\frac{V(B_{rr})}{V(B)}B_RA_{Rr}M_{rR}+\frac{V(B_{rr})}{V(B)}B_RA_{Rr}M_{rr}A_{rR}B_R \tag{32}$$

A more detailed analysis would focus on establishing connections between the economic landscapes generated by the intraregional Leontief inverses $B_r=(I-A_{rr})^{-1}$, $B_R=(I-A_{RR})^{-1}$ of the isolated region r and the isolated rest of economy and their extended regional Leontief inverses $B_{rr}=(I-A_{rr}-A_{rR}B_RA_{Rr})^{-1}$; $B_{RR}=(I-A_{RR}-A_{Rr}B_rA_{rR})^{-1}$.

Initially, comparison will be made of the multiplier product matrices of the isolated region M_r and the region within the economy M_{rr}. Equation (17) implies that the extended Leontief inverse can be decomposed into the products of internal and external multipliers describing direct and induced self-influences (Miyazawa, 1966, 1976):

$$B_{rr}=B_rB_{rr}^R=B_{rr}^LB_r \tag{33}$$

where

$$B_{rr}^L=(I-B_rA_{rR}B_RA_{Rr})^{-1};\ B_{rr}^R=(I-A_{rR}B_RA_{Rr}B_r)^{-1} \tag{34}$$

are the left and right Miyazawa external multipliers for the region r.

For the intraregional Leontief inverse B_r, the economic landscape of the isolated region r corresponds to the following multiplier product matrix:

$$M_r=M(B_r)=\frac{1}{V(B_r)}M_r(B_r)M_c(B_r) \tag{35}$$

The economic landscape, $M_{rr}=M(B_{rr})$, of the extended Leontief inverse corresponds to the following multiplier product matrix

$$M_{rr}=M(B_{rr})=\frac{1}{V(B_{rr})}M_r(B_{rr})M_c(B_{rr}) \tag{36}$$

Using (31) and (13) one obtains:

$$\begin{aligned}M_{rr}&=\frac{1}{V(B_{rr})}M_r(B_{rr}^LB_r)M_c(B_rB_{rr}^R)=\\&=\frac{1}{V(B_{rr})}B_{rr}^LM_r(B_r)M_c(B_r)B_{rr}^R=\frac{V(B_r)}{V(B_{rr})}B_{rr}^LM_rB_{rr}^R\end{aligned} \tag{37}$$

Analogously, for the rest of economy:

$$M_{RR}=\frac{V(B_R)}{V(B_{RR})}B_{RR}^LM_RB_{RR}^R \tag{38}$$

Further, let us introduce the isolated interregional multiplier product matrices in the form:

$$
\begin{aligned}
M_{r/R} &= \frac{1}{V(B_r)} M_r(B_r) M_c(B_R) \\
M_{R/r} &= \frac{1}{V(B_R)} M_r(B_R) M_c(B_r)
\end{aligned}
\tag{39}
$$

Therefore, analogously to (9) and (38) one obtains:

$$
M_{rR} = \frac{V(B_r)}{V(B_{rr})} B^L_{rr} M_{r/R} B^R_{RR}; \quad M_{Rr} = \frac{V(B_R)}{V(B_{RR})} B^L_{RR} M_{R/r} B^R_{rr} \tag{40}
$$

Introducing (37), (38) and (40) into (25) we have

$$
\begin{aligned}
M(B) &= M(B)[rr] + M(B)[rR] + M(B)[Rr] + M(B)[RR] \\
&= \frac{V(B_r)}{V(B)} \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} B^L_{rr} M_r B^R_{rr} \begin{bmatrix} I & A_{rR} B_R \end{bmatrix} + \frac{V(B_r)}{V(B)} \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} B^L_{rr} M_{r/R} B^R_{RR} \begin{bmatrix} A_{rR} B_R & I \end{bmatrix} + \\
&= \frac{V(B_R)}{V(B)} \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} B^L_{RR} M_{R/r} B^R_{rr} \begin{bmatrix} I & A_{rR} B_R \end{bmatrix} + \frac{V(B_R)}{V(B)} \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} B^L_{RR} M_R B^R_{RR} \begin{bmatrix} A_{Rr} B_r & I \end{bmatrix}
\end{aligned}
\tag{41}
$$

Using (17) one obtains:

$$
\begin{aligned}
M(B) &= M(B)[rr] + M(B)[rR] + M(B)[Rr] + M(B)[RR] = \\
&= \frac{V(B_r)}{V(B)} \begin{bmatrix} B^L_{rr} \\ B_R A_{Rr} B^L_{rr} \end{bmatrix} M_r \begin{bmatrix} B^R_{rr} & B^R_{rr} A_{rR} B_R \end{bmatrix} + \frac{V(B_r)}{V(B)} \begin{bmatrix} B^L_{rr} \\ B_R A_{Rr} B^L_{rr} \end{bmatrix} M_{r/R} \begin{bmatrix} B^R_{RR} A_{Rr} B_r & B^R_{RR} \end{bmatrix} + \\
&+ \frac{V(B_r)}{V(B)} \begin{bmatrix} B_r A_{rR} B^L_{RR} \\ B^L_{RR} \end{bmatrix} M_{R/r} \begin{bmatrix} B^R_{rr} & B^R_{rr} A_{rR} B_R \end{bmatrix} + \frac{V(B_R)}{V(B)} \begin{bmatrix} B_r A_{rR} B^L_{RR} \\ B^L_{RR} \end{bmatrix} M_R \begin{bmatrix} B^R_{RR} A_{Rr} B_r & B^R_{RR} \end{bmatrix}
\end{aligned}
\tag{42}
$$

Using the relationships $B_r(I-A_{rr})=I$; $B_R(I-A_{RR})=I$, the following expression may be presented:

$$
\begin{aligned}
M(B) = &\frac{V(B_r)}{V(B)} \begin{bmatrix} B_{rr} \\ B_R A_{Rr} B_{rr} \end{bmatrix} (I-A_{rr}) M_r (I-A_{rr}) \begin{bmatrix} B_{rr} & B_{rr} A_{rR} B_R \end{bmatrix} + \\
&+ \frac{V(B_r)}{V(B)} \begin{bmatrix} B_{rr} \\ B_R A_{Rr} B_{rr} \end{bmatrix} (I-A_{rr}) M_{r/R} (I-A_{RR}) \begin{bmatrix} B_{RR} A_{Rr} B_r & B_{RR} \end{bmatrix} + \\
&+ \frac{V(B_R)}{V(B)} \begin{bmatrix} B_r A_{rR} B_{RR} \\ B_{RR} \end{bmatrix} (I-A_{RR}) M_{R/r} (I-A_{rr}) \begin{bmatrix} B_{rr} & B_{rr} A_{rR} B_R \end{bmatrix} + \\
&+ \frac{V(B_R)}{V(B)} \begin{bmatrix} B_r A_{rR} B_{RR} \\ B_{RR} \end{bmatrix} (I-A_{RR}) M_R (I-A_{RR}) \begin{bmatrix} B_{RR} A_{Rr} B_r & B_{RR} \end{bmatrix}
\end{aligned}
\tag{43}
$$

or

$$
\begin{aligned}
M(B) = &\frac{V(B_r)}{V(B)} \begin{bmatrix} B_{rr} \\ B_{Rr} \end{bmatrix} (I-A_{rr}) M_r (I-A_{rr}) \begin{bmatrix} B_{rr} & B_{rR} \end{bmatrix} + \\
&+ \frac{V(B_r)}{V(B)} \begin{bmatrix} B_{rr} \\ B_{Rr} \end{bmatrix} (I-A_{rr}) M_{r/R} (I-A_{RR}) \begin{bmatrix} B_{Rr} & B_{RR} \end{bmatrix} +
\end{aligned}
\tag{44}
$$

$$+\frac{V(B_R)}{V(B)}\begin{bmatrix}B_{rR}\\B_{RR}\end{bmatrix}(I-A_{RR})M_{R/r}(I-A_{rr})\begin{bmatrix}B_{rr} & B_{rR}\end{bmatrix}+$$
$$+\frac{V(B_R)}{V(B)}\begin{bmatrix}B_{rR}\\B_{RR}\end{bmatrix}(I-A_{RR})M_R(I-A_{RR})\begin{bmatrix}B_{Rr} & B_{RR}\end{bmatrix}$$

IV. *Interpretations.*

The following three-tier interpretation of the (25), (41) and (44) may now be presented: the multiregional multiplier product matrix, $M(B)$, which reveals the spread of point-wise changes in the multiregional economy is the sum (superposition) of the four block matrices $M(B)[rr]$; $M(B)[rR]$; $M(B)[Rr]$; $M(B)[RR]$. It is important to realize that each matrix starts from the micro-level isolated intraregional and interregional multiplier product matrices M_r, M_R, $M_{r/R}$, $M_{R/r}$. Thereafter, the system focuses at the meso-level, where attention is directed to the spread of changes to the rest of the economy through the extended Leontief inverses which are themselves decomposed into left and right Miyazawa external multipliers. Finally, at the macro-level, the spread of changes proceeds though the mechanism of the block fields of influence of changes in each block.

This interpretation can be revealed for the block matrix $M(B)[rr]$ corresponding to the region r; other block matrices can be considered in an analogous fashion. The decompositions will follow the path of interactions between method outlined in table 1, but with an explicit focus on the interrelationships between a region and the rest of the economy in which it is nested; the role of internal and external effects in the spirit of Miyazawa (1966) will be readily apparent. To further assist in the interpretation, one might consider using this approach to reinterpret the structural path of relationships between the Kobe region and the rest of Japan in the wake of the recent Great Hanshin earthquake (see Okuyama et al., 1998).

Micro-level of isolated region:

The analysis begins at a very micro-level, with attention focused on individual sectors or even sets of coefficients. Starting from (41), the block matrix

$$M(B)[rr]=\frac{V(B_r)}{V(B)}\begin{bmatrix}I\\B_RA_{Rr}\end{bmatrix}B^L_{rr}M_rB^R_{rr}\begin{bmatrix}I & A_{rR}B_R\end{bmatrix}$$

includes the multiplier product matrix, M_r, that represents the micro-level economic landscape of the isolated region r defining, with the help of the Leontiefinverse $B_r=(I-A_{rr})^{-1}$, the hierarchy of the backward and forward linkages between the sectors of the region r and the key sectors of this region interpreted independently of the influence of the rest of economy. Moreover, M_r represents a matrix of first order intensities of the fields of influence of individual changes in direct regional inputs, i.e., the gradient of first order changes in the Leontief inverse B_r. Note that the analysis essentially ignores the rest of the economy, focusing on the interplay of fields of influence in the context of internal multiplier effects. It is as though Kobe were treated as an island economy, with no linkages to the rest of the world.

Meso-level of the influence of the economy on the region:

Now, the impact on and received from the rest of the economy is considered. Structural changes in the Kobe economy generated by the earthquake would affect distribution of goods and services between Kobe and the rest of Japan. These bilateral exchanges are handled in the following fashion. The interpretation begins with the block matrix presented in (27):

$$M(B)[rr] = \frac{V(B_{rr})}{V(B)} \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_{rr} \begin{bmatrix} I & A_{rR} B_R \end{bmatrix} = \frac{V(B_{rr})}{V(B)} \begin{bmatrix} M_{rr} & M_{rr} A_{rR} B_R \\ B_R A_{Rr} M_{rr} & B_R A_{Rr} M_{rr} A_{rR} B_R \end{bmatrix}$$

that is based on the economic landscape

$$M_{rr} = \frac{V(B_r)}{V(B_{rr})} B_{rr}^L M_r B_{rr}^R$$

defined with the help of the extended Leontief inverse of the region r within the economy $B_{rr} = (I - A_{rr} - A_{rR} B_R A_{Rr})^{-1}$. The matrix, M_{rr}, provides insights into a number of properties of the regional structure. First, it describes a hierarchy of the backward and forward intraregional sectoral linkages and the set of regional key sectors. Secondly, it indicates the changes in these hierarchies and in the set of key sectors caused by the economic interconnections of the region with the rest of economy. The mechanism through which these interconnections occur is described by the left and right Miyazawa external multipliers of the region r.

$$B_{rr}^L = (I - B_r A_{rR} B_R A_{Rr})^{-1}; B_{rr}^R = (I - A_{rR} B_R A_{Rr} B_r)^{-1}$$

These external multipliers may be interpreted as a meso-level feedback loop (see Sonis *et al.*, 1997b) for the transfer of economic changes between the region and the rest of economy generated by the components $B_r A_{rR} B_R A_{Rr}$, $A_{rR} B_R A_{Rr} B_r$. Schematically, these transfers of influence may be considered to follow this pattern:

$$[r] \xrightarrow{A_{Rr}} [R] \xrightarrow{B_R} [R] \xrightarrow{A_{Rr}} [r] \xrightarrow{B_r} [r]$$

Hence, the interactions between the two regions involve consideration of external multiplier effects that can be reinterpreted as self-influence feedback loops. Disruption in the Kobe economy generates disruption in the rest of Japan and this second round of impacts return to effect the Kobe economy yet again. Thus, suppliers in a commodity chain of production located in Kobe would have been unable to ship their products to firms in the rest of Japan; these firms without needed inputs would have had to reduce production levels and thereby would have been unable to send the enhanced products back to the Kobe region for further use in intermediate production or final consumption.

Macro-level transfer of changes in the region through the rest of economy:

An alternative perspective is provided here as a way of interpreting the impact of changes in a region and the rest of the economy. The economic landscape of the extended regional Leontief inverse, B_{rr}, can also be presented in a form:

$$M_{rr}=\frac{V(B_r)}{V(B_{rr})}B^L_{rr}M_rB^R_{rr}=\frac{V(B_r)}{V(B_{rr})}B^L_{rr}B_r(I-A_{rr})M_r(I-A_{rr})B_rB^R_{rr}$$

Here, the matrices, $B^L_{rr}B_r$, $B_rB^R_{rr}$, are parts of the block field of influence of changes in the region r; this can be seen from the following chain of expressions derived from (41) - (44):

$$\begin{bmatrix} I \\ B_RA_{Rr} \end{bmatrix}B^L_{rr}=\begin{bmatrix} B^L_{rr} \\ B_RA_{Rr}B^L_{rr} \end{bmatrix}=\begin{bmatrix} B_{rr} \\ B_RA_{Rr}B_{rr} \end{bmatrix}(I-A_{rr})=\begin{bmatrix} B_{rr} \\ B_{Rr} \end{bmatrix}(I-A_{rr});$$

$$B^R_{rr}\begin{bmatrix} I & A_{rR}B_R \end{bmatrix}=\begin{bmatrix} B^R_{rr} & B^R_{rr}A_{rR}B_R \end{bmatrix}=(I-A_{rr})\begin{bmatrix} B_{rr} & B_{rr}A_{rR}B_R \end{bmatrix}=(I-A_{rr})\begin{bmatrix} B_{rr} & B_{rR} \end{bmatrix}$$

The block vectors $\begin{bmatrix} \mathrm{B}_{rr} \\ \mathrm{B}_{Rr} \end{bmatrix}$; $\begin{bmatrix} \mathrm{B}_{rr} & \mathrm{B}_{rR} \end{bmatrix}$ are the components of the block fields of influence of changes that originate within the regional inputs; therefore, from (37) the block matrix

$$M(B)[rr]=\frac{V(B_r)}{V(B)}\begin{bmatrix} B_{rr} \\ B_{Rr} \end{bmatrix}(I-A_{rr})M_r(I-A_{rr})\begin{bmatrix} B_{rr} & B_{rR} \end{bmatrix}$$

represents the macro-level spread of changes within the region r over all the economy. The approach here focuses at a more macro-level than the first one that was presented, but the underlying organization is very similar.

The region versus the rest of economy:

Here, a modification of an earlier approach to the region versus the rest of the economy is provided that extends the interpretation to a broader context (see Sonis *et al.*, 1996). If attention was directed only to the regional part, $M(B)[rr]$, of the economic landscape, $M(B)$, then (32) may be shown as:

$$[M(B)]_{rr}=\frac{V(B_{rr})}{V(B)}M_{rr}+\frac{V(B_{rr})}{V(B)}M_{rR}A_{Rr}B_r+\frac{V(B_{RR})}{V(B)}B_rA_{rR}M_{Rr}+\frac{V(B_{RR})}{V(B)}B_rA_{rR}M_{RR}A_{Rr}B_r$$

This part of (32) describes the spread of changes within the region r caused by (i) the changes in direct inputs within the region, $\frac{V(B_{rr})}{V(B)}M_{rr}$; (ii) changes in regional forward linkages, $\frac{V(B_{rr})}{V(B)}M_{rR}A_{Rr}B_r$; (iii) changes of the regional backward linkages, $\frac{V(B_{RR})}{V(B)}B_rA_{rR}M_{Rr}$ and, finally, (iv) changes in the direct inputs within the isolated rest of economy, $\frac{V(B_{RR})}{V(B)}B_r A_{rR}M_{RR}A_{Rr}B_r$. This decomposition provides a summary of the changes focusing on a decomposition into internal, forward, backward and external linkages.

V. *Conclusions*

While many decomposition techniques for interpreting structure and structural change have been proposed and subsequently modified, there have been few attempts to explore the links between sets of methodologies. This paper has tried to provide a mapping between several alternative, yet complementary approaches and, in the previous section, provide a summary interpretation of the insights that they offer. Particular attention is paid to the distinction, first articulated by Miyazawa (1966), between internal and external effects. Overlying this underpinning is the strong presence of hierarchical influences and the superposition of different intersectoral and spatial mechanisms creating change. The methodology focused on a two region (region versus the rest of the economy) context; however, the extension to the n-region case, while more complicated, would follow similar paths.

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