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FAIRNESS IN MARKETS AND GOVERNMENT POLICIES: A WEAK EQUITY CRITERION FOR ALLOCATION MECHANISMS

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Abstract

An allocation is said to be "weakly envy-free" if no individual envies others with the same or smaller endowments. We show that competitive equilibrium allocations are weakly envy-free, i.e., this weak notion of fairness is achieved in markets. Since government policies are expected to improve fairness in market economies, we require they satisfy at least this weak equity criterion in any economy. We show that many well-known equity criteria for taxation (e.g., anonymity, incentive-preservation, horizontal equity, ability-to-pay principle to finance pure public goods, and benefit principle for impure public goods) are implied by this weak criterion.

I. *Introduction*

It is widely perceived that markets cannot attain "fairness"[1] and thus some forms of government policies are necessary.[2] At the same time, markets are perceived to achieve some notion of fairness. For example, the Japanese government is sometimes considered to be unfair for its regulations or for not opening the economy, which suggests that markets can attain some notion of equity. The purpose of this paper is to clarify a notion of fairness achieved in exchange markets[3] and study its implications for government policies.

In 1967, Foley introduced a famous equity criterion for allocations in his discussion on the optimal government policies in a general equilibrium model: An allocation is said to be envy-free, if no one envies others under the allocation.[4] Since the equity criterion allows individuals to judge fairness of allocations based on their own preferences and does not require them to know preferences of others, this concept is likely to be accepted as an equity

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[1] In this paper, by "markets" we always mean competitive markets (markets under perfect competition). We also use the term "fairness" and "equity" interchangeably (cf. Remark 1).

[2] For example, Atkinson-Stiglitz (1980, p.6) states as follows: "Pareto efficiency does not ensure that the distribution that emerges from the competitive process is in accord with the prevailing concepts of equity (whatever these may be). One of the primary activities of the government is indeed redistribution."

[3] An extension to production economies is attempted in Section IV. This extension is important because of a well-known non-existence problem in production economies [Pazner-Schmeidler (1974)].

[4] Besides its intuitive appeal as an equity criterion, it is known to have some nice properties: It does not require interpersonal comparability nor cardinality of welfare [cf. Varian (1974)].

criterion in economies where it is practically impossible to know preferences of others [cf. Pazner (1977) and Yamashige (1995)].[5]

In this paper, we introduce a weaker version of this equity criterion:[6] An allocation is said to be "weakly envy-free" if no individual envies others with the same or smaller endowments. The condition is a necessary condition for an allocation to be envy-free and essentially requires that the envy-structure in an initial state should not be overturned in the final state. Since the endowment of each individual can be seen as his/her contribution to a society, the condition may be also seen as a requirement that each individual should not envy those who make the same or smaller contributions to the society.[7] We show that competitive equilibrium allocations in exchange economies are weakly envy-free, i.e., this weak notion of fairness is achieved in markets.[8] This fact may be a reason why the market system has been a socially stable allocation mechanism in many countries.

Since the competitive equilibria satisfy this weak notion of fairness, it seems natural to expect that government policies, which are supposed to improve fairness in market economies, should not violate this minimum requirement for fairness. A government policy is said to be weakly envy-free if under the policy no individual envies others with the same or smaller endowments. We require that any government policy satisfy this criterion in arbitrary economy because governments usually do not know preferences and endowments of individuals. This requirement indeed seems natural in the sense that it requires that the envy-structure without a government should not be overturned in the presence of the government.[9]

We take this equity criterion as the weakest requirement that every government policy must satisfy, and we study how this criterion restricts the set of admissible government policies. The results indicate that our new equity criterion for government policies implies many well-known equity criteria for tax systems such as anonymity, incentive-preservation[10], horizontal equity, ability-to-pay principle to finance pure public goods. In relation to the benefit principle, we also show that the weak equity criterion requires the benefit tax to

[5]Yamashige (1995) argued that when each individual has some information on well-being of some individuals, for example, of those with handicaps, the appropriate notion of fairness will not be the envy-free principle but will be similar to the one discussed by Rawls (1971). Hence, the equity criterion for government policies discussed in this paper should be applied only to economies in which individuals do not have any special welfare characteristics that others can identify.

[6]An equity criterion reduces the number of admissible allocations. We say that an equity criterion is stronger than another if the set of admissible allocations under the criterion is a subset of the one under the other criterion.

[7]A sophist Hippias of Elist in the ancient Greek, according to de la Mora (1984, p.4), classified "envy" into two types, which "Aristotle later on will establish as the proper definition of indignation and envy": The "just" one is "the envy towards undeserving people when they receive honors" and the "unjust" one is the envy "of those who envy good people". Hence, we may be able to say, roughly, that our "weakly envy-free" criterion is a principle to prohibit causing the "just" envy and let the "unjust" one be as it is.

[8]This observation shows that the concept of fairness attained in markets does depend on each individual's initial state and therefore differs from those proposed by Rawls (1971) and Harsanyi (1955). See, e.g., Sen (1970, Chapter 9) for more details.

[9]A similar requirement is considered in Feldstein (1976) and King (1983), in which they require that "the tax system should not lead to changes in the ranking of utilities". This notion of fairness, however, requires interpersonal comparison of utilities, which in general is difficult.

[10]This term, which requires that the tax should not reverse the relative income ranks to preserve incentives for individuals to earn higher incomes, is adopted from Fei (1981).

finance impure public goods, although it is not a fair tax in our sense to finance pure public goods. Since these properties of tax systems are observed in many countries, it seems that the weak no-envy equity criterion is indeed adopted by many governments.

The paper is organized as follows. In the next section, we discuss equity criteria for general allocation mechanisms and show that competitive equilibrium allocation mechanisms are weakly envy-free. In Section III, we introduce a government, define our no-envy criterion, and discuss its relationship with well-known equity criteria for tax systems. Section IV discusses two extensions of the model. Section V concludes the paper.

II. *Fairness in Markets*

Markets usually do not treat individuals symmetrically. The more an individual is endowed the better off he/she becomes. This asymmetry seems to be a reason why people consider that markets fail to attain equity. In order to analyze the asymmetry in markets, we define allocation mechanisms each of which assigns each economy an allocation of commodities, and introduce equity criteria for such allocation mechanisms.[11] Then, we define the competitive equilibrium allocation mechanism and study its property.

1. Preliminaries

Let $N = \{1, \dots, n\}$ be a set of individuals. Let $X \subseteq \mathbf{R}^m_+$ be a commodity space. Let $u_i : X \to \mathbf{R}$ be a utility function of an individual $i \in N$. We assume that u_i is non-decreasing in each argument. Let $\mathcal{U}$ be a space of such utility functions. Let $\omega_i \in X$ be an endowment of $i \in N$. Since an individual i is characterized by a pair (ω_i, u_i), an exchange economy is defined by $e = (\omega_1, \dots, \omega_n; u_1, \dots, u_n) \in X^n \times \mathcal{U}^n$. Let $\mathbf{E}$ be a space of exchange economies.

We consider assignment rules which are dependent on each individual's name and endowment. Formally, for each $i \in N$, an *assignment function* is defined by a mapping $\tilde{x}_i : X \times \mathbf{E} \to X$ such that $\tilde{x}_i(\hat{\omega}_i; e) \in X$ represents a commodity bundle of individual $i \in N$ when he/she has an endowment $\hat{\omega}_i \in X$ in an economy $e = (\omega_1, \dots, \omega_i, \dots, \omega_n; u_1, \dots, u_i, \dots, u_n)$. Given a collection of assignment functions $\{\tilde{x}_i\}_{i \in N}$, a mapping $\tilde{x} : \mathbf{E} \to X^n$ defined by $\tilde{x}(e) = (\tilde{x}_1(\omega_1; e), \dots, \tilde{x}_n(\omega_n; e))$ for each $e \in \mathbf{E}$ is called an *allocation mechanism* if it satisfies the resource constraint $\sum_{i \in N} \tilde{x}_i(\omega_i; e) \leq \sum_{i \in N} \omega_i$. Hence, an allocation mechanism is associated with a collection of assignment functions which satisfies the resource constraint in each economy.[12]

First, we introduce two equity criteria for allocations.

[11] This approach is similar to the one taken by Thomson (1983) who studies various notions of fairness by analyzing allocation mechanisms (which he calls choice correspondences) with permutations of endowment vectors. Our goal in this section is to study allocation mechanisms to figure out the fairness embodied in the competitive equilibrium allocation mechanism.

[12] We find that this definition of an allocation mechanism is useful to study circumstances in which individuals consider effects of changes in their own initial endowments in a given economy (e.g., competitive markets where individuals behave as "price-takers").

Definition 1 *In an economy $e \in \mathbf{E}$, an allocation $\tilde{x}(e)$ is said to be envy-free if and only if*

$$u_i(\tilde{x}_i(\omega_i; e)) \geq u_i(\tilde{x}_j(\omega_j; e)) \quad \forall\, i, j \in N.$$

Definition 2 *In an economy $e \in \mathbf{E}$, an allocation $\tilde{x}(e)$ is said to be weakly envy-free if and only if*

$$\omega_i \geq \omega_j \Rightarrow u_i(\tilde{x}_i(\omega_i; e)) \geq u_i(\tilde{x}_j(\omega_j; e)) \quad \forall\, i, j \in N.$$

The second criterion is clearly weaker than the first one. Under the envy-free allocation, no one envies commodity bundles of others. Under the weakly envy-free allocation, someone may envy commodity bundles of others, but each individual never envies commodity bundles of other individuals with the same or smaller endowments.

We now introduce two equity criteria for allocation mechanisms. Notice that if an allocation mechanism $\tilde{x}$ is (weakly) envy-free then an allocation $\tilde{x}(e)$ is (weakly) envy-free in any economy $e \in \mathbf{E}$.

Definition 3 *An allocation mechanism $\tilde{x}$ is said to be envy-free (in $\mathbf{E}$)*[13] *if and only if, for all $e \in \mathbf{E}$, $i, j \in N$, and $\hat{\omega}_i, \hat{\omega}_j \in X$,*

$$u_i(\tilde{x}_i(\hat{\omega}_i; e)) \geq u_i(\tilde{x}_j(\hat{\omega}_j; e)).$$

Definition 4 *An allocation mechanism $\tilde{x}$ is said to be weakly envy-free (in $\mathbf{E}$) if and only if, for all $e \in \mathbf{E}$, $i, j \in N$, and $\hat{\omega}_i, \hat{\omega}_j \in X$,*

$$\hat{\omega}_i \geq \hat{\omega}_j \Rightarrow u_i(\tilde{x}_i(\hat{\omega}_i; e)) \geq u_i(\tilde{x}_j(\hat{\omega}_j; e)).$$

Remark 1. Consider the following weaker equity criteria for allocation mechanisms, which require that $\tilde{x}(e)$ be (weakly) envy-free in any economy $e \in \mathbf{E}$. An allocation mechanism $\tilde{x}$ is said to be envy-free* (in $\mathbf{E}$) if and only if, for all $e \in \mathbf{E}$ and $i, j \in N$, $u_i(\tilde{x}_i(\omega_i; e)) \geq u_i(\tilde{x}_j(\omega_j; e))$. An allocation mechanism $\tilde{x}$ is said to be weakly envy-free* (in $\mathbf{E}$) if and only if, for all $e \in \mathbf{E}$ and $i, j \in N$, $\omega_i \geq \omega_j \Rightarrow u_i(\tilde{x}_i(\omega_i; e)) \geq u_i(\tilde{x}_j(\omega_j; e))$. The original definitions are stronger than new ones because the "no-envy" condition must be satisfied not only in each economy but also "off-economy" of each economy (as in the subgame perfect equilibrium in game theory). We think that our original definitions are useful since individuals often think of changes in their own endowments in a given environment.

2. Fairness of Competitive Equilibrium Allocation Mechanism

We are ready to study an important allocation mechanism which actually motivated various definitions in the last subsection. Let $\Delta = \{(q_1, \ldots, q_m) \in \mathbf{R}^m_+ : \sum_{k=1}^m q_k = 1\}$ be a space of price vectors. Define a budget correspondence $B : \Delta \times X \longrightarrow\!\!\!\rightarrow X$ by $B(q, \omega_i) = \{x \in X : q \cdot x \leq q \cdot \omega_i\}$.

[13] We often omit the phrase "in $\mathbf{E}$" as long as the class of economies we consider is obvious.

Definition 5 *An allocation mechanism $\tilde{x}$ is said to be a competitive equilibrium allocation mechanism if and only if, for any economy $e \in \mathbf{E}$, there is $q_e \in \Delta$ such that, for all $i \in N$ and $\hat{\omega}_i \in X$,*

$$\tilde{x}_i(\hat{\omega}_i; e) \in \arg \max_{x \in B(q_e, \hat{\omega}_i)} u_i(x).$$

Namely, a competitive equilibrium allocation mechanism is associated with demand functions given an equilibrium price system (as assignment functions). It is well known that under reasonable conditions this mapping is well defined [e.g., Debreu (1959)]. Now, we have the following result which can be seen as a claim that markets are fair to some degree.[14]

Proposition 1 *Every competitive equilibrium allocation mechanism is weakly envy-free.*

Proof. Let $e \in \mathbf{E}$ be arbitrary and $(\tilde{x}(e), q_e)$ be a competitive equilibrium. Let $\hat{\omega}_i \geq \hat{\omega}_j$. Then, $B(q_e, \hat{\omega}_i) \supseteq B(q_e, \hat{\omega}_j)$ because if $x \in B(q_e, \hat{\omega}_j)$ then $q_e \cdot x \leq q_e \cdot \hat{\omega}_j \leq q_e \cdot \hat{\omega}_i$, i.e., $x \in B(q_e, \hat{\omega}_i)$. Therefore,

$$u_i(\tilde{x}_i(\hat{\omega}_i; e)) = \max_{x \in B(q_e, \hat{\omega}_i)} u_i(x) \geq \max_{x \in B(q_e, \hat{\omega}_j)} u_i(x) = u_i(\tilde{x}_i(\hat{\omega}_j; e)) \geq u_i(\tilde{x}_j(\hat{\omega}_j; e)),$$

where the last inequality is due to $\tilde{x}_j(\hat{\omega}_j; e) \in B(q_e, \hat{\omega}_j)$. Namely, the allocation mechanism is weakly envy-free. ∎

As far as the competitive equilibrium is concerned, the following result is also useful. (Since the proof is similar to the one for Proposition 1 above, we omit it.)

Proposition 2 *Let $(\tilde{x}(e), q_e)$ be a competitive equilibrium in an economy $e \in \mathbf{E}$. Then, for all $i, j \in N$ and $\hat{\omega}_i, \hat{\omega}_j \in X$,*

$$q_e \cdot \hat{\omega}_i \geq q_e \cdot \hat{\omega}_j \Rightarrow u_i(\tilde{x}_i(\hat{\omega}_i; e)) \geq u_i(\tilde{x}_j(\hat{\omega}_j; e)).$$

The condition above is stronger than the one for the weakly envy-free allocation mechanism because $\hat{\omega}_i \geq \hat{\omega}_j$ implies $q_e \cdot \hat{\omega}_i \geq q_e \cdot \hat{\omega}_j$ but not vice versa. This idea is used in Section IV.1 to strengthen the definition of the weak no-envy criterion for government policies.

Despite these facts about the competitive equilibrium allocation mechanism, we sometimes experience bad feelings that others with the same or smaller endowments get better off. This may be because an allocation is determined not only by markets but also by governments. If such a situation indeed happened, then we would feel unfair. In other words, this weak notion of fairness seems to be a natural requirement that government policies satisfy. In the next section, we adopt this concept as an equity criterion for government policies and discuss its relationship with other well-known equity criteria for taxation.

Remark 2. In this paper, we consider that the fairness of markets is captured by the weak no-envy property. In relation to the benefit tax discussed later, we may be able to

[14] Schmeidler-Vind (1972) has reported a similar result: In the competitive equilibrium each individual's net trade is at least as good as the net trade of any other individual.

argue that markets are fair because each individual is paying a price equal to the benefit from the (last unit of) commodity. Since this property does not require any comparison among individuals, it may be useful to call this property a concept of *fairness* and call a no-envy property a notion of *equity* because of its "equalization" property in individual comparisons.[15] Probably, our feeling that markets are "fair" relies on these two aspects of the competitive equilibrium allocation and it is a kind of miracle that markets in economies with pure private goods possess these two properties. As we will see later in Section III.3, this miracle need not happen when there are public goods.[16]

III. *Fairness in Government Policies*

In this section, we introduce a government in the exchange economy above and define a concept of weakly envy-free government policies. Then, we study this equity criterion in relation to other well-known equity criteria for taxation.

1. Preliminaries

As before, there are m commodities. Let $\{1, 2, \ldots, \ell, \ell+1, \ldots, m\}$ be a set of commodities. We assume that the first ℓ commodities are publicly provided but also can be individually provided and purchased by each agent. (For example, 'policing' is not only publicly provided but also privately purchased.) First, we assume that public goods are pure, i.e., equally consumed by each individual. We relax this assumption in Section III.3.

We assume that a government policy can be divided into two parts: supply of public goods (policies which directly affect utilities) and taxation (policies which affect budget constraints).[17] Let $Y \subseteq \mathbf{R}^{\ell}_{+}$ $(\ell \leq m)$ be a space of publicly provided goods and T be an abstract space of tax policies for each individual. By letting $Z \subseteq \mathbf{R}^{m-\ell}_{+}$ be a space of purely private goods, we have the commodity space $X = Y \times Z$. Since the utility from publicly provided goods and privately purchased goods may be different for each commodity $\{1, \ldots, \ell\}$, we define utility functions for each vector of public goods. Let $u_i : X \times Y \to \mathbf{R}$ be a utility function of $i \in N$ such that $u_i(x; g)$ represents i's utility when he/she has $x \in X$ under a vector of public goods $g \in Y$. We assume that $u_i(x; g)$ is non-decreasing in x and g. Let $\mathcal{U}$ be a space of such utility functions. Let $\omega_i \in X$ be an endowment of $i \in N$.

Let $\Gamma = Y \times T^n$ be a space of possible policies. Let $G = (g, t_1, \ldots, t_n) \in \Gamma$ be a generic notation for a government policy. A budget correspondence of each individual is given by $B : \Delta \times X \times T \longrightarrow\!\!\!\!\rightarrow X$ such that $B(q, \omega_i, t_i) \subset X$ represents the set of affordable bundles for individual $i \in N$ when the market price is $q \in \Delta$, the initial endowment is $\omega_i \in X$ and the tax policy to this individual is $t_i \in T$. We assume free-disposal of commodities, by which we mean that if $x \in B(q, \omega_i)$ then $x' \in B(q, \omega_i)$ for all $x' \in X$ such that $x' \leq x$.

[15] Incidentally, Foley (1967) called an allocation "equitable" if it is envy-free and an allocation "fair" if it is envy-free and Pareto optimal, which we think is of little use.

[16] Sato (1985, 1987) studied notions of fairness for government policies based on the benefit principle.

[17] Even though the government's regulations are not explicitly considered, some forms of regulations may be analyzed as long as they have similar effects as the policies considered here.

We consider an assignment rule which is dependent on an individual's name, endowment and a government policy. Namely, for each $i \in N$, an *assignment function under a government policy* $G \in \Gamma$ is a mapping $\tilde{x}_i^G : X \times T \times \mathbf{E} \to X$ such that $\tilde{x}_i^G(\hat{\omega}_i, t_i; e)$ represents the commodity bundle of individual $i \in N$ in an economy $e \in \mathbf{E}$ under a government policy G when his/her endowment is $\hat{\omega}_i$ and the tax policy on him/her is t_i. Given a collection of assignment functions $\{\tilde{x}_i^G\}_{i \in N}$, a mapping $\tilde{x}^G : \mathbf{E} \to X^n$ defined by $\tilde{x}^G(e) = (\tilde{x}_1^G(\omega_1, t_1; e), \ldots, \tilde{x}_n^G(\omega_n, t_n; e))$ for each $e \in \mathbf{E}$ is called an *allocation mechanism under a government policy* G if it satisfies the resource constraints: $\sum_{i \in N} \tilde{x}_{ik}^G(\omega_i, t_i; e) + g_k \leq \sum_{i \in N} \omega_{ik}$ for all $k \in \{1, \ldots, \ell\}$, and $\sum_{i \in N} \tilde{x}_{ik}^G(\omega_i, t_i; e) \leq \sum_{i \in N} \omega_{ik}$ for all $k \in \{\ell + 1, \ldots, m\}$.

Definition 6 *An allocation mechanism $\tilde{x}^G$ is said to be a competitive equilibrium allocation mechanism under a government policy $G \in \Gamma$ if and only if, for any economy $e \in \mathbf{E}$, there exists a price system $q_e^G \in \Delta$ such that, for all $i \in N$ and $\hat{\omega}_i \in X$,*

$$\tilde{x}_i^G(\hat{\omega}_i, t_i; e) \in \arg \max_{x \in B(q_e^G, \hat{\omega}_i, t_i)} u_i(x; g).$$

2. Weak No-Envy Criterion for Government Policies

Fairness of a government policy G can be judged by the properties of the allocation mechanism $\tilde{x}^G : \mathbf{E} \to X^n$. In this subsection, we propose our equity criterion for government policies, which is based on observations about the competitive equilibrium allocation mechanism studied in the previous section.

Definition 7 *A government policy G is said to be weakly envy-free, or said to satisfy weak no-envy criterion, (in $\mathbf{E}$) if and only if, for all $e \in \mathbf{E}$, $i, j \in N$, and $\hat{\omega}_i, \hat{\omega}_j \in X$,*

$$\hat{\omega}_i \geq \hat{\omega}_j \Rightarrow u_i(\tilde{x}_i^G(\hat{\omega}_i, t_i; e); g) \geq u_i(\tilde{x}_j^G(\hat{\omega}_j, t_j; e); g).$$

Namely, the weak no-envy criterion for a government policy requires that under the government policy any individual should not envy others with the same or smaller endowments. Notice that the criterion does not require interpersonal comparability nor cardinality of welfare. Notice also that we require each government policy satisfy the weak no-envy condition in any economy because governments usually do not know preferences and endowments of individuals.

Now, we want to present our main result of this section, from which we derive a series of implications for tax systems. The proposition shows that the weak no-envy criterion for a government policy can be characterized by its effect on individual budget sets. It says that a government policy is weakly envy-free if and only if after-tax budget set of each individual is no smaller than those of others with the same or smaller endowments.

Proposition 3 *A government policy G is weakly envy-free if and only if, for all $e \in \mathbf{E}$, $i, j \in N$, and $\hat{\omega}_i, \hat{\omega}_j \in X$,*

$$\hat{\omega}_i = \hat{\omega}_j \quad \Rightarrow \quad B(q_e^G, \hat{\omega}_i, t_i) = B(q_e^G, \hat{\omega}_j, t_j) \text{ and}$$
$$\hat{\omega}_i > \hat{\omega}_j \quad \Rightarrow \quad B(q_e^G, \hat{\omega}_i, t_i) \supseteq B(q_e^G, \hat{\omega}_j, t_j).$$

Proof. First of all, it is easy to check that the two conditions above are equivalent to

$$\hat{\omega}_i \geq \hat{\omega}_j \Rightarrow B(q_e^G, \hat{\omega}_i, t_i) \supseteq B(q_e^G, \hat{\omega}_j, t_j).$$

Hence, we show that G is weakly envy-free if and only if the condition above holds.

First, we show sufficiency. Let $e \in \mathbf{E}$ be arbitrary, and let $\hat{\omega}_i \geq \hat{\omega}_j$. Then, we can assume $B(q_e^G, \hat{\omega}_i, t_i) \supseteq B(q_e^G, \hat{\omega}_j, t_j)$. Since $\tilde{x}_i^G(\hat{\omega}_i, t_i; e)$ is the optimal commodity bundle for i, together with the assumption, we have

$$u_i(\tilde{x}_i^G(\hat{\omega}_i, t_i; e); g) \geq u_i(x; g) \quad \forall\, x \in B(q_e^G, \hat{\omega}_j, t_j),$$

which implies $u_i(\tilde{x}_i^G(\hat{\omega}_i, t_i; e); g) \geq u_i(\tilde{x}_j^G(\hat{\omega}_j, t_j; e); g)$ because $\tilde{x}_j^G(\hat{\omega}_j, t_j; e) \in B(q_e^G, \hat{\omega}_j, t_j)$. Namely, G is weakly envy-free.

Next, we show necessity. Let $\hat{\omega}_i \geq \hat{\omega}_j$ and suppose $B(q_e^G, \hat{\omega}_i, t_i) \not\supseteq B(q_e^G, \hat{\omega}_j, t_j)$. Then, there exists $\bar{x} \neq 0$ such that $\bar{x} \in B(q_e^G, \hat{\omega}_j, t_j)$ and $\bar{x} \notin B(q_e^G, \hat{\omega}_i, t_i)$. Suppose that i and j have a (Leontief-type) utility function[18]

$$u(x; g) = \min\{\frac{x_1}{\bar{x}_1}, \frac{x_2}{\bar{x}_2}, \ldots, \frac{x_n}{\bar{x}_n}\} \quad \forall\, g \in Y.$$

Then, $u \in \mathcal{U}$. Since $\bar{x} \in B(q_e^G, \hat{\omega}_j, t_j)$ and $u(\bar{x}, g) > u(x, g)$ for all $x < \bar{x}$, we have

$$\begin{aligned} u(\tilde{x}_j^G(\hat{\omega}_j, t_j; e); g) &= \max_{x \in B(q_e^G, \hat{\omega}_j, t_j)} u(x; g) = u(\bar{x}; g) = \max_{x \leq \bar{x}} u(x; g) \\ &> \max_{x \in B(q_e^G, \hat{\omega}_i, t_i)} u(x; g) = u(\tilde{x}_i^G(\hat{\omega}_i, t_i; e); g), \end{aligned}$$

which implies that individual i (with $\hat{\omega}_i \geq \hat{\omega}_j$) envies j. Namely, $B(q_e^G, \hat{\omega}_i, t_i) \supseteq B(q_e^G, \hat{\omega}_j, t_j)$ is necessary (when $\hat{\omega}_i \geq \hat{\omega}_j$) for a government policy to satisfy the weak no-envy criterion. ∎

The result has important implications for tax policies. First, the weak no-envy criterion requires the "ability-to-pay principle" to finance pure public goods in the sense that the initial endowment must be a tax base.[19] Secondly, the weak no-envy criterion requires the "horizontal equity" in the sense that individuals with the same endowment must have the same after-tax budget set (independent of the preferences). Moreover, the weak no-envy criterion requires "incentive-preservation [Fei (1981)]" in the sense that the after-tax budget set must be increasing in (the supply of) initial endowments. Finally, we can show that the weak no-envy criterion requires anonymity of tax systems. We say that a tax system $(t_1, \ldots, t_n)$ (in G) is *anonymous* if and only if, for any $e \in \mathbf{E}$ and for all $i, j \in N$, $B(q_e^G, \omega_i, t_i) = B(q_e^G, \omega_i, t_j)$.

[18] We use a convention $x/0 = \infty$ for all $x \in \mathbf{R}_+$.

[19] Notice that initial endowment is considered to represent an ability-to-pay here, while traditionally income and/or consumption have been regarded as an ability-to-pay. Note also that, in the traditional ability-to-pay approach, it is implicitly required that individuals with more ability pay more taxes, which is termed as "minimal progressiveness" in Fei (1981), but the weak no-envy criterion does not require this simple property. Hence, it may be slightly misleading to say that our equity criterion demands the ability-to-pay principle to finance pure public goods.

Corollary 1 *A government policy satisfies the weak no-envy criterion only if the tax system is anonymous.*

Proof. Let G satisfy the weak no-envy criterion. Suppose that the tax system is not anonymous. Then, there are an economy $e \in \mathbf{E}$ and $i, j \in N$ such that $B(q_e^G, \omega_i, t_i) \neq B(q_e^G, \omega_i, t_j)$. Now, suppose $\hat{\omega}_j = \omega_i$. Then, by Proposition 3, we must have

$$B(q_e^G, \hat{\omega}_j, t_j) = B(q_e^G, \omega_i, t_i) \neq B(q_e^G, \omega_i, t_j) = B(q_e^G, \hat{\omega}_j, t_j).$$

The contradiction shows that the anonymity is a necessary condition. ∎

Notice that these properties hold under the standard income tax system, which suggests that the weak no-envy criterion may be indeed adopted by many governments. Although the weak no-envy criterion is satisfied under the standard income tax system, the criterion does not require incomes to be the tax base. For example, even if two individuals have the same income the weak no-envy criterion does not require that they have the same after-tax budget set unless they have the same endowment. We will see in Section IV.1 that, under a slightly stronger equity criterion, incomes are required to be the tax base; but we also see some subtle argument that the equitable tax system is probably a little more complicated than the standard income tax system.

In the following two subsections, we study this weak equity criterion in relation to the benefit principle for taxation and to the Pareto optimality of government policies.

3. Relationship with the Benefit Principle

It seems worth studying the weak no-envy criterion in relation to the benefit tax (Lindahl tax) [e.g., Wicksell (1896), Lindahl (1919) and Atkinson-Stiglitz (1980, Ch.16)]. As we have already pointed out in Section I, the no-envy criterion can be seen as an equity criterion which allows each individual (with incomplete information on preferences of others) to judge fairness of allocations from his/her own point of view (using his/her own utility function). Put differently, the no-envy criterion may not be a suitable concept of fairness if everyone knows preferences of others.[20] To see this, let us consider the following example.

Suppose that you and your close friend have the same endowment but have different tastes over pure public goods. You do not appreciate the public goods at all, but you know that your friend appreciates them very much. If you pay the same amount of tax, the final allocation is envy-free because the public goods are equally consumed. If you pay the Lindahl tax which requires you pay for the benefit from the public goods, then the final allocation is not likely to be envy-free (cf. Example 1 below). Which tax policy would be called fair? Since you know that your friend appreciates the public goods more than you do, you may feel that he should pay more taxes, i.e., Lindahl tax (benefit tax) sounds fair to you. Hence, if each individual knows preferences of others, the no-envy criterion may not be an appropriate notion of equity.

[20] For example, the arguments by Wicksell (1896) and Lindahl (1919) seem to be based on an implicit assumption that governments and individuals know preferences of all individuals over public goods.

Our point of view in this paper is that, in economies where it is practically impossible for individuals and governments to know preferences of all individuals, a concept of fairness which allows individuals to make judgments based on their own preferences is more likely to be accepted [cf. Yamashige (1995)]. Hence, even if you feel that your friend should pay more taxes, you would probably agree to pay the same tax as your friend does if you know that it is impossible for governments to know the preferences of all individuals.[21]

Example 1. The following example shows that the Lindahl tax system need not satisfy the weak no-envy criterion. Consider an economy with two individuals and one commodity whose price is equal to 1. Each has one unit of endowment, and the utility functions are given by $u_i(x;g) = x^{\frac{1}{4}}g^{\frac{3}{4}}$ and $u_j(x;g) = x^{\frac{3}{4}}g^{\frac{1}{4}}$, i.e., i appreciates the public good more than j. A Lindahl tax equilibrium is a vector $(x_i, x_j; g, t_i, t_j)$ which is a solution to the problems

$$\max_{(x_i;g)} u_i(x_i;g) \quad s.t. \;\; x_i + t_i g = 1 \quad \text{and} \quad \max_{(x_j;g)} u_j(x_j;g) \quad s.t. \;\; x_j + t_j g = 1$$

with the constraints $t_i g + t_j g = g$ and $x_i + x_j + g = 2$. A simple calculation shows that the equilibrium is given by $(x_i, x_j; g, t_i, t_j) = (\frac{1}{4}, \frac{3}{4}, 1, \frac{3}{4}, \frac{1}{4})$, i.e., individual i who appreciates the public good more pays more $(t_i = \frac{3}{4} > \frac{1}{4} = t_j)$. The Lindahl tax system, however, does not satisfy the weak no-envy equity criterion because $u_i(\frac{1}{4};1) = (\frac{1}{4})^{\frac{1}{4}} < (\frac{3}{4})^{\frac{1}{4}} = u_i(\frac{3}{4};1)$, i.e., i envies j.

In the theory of public finance, the Lindahl tax has been perceived as an ideal tax (in terms of efficiency and equity) but a difficult one to implement due to the lack of information about individual preferences. The discussion above, however, indicates that governments need not have the pessimistic sentiment for not being able to use the Lindahl tax because it is not a fair tax system in economies where individuals judge fairness using their own preferences (due to the lack of their knowledge about preferences of others).

The argument above, however, does not hold if the public goods are not pure. In fact, we show that if the public goods are not pure then the benefit principle must be introduced for a government to satisfy the weak no-envy criterion. This case is important because the assumption that everyone uses the same amount of public goods is sometimes restrictive.[22]

To analyze the problem, we assume that there is a constraint function $\nu_i : Y \to Y$ such that $\nu_i(g) \in Y$ represents the amount of public goods an individual i can use and that this constraint is a result of some government policy, i.e., ν_i is part of a policy variable. For example, some government policies may be age-specific, location-specific, etc.[23] The utility function is now modified as $u_i(x; \nu_i(g))$. We need not change other structures (e.g., resource constraints) at all. We say that public goods g are *impure* if $\nu_i(g) \neq \nu_j(g)$ for some $i, j \in N$.

[21] This conjecture would be more likely to be accepted especially if you and your friend could have the same influence on the decision on public goods.

[22] When we consider fairness among generations, even the pure public goods do not have a nature of equal consumption among individuals in different generations. Hence, the consideration on the impure public goods is useful to study intertemporally fair tax systems. As the result in this section suggests, the intertemporally fair tax system comes to have a flavor of the benefit tax [cf. Yamashige (1996)].

[23] It is very important that ν_i is determined by the government because otherwise, e.g., if it reflects individual decisions on the use of public goods, the constraint functions should not be used.

Under this modification, it seems natural to use the following definition of the weak no-envy criterion for government policies with impure public goods. Note that ν_i is replaced by ν_j in i's utility function to be compared because the constraint is individual-specific, which also implies that individuals are assumed to know the levels of public goods used by others.

Definition 8 *A government policy G is said to satisfy weak no-envy criterion (in $\mathbf{E}$) if and only if, for all $e \in \mathbf{E}$, $i, j \in N$, and $\hat{\omega}_i, \hat{\omega}_j \in X$,*

$$\hat{\omega}_i \geq \hat{\omega}_j \Rightarrow u_i(\tilde{x}_i^G(\hat{\omega}_i, t_i; e); \nu_i(g)) \geq u_i(\tilde{x}_j^G(\hat{\omega}_j, t_j; e); \nu_j(g)).$$

The following example shows that Proposition 3 does not hold if public goods are not equally consumed by everyone.

Example 2. Consider two individuals who are exactly the same except for their constraint functions ν_i and ν_j. Suppose $X = \mathbf{R}_+^2$ and $Y = \mathbf{R}_+$, i.e., there are two commodities one of which is purely private and the other is publicly (and also privately) provided. Suppose that the utility function is given by $u(x; \nu(g)) = x_1 + x_2 + \nu(g)$ for each i and j, and the constraint functions are given by $\nu_i(g) = \frac{2}{3}g$ and $\nu_j(g) = \frac{1}{3}g$. Let $\omega_i = \omega_j = (1, 1)$. Assume $g = 2$ and that the tax on i and j is 1. Assume that the price of each commodity is equal to 1. Then, the budget constraint is given by $x_1 + x_2 = 1$ for each individual. According to Proposition 3, this policy would satisfy the weak no-envy criterion if the constraint functions were the same. But this is not the case here because in a competitive equilibrium allocation under G, $u(x; \nu_i(g)) = \frac{7}{3} > \frac{5}{3} = u(x; \nu_j(g))$, i.e., j envies i despite the fact that they both had the same endowment and preference. Hence, Proposition 3 does not hold when public goods are not pure. To see a nature of the government policy which satisfies the no-envy criterion, suppose that the tax on i is 2 and the tax on j is 1 (with $g = 3$). Then, it satisfies the no-envy criterion because $u(x; \nu_i(g)) = 2 = u(x; \nu_j(g))$, i.e., by imposing a higher tax on the individual who uses public goods more, the no-envy equity criterion was satisfied.

The example also suggests that the benefit tax must be introduced for each individual not to envy others who can use public goods more.[24] In general, however, if the usable levels of public goods are different among individuals then the government has to know preferences of all individuals to satisfy the no-envy criterion. Since it is not easy to know individual preferences, if an impure public good can be supplied in competitive markets, then from the viewpoint of fairness, it may be better to let it be traded in the markets because the competitive equilibrium allocations are weakly envy-free (Proposition 1).

4. Weak No-Envy Criterion and Pareto Optimality

It was shown in the last subsection that the Lindahl tax, which is known to achieve Pareto optimality, does not necessarily satisfy the no-envy equity criterion. One may ask if

[24] It is not difficult to show that if $w_i = w_j$ and $v_i(g) \geq v_j(g)$ then $B(q_e^G, \omega_i, t_i) \subseteq B(q_e^G, \omega_j, t_j)$ must hold to satisfy the weak no-envy criterion, which we want to interpret as a result that the benefit tax must be introduced to finance impure public goods.

there exists a government policy which satisfies the weak no-envy criterion and the Pareto optimality. To this question, it is useful to point out that Foley (1967) has already shown such an existence result. We think that this result is useful because it provides us with a foundation for calculating the optimal supply of public goods and taxes which satisfy the weak no-envy criterion. Unfortunately, however, in order to find such a government policy, governments also need to have information about individual preferences.

In our notations, the theorem can be stated as follows.[25]

Theorem [Foley (1967, Theorem 3.29)]

Under conditions in Foley (1967, Theorem 3.29), there exist Pareto optimal policy $G = (g, t_1, \ldots, t_n)$ *and* $s \in \mathbf{R}$ *such that* $t_i = s\, q_e^G \cdot \omega_i$ *for all* $i \in N$ *and* $\sum_{i \in N} t_i = \sum_{k=1}^{\ell} q_{ek}^G g_k$.

Namely, it is shown that a Pareto optimal supply of public goods is possible under a proportional tax on each individual's income. Now it is easy to check that under the proportional income tax, $\hat{\omega}_i = \hat{\omega}_j$ implies $B(q_e^G, \hat{\omega}_i, t_i) = B(q_e^G, \hat{\omega}_j, t_j)$, and $\hat{\omega}_i > \hat{\omega}_j$ implies $B(q_e^G, \hat{\omega}_i, t_i) \supsetneq B(q_e^G, \hat{\omega}_j, t_j)$; and thus, by Proposition 3, the government policy above satisfies the weak no-envy criterion.[26]

Example 3. Consider an economy in Example 1. We find a Foley's equilibrium $(x_i, x_j; g, s)$. An equilibrium can be obtained by solving the following maximization problem:

$$\max_{(x_i, x_j; g)} u_i(x_i; g) + u_j(x_j; g) \quad s.t. \;\; x_i + x_j + g = 2$$

with the constraint $x_i = (1 - s) \cdot 1$ and $x_j = (1 - s) \cdot 1$ (proportional income tax). Another easy calculation shows that the Foley's equilibrium is $(x_i, x_j; g, s) = (\frac{3}{8}, \frac{3}{8}, \frac{5}{4}, \frac{5}{8})$ and that the Pareto optimality condition (Samuelson condition) is given by $3x_i + \frac{1}{3}x_j = g$ which is clearly satisfied in the Foley's equilibrium (as well as in the Lindahl tax equilibrium). In the Foley's equilibrium, each individual pays the same tax $(\frac{5}{8})$ because both have the same endowment. Notice that the government needs information about preferences of all individuals, and it cannot separate the decision on taxation and the decision on public goods to find the optimal policy.

As Foley's theorem and the example above show, under reasonable assumptions, as long as public goods are pure ("uniform" in Foley's term), there are a Pareto optimal level of public goods and a tax system which satisfy our weak no-envy criterion. As Foley himself suggests, in that class of government policies, there may be a progressive income tax system which is consistent with some stronger notion of equity and Pareto optimality. The inquiry for this question may be an interesting direction to pursue.

[25] In relation to an extension to production economies in Section IV.2, it may be useful to point out that the theorem applies to a general production economy and that a condition in the Foley's theorem requires that each individual have strictly positive endowments. See also Diamantaras (1992) for a recent study on the existence of a Pareto optimal envy-free allocation in an economy with public goods.

[26] Foley (1967, p.76) showed that the competitive equilibrium allocation under such G is envy-free when the initial endowment is the same, which seems to be his original intention for introducing his famous envy-free equity criterion.

IV. *Extensions*

In this section, we discuss two extensions. We first consider an extension of the weak no-envy criterion, and then we sketch an extension of our model to production economies.

1. Income-Based Weak No-Envy Criterion

At the end of Section III.2, we argued that the endowment-based weak no-envy criterion is too weak to require incomes to be the tax base. In this section, we strengthen the no-envy criterion by virtue of Proposition 2 in Section II. Let $G = 0$ mean that $g = 0$ and there is no tax policy.[27]

Definition 9 *A government policy G is said to satisfy income-based weak no-envy criterion (in $\mathbf{E}$) if and only if, for all $e \in \mathbf{E}$, $i, j \in N$, $\hat{\omega}_i, \hat{\omega}_j \in X$,*

$$q_e^0 \cdot \hat{\omega}_i \geq q_e^0 \cdot \hat{\omega}_j \Rightarrow u_i(\tilde{x}_i^G(\hat{\omega}_i, t_i; e); g) \geq u_i(\tilde{x}_j^G(\hat{\omega}_j, t_j; e); g).$$

Since the competitive equilibrium satisfies the corresponding criterion (Proposition 2), it would be also justifiable to impose this condition on government policies.

A subtle issue is the choice of a price system in the definition of "income". Although we adopted q_e^0, the price system q_e^G, for example, might have been also an adequate choice under which the tax base becomes $q_e^G \cdot \omega_i$ as in the standard income tax system. To explain why we chose q_e^0 (and thus $q_e^0 \cdot \omega_i$ as the tax base), consider a government policy which changes an income distribution via a change in the price system. For instance, suppose that, before any government policy, j had more income than i but, given a government policy, i now has more income than j. Then, j would envy i and feel that the policy is unfair. Since the spirit of the weak no-envy criterion is that the government should not overturn the envy-structure, the best choice seems to be q_e^0, the price before any policy is implemented.

Given such a definition, we can derive the following characterization of the income-based no-envy criterion, which can be easily proved by modifying the proof for Proposition 3.

TABLE 1. Income-Based Weak No-Envy Criterion

Initial Condition	Required Condition
$q_e^0 \cdot \hat{\omega}_i = q_e^0 \cdot \hat{\omega}_j$	$B(q_e^G, \hat{\omega}_i, t_i) = B(q_e^G, \hat{\omega}_j, t_j)$
$q_e^0 \cdot \hat{\omega}_i > q_e^0 \cdot \hat{\omega}_j$	$B(q_e^G, \hat{\omega}_i, t_i) \supseteq B(q_e^G, \hat{\omega}_j, t_j)$

Although the requirement that we use q_e^0 will not be a problem for governments when they *design* policies, it may be a problem when they *check* the fairness of the current policy.

[27]Notice that the following criterion is indeed stronger than the endowment-based no-envy criterion. First, under the income-based no-envy criterion, all individuals are compared (i.e., individuals are totally ordered), whereas, under the endowment-based no-envy criterion, individuals are compared if endowments are comparable (i.e., individuals are partially ordered). Secondly, given a price system q_e^0, $\omega_i \geq \omega_j$ implies $q_e^0 \cdot \omega_i \geq q_e^0 \cdot \omega_j$; and thus if G satisfies the endowment-based no-envy criterion then it must satisfy the income-based no-envy criterion. Hence, the set of admissible government policies is smaller under the income-based no-envy criterion.

Furthermore, this requirement complicates the tax system which satisfies the no-envy equity criterion. For example, if q_e^G were the price to evaluate incomes then the standard income tax system could have been the one that satisfies the weak no-envy criterion. This, however, would not be true any more when q_e^0 must be used.

This last point actually has some interesting policy implications. For example, we may be able to argue that the government should tax all gains that individuals obtain from public projects, or compensate for losses that individuals incur from public projects. Such a policy is fair in the sense that if this is not done individuals who get no gain form the policy or who suffer from the policy will feel that the policy is unfair.[28] Since potential economic gains from government policies are often sources of political corruption, we believe that this consideration is important.

2. An Extension to Production Economies

Here we sketch a possible extension of our model to production economies. We think that this extension is interesting because in production economies a non-existence result of an envy-free Pareto optimal allocation is well known [Pazner-Schmeidler (1974)]. Since a competitive equilibrium allocation in production economies is Pareto optimal, the following Proposition 4 suggests that the non-existence problem is not a problem as long as we are interested in the weakly envy-free allocation.

We consider simple production economies. First, we assume that each individual is endowed with H units of time, and that the income is defined by $w_i(H - \ell_i) + z_i$, where w_i is an endogenously determined wage, ℓ_i is leisure, and z_i is an initial endowment of individual i. Let $W \subseteq \mathbf{R}_+$ and $X \subseteq \mathbf{R}_+$ be a space of wages and a space of endowments. The utility function of i is now modified as $u_i(c_i, \ell_i)$, a non-decreasing function of consumption c_i and leisure ℓ_i. Let $\mathcal{U}$ be a space of such utility functions. Let $y \subseteq \mathbf{R}^{n+1}$ be a production possibility set. Let $\mathcal{Y}$ be a space of production possibility sets and let $\mathbf{E} = \mathcal{U}^n \times X^n \times \mathcal{Y}$. The budget constraint is given by $B(w_i, z_i) = \{(c_i, \ell_i) : c_i \leq w_i(H - \ell_i) + z_i\}$. Let $x_i = (c_i, \ell_i)$. A vector $(x_1, \ldots, x_n)$ is said to be an allocation if $(\sum_{i=1}^n (c_i - z_i), \ell_1 - H, \ldots, \ell_n - H) \in y$.

Definition 10 *An allocation mechanism $\tilde{x}$ is said to be a competitive equilibrium allocation mechanism in production economies if and only if, for any economy $e \in \mathbf{E}$, there is a wage vector $(w_1(e), \ldots, w_n(e)) \in W^n$ such that, for each $i \in N$ and $\hat{z}_i \in X$,*

$$\tilde{x}_i(w_i(e), \hat{z}_i; e) \in \arg \max_{x \in B(w_i(e), \hat{z}_i)} u_i(x).$$

In order to capture an idea that the equilibrium wage can be seen as a reflection of i's initial endowment of ability, we introduce the following notion of weak envy-free allocation in production economies. To simplify notations, let $\hat{\omega}_i = (\hat{w}_i, \hat{z}_i) \in W \times X$.

[28]This point of view has been also argued, for example, by Wicksell (1896, Section VII); "my proposal (taxation on unearned increments of wealth) would also have to apply to the increase in the value of land adjacent to newly opened transportation routes".

Definition 11 *A competitive equilibrium allocation mechanism $\tilde{x}$ in production economies is said to be weakly envy-free (in $\mathbf{E}$) if and only if, for all $e \in \mathbf{E}$, $i, j \in N$ and $\hat{\omega}_i, \hat{\omega}_j \in W \times X$,*

$$\hat{\omega}_i \geq \hat{\omega}_j \Rightarrow u_i(\tilde{x}_i(\hat{\omega}_i; e)) \geq u_i(\tilde{x}_j(\hat{\omega}_j; e)).$$

Proposition 4 *Any competitive equilibrium allocation mechanism in production economies is weakly envy-free.*

Proof. Let $\tilde{x}$ be a competitive equilibrium allocation mechanism and, let $e \in \mathbf{E}$ be arbitrary. Let $\hat{\omega}_i \geq \hat{\omega}_j$. Then, $B(\hat{\omega}_i) \supseteq B(\hat{\omega}_j)$ because if $(c, \ell) \in B(\hat{\omega}_j)$ then we have $c \leq \hat{w}_j(1-\ell) + \hat{z}_j \leq \hat{w}_i(1-\ell) + \hat{z}_i$, i.e., $(c, \ell) \in B(\hat{\omega}_i)$. Hence,

$$u_i(\tilde{x}_i(\hat{\omega}_i; e)) = \max_{x \in B(\hat{\omega}_i)} u_i(x) \geq \max_{x \in B(\hat{\omega}_j)} u_i(x) = u_i(\tilde{x}_i(\hat{\omega}_j; e)) \geq u_i(\tilde{x}_j(\hat{\omega}_j; e)),$$

where the last inequality is due to $\tilde{x}_j(\hat{\omega}_j; e) \in B(\hat{\omega}_j)$. Namely, the allocation mechanism is weakly envy-free. ∎

This result, together with the existence result for a competitive equilibrium in production economies [e.g., Debreu (1959)], suggests that, as far as the weakly envy-free allocation is concerned, there is a Pareto optimal weakly envy-free allocation in each production economy under reasonable assumptions.

The competitive equilibrium allocation mechanism for production economies under a government policy and the weak no-envy criterion are similarly defined as in Section III. We assume that the public good is given by $g \in \mathbf{R}_+$. Then, given a production possibility set $y \subseteq \mathbf{R}^{n+1}$, the resource constraint is given by $(\sum_{i=1}^n (c_i - z_i) + g, \ell_1 - H, \ldots, \ell_n - H) \in y$.

Now, we have the following characterization of the weakly envy-free government policy in production economies. Since the proof is similar to the one for Proposition 3, we leave it for readers.

Proposition 5 *A government policy G is weakly envy-free if and only if, for all $e \in \mathbf{E}$, $i, j \in N$, and $\hat{\omega}_i, \hat{\omega}_j \in W \times X$,*

$$\hat{\omega}_i = \hat{\omega}_j \quad \Rightarrow \quad B(\hat{\omega}_i, t_i) = B(\hat{\omega}_j, t_j) \text{ and}$$
$$\hat{\omega}_i > \hat{\omega}_j \quad \Rightarrow \quad B(\hat{\omega}_i, t_i) \supseteq B(\hat{\omega}_j, t_j).$$

As for the existence of a Pareto optimal weakly envy-free government policy, Foley's theorem in Section III.4 strongly suggests an existence result in which a proportional tax on the value of initial endowments ($w_i H + z_i$ in our model) achieves Pareto optimal weakly envy-free allocation with public goods. There are, however, two caveats for such a result. First, unlike Foley's theorem, in the model above, each individual is assumed to be endowed with only one type of labor (i.e., individualistic input). Hence, a formal proof must be attempted. Secondly, since it is usually hard to observe wages, the government may have to give up the proportional income tax and must introduce the labor income tax, which generally causes some efficiency losses. In such a case, the existence result is in question, and the second-best theory for weakly envy-free government policies may be required.

V. *Concluding Remarks*

Everyone would agree that the issue of fairness is an important one but also a difficult one in our society. Therefore, it is natural that we do not yet have a satisfactory equity criterion. In this paper, we proposed a weak equity criterion which requires that an institution (allocation mechanism) should not overturn the envy-structure of a society. We showed that this criterion is satisfied even in the competitive markets. The equity criterion has nice properties (no need for interpersonal comparison nor cardinality of welfare) and is appealing especially in economies where individuals do not know preferences of others and thus should be allowed to judge equity of allocations based on their own preferences.

We showed that when we apply this weak equity criterion to government policies many well-known equity criteria for taxation are implied by this weak criterion. The results are natural because, for instance, the spirit of the horizontal equity is indeed to prevent the occurrence of envies among "similar" individuals. Since the results are consistent with the tax systems in many countries, the criterion seems to be adopted by many governments, which may be of no wonder because politicians often act to reduce constituents' complaints and try to design socially stable government policies.

Although our results provide some explanations for the present tax systems in many countries, such as anonymous and incentive-preserving income tax systems to finance pure public goods and benefit taxes (user fees) for impure public goods, our criterion is very weak. It is so weak that the criterion may not be worth to be called a concept of fairness. For example, it does not impose any restriction on government policies which treat rich people favorably. We hope that our no-envy condition will be accepted as a necessary condition for fairness, and the stronger notion of equity, which may be truly called a concept of fairness, will be developed based on this principle.

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REFERENCES

Atkinson, A. B. and J. E. Stiglitz (1980) *Lectures on Public Economics*, Maidenhead, McGrow-Hill.

Debreu, G. (1959) *Theory of Value*, New Haven, Yale University Press.

de la Mora, F. G. (1987) Egalitarian Envy: The Political Foundations of Social Justice, New York, Paragon House.

Diamantaras, D. (1992) "On Equity with Public Goods," *Social Choice and Welfare* 9, pp.141-157.

Feldstein, M. S. (1976) "On the Theory of Tax Reform," *Journal of Public Economics* 6, pp.77-104.

Fei, J. C. H. (1981) "Equity Oriented Fiscal Programs," *Econometrica* 49, pp.869-881.

Foley, D. K. (1976) "Resource Allocation and the Public Sector," *Yale Economic Essays* 7.

Harsanyi, J. C. (1955) "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility," *Journal of Political Economy* 63, pp.309-321.

King, M. (1983) "An Index of Inequality: With Applications to Horizontal Equity and Social Mobility," *Econometrica* 51, pp.99-115.

Lindahl, E. (1919) "Just Taxation - A Positive Solution," in R. A. Musgrave and A. T. Peacock eds. (1958) *Classics in the Theory of Public Finance*, London, Macmillan, pp.168-176.

Pazner, E. (1977) "Pitfalls in the Theory of Fairness," *Journal of Economic Theory* 14, pp.458-466.

Pazner, E. and D. Schmeidler (1974) "A Difficulty in the Concept of Fairness," *Review of Economic Studies* 41, pp.441-443.

Rawls, J. (1971) *A Theory of Justice*, Cambridge, Harvard University Press.

Sato, T. (1985) "Equity and Fairness in an Economy with Public Goods," *Economic Review* 36, pp.364-373.

Sato, T. (1987) "Equity, Fairness and Lindahl Equilibria," *Journal of Public Economics* 33, pp.261-271.

Schmeidler, D. and K. Vind (1972) "Fair Net Trades," *Econometrica* 40, pp.637-642.

Sen, A. K. (1970) *Collective Choice and Social Welfare*, San Francisco, Holden-Day.

Thomson, W. (1983) "Equity in Exchange Economies," *Journal of Economic Theory* 29, pp.217-244.

Varian, H. R. (1974) "Equity, Envy, and Efficiency," *Journal of Economic Theory* 9, pp.63-91.

Wicksell, W. (1896) "A New Principle of Just Taxation," in R. A. Musgrave and A. T. Peacock eds. (1958) *Classics in the Theory of Public Finance*, London, Macmillan, pp.72-118.

Yamashige, S. (1995) "Subjectively Envy-Free Allocation: Characterization and Existence," University of Toronto Working Paper (UT-ECIPA-YAMASHIG-95-02).

Yamashige, S. (1996) "Intertemporal Equity and Sustainability of Tax System," mimeo.