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THE OPTIMAL INSURANCE AGAINST CONSUMPTION PRICE RISKS

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Abstract

This paper considers the optimal insurance contract between a risk-neutral agent and a risk-averse consumer against risks of variations in the prices of consumption goods. The theory is applicable to private insurance, social security, and wage contracts. Some basic properties of the optimal insurance are found including whether or not it is characterized by fixity of insurance payment or fixity of utility. In most cases, the optimum is characterized by random payment and random utility. The insuree's welfare under the optimal contract is larger than that achievable under price certainty.

I. *Introduction*

The problem of insurance against risks of variations in the prices of consumption goods has not been analysed in the theory of insurance to any significant extent in spite of its importance. Prices of consumption goods often change substantially even in a relatively short period of time. The prices of agricultural products vary over time, fuel prices can change drastically in a very short period of time, house prices can double in a few years, tuition and fees for higher education are altered frequently and quite independently of the general price level in many countries, and so on.

This lack of analysis is partly due to the fact that such insurance is not very prevalent in private insurance markets. In fact, many *older* standard textbooks of insurance state that consumption price risk is not insurable because it is a speculative or market risk rather than a pure risk. In this context pure risk refers to uncertainty as to whether the destruction of an object will occur, while speculative risk relates to uncertainty about an event that produces either a loss or a gain [see e.g. Greene (1977)].

However, related matters have been analysed in the field of labour economics in the context of 'implicit' labour contracts.[1] According to this implicit contract theory, a risk-neutral firm plays the role of an insurance company and offers its risk-averse workers wages (and employment) which eliminate or reduce risks these workers would face. The

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[1] See Rosen (1985) and Hart and Holmstrom (1987) for surveys on the implicit contract theory.

risks considered initially by Baily (1974) and Azariadis (1975) are those which relate to the value of workers' productivity and these authors established the well-known proposition that workers will be offered fixed wages independently of the states of nature that will prevail. Polemarchakis (1979) and Newbery and Stiglitz (1987) then expanded on the theory and included other risks such as uncertainty regarding the prices of consumption goods. However, the aspect of insurance against consumption price risks has not yet been sufficiently analysed and the overall properties of the optimal wage contracts under consumption price uncertainty have not been discussed in detail. This is partly because previous authors have directed their attention more to the aspect of (un)employment than to wages.

The necessity to analyse insurance against consumption price risks is not limited to the field of labour contracts. Such insurance should also be an important element in social security [see Diamond (1977)]. A government that is interested in the welfare of certain groups or classes of citizens should devise a social security system that takes into account consumption price uncertainty. Some countries already have social security systems that do this to some extent. Private insurance companies may begin to sell such insuarnce in the future, since, according to *modern* authors on insurance, virtually all kinds of risks are insurable so far as they do not have serious moral hazard or adverse selection problems and agreements can be made between sellers and buyers on insurance contracts [Borch (1990)]. The same type of analysis is also necessary to consider this type of private insurance.

This paper elucidates the problem of optimal insurance against consumption price risks, which essentially applies to all of the above three cases. In particular, it analyses the basic properties that optimal insurance contracts should possess. Whether fixity of wages or utility is related to optimality in some situations is also examined in some detail.

Fixity of wages has attracted special attention in the theory of implicit labour contracts. The optimality of fixed wages was initially proved under the assumption that the overall price level is fixed [Baily (1974) and Azariadis (1975)[. Even when there is consumption price uncertainty, fixity of wages still holds if the worker's indirect utility function is separable between income and prices [Polemarchakis (1979)]. Fixity of wages under consumption price uncertainty is an interesting phenomenon. If the only risks for insurance are those of consumption goods prices, then the firm does not have to be risk-neutral to pay fixed wages. In fact, there is no risk shifting at all in this case, even though the worker is risk-averse and actually faces risks. Hence, it is necessary to understand why such a phenomenon arises.

This paper shows that fixity of wages is a rather special case for general utility functions. If a worker was given a fixed wage and the prices of some consumption goods happened to be high (low) with all the others remaining constant, then his/her level of satisfaction would be low (high). Thus a fixed wage does not seem to be optimal. A wage contract that is more akin in spirit to that in the initial implicit contract theory may be a fixed-utility wage contract, under which the wage is high (low) when prices are high (low). However, it can be shown that this contract is never optimal, though it is close to optimality if the worker is very risk-averse. Since neither of the two simple types of wage contracts is optimal (except in a special case), the optimal contract should be characterized in general by a random wage and random utility. If the worker is not so risk-averse, the optimal insurance contract will be characterized even by high (low) wages when prices are low (high) rather than high (low).

The following discussion relates primarily to the context of wage contracts, but essentially the same theory applies to social security provided by a government and insurance supplied by a private insurance company. In order to analyse only what is common among the three types of insurance, it is assumed that a firm does not have the option of layoffs, as there is no sufficiently comparable concept in either social security or private insurance. This implies that as far as wage contracts are concerned, our theory applies to internal labour markets where layoffs are rare. It also implies that the theory does not have a serious enforceability problem in the analysis, because the prices of consumption goods can be observed objectively by a third party much more easily than the value of workers' productivity, the observability of which is a complicating factor in the analysis of layoffs.

Section II discusses some basic properties of the optimal insurance in a general framework. Section III assumes that the consumer has a constant degree of relative risk aversion and characterizes explicitly the optimal insurance contract in detail. Section IV provides a brief summary of the results.

II. *The General Model*

This paper considers the wage contract between a risk-averse worker and a risk-neutral firm. The contract specifies the level of the wage to be paid in each state which is characterized by the level of consumption price. In the case of social security, the firm should be interpreted as a government, the worker as a citizen in a certain group, and the wage as a social security payment. In the case of private insurance, the firm should be interpreted as an insurance company, the worker as an insurance buyer, and the wage as an insurance payment.

The worker is a consumer who consumes goods Y and X from his/her wage as the only source of income. Y and X can be considered as composite goods. His/her preferences are assumed to be expressed by

$$U(y,x)=u[v(y,x)]=u[v], \tag{1}$$

where u is concave and strictly increasing, $v>0$ is strictly quasi-concave, strictly increasing, and linearly homogeneous, and y and x are respectively the amounts of goods Y and X consumed.

Without loss of the essence of argument, let the prices of Y and X be 1 and sp, respectively, where p is a positive constant and s is a positive random coefficient with a mean equal to unity. The price of X is assumed to be determined in the world market. The worker and his/her firm do not know the value of s when the wage contract is made, but both do know the probability distribution function of s. After they observe the realized value of s, the wage is paid according to the contract and the worker's consumption is determined in accordance with the amount of wage paid and the realized price of X.

Similarly, without loss of generality, assume that the (expected) marginal value product of the worker is 1. This is the main source of wage payment. The optimal contract determines the wage payment in each state so as to maximize the worker's expected utility subject to the condition that the expected wage payment equals 1. In other words, the optimum is the best contract to the worker in the class C of all different types of contracts for wage payments whose expected values equal 1. Alternatively, the problem can be formulated so as to maximize the firm's expected profit subject to the condition that the worker's expected utility is no less than some specified level. However, these two formulations lead to essentially the same properties of the optimal contracts. In the case of social security, the above marginal value product should be interpreted as the government's revenue to be allocated for the social security benefits of each person in the group under consideration. In the case of private insurance, it should be interpreted as the insurance premium.

This paper intends to obtain some basic properties of the optimal wage contracts. For this purpose, two simple types of contract are carefully examined. One is the contract that involves a fixed payment and the other is the contract that involves fixed utility for the worker. Of particular interest is whether or not they are optimal and, if so, under what conditions. The latter type of contract will be analysed in some detail in this section. The next section considers the former type in some depth. These two types of contracts also provide some insight into optimal contracts in general.

The fixed-wage contract is defined as the contract that guarantees the wage payment equal to 1 irrespective of the realized value of s. In order to understand the basic properties of this contract, we define

$$v_{sp}=\max\{v(y,x)\colon y+spx=1\}. \tag{2}$$

Note that v_{sp} is a random variable, since s is stochastic. The expected utility under the fixed-wage contract is given by

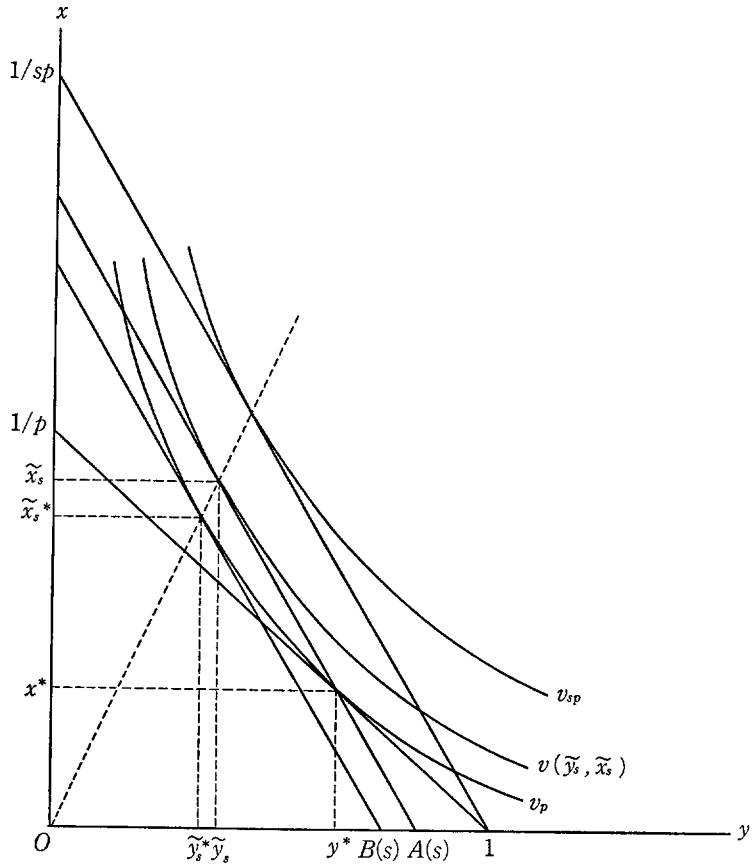
$$Eu[v_{sp}], \tag{3}$$

where E denotes an expectations operator with respect to s.

Though the wage is independent of the price level in the fixed-wage contract, this contract does not have the flavour of a contract, because the level of utility is random and the firm does not absorb the risk at all. A wage contract that seems more akin in spirit in this context to the contracts considered in the initial implicit contract theory is the one that is a member of C and guarantees a fixed level of utility on the part of the worker for any value of s. This is called *the fixed-utility wage contract.* Though this contract might seem to always dominate the fixed-wage contract (or even any other kind of contract) as long as the worker is risk-averse, it is not the case as will be shown below.

Let us derive some properties of the fixed-utility wage contract. This is done in several steps. First assume that s is equal to its mean, i.e., $s=1$ and that the wage paid is equal to one. Then the worker chooses $y=y^*$ and $x=x^*$, which maximize $v(y,x)$ or $u[v(y,x)]$ subject to $y+px=1$. See Figure 1. Define $v_p=v(y^*,x^*)$. Next let s be arbitrary and assume that the firm guarantees the wage that enables the worker to consume y^* and x^*. Then it is equal to $1+(s-1)px^*\equiv A(s)$. Note that since the expected value of $A(s)$ is equal to one, it is as equally costly to the firm as the fixed wage equal to one. If the worker is given this wage, however, he will not actually consume (y^*,x^*) when $s\neq 1$. His consumption will be $y=\tilde{y}_s$ and $x=\tilde{x}_s$ which maximize $v(y,x)$ or $u[v(y,x)]$ subject to $y+spx=A(s)$. Note that $v(\tilde{y}_s,\tilde{x}_s)>v_p$ if $s\neq 1$. If $s=1$, an equality holds. Now let $y=\tilde{y}_s{}^*$ and $x=\tilde{x}_s{}^*$ minimize $y+spx$ subject to $v(y,x)=v_p$. Define $D(s)=\tilde{x}_s{}^*/\tilde{x}_s$. Then linear homogeneity

FIGURE 1



of v implies that random wage $D(s)A(s) \equiv B(s)$ leads to fixed utility equal to $u[v_p]$. Note the position of $B(s)$ in Figure 1. Also note that $EB(s) < 1$ since the expected value of $A(s)$ equals one and $D(s) < 1$ except in the case of $s = 1$ for which $D(s) = 1$. Define $M = 1/EB(s) > 1$. Then the expected value of the random wage defined by

$$\tilde{w}_s = MB(s) \tag{4}$$

is equal to 1. Moreover, linear homogeneity of v and the fact that $B(s)$ leads to $u[v_p]$ imply that $\tilde{w}_s$ leads to the fixed utility level equal to

$$u[Mv_p]. \tag{5}$$

Therefore, (4) is the fixed-utility wage contract. Note that $\tilde{w}_s$ is stochastic and increasing in s, since $B(s)$ is increasing in s, as can be seen in Figure 1. Under the fixed-utility wage contract the wage is higher when the price of X is higher.

It is intuitively clear that if the worker is sufficiently risk-averse, the fixed-utility wage contract dominates the fixed-wage contract. However, the dominance relation is reversed when the worker is not so risk-averse. This can be understood by the following reasoning. Figure 1 indicates that $v_{sp}=v_p/B(s)$. Thus

$$Ev_{sp}=v_pE[B(s)]^{-1}>v_p[EB(s)]^{-1}=Mv_p, \tag{6}$$

where the inequality is due to convexity. The reason for this relation is that v becomes disproportionately large when s is small. The inequality implies that on the average the argument of u in (3) is larger than that of u in (5). Thus if u is sufficiently close to a linear function, (3) is larger than (5). This is the reason for the reversed dominance relation[2] and it already suggests that the fixed-utility wage contract is not optimal for some u.

An important result of this section is that the fixed-utility wage contract is in fact never optimal for any u with $u'>0$. In other words, the optimal wage contract is never characterized by fixed-utility. This is proved by considering a class of contracts of mixed wages $m+(1-m)\tilde{w}_s$ of the fixed wage and fixed-utility wage. When $m=1$, the mixed wage corresponds to the fixed-wage contract. When $m=0$, it corresponds to the fixed-utility wage contract. This class of contracts is denoted by C_m. Note that $C_m\subset C$, since the expected values of these mixed wages equal 1. The sub-class C_m provides some useful information about optimality of wage contracts in general. The worker's expected utility under a contract in C_m is given by

$$Eu[mv_{sp}+(1-m)Mv_p]. \tag{7}$$

Evaluating the derivative of (7) with respect to m at $m=0$ gives

$$u'[Mv_p]E(v_{sp}-Mv_p)>0 \tag{8}$$

by (6) or convexity. This implies that $m=0$ is not optimal in C_m for any u with $u'>0$. This fact in turn implies that $m=0$ is not optimal in C or that the fixed-utility wage contract is never optimal.

The same result would hold, even if v was related only to a single (composite) good. To show this, let $v=rx$, where r is a positive constant. If the price of X is uncertain and expressed as sp, then under the fixed-utility wage contract the wage equals s, the amount of X consumed by the worker equals $s/sp=1/p$, and his/her utility equals $u(r/p)$ in each s, while under the fixed-wage contract these are respectively 1, $1/sp$, and $u(r/sp)$. Thus the mixed wage equals $m+(1-m)s$, and its expected utility becomes $Eu[mr/sp+(1-m)r/p]$. The derivative of this function with respect to m evaluated at $m=0$ is positive, since $(r/p)u'(r/p)[Es^{-1}-1]>(r/p)u'(r/p)[(Es)^{-1}-1]=0$. Therefore the fixed-utility wage contract is never optimal in the single good case, as well.

The above discussion has already hinted at the reason for the nonoptimality of the fixed-utility wage contract. That is, the fixed-utility wage contract makes no use of the

[2] The relative magnitude between $Eu[v_{sp}]$ and $u[Mv_p]$ can be easily approximated by expanding u in (3) about Mv_p and retaining the first three terms, i.e.,

$$u[v_{sp}]\fallingdotseq u[Mv_p]+\{(v_{sp}/Mv_p-1)-R_r[Mv_p](v_{sp}/Mv_p-1)^2/2\}Mv_pu'[Mv_p],$$

where $R_r[\ \ ]$ is the degree of relative risk aversion of u. Taking the expectations of both sides, $Eu[v_{sp}]$ is (approximately) larger than $u[Mv_p]$ iff

$$R_r[Mv_p]<2E(v_{sp}/Mv_p-1)/E(v_{sp}/Mv_p-1)^2.$$

Note that the right-hand side of this inequality is positive because of convexity.

fact shown in (6). In other words, by making the wage slightly more rigid and v slightly more stochastic than under the fixed-utility wage contract, the expected value of v can be made larger than Mv_p, since v becomes disproportionately large when the price of X is low. The optimal value of m in C_m is determined so as to balance the two different kinds of benefits which derive from two conflicting sources, i.e., small variation in utility on the one hand and high expected return (with large variation in utility) on the other.

It should be noted that the optimal value of m can be greater than one in C_m for some u. Evaluating the derivative of (7) with respect to m at $m=1$ gives

$$Eu'[v_{sp}](v_{sp}-Mv_p)=cov\{u'[v_{sp}],\ v_{sp}-Mv_p\}+Eu'[v_{sp}]E(v_{sp}-Mv_p). \qquad (9)$$

If u is sufficiently close to a linear function, the first term on the right-hand side is sufficiently close to zero. On the other hand, the second term on the right-hand side is positive because u is strictly increasing and because of (6). Hence (9) is positive for u that is sufficiently close to a linear function. This implies that if the worker's preferences are sufficiently close to risk-neutrality, the optimal level of m is larger than one in C_m.[3]

Note that if the optimal level of m is larger than one, the wages are higher (lower) when the price of X is lower (higher). This is due to the fact that $\tilde{w}_s$ is increasing in s and the coefficient of the second term in $m+(1-m)\tilde{w}_s$ is negative. This result arises because an almost risk-neutral consumer wants to enjoy quite high levels of utility that are achievable when the price of X is low, a point which will be elaborated in the next section.

The above observation of the optimal levels of m in C_m suggests the following: If the worker is very risk-averse, the optimal level of m is close to 0 and the optimal contract is close to the fixed-utility wage contract. As his/her risk aversion becomes smaller, the optimal level of m approaches 1 and the optimal contract approaches the fixed-wage contract. If his/her risk aversion is very small, the optimal level of m is larger than 1 and the optimal contract is such that his/her utility fluctuates greatly. These phenomena actually arise in C, as will be seen in more detail in the next section.

The optimal wage contract has the property that the worker's welfare under it is larger than that achievable when there is no price uncertainty. The fact that $\tilde{w}_s$ is a member of C (see the discussion around (4)) implies that the risk-neutral firm is able to offer a wage contract under which the worker's expected utility is at least equal to $u[Mv_p]$. Since $M>1$, this lower limit is larger than $u[v_p]$, the level of utility attainable in the case of certainty where the price of X equals p or the expected value of the price of X under uncertainty. In short, the worker's welfare is larger under uncertainty than under certainty, if there is uncertainty in consumption price.[4] This kind of phenomenon never arises when the value of the worker's productivity is the only source of uncertainty. In such a case, risk absorption by a risk-neutral agent simply equates the levels of the worker's welfare under uncertainty and under certainty.

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[3] This can be also shown by the following approximation. By expanding u in (7) about Mv_p, we have
$$u[mv_{sp}+(1-m)Mv_p]\fallingdotseq u[Mv_p]+Mv_pu'[Mv_p]\{m(v_{sp}/Mv_p-1)-m^2R_r[Mv_p](v_{sp}/Mv_p-1)^2/2\}.$$
Because $Mv_pu'[Mv_p]$ is positive, maximizing the expected value of the above with respect to m is equivalent to maximizing the expected value of the expression inside the braces on the right-hand side. Hence the optimal m is larger than one iff
$$R_r[Mv_p]<E(v_{sp}/Mv_p-1)/E(v_{sp}/Mv_p-1)^2.$$

[4] This is reminiscent of Waugh (1944) who shows that consumers gain from price instability. Oi (1961) obtains similar result for producers.

The fact that $M>1$ in this model is due to the substitutability between Y and X. As shown above regarding $A(s)$, the possibility of substitution increases the worker's welfare keeping the firm's welfare constant. If there was no substitution as in a Leontief-type utility function or if v was a function of a single (composite) good, the fixed-utility wage contract would give the worker the same utility as that under certainty. To see this, let $v=rx$ as before. Then the worker's utility under certainty equals $u(r/p)$, since he/she consumes $1/p$ units of X when his/her wage equals one and the price of X equals p. This level of utility is the same as that under the fixed-utility wage contract under uncertainty as demonstrated above. Therefore, a different result would arise if substitution was not possible.

We have already noted, however, that even in the case of a single good there are contracts in C_m that dominate the fixed-utility wage contract. In other words, by choosing sufficiently small positive values for m in C_m, contracts can be devised that dominate the fixed-utility wage contract. Therefore, even if substitution was impossible, the worker's welfare would be larger under uncertainty than under certainty.

It is obvious that the result regarding higher welfare under uncertainty derives from two reinforcing factors: One is substitutability between consumption goods. As shown above, this makes the level of utility under the fixed-utility wage contract greater than that under certainty. The other is the convexity shown in (6). This factor generates contracts that are regarded as better than the fixed-utility wage contract. When substitution is possible, these two factors work together to produce this result. When it is not, only the second factor works.

III. *The Case of Constant Relative Risk Aversion*

This section develops more explicit properties of the optimal wage contracts by assuming that the worker's utility exhibits constant relative risk aversion. Let $v=v(y,x)$ and c denote the Arrow-Pratt measure of relative risk aversion of u, i.e., $c=-u''[v]v/u'[v]$. When this is constant for all v, $u[v]$ must be expressed by the following functions:

$$u[v]=v^{1-c} \qquad (0\leqq c<1), \tag{10}$$

$$u[v]=\log v \qquad (c=1), \tag{11}$$

$$u[v]=-\,v^{1-c} \qquad (1<c). \tag{12}$$

Since the worker has been assumed to be risk-averse, c should not equal 0, but this case is also considered in this section as a limiting situation.

Assume that s takes on a finite number $n\geqq 2$ of values s_i with probabilities $r_i>0$ and denote by w_i the levels of wage when $s=s_i$, where $i=1, 2, \ldots, n$. Assume further that $0<s_1<s_2<\ldots<s_n<+\infty$. Define

$$v_i=\max\{v(y,x): y+s_ipx=1\}, \quad (i=1, 2, \ldots, n). \tag{13}$$

Then the following relations hold:

$$v_1 > v_2 > \dots > v_n > 0. \tag{14}$$

Since v is assumed to be linearly homogeneous, the optimal wage contract can be obtained by choosing w_i ($i=1, 2, \dots, n$) so as to maximize

$$\sum r_i u[v_i w_i] \tag{15}$$

subject to

$$\sum r_i w_i = 1, \tag{16}$$

where the summations are undertaken from 1 to n. For $c \neq 0$ the first-order conditions are given by (16) and

$$u'[v_i w_i]v_i + \lambda = 0 \quad (i=1, 2, \dots, n), \tag{17}$$

where λ is a Lagrange multiplier.

Consider first the case of $0 \leqq c < 1$ or (10). In the limiting case of $c=0$, the maximization problem reduces to maximizing $\sum r_i v_i w_i$ subject to (16). Let

$$w_1 = 1/r_1 \quad \text{and} \quad w_i = 0 \text{ for } i=2, 3, \dots, n. \tag{18}$$

Then the maximand is equal to v_1. On the other hand, $\sum r_i v_i w_i \leqq v_1 \sum r_i w_i = v_1$. Thus the wage contract in (18) achieves the maximum. It is easy to show that it is the only wage contract that achieves the maximum.

For $0<c<1$, (17) becomes

$$(1-c)[v_i w_i]^{-c} v_i + \lambda = 0 \quad (i=1, 2, \dots, n). \tag{19}$$

This condition implies that

$$w_i = w_1 (v_i/v_1)^d \quad (i=2, 3, \dots, n), \tag{20}$$

where $d=(1-c)/c>0$. Using (20) and (16) gives

$$w_i = v_i^d / \sum r_j v_j^d \quad (i=1, 2, \dots, n), \tag{21}$$

where the summation in the denominator is undertaken from 1 to n.

Consider next the case in which $c=1$ or (11). In this case (17) becomes

$$1/w_i + \lambda = 0 \quad (i=1, 2, \dots, n), \tag{22}$$

which together with (16) implies that

$$w_i = 1, \quad (i=1, 2, \dots, n). \tag{23}$$

Finally, in the case of $c>1$ or (12), (17) becomes

$$(c-1)[v_i w_i]^{-c} v_i + \lambda = 0 \quad (i=1, 2, \dots, n). \tag{24}$$

This condition implies that

$$w_i = w_1(v_i/v_1)^d \quad (i=2, 3, \ldots, n), \tag{25}$$

where d is now negative. Using (25) and (16) gives

$$w_i = v_i^d/\sum r_j v_j^d \quad (i=1, 2, \ldots, n), \tag{26}$$

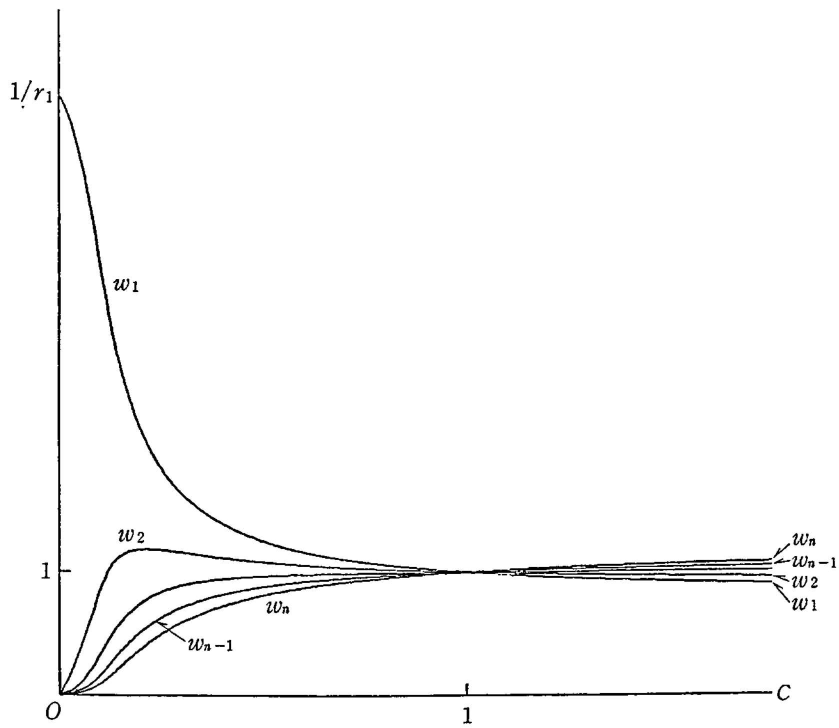
where the summation in the denominator is undertaken from 1 to n.

The solutions in (21), (23), and (26) give the optimal wage contract for each level of $c>0$. It will be helpful to develop some of its basic properties here. By the solution in (21) $w_i/w_k=(v_i/v_k)^d$. Thus $w_i>w_k$ for $i<k$ when $0<c<1$. See Figure 2. Furthermore, (21) implies that

$$w_i = [r_1(v_1/v_i)^d + \ldots + r_{i-1}(v_{i-1}/v_i)^d + r_i + r_{i+1}(v_{i+1}/v_i)^d + \ldots + r_n(v_n/v_i)^d]^{-1}$$
$$(i=1, 2, \ldots, n). \tag{27}$$

The definition of d suggests that as c approaches 0, d approaches $+\infty$. Therefore, (27) implies that as c approaches 0, the optimal wage contract approaches that under risk neutrality or (18). On the other hand, as c goes to 1, d goes to 0. Therefore, (21) implies that

FIGURE 2



Note: This Figure is based on the following data; $v=(yx)^{0.5}$, n=5, $s_1=0.6$, $s_2=0.8$, $s_3=1.0$, $s_4=1.2$, $s_5=1.4$, and $r_i=0.2$ for all i.

all of w_i go to 1. This limiting wage contract is the same as that under $c=1$ or (23). It can be seen in (27) that w_1 is strictly decreasing and w_n is strictly increasing in c.

It can be seen from (26) that $w_i/w_k=(v_i/v_k)^d$. Since d is negative in (26), $w_i<w_k$ for $i<k$ when $1<c$. Moreover, (26) implies that as c approaches 1, all of w_i approach 1, since d approaches 0. This limiting wage contract is again the same as that under $c=1$. Note that as c goes to infinity, d goes to -1. Hence, (26) implies that

$$w_i \rightarrow \tilde{w}_i \equiv 1/v_i[\sum r_j/v_j] \quad \text{as } c \rightarrow +\infty, \tag{28}$$

where the summation of the denominator of $\tilde{w}_i$ is undertaken from 1 to n. Note that $\tilde{w}_i < \tilde{w}_k$ for $i<k$. Since (27) also holds for $c>1$ with d being now negative, w_1 is strictly decreasing and w_n is strictly increasing in c again.

It can be seen in Figure 2 that w_1 is strictly decreasing and w_n is strictly increasing in c for $c>0$. Thus there is only one intersection of the two curves and it arises when $c=1$. In fact, (23) implies that all of w_i intersect at this value of c. This intersection corresponds to the fixed-wage contract. Therefore, the fixed-wage contract arises only when the worker's relative risk-aversion is exactly unity. For no other value of c, is the fixed-wage contract optimal.

It can be seen from (15) and (28) that $\tilde{w}_i$ brings about the same utility for all $i=1, 2, \ldots, n$. Moreover, it is easy to see that $\sum r_i\tilde{w}_i=1$. Hence, the set of $\tilde{w}_i$ $(i=1, 2, \ldots, n)$ corresponds to the fixed-utility wage contract. As the above observation shows, this contract is never optimal for any finite value of c, though it is close to the optimal wage contract when c is very large. This confirms one of the results in the previous section.

Since, strictly speaking, neither the fixed-wage contract nor the fixed-utility wage contract is optimal except in a very special case, the optimal wage contract is, in general, characterized by a random wage and random utility. When c is large, the worker tends to secure an almost equal standard of living in each state. This is achieved by receiving higher (lower) wages when consumption price is higher (lower). Since he/she is highly risk-averse, he/she tries to avoid the risk of falling into very low standards of living in some states. Such a contract that is almost riskless to him/her, however, does not so much make use of the benefit derived from the fact that the expected value of v is larger under more risky contracts. This benefit is used more for smaller values of c and the wage difference among different states narrows as c approaches 1. When c is less than 1, the wage is higher (lower) when consumption price is lower (higher) to make fuller use of such a benefit. This contract for $c<1$ may be called negative insurance, since the worker is actually 'buying' risks. The wage difference widens as c approaches 0. When the worker is completely risk-neutral, he/she bets everything on the best state.

This observation provides some insight into the nature of the fixed-wage contract. It arises when the worker is not highly risk-averse and also not close to risk-neutrality. The worker under this contract is obviously risk-averse. This type of contract arises when he/she is neither a 'risk seller' nor a 'risk buyer.' It arises because he/she balances the benefit derived from the almost equal standard of living in each state with that derived from a risky contract with a higher expected value of v. If he/she valued the former benefit more highly than the latter, the optimal contract would be closer to the fixed-utility contract. On the other hand, if he/she valued the latter benefit more highly than the former, the optimal contract would be closer to that under risk-neutrality.

Obviously, the firm does not have to be risk-neutral nor even less risk-averse than the worker to pay the fixed-wage, as long as there is no uncertainty in his/her marginal value product. Since there is no risk involved in wage payment, i.e., there is no risk shifting from the worker to the firm, the firm can, in fact, be much more risk-averse than the worker to pay the fixed-wage.

The fixed-wage contract may be an interesting phenomenon, because though the worker is risk-averse and faces a risk, he/she does not demand insurance. Strictly speaking, however, it arises only when his/her degree of risk aversion takes on a particular value. Moreover, it may not arise at all if a different functional form for u is considered.[5] A sufficient condition for existence of the fixed-wage contract is that the indirect utility function is separable between income and prices. The utility function that has been considered above satisfies this condition when $c=1$.

We have seen that the fixed-wage contract is a rare case as far as there is uncertainty in the prices of consumption goods. Thus efficient wage contracts should in general involve fluctuating wages. This is true even when the value of the worker's marginal product is also random. However, it will be helpful in understanding the reality to know under what conditions the optimal wage contract involves only small wage fluctuations. One obvious such condition is that the worker's degree of risk aversion is close to the value at which the fixed-wage contract is optimal. Another alternative condition is that the worker's degree of risk aversion is relatively high and the dispersion of probable price levels is small. In this case, the distance between the upper bound $\tilde{w}_n$ and the lower bound $\tilde{w}_1$ for wages is small. Still another is that the worker's degree of risk aversion is relatively high and the share of the good whose price is uncertain is small in his/her consumption expenditure. To see this, suppose v is a Cobb-Douglas utility function, i.e., that $v=y^b x^a$ where b and a are both positive and $b+a=1$. Then $\tilde{w}_n/\tilde{w}_1=v_1/v_n=(s_n/s_1)^a$. Hence, when the value of a is small, the distance between the two bounds is again small. A fourth condition is that the prices of many consumption goods are random and their effects almost cancel out.

IV. *Conclusions*

This paper has considered some basic properties of the optimal insurance against risks of variations in the prices of consumption goods. Most of the discussion has been undertaken in the context of the wage contract between a risk-neutral firm and a risk-averse worker/consumer, but essentially the same theory applies to the problems of social security and private insurance. We have examined in some detail whether or not the following two simple contracts are optimal, and if so, under what conditions: One is the fixed-wage contract, which pays the same amount of wage irrespective of the level of price. The other is the fixed-utility wage contract, which guarantees the fixed level of utility irrespective of the price level.

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[5] The author has tried experiments by assuming the functional form $u[v]=-\exp\{-c'v\}$, where $c'>0$ is a constant and by assigning several different values for c'. He has found that the optima are never characterized by fixed-wage contracts in any of the experiments. However, it is found that as c' increases the wage differences narrow and then widen as in the case of the text and the extent of the smallest differences is quite small.

It has been shown that the fixed-utility wage contract is never optimal, though it is close to optimality when the worker's degree of risk aversion is large. The fixed-wage contract becomes optimal only in a special case. When the worker's utility function exhibits constant relative risk aversion, the fixed-wage contract is optimal only when the degree of relative risk aversion is exactly unity. Therefore, the optimal wage contract is characterized in general by random wage and random utility. An interesting feature of the optimal contract is that the worker's welfare under it is greater than that achievable when there is no price uncertainty.

Figure 2 shows representative optimal wage contracts for different degrees of relative risk aversion. As this degree increases, the wage differences between different states narrow and then widen. In this process, the worker changes from being a 'risk buyer' to a 'risk seller' and the relative magnitude between the wages is reversed. The fixed-wage contract arises when these wage differences vanish. The worker under this contract is surely risk-averse and faces a risk, but does not demand insurance and thus there is no risk shifting from him/her to his/her firm.

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