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ASSET PRICING IN INCOMPLETE MARKETS*

DARRELL DUFFIE

Abstract

This is a brief review of some of the implications of incomplete markets for the pricing of financial securities.

I. *Introduction*

This is a brief review of some of the theory of asset pricing in a setting of incomplete markets. Recent work in this area has been largely motivated by a failure of the "standard theory," described below, to explain the empirical behavior of security prices. In particular, the standard theory is apparently inconsistent with the high volatility of security price processes, and with the spread between average rates of return on risky securities and riskless rates. This "unusual" spread was labeled the "equity premium puzzle" by Mehra and Prescott (1985), who considered incomplete markets as a possible explanation. The standard theory assumes the existence of complete markets for contingent claims. Intuition suggests that incomplete markets would prevent individuals from insuring themselves against unpredictable changes in income, and thereby cause their marginal rates of substitution across states and time to be more volatile than they would be in complete markets. Since the equilibrium price of a security is the expected weighted sum of its future dividends, with weights given by marginal rates of substitution, it follows that market incompleteness may increase security price volatility.

While this intuition may be correct, it does not speak to the spread between risky and riskless rates of return. In any case, the poor fit of the "standard model" may also be due, at least in part, to market "imperfections" beyond incomplete markets. For example, the volatility and spreads of security returns may be affected by:

a) short sales and borrowing constraints [Bewley (1982), Scheinkman and Weiss (1986), Lucas (1991), Heaton and Lucas (1992), Marcet and Singleton (1991), Cochrane and Hansen (1991), Hindy (1991), He and Pagès (1991)];
b) transactions costs [Grossman and Laroque (1989), Heaton and Lucas (1992), Aiyagari and Gertler (1991), Luttmer (1991)];
c) alternative utility functions [Constantinides (1990), Epstein and Zin (1989), Abel

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(1990), Sundaresan (1988), Ferson and Constantinides (1991)];

d) non-stationanty of the relevant time series;

e) failure of rational expectations of investment behavior [for example, Shiller (1981)];

f) noisy data, including time-aggregation [Grossman and Shiller (1982), Heaton (1988)].

Here, we will concentrate on the incompleteness of markets, but the presence of so many competing potential explanations for empirical asset price behavior leaves a cloudy picture of whether there will appear a new "standard model" with one, or at most a few, of these various "market imperfections" embedded as standard features.

In the vein of market incompleteness and asset pricing, the literature includes Bewley (1982), Mankiw (1986), Mehra and Prescott (1985), Duffie (1992), Mehrling (1990), Scheinkman (1989), Telmer (1991), He and Modest (1992), Lucas (1991), Heaton and Lucas (1992), Weil (1992), and Constantinides and Duffie (1991). This literature is certainly not distinct from studies, mentioned above, of the roles of portfolio constraints and transactions costs, in that these "frictions" also prevent agents from equating their marginal rates of substitution. There is a distinction, however. So long as there are no portfolio constraints or transactions costs, the volatility of security returns is limited in that all agents equate the projections of their marginal rates of substitution onto the span of security returns. Thus security prices are determined by discounting cash flows by the *average* of individual marginal rates of substitution, which is typically less volatile than a particular individual's marginal rates of substitution. With transactions costs or portfolio constraints, this modifying effect on volatility can be greatly reduced.

An important effect of incomplete markets that will not be a subject of this review is the severe indeterminancy of equilibria that is possible with securities whose cash flows are stated in units of account, rather than commodities. This effect is modeled by Cass (1986) and Geanakoplos and Mas-Colell (1987). If one fixes one of the many possible equilibria, then the issue of volatility due to indeterminancy disappeares, since the cash flows of a security can be converted at equilibrium prices of those of a commodity-paying security, and the usual arguments apply. If, however, indeterminancy is somehow connected with shifts in self-fulfilling expectations of security returns, then an increase in volatility may result. This effect is out of the scope of the usual perfect foresights rational expectations models, but may be worth pursuing . (See, for example, Jackson and Peck (1991).)

II. *A One-Period Example*

This section presents a static model of how incomplete markets might be responsible for systematic effects on the pricing of securities, relative to complete markets. Later, we extend to a dynamic model.

We fix a probability space $(\Omega,\mathscr{F},P)$ and a von Neumann-Morgenstern utility function $u:\mathbb{R}_+\rightarrow\mathbb{R}$ that is increasing, differentiable and strictly concave. The payoffs of the N available financial securities, in units of the single consumption good, are given by an $\mathbb{R}^N$-valued random variable D. There are m agents, all of whom have the same initial consumption endowment y_0. In the second period, the endowment of agent $i\in\{1,\dots,m\}$ is defined by a strictly positive random variable Y_i. Given some p in $\mathbb{R}^N$ defining security prices, agent i has the problem

$$\sup_{\theta \in \mathrm{R}^n} u(y_0 - \theta \cdot p) + \beta E[u(Y_i + \theta \cdot D)], \tag{1}$$

for some subjective discount factor β. An equilibrium is a security price vector p and portfolios $(\theta_1,\ldots,\theta_m)$ such that θ_i solves the portfolio problem of agent i for all i, and markets clear: $\sum_i \theta_i = 0$.

Assuming that the distribution of $(Y_1,\ldots,Y_m,D)$ does not depend on permutations of the order of $Y_1,\ldots,Y_m$, a symmetry condition, it is natural to assume the existence of an equilibrium in which the comsumption $C_i = Y_i + \theta_i \cdot D$ of agent i generates the same symmetry property for $(C_1,\ldots,C_m,D)$. Under this symmetry condition, and strict positivity of C_i, we have the first-order condition

$$p = E\left[\frac{\beta u'(C_i)D}{u'(y_0)}\right], \quad i \in \{1, \ldots, m\}. \tag{2}$$

From (2),

$$p = \beta E\left[\frac{[u'(C_1) + \ldots + u'(C_m)]D}{mu'(y_0)}\right]. \tag{3}$$

In the case of complete markets (defined by the fact that every consumption choice can be expressed in the form $\theta \cdot D$ for some portfolio θ), the symmetry condition, risk aversion (concavity of u), and Pareto optimality of equilibria (Arrow (1953)) implies that $C_i = \sum_{i=1}^{m} Y_i/m$ for all i, so that the complete markets security price vector p^* can be re-expressed from (3) as

$$p^* = \beta \frac{E\left[u'\left(\frac{C_1 + \ldots + C_m}{m}\right)D\right]}{u'(y_0)}. \tag{4}$$

For convex $u'(\cdot)$, a common assumption, Jensen's Inequality implies from (3) and (4) that

$$p^* \leq p.$$

Under these strong symmetry and convexity assumptions, it follows that a false assumption of complete markets will produce an unexpectedly low implied rate of time preference (high estimate of β). This is indeed consistent with the data. (For example, see Hansen and Singleton (1982).)

The Hansen-Jaganathan (1991) model proposes, as a diagnostic for volatility, a regression-style estimate of the variance of the projection of marginal rates of substitution onto the span of the security returns. For our setting, assuming that there is a riskless security (that is, that the span of D contains a non-zero constant), this projection is given by

$$\pi = E\left[\frac{\beta u'(C_i)D}{u'(y_0)}\right]^T [E(DD^T)]^{-1}D. \tag{5}$$

If we denote by π^* the corresponding projection in the complete-markets case, which is merely $\pi^* = \beta u'(C_i)/u'(y_0)$, the same Jensen's inequality arguments and the assumptions used previously imply that

$$var(\pi) \geq var(\pi^*). \tag{6}$$

(This is easily shown if, without loss of generality, one replaces D in (5) with an L^2–orthonormal basis for span (D).)

The fact that these implications of market incompleteness for asset price behavior depend on convex marginal utility obviously places the implications on a narrow footing. Concave marginal utility would reverse the direction of the impact of preferences on prices with market incompleteness. The "neutral" case of linear marginal utility is of some interest, despite the restrictiveness of quadratic utility. With linear marginal utility, it can be seen by re-tracing our last arguments that increasing the span of securities always increases the variance $var(\pi)$ of the projected "marginal rate of substitution" π. (For details, see Duffie (1992).)

III. *The Impact of Heterogeneous Uninsurable Permanent Income Shocks*

Most of the remainder of this lecture is devoted to a review of an example proposed by Constantinides and Duffie (1991) for illustrating the impact of market incompleteness on the behavior of security prices in a setting suitable for empirical issues. The example, simplified here for brevity, is a general equilibrium multi-period model in the spirit of single period partial equilibrium examples due to Mankiw (1986) and Weil (1992), among others.

The basic idea is to place oneself in a relatively restrictive "classical" setting of Lucas (1978), in which there would be no latitude for the behavior of security prices except for the incompleteness of markets.

Specifically, let $\{\mathscr{F}_0, \mathscr{F}_1, \mathscr{F}_2, \ldots\}$ be an increasing sequence of sub-σ-fields of $\mathscr{F}$, for some probability space $(\Omega, \mathscr{F}, P)$, and let L denote the consumption space of $\{\mathscr{F}_t\}$-adapted real-valued processes. Each agent has the utility function U on the space L_+ of non-negative processes in L defined, for some risk aversion coefficient $\alpha > 0$, by

$$U(C) = E\left[\sum_{t=0}^{\infty} e^{-\rho t} C_t^{1-\alpha}\right],$$

which is allowed to take the value $+\infty$.

The payoffs of n securities are defined by a dividend process d in L^n. The ex-dividend price process for the securities is given by some L^n-valued process p. As is well known (see, for example, Duffie (1992b)) mild technical conditions imply the equivalence between no arbitrage in security markets and the existence of a strictly positive process M in L, sometimes called a *pricing kernel*, such that, for all t,

$$p_t = \frac{1}{M_t} E\left[\sum_{s=t+1}^{\infty} M_s d_s \middle| \mathscr{F}_t\right]. \tag{7}$$

(See Clark (1993) and Delbaen and Schachermeyer (1992) for technical details.)

An aggregate consumption process $\bar{C}$ in L_+ is fixed. A measure space $(A, \mathscr{A}, \mu)$, with $\mu(A) = 1$, defines the space of agents. A consumption allocation is a collection $\{C_a : a \in$

$A\} \subset L_+$ of consumption processes, one for each agent, such that, for each t, $\int_A C_{at} d\mu(a)$ is a well defined random variable equal to the aggregate endowment $\bar{C}_t$ almost surely.

A portfolio process is a process θ in the space L^n of bounded processes in L^n, defining a portfolio θ_t held after trading at time t. Given a pricing kernel M defining security prices by (7) and an endowment process C_a in L, a budget-feasible choice by an agent a in A is a consumption-portfolio process pair (C,θ) in $L_+ \times L^n$ such that, for all $t \geq 0$.

$$C_t = C_{at} + \theta_{t-1} \cdot (p_t + d_t) - \theta_t \cdot p_t, \tag{8}$$

where θ_{-1} is the initial portfolio endowment of $(1,\ldots,1)$, common to all agents. (Since $\mu(A)=1$, the total supply of each security is thus 1.) A budget-feasible choice (C,θ) for agent a is optimal for a, given (M,C_a), if there is no other budget-feasible pair $(\hat{C},\hat{\theta})$ such that $U(\hat{C}) > U(C)$.

A portfolio allocation is a collection $\{\theta_a : a \in A\} \subset L_b^n$ of portfolio processes such that, for all t, $\int_A \theta_{at} d\mu(a)$ is a well defined random variable equal to $(1,\ldots,1)$ almost surely. Recall that $(\bar{C},d,U)$ is fixed throughout. An *equilibrium* for an endowment allocation $\{C_a : a \in A\}$ is a pricing kernel M with the property that there exists a consumption allocation $\{C_a^* : a \in A\}$ and a portfolio allocation $\{\theta_a^* : a \in A\}$ such that, for all a, (C_a^*, a_a^*) is optimal given (M,C_a). A pricing kernel M is *conceivable* if there is an endowment allocation for which M is an equilibrium. In other words, a pricing kernel is conceivable if there is some way to split up the total endowment $\bar{C}$ among the agents so that M defines equilibrium security prices. It is well understood since the aggregation theorem of Rubinstein (1976), and has been formalized under conditions by Kandori (1988), that with complete markets there is no latitude for the manner in which the pricing kernel sets prices: One must have, for all t,

$$\frac{M_{t+1}}{M_t} = \zeta_{t+1} \equiv e^{-\rho}\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\alpha}. \tag{9}$$

On the other hand, (9) is not required with incomplete markets and is easily rejected on the basis of standard quality-of-fit tests using U.S. data on security prices and aggregate consumption. (See, for example, Hansen and Singleton (1982).) More general utility models add degrees of freedom beyond the choice of (α,ρ) to better fit (9), as shown by Constantinides (1990), Ferson and Constantinides (1991), Epstein and Zin (1991), among others. Here, however, we will maintain the standard additive utility model but we will not impose complete markets. Instead, we will design an endowment allocation with persistent income shocks that cannot be hedged with the available securities. With that, it will be shown that any pricing kernel M is conceivable provided that, for all t,

$$\frac{M_{t+1}}{M_t} \geq e^{-\rho}\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\alpha}. \tag{10}$$

Specifically, given any pricing kernel M satisfying (10), consider the endowment allocation $\{C_a : a \in A\}$ defined by

$$C_{at} = \bar{C}_t \exp\left(\sum_{s=1}^{t} \eta_{as} y_s - \frac{y_s}{2}\right), \tag{11}$$

where

$$y_t=\sqrt{\frac{2}{\alpha^2+\alpha}}\left[\log\left(\frac{M_t}{M_{t-1}}\right)-\log\zeta_t\right]^{1/2}, \tag{12}$$

and where $\{\eta_{at}:a\in A, 0\leq t<\infty\}$ is a collection of jointly independent standard normal random variables, independent also of M, d, and $\bar{C}$. By suitable joint construction of $(\Omega,\{\mathscr{F}_t\},P)$ and $(A,\mathscr{A},\mu)$, as demonstrated by Green (1989) and others,[1] the law of large numbers can be effectively invoked to show that $\{C_a:a\in A\}$ is indeed a consumption allocation (that is, to show that $\int_A C_{at}d\mu(a)=\bar{C}_t$ almost surely for all t).

We can view (11) as a model of persistent idiosyncratic income shocks. The magnitude y_t of the shock at time t depends, according to (12), on the gap between the marginal rates of substitution implied by complete markets, as in (9), and the desired aggregate marginal rates of substitution M_t/M_{t-1} implied by the candidate pricing kernel M. The persistence of the income shocks is important for, as shown by D. Lucas (1991) and by Telmer (1991), transitory idiosyncratic income shocks are not sufficient to capture empirical measures of asset price volatility.

PROPOSITION. *If M is a pricing kernel satisfying (10) and $\lim_{t\to\infty}E(M_t)=0$, then M is conceivable.*

The "transversality" condition $E(M_t)\to 0$ is typical of infinite horizon models, but not necessary in general for equilibrium (See, for example, Kockerlakota (1990).) A complete proof of the proposition is given by Constantinides and Duffie (1991). The equilibrium demonstrated is actually one with no trade. That is, given M and $\{C_a:a\in A\}$ defined as above in terms of M, it is optimal for agent a to choose the endowed consumption C_a and to maintain the originally endowed portfolio (1,1,...,1). This choice is obviously budget-feasible and market clearing. For individual optimality, the key requirement is the "stochastic Euler equation"

$$p_t=E\left[\frac{M_{t+1}}{M_t}(p_{t+1}+d_{t+1})|\mathscr{F}_t\right]=E\left[e^{-\rho}\left(\frac{C_{a(t+1)}}{C_{a,t}}\right)^{-\alpha}(p_{t+1}+d_{t+1})|\mathscr{F}_t\right]. \tag{13}$$

The reader can verify by direct calculation that (13) is satisfied, using the construction (11) of C_a, the independence of η_a from $\{M,d,\bar{C}\}$, and the fact that if Z is normally distributed, then

$$E(e^Z)=\exp\left(E(Z)+\frac{var(Z)}{2}\right).$$

The additional optimality arguments beyond the "first-order condition" (13) are purely technical. For details, see the appendix of Constantinides and Duffie (1991).

Thus inequality (10), which is not easily relaxed, is enough to justify M as a pricing kernel, even in our restrictive setting. Condition (10) means pointwise-higher marginal rates of substitution than those implied with complete markets (an artifact of convex mar-

[1] See, also, Anderson (1990), Feldman and Gilles (1985), Judd (1985), and Uhlig (1990).

ginal utility), and allows significant latitude in the volatility of security prices. For example, one can easily attain the empirical volatility bounds of Hansen-Jaganathan by choosing "heterogeneity shocks" $\{y_t\}$ that are sufficiently large with sufficiently small probability. (See footnote 5 of Constantinides and Duffie (1991) for details.)

It can also be shown that it is impossible to relax (10) in this setting by a more clever choice for the distribution of the idiosyncratic income shock $X_{at}=C_{at}/\bar{C}_t$ than the normal distribution specified by (11). To see this, we note that market clearing requires that $\int_A X_{at} d\mu(a)=1$ almost surely. Jensen's Inequality then implies that $\int_A X_{at}^{-\alpha} d\mu(a)\geq 1$ almost surely, which can be combined with (13) to deduce the restriction (10) on marginal rates of substitution.

In summary, it is relatively easy to use uninsurable income shocks in this manner so as to justify a wide range of asset price behavior in equilibrium, even for fixed preferences, aggregate endowment, and security dividends. On the other hand, it may be difficult to do this and at the same time maintain an empirically reasonable cross-sectional distribution of consumption. This issue can be addressed by a slight extension of the above example which allows agents to die independently across time and among themselves each period, with some fixed probability, and to be replaced at death by heirs whose income shock (ratio of individual to per capita income) is reset to unity at birth. With this, one can use standard regenerative process theory to calculate, for given parameters, the steady-state cross-sectional consumption distribution implied by the model. (See Constantinides and Duffie (1991) for details.) It is then an as-yet-unresolved empirical issue whether asset price data and cross-sectional consumption data can both be statistically consistent with the sort of theoretical model proposed here (or perhaps with some reasonable extension of this model).

In principle, of course, any given arbitrage-free asset pricing behavior can always be justified in equilibrium by construction of a suitable preference assumption (say linear, with coefficients given by the security pricing kernel). Reasonable models, however, should include economically and empirically reasonable assumption on preferences, endowments, and trading restrictions, whether in the form of missing markets, short sale or credit constraints, transactions costs, or otherwise. For this purpose, there is as yet no standard research paradigm.

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