II. General Equilibrium and Uncertainty

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MARKET GAMES WITH ASYMMETRIC INFORMATION: VERIFICATION AND THE PUBLICLY PREDICTABLE INFORMATION CORE*

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Abstract

For cooperative (NTU) games generated by finite exchange economies with asymmetric information about common payoff-relevant states of the world, private information use is equivalent to the publicly predictable information sharing rule. This leads to balanced games which therefore have nonempty cores as well as Nash verifiability of a coalition member's ex ante contingent net trades. Conditions yielding Nash and strong verifiability for more general information sharing rules are also provided. In this way, a class of market games with partial commitment that are classified between cooperative and noncooperative games can be studied.

I. *Introduction*

This paper concerns the extent to which asymmetrically informed economic agents can be verified to have fulfilled contracts contingent on their private information. The term "verification" in the title refers to this property—contracts depend on information in a way that can be checked by others, so that deviations can be detected.

The setting is cooperative games induced by pure exchange economies with asymmetric information. I focus on the core as a solution concept because of its extensive use in economics and because one can argue that allocations not belonging to the core are not plausible as they can be blocked by some coalition. Thus, a statement applying to all core allocations includes those allocations of economic interest even if one does not wish to advocate the core, per se, as a solution concept. However, note that some results apply to all actions that are feasible for a coalition in terms of its resource constraint and its information.

Initially I permit coalition members each to use precisely their own (initial) private information. An important consequence of this choice is that it is equivalent to the use of publicly predictable information, as studied in another context by Blume and Easley (1990), Palfrey and Srivastava (1986), and Postlewaite and Schmeidler (1986, 1987). As a result, I can show that core allocations (which necessarily exist) are private Nash verifiable,

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which means that deviations by any agent can be detected. Moreover, the core with private information or publicly predictable information is nonempty as a consequence of the market games equivalence theorem asserting that the set of totally balanced games equals the set of cooperative games generated by exchange economies with concave utilities. This amounts to a substantial simplification of the proof that the games induced by economies with asymmetric information are well defined and have nonempty cores.

More generally, coalitions can be permitted to base their actions on any arbitrary information specification. Some restrictions on these information sharing rules yield strong verification. This means that deviations by any subset of a coalition can be detected by remaining coalition members. However, if one considers only singleton deviations, then modified Nash verification follows from the no free disposal assumption for any feasible allocation. Hence strong verifiability is significantly more restrictive than Nash verifiability.

All of this is demonstrated for a model involving cooperative games with nontransferable utility generated by economies in which strategies consist of ex ante contingent net trades of commodities.[1] Endowments and utilities are state dependent and "information" means the ability to condition one's net trade on a partition (actually, a sub-σ-field) of the set of states of the world. Parallel results also apply for games with transferable utility; compare Allen (1991b) to Allen (1991c).

The work most closely related to this paper is probably the article by Yannelis (1991) analyzing the private information core of an exchange economy. Banach lattice methods are used to obtain a core nonemptiness result under the assumption that every event occurs with strictly positive probability. The antecedent is the seminala rticle of Wilson (1978) which proposed definitions of the core (and also for the efficient allocations) of a finite exchange economy with asymmetric information and finitely many states of the world. Wilson discuses the notion of communication structures and focuses on two extreme cases: no use of asymmetric information (termed the null communication structure) within a coalition, which leads to a core concept in which blocking coalitions can use only that information which is common to all of their members, and complete information sharing (the full communication structure), which gives rise to a possibly empty core in which blocking can be based on any information held by one or more members of the coalition.[2,3]

A recent related paper is the study by Marimon (1990) of the incentive properties of the core of an economy with moral hazard. In his model, a continuum of agents have private information about their endowments or abilities and there are only a finite number of types. Efficient and core allocations are defined with an incentive compatibility constraint. A major contrast to my work is that Marimon focuses on adverse selection/moral

[1] If one interprets the basic economic model as the realization of a distribution on players' types, where the description of a type specifies the agent's economic characteristics—in this context, state-dependent cardinal utility functions, state-dependent initial endowment mappings, and initial information—then I use an interim concept.

[2] Kobayashi (1980) relaxed the assumption of finitely many states of the world and proved nonemptiness of the Wilson coarse core (under the assumptions of balancedness and strictly positive probability of every event) using techniques reminiscent of Bewley (1972). He also demonstrated core equivalence for the set of competitive equilibrium allocations in the sense of Radner (1968).

[3] Mailath and Zemsky (1991) examine superadditivity, balancedness, and the core with asymmetric information of the game they derive from a "divide the collusive surplus" problem in second price auctions.

hazard issues; this means that his agents draw states of the world independently (and he appeals to the law of large numbers). Hence, he analyzes idiosyncratic risk while I study systematic risk, so that my economic agents all care about a common state of the world.[4]

A more explicit game-theoretic approach is taken by Myerson (1984) and Rosenmüller (1990) in their work incorporating incomplete information into cooperative games. They use the Harsanyi formalism, so that the incomplete information concerns the finite set of agents' types.

My approach lies between these games-based analyses and the economies-based literature cited above in that I utilize the relationship between markets and the games that they generate to gain insight into asymmetric information problems. In fact, much of my contribution consists of modelling coalitions' information structures and deriving the induced game, thereby avoiding potentially heavy general equilibrium methods (cf. Yannelis (1991)).

The Harsanyi doctrine also forms the basis of the study by d'Aspremont and Gerard-Varet (1979) of incentives in noncooperative games with incomplete information, which is less directly related to my paper in that it takes a noncooperative approach, as does the huge literature on implementation and mechanism design. A systematic review is beyond the scope of this paper, but see especially Holmstrom and Myerson (1983) for the definition of alternative efficiency concepts with incomplete information and incentive constraints and the recent survey of Palfrey (1990). This paper is also related to the literature on incomplete contracts; see the survey by Hart and Holmstrom (1987), who point out that asymmetry of information between players and enforcers is the source of the difficulty in conveying information to others.

The remainder of this paper is organized as follows: Section 2 presents the model of pure exchange economies with asymmetric information. Section 3 derives the induced market games with private information and demonstrates that they have nonempty cores. The relation between private information and publicly predictable information is examined in Section 4, where it is demonstrated that these concepts yield (private) Nash verifiability. Then Section 5 proceeds to define strong verifiability and to analyze its relation to various information assignments to coalitions (formalized as information sharing rules). Section 6 concludes the paper with some comments on partially enforceable commitments and models that are between cooperative and noncooperative games.

II. *Exchange Economies with Private Information*

In this section, I model uncertainty and private information. Both pure exchange

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[4] Some recent work on optimal taxation has also used the core with asymmetric information. For example, Berliant (1992) proves that the fine core analogue of the core without asymmetric information contains only head taxes, while his coarse 1C-core may be empty. However, this public finance question again involves uncertainty and asymmetric information which is diametrically opposed to the type which interests me. In particular, I am concerned with information about systematic risks, so that there is a single drawing of a state of the world which then becomes an argument of every consumer's utility function. For optimal taxation, idiosyncratic risk is a better description although consumers completely know their own individual drawing of the state of the world. Incomplete information characterizes only the government, which cannot recognize an individual's type and thus cannot necessarily impose type-dependent tax rules. Instead, the government can observe only the distribution of types within a coalition (and not individual identities).

economies and the cooperative games (with nontransferable utility) that they generate are considered in this paper. To the extent possible, the same notation will be used for the economies and the games.

To begin, specify an abstract probability triple $(\Omega,\mathbf{F},\mu)$ to describe the uncertainty. The set of states of the world is denoted Ω, with typical element ω. Let $\mathbf{F}$ be a σ-field of subsets of Ω, interpreted as the measurable events that economic agents eventually learn, so that events in $\mathbf{F}$ may be payoff relevant for ex post utilities. The σ-additive probability measure μ defined on $(\Omega,\mathbf{F})$ represents agents' ex ante subjective probabilities attached to the occurrence of various events. To simplify notation, assume that these subjective probability assessments are the same for all agents; this could easily be generalized.

Let I denote the set of economic agents (consumers) in the pure exchange economy. No confusion will result from taking I also to be the set of players in the games examined here. An individual player or trader is denoted by $i\in I$. The set I is assumed to be finite; write $\sharp I$ for its cardinality. Let 2^I denote the set of subsets of I. Nonempty subsets of the player set I are termed *coalitions* in the game. A *submarket* is a pure exchange economy consisting of only those traders $i\in I'$ for some $I'\subseteq I$, $I'\neq\phi$.

Suppose that there is a finite number l of commodities (numbered $1,2,\ldots,l$) available in the economy. To summarize endowments, let $e:I\times\Omega\rightarrow\mathbb{R}^l_+$ denote an arbitrary measurable function which is uniformly bounded and write $e_i:\Omega\rightarrow\mathbb{R}^l_+$ for consumer i's random (state dependent) initial allocation function. Define the set E of allocations by

$$E=\{(x_1(\cdot),\ldots,x_{\sharp I}(\cdot))|\text{ for each } i\in I,\ x_i:\Omega\rightarrow\mathbb{R}^l \text{ is } \mathbf{F}\text{-measurable and}$$
$$-\sum_{j\in I}e_j(\omega)\leq x_i(\omega)\leq\sum_{j\in I}e_j(\omega)\ \text{ for almost all } \omega\in\Omega\}.$$

Interpret E as a convenient closed and bounded subset of measurable functions which contains all state-dependent individual allocation functions or state-dependent individual net trade functions that could ever be feasible for the economy.

Consumer i's preferences are specified by a state-dependent cardinal utility function $u_i:\mathbb{R}^l_+\times\Omega\rightarrow\mathbb{R}$ which is continuous and concave on $\mathbb{R}^l_+$ and $\mathbf{F}$-measurable as a function of Ω, so that it's jointly measurable (for the Borel σ-field $\mathbf{B}(\mathbb{R}^l_+)$ on i's consumption set $\mathbb{R}^l_+$). Assume also that there is some compact convex subset K of $C(\mathbb{R}^l_+,\mathbb{R})$ endowed with the (compact-open) topology of uniform convergence on compact subsets of $\mathbb{R}^l_+$ such that for (almost) all $\omega\in\Omega$, $u_i(\cdot;\omega)\in K$. This implies that all state-dependent utilities are uniformly equicontinuous and take uniformly (above and below) bounded values on any compact subset of $\mathbb{R}^l_+$.

Initial information is represented by sub-σ-fields of $\mathbf{F}$. For $i\in I$, write $\mathbf{G}_i$ for i's initial information. "Private information" means that each agent $i\in I$ can use precisely his initial information $\mathbf{G}_i$ as a member of any coalition. Assume that, for all $i\in I$, $e_i(\cdot)$ is $\mathbf{F}$-measurable and this function is known to trader i.

Finally, a trader's goal is to maximize state-dependent conditional expected utility (which is a $C(\mathbb{R}^l_+,\mathbb{R})$-valued random variable—or measurable function—defined on Ω) given the available information. This information can be analyzed by incorporating it into the consumer's objective function (i.e., by calculating conditional expected utilities given the information). However, a better alternative for the game-theoretic analysis is, whenever possible, to place the information into a measurability constraint on the agent's state-dependent allocation (demand, excess demand, individual net trade, etc.) functions because then the information enters into the definition of commodity spaces but not utilities in our market games. [This trick works well for private information sharing. See Section 3.] The insight comes from VanZandt (1988), who proposes to insert the asymmetric information into traders' budget constraints and observes that the dependence of his "measurability approach" budget correspondence then is lower hemicontinuous (and the correspondence has a closed graph) in information sub-σ-fields. Needless to say, payoffs to players in my games are taken to equal the expected utilities of final state-dependent commodity allocations.

Where necessary to define conditional expected utilities, I analyze the image measures $\mu_0 u_i^{-1}$ on the Frechet space $C(\mathbb{R}^l_+,\mathbb{R})$ induced by the vector-valued random variables u_i: $(\Omega,\mathbf{F},\mu)\rightarrow(C(\mathbb{R}^l_+,\mathbb{R}),B(C(\mathbb{R}^l_+,\mathbb{R})))$. Then proper versions of regular conditional distributions exist and conditional expected utilities are $C(\mathbb{R}^l_+,\mathbb{R})$-valued random variables that take values (almost surely) in the compact convex set K. In particular, conditional expected utility is (almost surely) continuous and concave on $\mathbb{R}^l_+$. (See Rudin (1973, pp. 73–78) for technical details on integration in Frechet spaces.)

Let $EU_i{:}L^\infty(\Omega,\mathbf{F},\mu{:}\mathbb{R}^l_+)\rightarrow\mathbb{R}$ denote player i's (unconditional) expected utility function, defined by

$$EU_i(x_i(\cdot))=\int_\Omega u_i(x_i(\omega);\,\omega)d\mu(\omega),$$

where the state-dependent individual allocation $x_i{:}\Omega\rightarrow\mathbb{R}^l_+$ can be assumed to be (μ-a.s.) bounded to ensure that the integral is well defined. Note that no unbounded allocation can be feasible.

Lemma 2.1. The expected utility function $EU_i{:}L^\infty(\Omega,\mathbf{F},\mu;\mathbb{R}^l_+)\rightarrow\mathbb{R}$ is concave. It is continuous for either the $\mathbf{L}^1$ or $\mathbf{L}^\infty$ norm on (almost surely) uniformly bounded state-dependent allocation functions $x_i(\cdot){:}\Omega\rightarrow\mathbb{R}^l_+$.

Proof. Let $x_i,x_i'\in\mathbf{L}^\infty(\Omega,\mathbf{F},\mu;\mathbb{R}^l_+)$ and take $\lambda\in[0,1]$. Then

$$\begin{aligned}
&EU_i(\lambda x_i+(1-\lambda)x_i')\\
&=\int_\Omega u_i(\lambda x_i(\omega)+(1-\lambda)x_i'(\omega);\,\omega)d\mu(\omega)\\
&\geq\int_\Omega[\lambda u_i(x_i(\omega);\,\omega)+(1-\lambda)u_i(x_i'(\omega);\,\omega)]d\mu(\omega)\\
&=\lambda\int_\Omega u_i(x_i(\omega);\,\omega)d\mu(\omega)+(1-\lambda)\int_\Omega u_i(x_i'(\omega);\,\omega)d\mu(\omega)\\
&=\lambda EU_i(x_i(\cdot))+(1-\lambda)EU_i(x_i'(\cdot)),
\end{aligned}$$

where the inequality follows from the concavity of $u_i(\cdot;\omega)$ for almost all $\omega\in\Omega$. This proves that $EU_i(\cdot)$ is concave, as desired.

To verify continuity, let $x_i^n(\cdot){:}\Omega\rightarrow\mathbb{R}^l_+$ be a sequence of uniformly bounded state-dependent allocation functions. Assume first that $x_i^n(\cdot)\rightarrow x_i(\cdot)$ for the $\mathbf{L}^\infty$ norm as $n\rightarrow\infty$.

Then $x_i^n(\cdot)\rightarrow x_i(\cdot)$ for the L^1 norm also (see Ash (1972, p. 96)) and the two cases reduce to one. Because L_P convergence implies convergence in measure (Ash (1972, p. 92), we have $x_i^n(\cdot)\rightarrow x_i(\cdot)$ in measure and, because the $u_i(\cdot;\omega)$ are assumed to be uniformly equicontinuous on compact subsets of $\mathbb{R}_+^l$, we have $u_i(x_i^n(\cdot);\cdot)\rightarrow u_i(x_i(\cdot);\cdot)$ in measure also as $n\rightarrow\infty$. Uniform boundedness of the $u_i(\cdot;\omega)$ on compact subsets of $\mathbb{R}_+^l$ implies that we may use the extended dominated convergence theorem (Ash (1972, p. 96)) to conclude that $EU_i(x_i^n(\cdot))\rightarrow EU_i(x_i(\cdot))$ as $n\rightarrow\infty$. [Alternatively, convergence in measure implies convergence in distribution and the generalized Lebesgue convergence theorem applies. Uniform integrability follows from the fact that the $u_i(x_i^n(\cdot);\cdot)$ functions are dominated by an integrable function by uniform boundedness of $u_i(\cdot;\omega)$ on compact subsets of $\mathbb{R}_+^l$. See Hildenbrand (1974, pp. 51–52) for details.] [ ]

Remark 2.2. The same argument can be used to obtain concavity and continuity of conditional expected utility on the set of state-dependent allocation functions in $\mathbf{L}^\infty(\Omega,\mathbf{F},\mu;\mathbb{R}_+^l)$. Alternatively, we could observe that if $\mathbf{G}$ is a complete sub-σ-field of $\mathbf{F}$, then $\mathbf{L}^\infty(\Omega,\mathbf{G},\mu;\mathbb{R}_+^l)$ and $\mathbf{L}^1(\Omega,\mathbf{G},\mu;\mathbb{R}_+^l)$ are closed subspaces of $\mathbf{L}^\infty(\Omega,\mathbf{F},\mu;\mathbb{R}_+^l)$ and $\mathbf{L}^1(\Omega,\mathbf{F},\mu;\mathbb{R}_+^l)$ respectively because both $\mathbf{L}^1$ and $\mathbf{L}^\infty$ convergence imply pointwise almost everywhere convergence, which preserves measurability. However, this property is not needed here, because we view information as imposing a measurability constraint on allowable allocation functions rather than an argument in agents' objective functions.

III. *Cooperative Games with Private Information*

The goal of this section is to derive the NTU cooperative game in characteristic function form generated by a pure exchange economy with private information. I also define the private information core and prove that it is nonempty.

Formally, a (cooperative) *nontransferable utility (NTU) game* in characteristic function form is a correspondence $V:2^I\rightarrow\mathbb{R}^{\#I}$ satisfying $V(\phi)=\{0\}$ and, for all $S\subseteq I$, $V(S)$ is nonempty, closed, and comprehensive for $S\neq\phi$. Moreover, the sets $V(S)$ are "cylinder sets" in that if $(\bar{u}_1,\ldots,\bar{u}_{\#I})\in V(S)$ and $\bar{u}_i=\bar{w}_i$ for all $i\in S$, then $(\bar{w}_1,\ldots,\bar{w}_{\#I})\in V(S)$. Comprehensiveness $(V(S)\supseteq V(S)-\mathbb{R}_+^{\#I})$ can be interpreted as "free disposability" of utility. Note that I do not require superadditivity as part of the definition of a cooperative game.[5]

To derive the NTU game associated with a pure exchange economy with private information, I must define its characteristic function $V:2^I\rightarrow\mathbb{R}^{\#I}$ based on the data describing the economy. Accordingly, set $V(\phi)=\{0\}$ and for each coalition $S(\phi\neq S\subseteq I)$, define

$$V(S)=\{(w_1,\ldots,w_{\#I})\in\mathbb{R}^{\#I}\mid \text{for } i\in S, \text{ there exist } x_i:\Omega\rightarrow\mathbb{R}_+^l \text{ with } w_i\leq\int_\Omega u_i(x_i(\omega);\omega)d\mu(\omega) \text{ such that } z_i(\cdot)=x_i(\cdot)-e_i(\cdot) \text{ is } \mathbf{G}_i\text{-measurable and } \sum_{i\in S}x_i(\omega)=\sum_{i\in S}e_i(\omega) \text{ for almost all } \omega\in\Omega\}.$$

The study of market games was initiated by Shapley and Shubik (1969). The main

[5] Hildenbrand and Kirman (1976, Chapter 3) is a good reference for economists who are not familiar with these concepts.

results in this literature establish an equivalence between market games (defined to be cooperative games in characteristic function form which satisfy total balancedness) and finite pure exchange economies in which traders have concave utility functions. Strictly speaking, a *market game* is a cooperative game having a characteristic function that can be generated by an economy with continuous concave utilities; it is *representable* by a market. I'm interested in the pure exchange case, although sharper results are possible with nontransferable utility if production sets can be added arbitrarily.

Following Billera (1974), Billera and Bixby (1974), and Mas-Colell (1975), define balancedness for NTU games as follows (where, for $S \subseteq I$, $V(S)_S = \{(w_1,\ldots,w_{\#I}) \in V(S) | w_i = 0 \text{ if } i \notin S\} = V(S) \cap \{(w_1,\ldots,w_{\#I}) \in \mathbb{R}^{\#I} | w_i = 0 \text{ if } i \notin S\}$):

Definition 3.1. A family $\mathbf{B}(S)$ of subsets of S is *balanced* (or a *balanced collection on* S) if there are nonnegative weights $w_T \geq 0$ for all $T \subseteq S$ such that $\sum_{\substack{T \subset S \\ T \ni i}} w_T = 1$ for all $i \in S$. For $S \subseteq I$, let $B(S)$ denote the set of all *balancing weights* for balanced collections on S; i.e., $B(S) = \{w : 2^S \to \mathbb{R}_+ | \sum_{T \ni i} w_T = 1 \text{ for all } i \in S \text{ and } w_T = 0 \text{ if } T \notin \mathbf{B}(S)\}$.

Definition 3.2. The NTU game $V : 2^I \to \mathbb{R}^{\#I}$ is *balanced* if $V(I) = \cup \{\sum_{T \subseteq I} w_T V(T)_T | w \in B(I)\}$. Equivalently, it is *balanced* if $V(I) \supseteq \sum_{T \subseteq I} w_T V(T)_T$ for every $w \in B(I)$.

Definition 3.3. An NTU game is *totally balanced* if all of its subgames are balanced. In symbols, $V : 2^I \to \mathbb{R}^{\#I}$ is *totally balanced* if $V(S) = \cup \{\sum_{T \subseteq S} w_T V(T)_T | w \in \mathbf{B}(S)\}$ for every $S \subseteq I$, $S \neq \phi$. Equivalently, it is *totally balanced* if $V(S) \supseteq \sum_{T \subseteq S} w_T V(T)_T$ for every $w \in B(S)$ and every $S \subseteq I$ with $S \neq \phi$.

The second version (in Mas-Colell (1975)) of Definitions 3.2 and 3.3 facilitates the demonstration that concave utilities lead to (totally) balanced games. To see the equivalence, select a coalition S arbitrarily ($\phi \neq S \subseteq I$). Clearly S itself is a balanced collection with balancing weights $w_S = 1$ and $w_T = 0$ for all $T \neq S$. Hence $V(S) \supseteq \sum_{T \subseteq S} w_T V(T)_T$ implies $V(S) = \sum_{T \subseteq S} w_T V(T)_T$ for some choice of weights. Therefore $V(S) \supseteq \sum_{T \subseteq S} w_T V(T)_T$ for all $w \in B(S)$ implies $V(S) = \cup \{\sum_{T \subseteq S} w_T V(T)_T | w \in B(S)\}$. Conversely, if $V(S) = \cup \{\sum_{T \subseteq S} w_T V(T)_T | w \in B(S)\}$ then trivially $V(S) \supseteq \sum_{T \subseteq S} w_T V(T)_T$ for all $w \in B(S)$.

The approximate equivalence (see Proposition 3.4 below) between market games and totally balanced games can be extended to situations with uncertainty and asymmetric information. Note that the direction of the result that any totally balanced game can be generated by a suitably chosen exchange economy can be bootstrapped from the theorems for economies without uncertainty, so that the "hard" direction becomes easy here. The converse claim that economies with private information lead to totally balanced games is difficult due to the necessity of showing that the $V(S)$ above define a genuine game. This follows from the Billera and Bixby (1974) theorem for the private information case (Proposition 3.6 and Theorem 3.8) and requires more work otherwise (see Allen (1991a)).

Proposition 3.4. Fix I, $(\Omega, \mathbf{F}, \mu)$, and $\mathbf{G}_1, \ldots, \mathbf{G}_{\#I}$. Then every totally balanced NTU game is a market game for a pure exchange economy with private information having countably many commodities or with $l=2\times(\sharp I)$ and utilities that may fail to be continuous. A dense subset (for the Hausdorff topology) can be generated by economies under uncertainty with asymmetric information with $l=(\sharp I)\times(\sharp I-1)/2$ and continuous concave utilities. The open and dense subset of NTU games which are balanced with slack[6] are generated by economies with private information having continuous, concave and nondecreasing utilities and finitely many commodities.

Proof. If, in our model, all traders' endowments and utilities are taken to be constant functions of the state of the world [i.e., if for every $i\in I$ and all $\omega, \omega'\in\Omega$, we have $e_i(\omega)=e_i(\omega')$ and $u_i(\cdot;\omega)=u_i(\cdot;\omega')$], then any pure exchange economy becomes a pure exchange economy with private information and, in fact, is equivalent to the desired economy without uncertainty or asymmetric information for any arbitrary specification of the initial private information sub-σ-fields $\mathbf{G}_i$, $i\in I$. To obtain the NTU Representation Theorem, appeal to Billera (1974), Billera and Bixby (1974) and Mas-Colell (1975) in addition to the above observation. [ ]

Remark 3.5. Note that the discontinuous utilities in Proposition 3.4 arise from Rader's (1972) transformation of an economy with production to a pure exchange economy. The discontinuity is unrelated to information.

For the purposes of this paper, the converse of the representation theorem is more useful and interesting. It claims that the cooperative games arising from markets are well behaved (in exactly the sense of the assumptions needed for the representation theorems). This is the easy direction of the equivalence theorems. However, with uncertainty and private information, the hypotheses must be checked (or the claimed properties of the resultant games must be verified directly). In general, the hardest part is to derive closedness of the induced game in characteristic function form; see Allen (1991a). While one can easily show that private information implies (a) that submarkets correspond to subgames and hence balancedness of all such games implies that any game in the class is totally balanced and (b) that convex combinations of (private information) measurable allocations for players in various coalitions are indeed measurable with respect to the information that may be used by players in the grand coalition, the market games equivalence theorems permit one to bypass, for the special case of private information, the direct demonstration that the NTU game is well defined.

However, before following this program, I pause to demonstrate that things do indeed work well for the special case of finitely many or countably many states of the world.

Proposition 3.6. If every $\sigma(\bigcup_{i\in I}\mathbf{G}_i)$-measurable subset F of Ω satisfies $\mu(F)>0$, then the NTU game generated by an economy with private information is well defined and totally balanced. In particular, the induced cooperative game $V:2^I\rightarrow\mathbb{R}^{\sharp I}$ is such that for all $S\neq\phi$, $V(S)$ is a nonempty, closed, comprehensive cylinder set.

Proof. I appeal to Theorem 2.1 of Billera and Bixby (1974). To apply their equivalence result for totally balanced games and market games, I need to check that I have, for

[6] For the definition, see Mas-Colell (1975).

each trader $i \in I$, (1) a consumption set X_i which is a nonempty convex compact subset of a real Hausdorff linear topological space, (2) a utility function $\hat{u}_i$ defined on X_i which is concave and upper semicontinuous, and (3) an initial endowment $\hat{e}_i \in X_i$; since we have a pure exchange economy, $Y_i = \{0\}$ for all $i \in I$. For every $i \in I$, set $X_i = \{x_i : \Omega \to \mathbb{R}^l_+ \mid x_i(\cdot) - e_i(\cdot)$ is $\mathbf{G}_i$-measurable and for μ-almost all $\omega \in \Omega, x_i^1(\omega) \leq \sum_{j \in I} e_j^1(\omega), \ldots, x_i^l(\omega) \leq \sum_{j \in I} e_j^l(\omega)\} \subseteq \mathbf{L}^1(\Omega, \mathbf{F}, \mu; \mathbb{R}^l)$. More precisely, X_i consists of equivalence classes (under the equivalence relation of equality μ-almost surely) of measurable functions in the above set, so that X_i is indeed Hausdorff for the topology induced by the $\mathbf{L}^1$ metric. Moreover, X_i is clearly a nonempty closed convex subset of the real topological vector space $\mathbf{L}^1(\Omega, \mathbf{F}, \mu; \mathbb{R}^l)$ which is, in fact, normed and hence a metric space. Obviously $\hat{e}_i : \Omega \to \mathbb{R}^l_+$ defined by $\hat{e}_i(\omega) = e_i(\omega)$ for all $\omega \in \Omega$ specifies trader i's initial endowment and $\hat{e}_i \in X_i$.

To verify compactness, it suffices [for instance, see Theorem I.6.15 of Dunford and Schwartz (1958, p. 22)] to establish that X_i is contained in a totally bounded set—i.e., that for any $\varepsilon > 0$, X_i can be covered by finitely many open balls of radius ε. Pick $\varepsilon > 0$ arbitrarily and let $|\cdot|$ denote the sum norm on $\mathbb{R}^l$. Because μ is a probability and $X_i \subseteq \mathbf{L}^1(\Omega, \mathbf{F}, \mu; \mathbb{R}^l_+)$, for any $\delta > 0$ there is a finite integer $N(\delta)$ and an ordering of the (at most countable) set Ω such that $\mu(\Omega \backslash \{\omega_1, \ldots, \omega_{N(\delta)}\}) < \delta$. [If Ω is uncountable but $\sigma(\bigcup_{i \in I} \mathbf{G}_i)$ is generated by a countable partition—which must be true if all measurable events receive strictly positive probability—rename the sets in the partition as points in Ω to simplify notation in the argument below.] Take $\delta = \varepsilon / 4 \max_{\omega \in \Omega} |\sum_{j \in I} e_j(\omega)|$. Think of $\mathbf{L}^1(\Omega, \mathbf{F}, \mu; \mathbb{R}^l)$ as the product of a Euclidean space $\mathbb{R}^{N(\delta)l}$ (corresponding to the components of the sequence x_i representing the evaluations at $\omega_1, \ldots, \omega_{N(\delta)}$) and a space T_i of "tail" sequences. We will exhibit a finite cover for each component consisting of open sets of radius $\varepsilon/2$ so that their pairwise products cover X_i and are contained in open balls of radius ε. The projection of X_i onto its $\omega_1, \ldots, \omega_{N(\delta)}$ components is a (closed) subset of $\mathbb{R}^{N(\delta)l}_+$ which is bounded because for all $n = 1, \ldots, N(\delta)$, we have $0 \leq x_i(\omega_n) \leq \sum_{j \in I} e_j(\omega_n)$. Therefore it is compact and hence totally bounded, so that there is a finite cover $U_1, \ldots, U_N$ consisting of open balls of radius $\varepsilon/2$. Upon examining the tail, we see that if $x_i, x_i' \in X_i$, then $\sum_{N(\delta)+1}^{\infty} \mu(\omega_n) |x_i(\omega_n) - x_i'(\omega_n) \leq \delta \cdot 2 |\sum_{j \in I} e_j(\omega_n)| \leq \varepsilon/2$. This means that T_i can be covered by a single open ball of radius $\varepsilon/2$. Therefore the open sets $U_1 \times T_i, \ldots, U_N \times T_i$ form a finite open cover for X_i and each $U_n \times T_i$ is contained in an open ball in X_i of radius ε. This completes the proof that X_i is totally bounded and thus compact.

It remains to exhibit upper semicontinuous and concave utility functions on X_i. Consider the expected utilities

$$EU_i(x_i) = \int_{\Omega} u_i(x_i(\omega); \omega) d\mu(\omega) = \sum_{n=1}^{\infty} u_i(x_i(\omega_n); \omega_n) \mu(\omega_n)$$

defined for $x_i \in X_i$. [Note that $\mathbf{G}_i$-measurability of $x_i(\cdot) - e_i(\cdot)$ is built into the definition of X_i, so that we need not worry about conditional expected utility here.] Since each $u_i(\cdot; \omega)$ is assumed to be concave on $\mathbb{R}^l_+$ (see Section 2), the average must also be concave. More precisely, if $x_i, x_i' \in X_i$ and $0 \leq \lambda \leq 1$, then

$$\begin{aligned}EU_i(\lambda x_i+(1-\lambda)x_i') &= \sum_{n=1}^{\infty} u_i(\lambda x_i(\omega_n)+(1-\lambda)x_i'(\omega_n);\omega_n)\mu(\omega_n)\\ &\geq \sum_{n=1}^{\infty}[\lambda u_i(x_i(\omega_n);\omega_n)+(1-\lambda)u_i(x_i'(\omega_n);\omega_n)]\mu(\omega_n)\\ &= \lambda\sum_{n=1}^{\infty} u_i(x_i(\omega_n);\omega_n)\mu(\omega_n)+(1-\lambda)\sum_{n=1}^{\infty} u_i(x_i'(\omega_n);\omega_n)\mu(\omega_n)\\ &= \lambda EU_i(x_i)+(1-\lambda)EU_i(x_i'),\end{aligned}$$

where the inequality follows from the concavity of each $u_i(\cdot;\omega_n)$. This shows that $EU_i: X_i \to \mathbb{R}$ is a concave function. To check that $EU_i(\cdot)$ is u.s.c. on X_i, we show the stronger property that $EU_i: X_i \to \mathbb{R}$ is continuous for the $\mathbf{L}^1$ topology. Accordingly, let $\{x_i^n\}_{n=1}^{\infty}$ be a sequence in X_i with $x_i^n \to x_i$ (for the $\mathbf{L}^1$ norm) as $n\to\infty$. Then $x_i^n \to x_i$ in measure as $n\to\infty$ and, because the $u_i(\cdot;\omega)$ are assumed to be uniformly equicontinuous on compact subsets of $\mathbb{R}^l_+$, we have $u_i(x_i^n(\cdot);\cdot) \to u_i(x_i(\cdot);\cdot)$ in measure also. Since the $u_i(\cdot;\omega)$ are also uniformly bounded on compact subsets of $\mathbb{R}^l_+$, the values taken by u_i on X_i are bounded. Hence, by the extended dominated convergence theorem (see Ash (1972, p. 96)), $EU_i(x_i^n) \to EU_i(x_i)$ as $n\to\infty$. See Lemma 2.1. [Alternatively, we could observe that convergence in measure implies convergence in distribution and use the generalized Lebesgue convergence theorem; uniform integrability holds because the $u_i(x_i^n(\cdot);\cdot)$ functions are dominated by an integrable function as above from uniform boundedness of $u_i(\cdot;\omega)$ on compact subsets. See Hildenbrand (1974, pp. 51–52).] [ ]

Remark 3.7. Because we can systematically disregard a subset of states of the world of measure zero, we could generalize the hypothesis of Proposition 3.6 to permit Ω to contain some subsets of μ-measure zero whose union is also of μ-measure zero. Thus, by countable additivity, we could permit $\Omega=(\bigcup_{n=1}^{\infty} F_n)\cup(\bigcup_{n=1}^{\infty} F_n')$, where $\mu(F_n)>0$ and F_n is an atom for all $n=1,2,\ldots$, and $\mu(F_n')=0$ for all n, since then $\mu(\bigcup_{n=1}^{\infty} F_n')=0$. Note that at most countably many disjoint events can receive positive probability. Moreover, the hypothesis is indeed needed in the proof. Even if $\Omega=[0,1]$ and μ is Lebesgue measure, the $\mathbf{L}^\infty$ unit ball fails to be compact for the $\mathbf{L}^1$ topology so that we lose the required compactness of the consumption set X_i in the proof of Proposition 3.6 whenever Ω is not "essentially" finite or countable.

Proposition 3.6, combined with the NTU Representation Theorem (Proposition 3.4) establishes the (approximate) equivalence between totally balanced games and market games with private information under the additional assumption that all events occur with strictly positive probability. The result below removes this restriction on the set of measurable events.

Theorem 3.8. For arbitrary $(\Omega,\mathbf{F},\mu)$, the NTU game generated by an economy with private information is well defined [i.e., $V: 2^I \to \mathbb{R}^{\sharp I}$ is such that $V(S)$ is a nonempty closed comprehensive cylinder set for any $S\neq\phi$] and totally balanced.

Proof.[7] Billera and Bixby (1974, Theorem 2.1) give sufficient conditions for the induced

[7] A conversation with Jean-François Mertens helped me to see a way to shorten this proof.

NTU game derived from an economy to be well defined and totally balanced. I must exhibit, for each $i \in I$, (1) a consumption set X_i which is a nonempty convex compact subset of a real Hausdorff linear topological space, (2) a utility function $\hat{u}_i$ on X_i which is concave and upper semicontinuous, and (3) an initial endowment $\hat{e}_i \in X_i$. Because I have a pure exchange economy, I can define the production sets by $Y_i = \{0\}$ for all $i \in I$.

For $i \in I$, set $X_i = \{x_i : \Omega \to \mathbb{R}^l_+ | x_i(\cdot) - e_i(\cdot)$ is $\mathbf{G}_i$-measurable and for μ-almost every $\omega \in \Omega$, $x_i(\omega) \leq \sum_{j \in I} e_j(\omega)\} \subseteq \mathbf{L}^1(\Omega, \mathbf{F}, \mu; \mathbb{R}^l)$ endowed with the *weak* topology. Because endowments are nonnegative, X_i is clearly nonempty. By Dunford and Pettis (1940) [see Dunford and Schwartz (1958, Exercise IV.13.68, pp. 349–350)], since X_i is a bounded subset of $\mathbf{L}^\infty(\Omega, \mathbf{F}, \mu; \mathbb{R}^l)$, its (weak) closure is a weakly sequentially compact subset of $\mathbf{L}^1(\Omega, \mathbf{F}, \mu; \mathbb{R}^l)$. However, X_i is clearly closed for the $\mathbf{L}^1$ norm topology of the locally convex space $\mathbf{L}^1(\Omega, \mathbf{F}, \mu; \mathbb{R}^l)$, so that by Theorem 3.12 of Rudin (1973, p. 64), X_i is also weakly closed because it's convex. By the Eberlein-Smulian Theorem (see Dunford and Schwartz (1958, Theorem V.6.1, p. 430)), because $\mathbf{L}^1(\Omega, \mathbf{F}, \mu; \mathbb{R}^l)$ is a Banach space, weak compactness and weak sequential compactness are equivalent. Hence X_i is nonempty, compact, and convex. Moreover, $\mathbf{L}^1(\Omega, \mathbf{F}, \mu; \mathbb{R}^l)$ is a real Hausdorff [see Rudin (1973, (b), p. 61) and observe that the continuous linear functionals separate points by considering integration against the $\mathbf{L}^\infty$ indicator function of the set of positive measure on which two distinct elements of $\mathbf{L}^1$ differ or note that the Banach space $\mathbf{L}^1(\Omega, \mathbf{F}, \mu; \mathbb{R}^l)$ is locally convex] linear topological space.

Set $\hat{e}_i = e_i$, where $e_i : \Omega \to \mathbb{R}^l_+$ is $\mathbf{F}$-measurable. Then $\hat{e}_i \in X_i$ as required.

The utilities $\hat{u}_i = EU_i : X_i \to \mathbb{R}$ are concave and $\mathbf{L}^1$ continuous by Lemma 2.1. I claim that they are weakly upper semicontinuous, which means that for any $\bar{u} \in \mathbb{R}$, $\{\hat{x}_i \in X_i | \hat{u}_i(\hat{x}_i) \geq \bar{u}\}$ is weakly closed. By the $\mathbf{L}^1$ continuity of $\hat{u}_i$, $\{\hat{u}_i \in X_i | \hat{u}_i(\hat{x}_i) \geq \bar{u}\}$ is closed for the $\mathbf{L}^1$ norm topology; it's convex by the concavity of $\hat{u}_i$. By Theorem 3.12 of Rudin (1973, p. 64), $\{\hat{x}_i \in X_i | \hat{u}_i(\hat{x}_i) \geq \bar{u}\}$ is therefore weakly closed. Hence $\hat{u}_i$ is weakly u.s.c. and concave on X_i. [ ]

Remark 3.9. Theorem 3.8 subsumes Proposition 3.6 in that the theorem does not exclude the case of at most countably many states of the world. The fact that the finite dimensional space $\mathbb{R}^{\#\Omega l}$ admits a unique topology which makes it into a real topological vector space implies that, for finite Ω, the proofs must be equivalent in that the topological space X_i is uniquely determined. Similarly, if Ω is countable (or, more generally, if the probability space $(\Omega, \mathbf{F}, \mu)$ consists of countably many atoms plus a set of μ-measure zero), the proofs are equivalent because the weak and strong $\mathbf{L}^1$ topologies are the same. See Dunford and Schwartz (1958, IV.8.13, p. 295 and IV.13.49, pp. 346–347).

Now return to the issue of the core with private information. The *core* of an NTU game $V : 2^I \to \mathbb{R}^{\#I}$ is the set of all payoff vectors $(w_1, \ldots, w_{\#I}) \in \mathbb{R}^{\#I}$ such that $(w_1, \ldots, w_{\#I}) \in V(I)$ (feasibility) and there does not exist a coalition $S \subseteq I$ and $(w'_1, \ldots, w'_{\#I}) \in V(S)$ such that $w'_i > w_i$ for all $i \in S$ (coalition S cannot block).

Definition 3.10. The *core* of an economy with private information consists of all state-dependent allocations $(x_1, \ldots, x_{\#I})$ where $x_i : \Omega \to \mathbb{R}^l_+$ for each $i \in I$ such that

(i) $\sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i(\omega)$ for (almost) all $\omega \in \Omega$.

(ii) each $x_i(\cdot)$ is such that $x_i(\cdot) - e_i(\cdot)$ is $\mathbf{G}_i$-measurable, and

(iii) there does not exist a coalition S ($\phi \neq S \subseteq I$) and allocations $x'_i : \Omega \to \mathbb{R}^l_+$ for $i \in S$ such that $\sum_{i\in S} x_i'(\omega)=\sum_{i\in S} e_i(\omega)$ for (almost) all $\omega\in\Omega$, each $x_i'(\cdot)-e_i(\cdot)$ is $\mathbf{G}_i$-measurable, and $EU_i(x_i')>EU_i(x_i)$ for all $i\in S$.

This is the usual concept of the core of a (pure exchange) economy except for the informational constraints that each net trade defining a core allocation must be $\mathbf{G}_i$-measurable and that blocking must be accomplished via net trades that are $\mathbf{G}_i$-measurable for each member i of the blocking coalition S. Recall that my commodity space consists of (ex ante) state-dependent commodity bundles derived from state-dependent net trades that are measurable with respect to private information. Moreover, payoffs are given by the ex ante expected utilities associated with these state-dependent allocations.

The set of *core imputations* consists of those payoff vectors $(EU_1(x_1(\cdot)),\ldots,EU_{\sharp I}(x_{\sharp I}(\cdot)))\in\mathbb{R}^{I\sharp}$ where $(x_1,\ldots,x_{\sharp 1}):\Omega\rightarrow\mathbb{R}_+^{\sharp Il}$ belongs to the core. Note that the set of core imputations of an economy equals the core of the market game induced by the economy.

Proposition 3.11. The private information core is nonempty.

Proof. By Theorem 3.8 (or Proposition 3.6 if Ω is at most countable) the market game is totally balanced. This implies that, by Scarf's (1967) theorem, the core of any private information market game is nonempty. [ ]

As a consequence of the market games approach, I then have an alternative proof of Yannelis's (1991) core nonemptiness theorem which does not directly require infinite-dimensional commodity space techniques. Note that Yannelis (1991) assumes that every event occurs with strictly positive probability; I do not need this assumption.

IV. *Publicly Predictable Information*

The concept of publicly predictable information (p.p.i.) was introduced by Blume and Easley (1990), Postlewaite and Schmeidler (1986, 1987), and Palfrey and Srivastava (1986) to study the possibilities for implementation of rational expectations equilibrium.[8] While these authors assume finitely many states of the world and use somewhat different definitions (and focus on necessary versus sufficient conditions for implementation), p.p.i. means that a given agent's (private) information can be learned from the pooled information of all other agents. If this is satisfied, one can clearly force truthful disclosure of the information by threatening to kill the trader if he lies or if he fails to tell everything that he knows (although agents are assumed not to attempt to pretend to know more than they actually do). Thus, p.p.i. appears to take at least a crude first step toward addressing agents' decisions to fully and truthfully reveal information to their coalitions, as publicly predictable information is verifiable in a sense to be defined below.

I advocate that the information available to a coalition be taken to consist of the publicly predictable information. More precisely, each member may use exactly that portion of his private information that is contained in the pooled information of all other coalition

[8] They do not consider the core or other cooperative game-theoretic concepts. Note also that some of this literature uses the term nonexclusivity.

members. In symbols, *p.p.i.* is defined to be $\mathbf{G}_i \cap \sigma(\bigcup_{\substack{j \in S \\ j \neq i}} \mathbf{G}_j)$ for the coalition S containing player i. As will be shown later, this has attractive verifiability properties. Moreover, publicly predictable information implies that coalitions are generally asymmetrically informed. The publicly predictable information structure lies between those defining the coarse and fine cores. It also gives an alternative characterization of the Yannelis (1991) private information core. This observation immediately leads to nice incentive properties for the private information core.

Definition 4.1. The *p.p.i. core* of a market game is the core of the NTU game induced by the pure exchange economy with asymmetric information in which each member of a coalition is restricted to publicly predictable information. The *p.p.i. core of an economy* with asymmetric information consists of those state dependent allocation functions $(x_1, \ldots, x_{\sharp I})$, where $x_i : \Omega \to \mathbb{R}^l_+$ for $i \in I$, such that

(i) $\sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i(\omega)$ for (almost) all $\omega \in \Omega$,

(ii) each $x_i(\cdot) - e_i(\cdot)$ is $\mathbf{G}_i \cap \sigma(\bigcup_{\substack{j \in I \\ j \neq i}} \mathbf{G}_j)$-measurable, and

(iii) there does not exist a coalition $S(\phi \neq S \subseteq I)$ and allocations $x'_i : \Omega \to \mathbb{R}^l_+$ for $i \in S$ such that $\sum_{i \in S} x'_i(\omega) = \sum_{i \in S} e_i(\omega)$ for almost all $\omega \in \Omega$, each $x'_i(\cdot) - e_i(\cdot)$ is $\mathbf{G}_i \cap \sigma(\bigcup_{\substack{j \in S \\ j \neq i}} \mathbf{G}_j)$-measurable, and $EU_i(x'_i) > EU_i(x_i)$ for all $i \in S$.

An important insight is the equality of the private information core (my term) studied by Yannelis (1991) and the p.p.i. core. The p.p.i. core is defined by the property that any feasible allocation to a coalition S (and therefore any core allocation or any allocation that blocks) must have the property that, for each member $i \in S$, i's net trade $x_i(\cdot) - e_i(\cdot)$ at the allocation $x_i : \Omega \to \mathbb{R}^l_+$ is measurable with respect to the p.p.i. sub-σ-field $\mathbf{G}_i \cap \sigma(\bigcup_{\substack{j \in S \\ j \neq i}} \mathbf{G}_j)$ of $\mathbf{F}$. To see the equivalence, note that feasibility (implicitly, we assume no free disposal which doesn't seem to be a loss whenever utilities are strictly monotone on $\mathbb{R}^l_+$ for all $i \in I$ and all $\omega \in \Omega$ requires $\sum_{j \in S} x_i(\omega) = \sum_{j \in S} e_i(\omega)$ so that we must have

$$x_i(\omega) - e_i(\omega) = \sum_{\substack{j \in S \\ j \neq i}} e_j(\omega) - \sum_{\substack{j \in S \\ j \neq i}} x_j(\omega).$$

In other words, the left side is $\mathbf{G}_i$-measurable and the right side is $\sigma(\bigcup_{\substack{j \in S \\ j \neq i}} \mathbf{G}_j)$-measurable, so that both sides are $\mathbf{G}_i \cap \sigma(\bigcup_{\substack{j \in S \\ j \neq i}} \mathbf{G}_j)$-measurable (publicly predictable information). Feasibility requires that each $i \in S$ may use only the portion of his initial information which can be verified from the pooled private information of the rest of the coalition S. More formally, we have argued the following:

Proposition 4.2. The private information core is characterized by publicly predictable information.

Corollary 4.3. The p.p.i. core is nonempty for arbitrary sets of states of the world and arbitrary σ-fields.

Proof. Combine Proposition 4.2 with Proposition 3.11 [ ]

A major reason for my interest in publicly predictable information and its equivalence to the private information scheme (in which each agent may use only his own initial information, regardless of the coalition to which he belongs) is its verification feature. Under p.p.i. or private information, no unilateral deviation by a single coalition member can fail to be detected by the remaining coalition members. This means that such agreements—net trades that are measurable with respect to p.p.i. or, equivalently, private information—can be enforced by the coalition. Thus I have derived the set of binding agreements naturally for the cooperative game rather than imposing commitment as part of the game-theoretic paradigm.

Definition 4.4. The state-dependent $\mathbf{G}_j$-measurable net trade vector $z_j:\Omega\to\mathbb{R}^l$, where $j\in S$, is *private Nash verifiable* for coalition S if $z_j(\cdot)$ is $\sigma(\bigcup_{\substack{i\in S\\ i\neq j}}\mathbf{G}_i)$-measurable.

Proposition 4.5. Net trades in the p.p.i. or private information core (i.e., those $z_i:\Omega\to\mathbb{R}^l$ such that $x_i(\cdot)=z_i(\cdot)+e_i(\cdot)$ is a core allocation) satisfy private Nash verifiability for the grand coalition. Moreover, any blocking allocation must give rise to net trades that are private Nash verifiable for the blocking coalition.

Proof. By Proposition 4.2 and the definition of p.p.i., $z_i:\Omega\to\mathbb{R}^l$ satisfying (ii) in the definition of the core (Definitions 3.10 and 4.1) must be $\mathbf{G}_i\cap\sigma(\bigcup_{j\neq i}\mathbf{G}_j)$-measurable. Part (iii) of the definition yields the analogous requirement that if S blocks $x_1(\cdot),\ldots,x_{\#I}(\cdot)$ by the alternative allocation $x_i':\Omega\to\mathbb{R}^l_+$ for $i\in S$, then $z_i'(\cdot)=x_i'(\cdot)-e_i(\cdot)$ must be $\mathbf{G}_i\cap\sigma(\bigcup_{\substack{j\in S\\ j\neq i}}\mathbf{G}_j)$-measurable. Hence $z_i(\cdot)$ is $\sigma(\bigcup_{j\neq i}\mathbf{G}_j)$-measurable (because $\sigma(\bigcup_{j\neq i}\mathbf{G}_j)\supseteq\mathbf{G}_i\cap\sigma(\bigcup_{j\neq i}\mathbf{G}_j)$) and $z_i'(\cdot)$ must be $\sigma(\bigcup_{\substack{j\in S\\ j\neq i}}\mathbf{G}_j)$-measurable (since $\sigma(\bigcup_{\substack{j\in S\\ j\neq i}}\mathbf{G}_j)\supseteq\mathbf{G}_i\cap\sigma(\bigcup_{\substack{j\in S\\ j\neq i}}\mathbf{G}_j)$). [ ]

V. *General Information Sharing Rules and Strong Verification*

In place of deviations from the agreement by a single coalition member, one may wish to consider the possibility for an arbitrary subset to violate its commitments.[9] This motivates the definition of alternative versions of strong verification as well as the specification of information sharing rules for the results that follow. However, all of the verification concepts that I consider involve the issue of whether other players have access to at least as much (in the sense of set-theoretic containment of information sub-σ-fields) information as the information that deviating players have. If this ordering is satisfied, net trade commit-

[9] Cremer (1991) considers robustness to deviations by coalitions containing two or three players in a Groves mechanism that assumes enforcable agreements.

ments can also be verified because net trade strategies must be measurable with respect to the own information of (potentially deviating) players. In this sense, verifiability of information implies verifiability of contracts, so that agreements are indeed binding.

Notice first that, for arbitrary subsets of a coalition and more general coalition information conditions than private information (or p.p.i.), the choice of information assignments to a split coalition is not obvious. Can the defecting group use all of the information that was available to them in the larger coalition and similarly, can the remaining players use all of their previous information? The question did not arise in Section 4 because private information is defined without explicit reference to coalitions. Here this issue necessitates the consideration of several variants of strong verification and several versions of Nash verification.

The information available for use by members of a coalition can differ from their initial information. In particular, this can depend on the coalition to which a player belongs. To capture these generalizations in an n-player game, define 2^n-1 mappings associating m-tuples (for $0<m\leq n$) of initial information structures to m-tuples of information structures describing the information that coalition members may use. For notational completeness, I define the information of the empty set to be the trivial σ-field $\{\Omega,\phi\}$.

Definition 5.1. An *information sharing rule* is a collection $F=\{f(S)|S\subseteq I\}$ of $2^{\sharp I}$ mappings for an economy (or game) with asymmetric information. Let $\mathbf{F}^{**}$ denote the set of all sub-σ-fields of $\mathbf{F}$. Then, for $S\subseteq I$, $S\neq\phi$, we have

$$f(S):\underbrace{\mathbf{F}^{**}\times\ldots\times\mathbf{F}^{**}}_{\sharp S\text{ times}}\rightarrow\underbrace{\mathbf{F}^{**}\times\ldots\times\mathbf{F}^{**}}_{\sharp S\text{ times}}$$

written as, if $S=\{s(1),\ldots,s(\sharp S)\}$,

$$f(S)(\mathbf{G}_{s(1)},\ \ldots,\ \mathbf{G}_{s(\sharp S)})=(\mathbf{H}_{s(1)},\ \ldots,\ \mathbf{H}_{s(\sharp S)})$$

where, for each $i\in S$, $\mathbf{H}_i$ is a sub-σ-field of $\mathbf{F}$. For $S=\phi$, set $f(\phi)=\{\Omega,\phi\}$. Write $f(S)^i$ for player i's information in coalition S if $i\in S$. A *measurable allocation for S* is a state-dependent allocation $(x_{s(1)}(\cdot),\ldots,x_{s(\sharp S)}(\cdot))$, where each $x_i:\Omega\rightarrow\mathbb{R}^l_+$ for $i\in S$ is such that $z_i(\cdot)=x_i(\cdot)-e_i(\cdot)$ is $f(S)^i$-measurable.

Several concrete examples of information sharing rules are obvious and interesting. However, note that the definition permits all arbitrary possibilities. Moreover, there need not be any relation among the information sharing rules used by different coalitions, even those that are subsets or supersets of one another, and the definition does not require that a coalition's information be nontrivially related to the initial information of its members.

Example 5.2. The *private information sharing rule* is the (unique) information sharing rule $F_p=\{f_p(S)|S\subseteq I\}$ satisfying, for each $S\neq\phi$,

$$f_p(S)(\mathbf{G}_{s(1)},\ \ldots,\ \mathbf{G}_{s(\sharp S)})=(\mathbf{G}_{s(1)},\ \ldots,\ \mathbf{G}_{s(\sharp S)}).$$

In other words, $f(S)(\cdot)$ is the identity mapping for all $S\subseteq I(S\neq\phi)$ if and only if F_p is the private information sharing rule.

Example 5.3. The *p.p.i. information sharing rule* is the (unique) information sharing rule $F_{p.p.i.}=\{f_{p.p.i.}(S)|S\subseteq I\}$ satisfying, for each $S\neq\phi$,

$$f_{p.p.i.}(S)(\mathbf{G}_{s(1)},\ldots,\mathbf{G}_{s(\sharp S)})=(\mathbf{G}_{s(1)}\cap\sigma(\bigcup_{\substack{i\in S\\ i\neq s(1)}}\mathbf{G}_i),\ldots,\mathbf{G}_{s(\sharp S)}\cap\sigma(\bigcup_{\substack{i\in S\\ i\neq s(\sharp S)}}\mathbf{G}_i)).$$

Example 5.4. The *coarse information sharing rule* is the (unique) information sharing rule $F_c=\{f_c(S)|S\subseteq I\}$ satisfying, for each $S\neq\phi$,

$$f_c(S)(\mathbf{G}_{s(1)},\ldots,\mathbf{G}_{s(\sharp S)})=(\bigcap_{i\in S}\mathbf{G}_i,\ldots,\bigcap_{i\in S}\mathbf{G}_i).$$

This is a legitimate information sharing rule because the intersection of sub-σ-fields is itself a sub-σ-field of $\mathbf{F}$.

Example 5.5. The *constant or* **H**-*information sharing rule* is an information sharing rule $F_H=\{f_H(S)|S\subseteq I\}$ in which, for all $S\subseteq I$, $S\neq\phi$, $f(S)$ is the constant mapping equal to the $\sharp S$-tuple $(\mathbf{H},\ldots,\mathbf{H})$, or, in other words, $f_H(S)^i=\mathbf{H}$ for all $i\in S$. Special cases include *null* information $f_\phi(S)^i=\{\Omega,\phi\}$, *common* information $f(S)^i=\bigcap_{j\in I}\mathbf{G}_j$, pooled information $f(S)^i=\sigma(\bigcup_{j\in I}\mathbf{G}_j)$, and *full* information $f_F(S)^i=\mathbf{F}$.

The analogue of Proposition 3.8 stating that the NTU game is well defined holds for any information sharing rule. More precisely, if $F=\{f(S)|S\subseteq I\}$ is an information sharing rule, define the correspondence $V:2^I\rightarrow\mathbb{R}^{\sharp I}$ by $V(\phi)=\{0\}$ and for any $S\subseteq I$, $S\neq\phi$, $V(S)=\{(w_1,\ldots,w_{\sharp I})\in\mathbb{R}^{\sharp I}|$ for $i\in S$, there exist $x_i:\Omega\rightarrow\mathbb{R}^l_+$ with $w_i\leq\int_\Omega u_i(x_i(\omega);\omega)d\mu(\omega)$ such that $z_i(\cdot)=x_i(\cdot)-e_i(\cdot)$ is $f(S)^i$-measurable and $\sum_{i\in S}x_i(\omega)=\sum_{i\in S}e_i(\omega)$ for almost all $\omega\in\Omega\}$. Then for any $S\neq\phi$, $V(S)$ is a nonempty closed comprehensive cylinder set which is moreover convex. For a proof, see Theorem 4.1 in Allen (1991a).

The next series of definitions sets out several important properties of (some) information sharing rules. The last one leads to a sufficient condition for nonemptiness of the core of a pure exchange economy with asymmetric information and an exogenously given information sharing rule.

Definition 5.6. The information sharing rule $F=\{f(S)|S\subseteq I\}$ is *divisible* if for all $S\subseteq I$ and all nonempty $S'\subset S$, $\sigma(\bigcup_{i\in S'}f(S')^i)=\sigma(\bigcup_{i\in S}f(S)^i)$. In other words, $\sigma(\bigcup_{i\in S}f(S)^i)$ must be independent of $S\subseteq I$.

Definition 5.7. The information sharing rule $F=\{f(S)|S\subseteq I\}$ is *symmetric* if for all $S\subseteq I$ and all $i,j\in S$, $f(S)^i=f(S)^j$.

Definition 5.8. An information sharing rule $F=\{f(S)|S\subseteq I\}$ is *bounded* if $\bigcup_{S\ni i}f(S)^i\subseteq f(I)^i$ for all $i\in I$; it is *nested* if $\bigcup_{\substack{T\ni i\\ T\subseteq S}}f(T)^i\subseteq f(S)^i$ whenever $i\in S$.

Of the above examples, only constant information shairng is divisible. Both coarse and constant information sharing are symmetric whereas private and p.p.i. are not. In more general terms, only a "few" information sharing rules are symmetric. All except the coarse information sharing rule are nested (and therefore bounded).

Remark 5.9. Games generated by economies with bounded information sharing rules are balanced and those derived from nested information sharing rules are totally balanced. Hence either condition implies that the core is nonempty. However, coarse information sharing can lead to an empty core. See Allen (1991b) for details, including a discussion of the differences between these core concepts and the definition proposed by Wilson (1978).

Definition 5.10. The *core of an economy with asymmetric information* under the given information sharing rule $F=\{f(S)|S\subseteq I\}$ consists of those state-dependent allocations $(x_1(\cdot),\ldots,x_{\sharp I}(\cdot))$, where $x_i:\Omega\to\mathbb{R}^l_+$ for $i\in I$, satisfying

(i) $\sum_{i\in I} x_i(\omega)=\sum_{i\in I} e_i(\omega)$ for almost all $\omega\in\Omega$,

(ii) $z_i:\Omega\to\mathbb{R}^l_+$ defined by $z_i(\cdot)=x_i(\cdot)-e_i(\cdot)$ is $f(S)^i$-measurable, and

(iii) there does not exist a coalition S $(S\subseteq I, S\neq\phi)$ and state-dependent allocations $x_i':\Omega\to\mathbb{R}^l_+$ for $i\in S$ such that $\sum_{i\in S} x_i'(\omega)=\sum_{i\in S} e_i(\omega)$ for almost all $\omega\in\Omega$, $z_i'(\cdot)=x_i'(\cdot)-e_i(\cdot)$ is $f(S)^i$-measurable, and $\int_\Omega u_i(x_i'(\omega);\omega)d\mu(\omega)>\int_\Omega u_i(x_i(\omega);\omega)d\mu(\omega)$ for all $i\in S$.

With the above preliminaries complete, I now define verification with respect to deviations by an arbitrary subset of a coalition. Notice that these concepts generalize and extend private Nash verification in that the set of deviating players need not be a singleton and the information sharing rule need not be private or p.p.i.

Definition 5.11. The allocation $(x_{s(1)}(\cdot),\ldots,x_{s(\sharp S)}(\cdot))$ for coalition S, where $x_i:\Omega\to\mathbb{R}^l_+$ for each $i\in S$, satisfies *strong verifiability* if for all nonempty subsets S' and S'' of S with $S'\cap S''=\phi$ and $S'\cup S''=S$, for all $i\in S'$, $z_i(\cdot)=x_i(\cdot)-e_i(\cdot)$ is $\sigma(\bigcup_{j\in S''} f(S'')^j)$-measurable.

Definition 5.12. The allocation $(x_{s(1)}(\cdot),\ldots,x_{s(\sharp S)}(\cdot))$ for coalition S, where $x_i:\Omega\to\mathbb{R}^l_+$ for each $i\in S$, satisfies *modified strong verifiability* if for all nonempty subsets S' and S'' of S with $S'\cap S''=\phi$ and $S'\cup S''=S$, for all $i\in S'$, $z_i(\cdot)=x_i(\cdot)-e_i(\cdot)$ is $\sigma(\bigcup_{j\in S''} f(S)^j)$-measurable.

The difference between strong verification and modified strong verification lies in the information assigned to remaining players. For strong verification, the reduced coalition may only use that information available to it under the information sharing rule. On the other hand, modified strong verification permits nondeviating players to use all of the information that they have in the larger coalition. If the information sharing rule is monotonic in the sense that $f(S)^i\subseteq f(T)^i$ whenever $i\in S\subset T$, then modified strong verification is weaker than strong verification in the sense that strong verification implies modified strong verification in this case. For deviations by a singleton with private information sharing or p.p.i., both definitions reduce to private Nash verification; they are the same because private is equivalent to publicly predictable information and $f(S)^i=\mathcal{G}_i$ whenever $i\in S$, so that $f(S)^i$ is independent of $S\ni i$ for private information sharing. However, in general, private or p.p.i. violates both strong and modified strong verifiability [which are the same for private information sharing because $f(S)^i$ does not depend on S $(S\ni i)$] for larger coalitions so that these properties are genuinely distinct from private Nash verifiability.

Proposition 5.13. For any coalition S, constant information sharing leads to measurable allocations for S that satisfy strong verifiability and modified strong verifiability.

Proof. Trivial from the definitions, as all measurability requirements involve the same σ-field. [ ]

Proposition 5.14. For any coalition S, measurable allocations for S under coarse information sharing satisfy strong verification and modified strong verification.

Proof. For any $i \in S$, $i \in S'$ with $S''=S \backslash S'$, $f(S)^i = \bigcap_{j \in S} \mathbf{G}_j$, $\sigma(\bigcup_{j \in S''} f(S)^j) = \bigcap_{j \in S} \mathbf{G}_j$, and $\sigma(\bigcup_{j \in S''} f(S'')^j) = \bigcap_{j \in S''} \mathbf{G}_j$. Since $\bigcap_{j \in S''} \mathbf{G}_j \supseteq \bigcap_{j \in S} \mathbf{G}_j$ whenever $S'' \subset S$, strong verifiability holds. Modified strong verifiability follows from $f(S)^i = \bigcap_{j \in S} \mathbf{G}_j = \sigma(\bigcup_{k \in S''} \bigcap_{j \in S} \mathbf{G}_j) = \sigma(\bigcup_{k \in S''} f(S)^k)$. [ ]

Proposition 5.15. If F is divisible, or if F is symmetric and also nonincreasing in the sense that $S \subset T$ implies $f(S)^i \supseteq f(T)^i$ for all $i \in S$, then for any coalition S, all measurable allocations for S are strongly verifiable.

Proof. Strong verification requires $f(S)^i \subseteq (\bigcup_{j \in S''} f(S'')^j)$ for all $i \in S$ and all $S'' \subset S$ with $i \notin S''$. This is clearly satisfied whenever $\sigma(\bigcup_{j \in T} f(T)^j)$ is independent of T. If F is symmetric, the condition reduces to $f(S)^i \subseteq f(S'')^j$ for $S'' \subset S$ with $i \in S$ and $j \in S''$, which is equivalent to nonincreasingness whenever F is symmetric. [ ]

Remark 5.16. Since constant information sharing is divisible and coarse information sharing is both symmetric and nonincreasing, the strong verification conclusions of Propositions 5.13 and 5.14 can be regarded as corollaries to Proposition 5.15.

Proposition 5.17. Symmetry of F is a necessary and sufficient condition for the modified strong verifiability of all measurable allocations for any coalition S.

Proof. Modified strong verification requires $f(S)^i \subseteq \sigma(\bigcup_{j \in S''} f(S)^j)$, which is automatically satisfied whenever F is symmetric. Hence, symmetry of F is sufficient. For necessity, note that if F is not symmetric, then there is a coalition S, a nonempty subset $S'' \subset S$ and a player $i \in S \backslash S''$ such that $f(S)^i \not\subset \sigma(\bigcup_{j \in S''} f(S)^j)$. Therefore one can construct a $f(S)^i$-measurable allocation for i which fails to be $\sigma(\bigcup_{j \in S''} f(S)^j)$-measurable. [ ]

Remark 5.18. The analogue to Remark 5.16 applies to modified strong verification, so that this property holds for constant and coarse information sharing (in Propositions 5.13 and 5.14) as a consequence of Proposition 5.17. However, note that many additional information sharing rules satisfy symmetry; for example, consider $f(S)^i = \sigma(\bigcup_{j \in S} \mathbf{G}_j)$ for all $i \in S \subseteq I$ (fine information sharing).

Corollary 5.19. With p.p.i. or private information sharing, there exist pure exchange economies with asymmetric information, coalitions, and measurable allocations which violate strong verifiability and modified strong verifiability.

Proof. Private information sharing and p.p.i. are not symmetric except in trivial cases.

Hence the necessity for modified strong verification in Proposition 5.17, combined with the observation that strong verification implies modified strong verification because the private information sharing rule is monotonic, yields the result. [ ]

Returning now to the issue of singleton deviators in Section 4, one can extend the notion of private Nash verifiability to Nash verifiability and modified Nash verifiability. These concepts consist simply of strong and modified strong verification applied to single player deviations.

Definition 5.20. The allocation $(x_{s(1)}(\cdot),\dots,x_{s(\sharp S)}(\cdot))$ for coaliation S, where $x_i:\Omega\to\mathbb{R}^l_+$ for each $i\in S$, satisfies *Nash verifiability* if for all players $i\in S$, $z_i(\cdot)=x_i(\cdot)-e_i(\cdot)$ is $\sigma(\bigcup_{j\in S\setminus i} f(S\setminus i)^j)$-measurable and *modified Nash verifiability* if $z_i(\cdot)$ is $\sigma(\bigcup_{j\in S\setminus i} f(S)^j)$-measurable for all $i\in S$.

The next result shows that modified Nash verifiability is indeed an extremely weak requirement for feasible (and hence core and blocking) allocations whenever there is no free disposal. This follows from the observation that if net trades must sum to zero, then any individual's net trade is measurable with respect to the same information as the total net trade of all other players (all other coalition members for blocking allocations) since these functions differ precisely by a factor of multiplication by minus one.

Proposition 5.21. For any information sharing rule, all core allocations are modified Nash verifiable for the grand coalition and all blocking allocations are modified Nash verifiable for the blocking coalition. In fact, all feasible allocations (with no free disposability) are modified Nash verifiable. Moreover, symmetry of the information sharing rule is a sufficient condition for the modified Nash verifiability of all measurable (and not necessarily feasible) allocations for any coalition S.

Proof. The first two statements follow from the observation that $\sum_{i\in S} z_i(\omega)=0$ for almost all $\omega\in\Omega$ implies that both $z_i(\cdot)$ and $\sum_{j\in S\setminus i} z_j(\cdot)$ must be measurable with respect to the same σ-field. Hence modified Nash verifiability of core and blocking allocations is automatic, and similarly for any feasible allocation. The third statement follows from Proposition 5.17. [ ]

Remark 5.22. An interpretation of the first statement of Proposition 5.21 and its proof is to think of publicly predictable information with agents' initial information $\mathbf{G}_i$ replaced by $f(S)^i$ for a given coalition S. Then the equivalence between the p.p.i. and private information cores gives the result. Alternatively, it follows from the results in Section 4 for private Nash verification.

Remark 5.23. Symmetry is not necessary for the modified Nash verifiability of all measurable allocations for all coalitions. Indeed, a weaker sufficient condition is that in any coalition S with $\sharp S\geq 2$ there are players $i,j\in S$, $i\neq j$, with $f(S)^i=f(S)^j$; every coalition (except singletons) has duplicate information. More generally, the information sharing rule must satisfy a publicly predictable information property in the form that $f(S)^i=$

$f(S)^i \cap \sigma(\bigcup_{\substack{j\in S\\ j\neq i}} f(S)^j)$ for all $i \in S \subseteq I$. This is automatic for core and blocking allocations. However, note that p.p.i. is not equivalent to private information for arbitrary (measurable) allocations, so that this variant of p.p.i. does not hold automatically for all information sharing rules (except at a core or blocking allocation or for feasible allocations with no free disposal). Hence the proposition cannot be strengthened to assert modified Nash verifiability at arbitrary measurable allocations.

Proposition 5.24. If the information sharing rule is divisible or if it is symmetric and nonincreasing, then all measurable allocations for any coalition S are Nash verifiable.

Proof. This follows as a corollary to Proposition 5.15. [ ]

Remark 5.25. In Proposition 5.24, the hypothesis of a divisible information sharing rule could be replaced by the weaker condition that $f(S)^i \subseteq \sigma(\bigcup_{\substack{j\in S\\ j\neq i}} f(S\backslash i)^j)$ for all $i \in S$ or, equivalently, $f(S\cup\{i\})^i \subseteq \sigma(\bigcup_{j\in S} f(S)^j)$ whenever $i \notin S$. When the requirements for divisibility are weakened in this way by considering only sets S' which are singletons, $\sigma(\bigcup_{i\in S} f(S)^i)$ need not be independent of S (as it was in Definition 5.6). For example, consider $I=\{1,2,3\}$, $\Omega=\{a,b\}$, and $\mathbf{G}_1=\mathbf{G}_2=\{\Omega,\{a\},\{b\},\phi\}$, $\mathbf{G}_3=\{\Omega,\phi\}$, with $f(S)^i=\sigma(\bigcup_{j\in S}\mathbf{G}_j)$, so that $\mathbf{G}_i \subset \mathbf{G}_j \cup \mathbf{G}_k$ for all $i\neq j$, $j\neq k$, $i\neq k$ but $\{\Omega,\phi\}=f(\{3\})^3 \neq f(S)^i=\{\Omega,\{a\},\{b\},\phi\}$ for all $i\in S$ and $S\neq\{3\}$.

Remark 5.26. If one wishes to ignore events of measure zero in the definition of verification, then the properties of information sharing rules should all be stated in terms of completions of the various sub-σ-fields of the (complete) σ-field $\mathbf{F}$.

VI. *Discussion*

This paper suggests a first step toward the inclusion of partial enforcement or limited commitment in games. With Nash verifiability, compliance by singletons can be monitored and thus any coalition can detect deviations from the agreed strategies by a single member. Consequently, private information or publicly predictable information sharing has an attractive decentralization property. On the other hand, if one desires to strengthen the requirement to strong verifiability, then few (interesting) information sharing schemes can meet the test in general, although specific examples may exhibit better performance. This phenomenon is reminiscent of the incentives situation, in that Nash implementation is relatively easy while strong implementation tends to be unlikely, just as the existence of Nash equilibria is fairly easy while it is difficult to have strong equilibria.

My analysis deliberately mixes cooperative and noncooperative concepts. The notions of Nash and strong verifiability are inherently noncooperative in nature as they inquire about strategic deviations given players' strategy choices agreed upon by the coalition, so that they are the analogues of Nash equilibrium and strong equilibrium in a noncooperative game. Yet the ideas of coalitions and agreements only make sense in cooperative games as they rely on the ability to communicate and to form commitments.

Cooperative games with asymmetric information seem to lead naturally to such games that fall strictly between the cooperative and noncooperative paradigms. Contracts made under asymmetric information tend to be incomplete or limitedly enforceable. These are interesting problems for which the restriction to publicly predictable information use offers some help.

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