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SPECULATIVE EQUILIBRIA AND TECHNICAL ANALYSIS*

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Abstract

The existence of speculative equilibria is reviewed in a simple overlapping generations, infinite horizon economy. In equilibrium, all agents bid for assets based on increasing functions of non-fundamental, private information. This is a unique best response to the strategies of the other agents, which implies that speculative information is valuable. Technical analysis is also demonstrated. Agents chart past prices and change their bidding behavior if they observe two consecutive large upward or donwward price movements. This leads to large bubbles and crashes in the market with small probability.

I. *Introduction*

> They are concerned, not with what an investment is really worth to a man who buys it "for keeps," but with what the market will value it at, under the influence of mass psychology, three months or a year hence. Moreover, this behavior is not the outcome of a wrong-headed propensity.
>
> John Maynard Keynes, *The General Theory* (1936)

It has long been recognized that it may be rational to trade based on non-fundamental information. This idea has more recently been explored in some detail by the sunspots literature.[1] A model developed in Jackson and Peck (1991) extends the sunspots literature by decentralizing the information process. Rather than requiring that all agents observe the same "sunspot," agents have their own perceptions of market psychology. Imperfect correlation among this information leads information to be valuable to individual agents. For such information structures there exist equilibria which are *strict*, meaning that each agent's unique best response to the actions of the other agents is to bid based on the private "speculative" information. Thus, information results in positive rents to individual agents.

In equilibrium, prices increase over time, even when agents are risk neutral and there

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[1] An incomplete list includes Shell (1977), Azariadias (1981), Cass and Shell (1983), Peck (1988), Spear (1989), Peck and Shell (1991).

is no fundamental value to the asset. This fact sheds some light on the equity premium puzzle identified in the literature [see, for instance, Mehra and Prescott (1985) and Hansen and Jaganathan (1991)]. Speculative information is valuable since it helps agents predict the equilibrium bids of other agents. This value is reflected in an expected return which is unrelated to the fundamental value. The use of speculative information also raises the variance of prices above their fundamental variation, which helps shed light on another puzzle: excess volatility [see Flood and Hodrick (1990) for a recent survey].

The motivation behind this model is to provide a paradigm for the study of the interrelation of fundamental information, speculative information, and information extracted from prices. A number of characteristics are important in order to have a model in which agents trade based on speculative information, and at the same time are fully rational and earn economic rents from their speculation.

First, it is important that the economy has overlapping generations and an infinite time horizon. Together these imply that the initial endowments are Pareto inefficient and that goods should be transferred across generations. This means that no-trade theorems [Milgrom and Stockey (1982) and Tirole (1982)] do not apply. Rational trade based on one's perceptions of market psychology can take place and result in positive rents, without the possibility of arbitrage.

Second, it is essential to carefully model price formation in order to understand the role of information in a market. If one does not model price formation explicitly and instead relies on a limiting notion such as rational expectations equilibria, then one runs into difficulties such as the paradox identified by Grossman (1976) and Grossman and Stiglitz (1980). That paradox arises because equilibrium prices reveal information and agents can condition their demand on those prices. This renders private information useless, since it can be inferred from prices. This sort of analysis misses out on the value of information, because information is valuable during the process through which prices are formed, and not in the limit after they have been formed. [This issue is discussed in detail in Dubey, Geanokoplos, and Shubik (1987), and Jackson (1991).] This suggests that one must be careful to model the trading process through which prices are formed. This permits an analysis of the role of speculative information without reliance on noise traders on other such constructs.[2] Here, a Vickrey auction is used to model trade. [See Milgrom (1981) for more on Vickrey auctions.] The Vickrey auction is a natural choice since it retains the flavor of a competitive model: agents cannot manipulate the price unless they fail to procure an asset, in which case they have no incentive to change the price.

The first part of the paper reviews results from Jackson and Peck (1991) and Jackson (1992) on the existence of speculative equilibria. Equilibria exist in which all agents (of every generation) bid based on their private and non-fundamental information. The equi-

[2] De Long, Shleifer, Summers, and Waldmann (1990) and Allen and Postlewaite (1991) examine speculation in rational expectations settings. The De Long, Shleider, Summers, and Waldmann (1990) article relies on noise traders who are irrational in that they act based on incorrect probability assessments. The Allen and Postlewaite (1991) model has fully rational agents, but a breakdown in common knowledge. By modeling price formation, the results presented here are consistent with full rationality and common knowledge. The point is not that full rationality and common knowledge are realistic assumptions; but instead that speculation is not something which depends on irrationality or a breakdown in information. This is consistent with the view expressed by Allen and Postlewaite, only taken further.

libria are strict Nash equilibria. This means that an agent would be worse off not paying attention to private speculative (non-fundamental) information and that agents will expend resources developing that information. In some environments, the equilibria are stationary so that all agents use the same bidding functions.

The second part of the paper provides new results on technical analysis. An equilibrium is constructed in which agents bid based on their signals and on the relative movements of past prices. If two consecutive large upward price movements have occurred in the past, then agents bid prices up to a new higher level, taking this as good news. If two consecutive large downward price movements have occurred in the past, then agents bid prices down to a new lower level, taking this as bad news. Again, this is a strict equilibrium and so agents have an incentive to chart past prices. The resulting equilibrium will exhibit large upward and downward price movements with very small probability.

II. *The Economy*

Consider the overlapping generations model from Jackson and Peck (1991), which is described as follows. A new generation of agents is born at each time $t \in \{0,1,2,3,\ldots\}$. Each generation consists of a finite number n of individuals who live for two periods.

There is a single consumption good in the model which agents consume in the second period of their life. [The model is easily extended to allow for consumption each period, but with substantial notational complication and little added insight.] Each individual born at time t is endowed with e_t units of the consumption good. Individuals' have von Neumann-Morgenstern preferences for consumption, which are represented by an increasing and concave utility function U_t.

There are k ($n>k>0$) indivisible, infinitely lived assets which pay dividends (of the consumption good) each period. The dividend paid to the holder of an asset at time t is denoted d_t. Assets are initially held by agents born at time $t=0$. The first opportunity for trade occurs at time $t=1$. Markets are incomplete. The only market which exists is for the trade of the infinitely lived asset. At each time t, agents in the second period of their life (those born at time $t-1$) may sell their assets to agents in the first period of their life (those born at time t).

An agent of generation t who does not purchase an asset has final consumption

$$e_t(1+r),$$

where $r \geq 0$ is a rate of return on stored consumption goods. An agent of generation t who purchases an asset at time t and sells it at time $t+1$ has final consumption.

$$(e_t - p_t)(1+r) + d_{t+1} + p_{t+1}.$$

The timing of the model is as follows. Period t begins with the birth of new agents and the death of old agents (those born at time $t-2$). Next, current holders of the infinitely lived assets receive their dividends d_t. After dividends have been paid, the sale of assets takes place. The agents of generation $t-1$ who hold assets can sell them to agents of generation t. Period t ends with agents of generation $t-1$ consuming, and the agents of generation t storing their goods and assets.

Trade

Assets are sold through a Vickrey auction. Young agents simultaneously submit bids $0 \leq b_t^i \leq e_t$. Each of the k highest bidders obtain one of the k assets. [Ties are broken according some fixed rule. The particular rule is unimportant since ties will occur with zero probability.] The winning bidders pay the same price, which is the value of the $k+1$-th highest bid. Each of the old agents who held an asset receives the price.

As discussed in Jackson and Peck (1991) [see also Milgrom (1981)], the choice of a Vickrey auction makes it easy to analyze price formation and the value of speculative information. This follows from the competitive features of the Vickrey auction. The k winning bidders pay a price equal to the $k+1$-th highest bid, and thus the winning bidders cannot affect the price without losing the asset. The agent who had the $k+1$-th highest does not purchase an asset and hence has no incentive to manipulate the price.

Information

Individuals of each generation have symmetric information about dividends. They also see the history of prices up to the present. In addition, each agent i receives a real-valued private signal s_t^i. These private signals are variables and defined on a common probability space (Ω, F, μ). The signals are independent of the information about dividends. Specific assumptions about the joint distribution of the signals across generations and among a generation are outlined in (C1)–(C3) below. Roughly, these assumptions imply that signals are correlated across generations and that higher values of signals today are "good news," in the sense that they correspond to higher expectations about signals tomorrow. This leads to the interpretation of the private signals as representing agents' private perceptions or information about market psychology.

The following notation will be useful: s_t^* denotes the $k+1$-th highest signal at time t, y_t^i denotes the k-th highest signal among the set $\{s_t^j\}_{j \neq i}$, and $h_t = (s_{t-1}^*, \ldots, s_2^*, s_1^*)$ is the history of s^*'s.

III. *The Existence of Speculative Equilibria*

The following conditions assure the existence of speculative equilibria. Let $\mu(s_{t+1}^* | s_t^i = s, y_t^i = y, h_t = h)$ be a version of the conditional probability of s_t^*, given agent i of generation t's signal, the order statistic of other agents' signals, and the history of s^*. Since this is only one version of the conditional probability, all statements which follow should be read as holding almost always, even though this notation is suppressed.

(C1) For all t, i and h, $\mu(s_{t+1}^* | s_t^i = s, y_t^i = y, h_t = h)$ first order stochastic dominates $\mu(s_{t+1}^* | s_t^i = \tilde{s}, y_t^i = \tilde{y}, h_t = \tilde{h})$ whenever $s \geq \tilde{s}$, $y \geq \tilde{y}$, and $h \geq \tilde{h}$.[3] This dominance is strict when $s > \tilde{s}$.

This first condition states that signals are correlated across generations and that higher signals are "good news" about future signals.

[3] $h \geq \tilde{h}$ means that each component of h is at least as large as the corresponding component of $\tilde{h}$.

(C2) For all t, i, j, s, y and h

$$\mu(s^*_{t+1}|s^i_t=s, y^i_t=y, h_t=h)=\mu(s^*_{t+1}|s^j_t=s, y^j_t=y, h_t=h)$$

This second condition states that agents have comparable information. Although they may obtain different realizations of signals, they are equivalent in their precision. This assumption is made for technical convenience. It permits us to concentrate on finding an equilibrium which is symmetric among agents of a given generation.

(C3) $\mu(y^i_t=s|s^i_t=s, h_t=h)=0$ for all t, i, s and h.

This third condition will assure that the probability of an agent observing exactly the same signal as the k-th highest of the other agents is zero. This assumption is also made for technical convenience, since the analysis of an auction is substantially more complicated when ties occur with positive probability.

(C4) $D_t=E_t[\sum_{\tau=t+1}^{\infty} d_\tau/(1+r)^{\tau-t}]$ is well defined for all t and $D_{t+1}=D_t-E_t[d_{t+1}]$. Endowments grow so that either $e_{t+1}\geq(1+r)e_t$ or $E_t[U_t(e_t+d_{t+1})]\geq U_t(e_t(1+r))$ and $e_t\geq e_1$ for all t. And, $e_1>D_1$.

This last condition assures that agents can afford to bid the fundamental value of the assets.

The following Proposition is from Jackson (1992).

PROPOSITION: THE EXISTENCE OF SPECULATIVE EQUILIBRIA. *If (C1)–(C4) are satisfied and agents of each generation observe the history of past prices, then there exists an equilibrium of the infinite horizon economy in which all agents bid according to an increasing (and linear) function of their signals. Furthermore, if the support of an agent's signal is connected, then the equilibrium bidding strategy is the unique best reponse to the strategies of the other agents for almost all signals.*

The proposition above establishes the existence of an equilibrium in which all agents of every generation use their signals in selecting a bidding strategy. The last statement says that the equilibrium is strict. This has two implications. First, if other agents are bidding according to their private signals, then an agent's only choice is to do the same. Even though this information is unrelated to the fundamental structure of the economy, it is valuable to the agent since it is useful in predicting competing agents' bids and the next period price for the asset. Second, the equilibrium will be immune to refinements of the Nash equilibrium concept (such as perfect, proper, etc.).

Under additional assumptions which assure that the environment is stationary, there exist equilibria in which all individuals use the same bidding function. [See Jackson (1992) for details.] This would seem to make the equilibrium more focal and also allow one generation to easily teach the next how to bid.

The following example illustrates the method used to prove the existence proposition.[4]

[4] See Jackson (1992) for a proof. This example is similar to an example in Jackson and Peck (1991). An important difference is that in the equilibrium here, all agents of all generations bid as increasing functions of their signals. In the Jackson and Peck example, agents only bid an functions of their signals as long as the price remained in certain bounds.

EAXMPLE 1. *Risk Neutral Traders and No Dividends.*

In this example there are two risk neutral agents in each generation and a single asset which pays no dividends. $r=0$ and $e_t \geq 1$ for all t. The signal structure is as follows

$$s_1^i = z_1^i + \frac{1}{2}$$

$$s_t^i = z_t^i + \frac{s_{t-1}^1 + s_{t-1}^2}{2}, \qquad (1)$$

where z_t^i are independently and identically distributed with support in $\left(0, \frac{1}{2^{t+1}}\right)$. Signals each period are the average of last period's signals plus an idiosyncratic random term. Notice that the support of s_t^i is a subset of $(1/2,1)$ for every i and t. The particular signal structure here assures that signals do not grow and that agents can bid straight functions of the signals. More generally, bids need to be scaled functions of signals to ensure feasibility. The details of scaling bids to ensure feasibility for general signal structures are given in Jackson (1992).

Let P_t denote the history of prices $\{p_{t-1}, \ldots, p_2, p_1\}$. An equilibrium is constructed as follows.[5] Consider bids defined by

$$b_1(s_1^i) = s_1^i,$$

$$b_t(s_t^i, P_t) = p_{t-1} + s_t^i - E[s_t^* | s_{t-1}^1 = y_{t-1}^1 = s_{t-1}(P_t)],$$

where[6] $s_{t-1}(P_t)$ is the price setting signal inferred from the price

$$s_{t-1}(P_t) = p_{t-1} - p_{t-2} + E[s_{t-1}^* | s_{t-2}^1 = y_{t-2}^1 = s_{t-2}(P_{t-1})]. \qquad (2)$$

Remark that the definitions given above are recursive: $s_{t-1}(\cdot)$ depends on $s_{t-2}(\cdot)$. This structure is well defined taking $s_1(P_2) = p_1$.

To verify that this is an equilibrium, consider an agent of generation $t-1$'s expected utility for purchasing an asset conditional on the values of s_{t-1}^i and y_{t-1}^i, and given that other agents follow the above bidding strategy. It is shown below that the bidding function defined above is the unique maximizer of this expected utility. This implies that it is also the unique maximizer even if it is not possible to condition on $y_t^i = y$. [Again see Jackson (1992) for details.] In order to purchase an asset, agent i would have to bid at least $b_{t-1}(y)$ and would then have an expected utility of

[5] In addition to the scaling of bids to ensure feasibility, the general proof differs from this example in that with general utility functions, a function of P_t substitutes in the place of $E[s_t^* | s_{t-1}^1 = y_{t-1}^1 = s_{t-1}(P_t)]$ (which simplifies in the case of risk neutrality).

[6] In this example, expectations condition only on current signals, since past signals are redundant, given current signals, in forming expectations concerning future signals.

$$E[e_{t-1}+s_t^*-E[s_t^*|s_{t-1}^1=y_{t-1}^1=s_{t-1}(b_{t-1}(y),P_{t-1})]|s_{t-1}^i=s,y_{t-1}^i=y].$$

Given that all other agents are bidding according to the prescribed functions, it follows that $s_{t-1}(b_{t-1}(y),P_{t-1})=y$ and so the expected utility simplifies to

$$e_{t-1}+E[s_t^*|s_{t-1}^i=s,y_{t-1}^i=y]-E[s_t^*|s_{t-1}^1=y_{t-1}^1=y]$$

or

$$e_{t-1}+\frac{s+y}{2}-y. \tag{3}$$

It is clear that the agent would prefer to purchase an asset in those situations in which $s>y$, and would prefer not to purchase an asset when $s<y$. Given the other agents bids, the only way to assure that this happens is to bid according to the prescribed bidding function.

Given the equilibrium structure, bids can be simplified to eliminate dependence on past prices. [It is straightforward to verify that the simplified bids also form an equilibrium.] Substituting from (1) and (2),

$$E[s_t^*|s_{t-1}^1=y_{t-1}^1=s_{t-1}(P_t)]=E_{t-1}[z_t^*]+p_{t-1}-p_{t-2}+E[s_{t-1}^*|s_{t-2}^1=y_{t-2}^1=s_{t-2}(P_{t-1})].$$

Solving recursively

$$E[s_t^*|s_{t-1}^1=y_{t-1}^1=s_{t-1}(P_t)]=p_{t-1}+\sum_{\tau=2}^{t}E[z_\tau^*].$$

Therefore, for $t\geq 2$ bids can be rewritten as

$$b_t(s_t^i,P_{t-1})=s_t^i-\sum_{\tau=2}^{t}E[z_\tau^*]$$

Notice that this is always between 0 and 1, since $1/2<s_t^i<1$ and $0<\sum_{\tau=2}^{t}E[z_\tau^*]<1/2$.

The Price Path and Value of Speculative Information

The price path which results from the above equilibrium is

$$\begin{aligned}E[p_{t+1}-p_t]&=E[z_{t+1}^*+\frac{s_t^1+s_t^2}{2}-\sum_{\tau=2}^{t+1}E(z_\tau^*)-s_t^*+\sum_{\tau=2}^{t}E(z_\tau^*)]\\&=E\left[\frac{s_t^1+s_t^2}{2}-s_t^*\right]>0.\end{aligned}$$

The expected price path shows an increase over time, even though agents are risk neutral and the assets have no fundamental value. There are no arbitrage opportunities since in order for some agent who does not have an asset to obtain an asset, the agent would have to bid at least

$$\max_i\{s_t^i\}-\sum_{\tau=2}^{t}E(z_\tau^*),$$

and so the price at time t would become

$$p_t' = \max\ \{s_t^i\} - \sum_{\tau=2}^{t} E(z_\tau^*),$$

and then

$$E[p_{t+1} - p_t'] = E\left[\frac{s_t^1 + s_t^2}{2} - \max_i\ \{s_t^i\}\right] < 0,$$

and so the agent would lose by attempting to purchase the asset.

The fact that prices are expected to rise over time reflects an equilibrium rent to the speculative information. The value of the speculative information to an individual can be determined by comparing an agent's ex-ante expected utility when the agent observes a signal and bids, to the utility when the agent does not observe a signal and does not bid. Based on observing a signal, the agent has a 1/2 probability of obtaining an asset with an expected value of $E\left[\frac{s_t^1 + s_t^2}{2} - s_t^*\right]$ and 1/2 probability of not obtaining an asset. Thus the expected utility is

$$e_t + \frac{1}{2} E\left[\frac{s_t^1 + s_t^2}{2} - s_t^*\right]$$

The value of the signal is thus $\frac{1}{2} E\left[\frac{s_t^1 + s_t^2}{2} - s_t^*\right]$. This value is greater than zero, which means that signals are valuable to an agent.

Notice that the above equilibrium can easily be extended to allow for $n > 2$ and $k \geq 1$. [There are many ways to extend the signal structure. The simplest, which requires no alterations in the bidding strategies, is to have a signal be an idiosyncratic term plus the average of the k-th and $k+1$-th highest signals from the last period.] In that case, the value of the signal depends on the expected value of the average of the k-th and $k+1$-th highest signals less the $k+1$-th signal. This value is decreasing as the number of agents increases. Thus the value of private information is related to the degree of imperfect competition.

IV. *Technical Analysis*

In this section, an example is developed in which agents chart past prices in addition to observing their signals. Each time they observe specific patterns, they anticipate that the equilibrium will move to a new level.[7] These anticipations become self-fulfilling. This results in relatively large changes in the price path, with very small probability.[8] Again, the equilibrium is strict and so agents have incentives to chart past prices, just as they have

[7] The term technical analysis is used since agents' bids are functions of past price patterns. The equilibria described here can also be thought of as offering some insight into how agents should "program" their trades to depend on certain prices being hit and that they should coordinate on these prices.

[8] For alternative models of "crashes," see Bulow and Klemperer (1991) and Bikhchandani, Hirshleifer, Welch (1991), and Hu (1993).

incentives to obtain speculative information.

In Jackson and Peck (1991) it was informally argued that agents' private signals might be interpreted as coming from the act of charting prices. Here, a different approach is taken. The act of charting prices is modeled directly, in addition to the act of obtaining speculative information. The speculative information provides the random price movements which makes the charting work. Without these random price movements, charting would be unecessary since the price path could be perfectly predicted.[9] It should be noted that, in order to have an incentive for agents to chart prices (as opposed to having agents be indifferent between charting or not), asymmetric information is necessary. In this model speculative information fills that role. One could also develop a charting equilibrium with uncertain *and asymmetric* fundamental information.

EXAMPLE 2. *Technical Analysis.*

In this example, if agents see two consecutive downward price movements, each of more than some pre-determined size d, then they adjust their bids downward by some level K. Similarly, if agents see two consecutive upward price movements, each of more than some pre-determined size d, then they adjust their bids upward by some level K.[10] The actual bidding functions will turn out to be a bit more complicated, since, for instance, agents whose actions will lead to a second large price drop should anticipate that the market will drop as a result of their actions and take this into account when choosing a bid. Thus, equilibrium bidding functions turn out to be discontinuous at certain points. For simplicity, assume that agents are risk neutral, assets pay no dividends, $n=2$, $r=0$, $e_t=1000$ and that signals are determined by

$$s_1^i = z_1^i + 500,$$

$$s_t^i = z_t^i + \frac{s_{t-1}^1 + s_{t-1}^2}{2},$$

where the z_t^i have an iid distribution so that z_t^* is uniformly distributed on (-1,1). It will be clear that similar examples can be constructed in the absence of these assumptions.

In the equilibrium the distance of what is considered a large price movement is denoted d. Since agents bids will be based on their signals, and signals move at most one unit from generation to generation d is a number close to 1. The amount of the price "jump" or "crash" in response to two consecutive large upward price movements or two consecutive large downward price movements is denoted K. The following bidding strategies, as functions of signals and past prices, form an equilibrium.[11]

Agents at time $t=1$ bid $b_1(s_1^i)=s_1^i$. Agents at time $t>1$ bid according to

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[9] This is related to the work of Brown and Jennings (1990) who show that agents will pay attention to past price paths since they contain information which is not redundant with agents current information. Their work concentrates on the revelation of fundamental information, whereas here we concentrate on the correlation of agents' actions.

[10] If three consecutive large price movements (in the same direction) are observed, this is interpreted as two sets of two movements.

[11] To make sure that bids are always between 0 and e_t, some adjustments are necessary. A quick way to do this is simply set bounds inside of 0 and e_t. As long as past price lies inside of these bounds agents bid according to the given functions. If the price ever exceeds the bounds then agents bid that same price forever after. An alternative adjustment would be to scale the bids over time, so that agents always bid as functions of their signals and charts. This second sort of approach is illustrated in Jackson (1992).

$$b_t(s_t^i)=s_t^i+f_t(P_{t-1}) \quad \text{if} \quad p_{t-1}-d<s_t^i+f_t(P_{t-1})<p_{t-1}+d \tag{4}$$

$$b_t(s_t^i)=s_t^i+f_t(P_{t-1})-x \quad \text{if} \quad s_t^i+f_t(P_{t-1})\leq p_{t-1}-d\not\leq p_{t-2}-d \tag{5}$$

$$b_t(s_t^i)=s_t^i+f_t(P_{t-1})+x \quad \text{if} \quad s_t^i+f_t(P_{t-1})\geq p_{t-1}+d\not\geq p_{t-2}+d \tag{6}$$

$$b_t(s_t^i)=s_t^i+f_t(P_{t-1})-x-K \quad \text{if} \quad s_t^i+f_t(P_{t-1})\leq p_{t-1}-d\leq p_{t-2}-d \tag{7}$$

$$b_t(s_t^i)=s_t^i+f_t(P_{t-1})+x+K \quad \text{if} \quad s_t^i+f_t(P_{t-1})\geq p_{t-1}+d\geq p_{t-2}+d \tag{8}$$

The function $f_t(P_{t-1})$ is defined to be K times the number of τ for which $p_\tau\geq p_{\tau-1}+d\geq p_{\tau-2}+d$ minus the number of τ for which $p_\tau\leq p_{\tau-1}-d\leq p_{\tau-2}-d$. To apply the above definitions to agents bidding at time $t=2$, set the bids as if $p_0=p_1$.

The above bids cover different situations. (4) applies if there were no large movements last period and the agent's bid will not lead to a large movement. In this situation, agents do not have to worry about their behavior resulting in a jump or crash next period. (7) and (8) apply to situations in which the agent knows that his or her bid will result in a second large price movement and that this will trigger a crash or jump. Thus the bids in (7) and (8) anticipate the jump or crash and incorporate it. (5) and (6) are intermediate cases where agents realize that their bids will result in a first large price movement. They then know that with some small probability that next period a second large movement will occur and that prices will jump or crash next period. Thus these bids incorporate an amount x which adjusts for that possibility. [(7) and (8) also incorporate x since it is possible that prices will see a large price movement yet a third (or mpre) time resulting in another jump or crash.] The value of x must satisfy certain restrictions in order for the above to form an equilibrium. Specifically, $d+x<1$ and

$$\frac{2(x+x^2)}{1-d-x}=K. \tag{9}$$

Not all d and K will allow for a feasible x and thus for an equilibrium to exist. An example of values which satisfy (9), is $x=.1$, $d=.89$ (so $x+d<1$) and $K=22$.

Again, the reason that the construction here is somewhat complicated, is that agents anticipate the effects charting has on prices. Agents who know that a large price movement has just occurred, also know that if their bids lead to a second large price movement, then future bids will adjust by K. This means that these agents should adjust by K now, as they bid. Likewise, agents who know that the present price will result in a large price movement, have to account for the probability of a second large price movement coming next.

Let us now verify that the bids defined in (4)–(8) form an equilibrium.

Consider an agent's expected utility bidding enough to obtain an asset, given that other agents bid according to (4)–(8), and conditional on knowing the value of $y=y_t^i$.

If (4) applies to y, then this utility is

$$e_t-y-f_t(P_{t-1})+E_t[b_{t+1}(s_{t+1}^*, P_t)|s_t^i=s, y_t^i=y].$$

In this case b_{t+1} will come from (4), (5), or (6), and $f_{t+1}=f_t$. Thus given the structure of signals this expected utility becomes

$$e_t-y+\int_{\max\left[\frac{y-s}{2}-d,-1\right]}^{\min\left[\frac{y-s}{2}+d,1\right]}\left(z^*+\frac{s+y}{2}\right)\frac{dz^*}{2}+\int_{-1}^{\max\left[\frac{y-s}{2}-d,-1\right]}\left(z^*+\frac{s+y}{2}-x\right)\frac{dz^*}{2}$$
$$+\int_{\min\left[\frac{y-s}{2}+d,1\right]}^{1}\left(z^*+\frac{s+y}{2}+x\right)\frac{dz^*}{2}$$

This is larger than e_t precisely when $s>y$. [To see this notice that if $s=y$, then this expression simplifies to e_t. Then notice that the expression is increasing in s.] Thus by bidding according to the suggested bid, the agent will win precisely when $s>y$, which is the unique best response.

Suppose that (5) applies to y. To win an asset in this case the agent's expected utility of bidding enough to obtain an asset (given that other agents bid according to (4)–(8)) and conditional on knowing the value of $y=y_t^i$ would be

$$e_t-y-f_t(P_{t-1})+x+E_t[b_{t+1}(s_{t+1}^*,P_t)|s_t^i=s,\,y_t^i=y].$$

In this case b_{t+1} will come from (4), (6), or (7). The expected utility becomes

$$e_t-y+x\int_{\max\left[\frac{y-s}{2}-x-d,-1\right]}^{\min\left[\frac{y-s}{2}-x+d,1\right]}\left(z^*+\frac{s+y}{2}\right)\frac{dz^*}{2}+\int_{-1}^{\max\left[\frac{y-s}{2}-x-d,\,-1\right]}\left(z^*+\frac{s+y}{2}-K-x\right)\frac{dz^*}{2}$$
$$+\int_{\min\left[\frac{y-s}{2}-x+d,1\right]}^{1}\left(z^*+\frac{s+y}{2}+x\right)\frac{dz^*}{2},$$

Again, this is larger than e_t precisely when $s>y$. [To see this notice that if $s=y$, then this expression simplifies to $e_t+x-(K+x)(1-x-d)/2+x(x-d+1)/2$, which by (9) simplifies to e_t. Then notice that the expression is increasing in s.] Thus by bidding according to the suggested bids, the agent will win precisely when $s>y$, which is the unique best response.

Checking the case where (6) applies is analogous to checking case (5) above, except with a few change of signs. Then checking the cases where (7) and (8) apply to y, is the same as checking (5) and (6), given the appropriate adjustments in f_{t+1}.

Price Movement

Since the equilibrium is strict, both speculative information and charting are valuable to agents. In this equilibrium, as in the previous example, there is a general upward drift in the price. The exact value of this drift will depend on the number of large price movements which have just occurred, as well as the specific distribution of the signals.

We can also say something about the probability of different price movements. If no recent price movements have occurred, then the chance of seeing one large upward price movement is

$$\left(1+\frac{\max_i\{s_t^i\}-\min_i\{s_t^i\}}{2}-d\right)\Big/2,$$

which is larger than the probability of seeing a large downward price movement

$$\left(1-\frac{\max_i\{s_t^i\}-\min_i\{s_t^i\}}{2}-d\right)\Big/2.$$

This means that jumps occur will with greater probability than crashes, which is consistent with the general upward drift of prices. Another interesting aspect of the equilibrium, is that the probability of first large price movements is different from subsequent (consecutive) large price movements. For instance, if a large price increase has just taken place, the chance of seeing another one next time is

$$\left(1+\frac{\max_i\{s_t^i\}-\min_i\{s_t^i\}}{2}-x-d\right)\Big/2.$$

This probability is less than that of seeing a first large price movement. This is due to the fact that agents who cause a first large price increase adjust their bids higher in anticipation that there might be another, consecutive price movement and hence a jump. This is makes it less probable that the second large movement will occur. It is also interesting to note that once a first large price increase (decrease) has occurred, the chance of seeing a fiist large price decrease (increase) rises. This happens since agents begin to bid higher (lower) than their signals, on the small probability of a large upward (downward) swing.

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REFERENCES

Allen, F. and A. Postlewaite (1991), "Rational Expectations and Stock Market Bubbles," CARESS WP #91-06, University of Pennsylvania.

Azariadis, C. (1981), "Self-Fulfilling Prophecies," *Journal of Economic Theory* 25, pp. 380–396.

Bikhchandani, S; D. Hirshleifer, and I. Welch (1991), "A Theory of Fads, Fashion, Custom and Cultural Change as Informational Cascades," UCLA AGSM WP#20-90.

Brown, D. and R. Jennings (1990), "On Technical Analysis," *Review of Financial Studies* 2, pp. 527–551.

Bulow, J. and P. Klemperer (1991), "Rational Frenzies and Crashes," Research Paper 1150, Standorf GSB.

Cass, D. and K. Shell (1983), "Do Sunspots Matter?" *Journal of Political Economy* 91, pp. 193–227.

De Long, J.B., A. Shleifer, L.H. Summers, and R.J. Waldmann (1990), "Noise Trader Risk in Financial Markets," *Journal of Political Economy* 98, pp. 703–738.

Dubey, P., J. Geanakoplos, and M. Shubik (1987), "The Revelation of Information in Strategic Market Games: A Critique of Rational Expectations Equilibria," *Journal of Mathematical Economics* 16, pp. 105–137.

Flood, R. and R. Hodrick (1990), "On Testing for Speculative Bubbles," *Journal of Economic Perspectives* 4, pp. 85–101.

Grossman, S. (1976), "On the Efficiency of Competitive Stock Markets Where Agents have Diverse Information," *Journal of Finance* 31, pp. 573–585.

Grossman, S. and J. Stiglitz (1980), "On the Impossibility of Informationally Efficient Markets," *The American Economic Review* 70, pp. 393–408.

Hansen, L. and R. Jaggannathan (1991), "Implications of Security Market Data for Models of Dynamic Economies," *Journal of Political Economy* 99, pp. 225–262.

Hu, J. (1993), "The Impact of Information Ambiguity and Information Asymmetry on Financial Markets," Ph. D Dissertation, Northwestern Univsrsity.

Jackson, M. (1991), "Equilibrium, Price Formation, and the Value of Private Information," *Review of Financial Studies* 4, pp. 1–16.

Jackson, M. (1992), "A Proof of the Existence of Speculative Equilibria," forthcoming: *Journal of Economic Theory*.

Jackson, M. and J. Peck (1991), "Speculation and Price Fluctuations with Private, Extrinsic Signals," *Journal of Economic Theory* 55, pp. 274–295.

Mehra, R. and E. Prescott (1985), "The Equity Premium: A Puzzle," *Journal of Monetary Economics* 15, pp. 154–161.

Milgrom, P. (1981), "Rational Expectations, Information Acquisition, and Competitive Bidding," *Econometrica* 49, pp. 921–944.

Milgrom, P. and N. Stokey (1982), "Information, Trade, and Common Knowledge," *Journal of Economic Theory* 26, pp. 17–27.

Peck, J. (1988), "On the Existence of Sunspot Equilibria in an Overlapping Generations Model," *Journal of Economic Theory* 44, pp. 19–42.

Peck, J. and K. Shell (1991), "Market Uncertainty: Correlated Equilibrium and Sunspot Equilibrium in Imperfectly Competitive Markets," *Review of Economic Studies* 58, pp. 1011–1031.

Shell, K. (1977), "Monnaie et Allocation Intertemporale," mimeo.

Spear, S.E. (1989), "Are Sunspots Necessary?" *Journal of Political Economy* 97, pp. 965–973.

Tirole, J. (1982), "On the Possibility of Speculation Under Rational Expectations," *Econometrica* 50, pp. 1163–1182.