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ASSET PRICING WITH INCOMPLETE MARKETS

DEBORAH J. LUCAS

Introduction

The consumption-based asset pricing model (CCAPM) of Merton (1973), Lucas (1978) and Breeden (1979) provides an intuitive and testable connection between consumption data and asset prices. If individuals prefer a smooth consumption stream, then securities whose payoffs are positively correlated with consumption should command a higher rate of return. This implies that observed return differentials across securities will depend on the differential correlations between asset payoffs and consumption. Most empirical tests, however, have rejected the representative consumer CCAPM (e.g., Hansen and Singleton (1983)). In a widely-cited paper, Mehra and Prescott (1985) show that the standard representative agent model has particular difficulty reproducing the equity premium (the difference between the return on equity and the return on risk-free bonds), and the low risk-free interest rate. Other puzzles in the context of the standard model include the variability of stock returns relative to dividends (Grossman and Schiller, 1981), and the steep slope of the term structure in early maturities (Backus, Gregory and Zin, 1989). Similar models in an international context cannot explain phenomena such as the volatility of exchange rates.

The empirical rejection of the CCAPM has been a major impetus for the rapidly growing literature on asset pricing with incomplete markets. One way to interpret the rejection of the representative consumer model is that there is too little volatility in the aggregate marginal rate of substitution of consumption (Hansen and Jagannathan, 1990). Market incompleteness has the potential to solve this problem because it severs the link between individual and aggregate consumption. In particular, it allows for the possibility that individual consumption growth is more volatile than in aggregate. For instance, if individuals cannot fully insure against spells of unemployment due to moral hazard or other contractual problems, individual consumption will fluctuate with employment status. Asset prices will be affected because individuals will be more reluctant to hold risky assets than if complete insurance were available.

This paper surveys the major findings in the literature on asset pricing with incomplete markets, with an emphasis on models that are empirically-directed. In working with incomplete markets, one must be aware of the possibility that a seemingly small change in assumptions can produce qualitatively different conclusions. I have tried to emphasize

* I wish to thank John Heaton for the many conversations that have shaped my understanding of this subject, and Andrew Winton for useful comments. A Spanish version of this paper is in *Cuardernos Economicos de ICE.*, 50, 1992.

results that appear to be fairly robust and the intuition behind them. Methodologically, the literature is divided between continuous time and discrete time models; both involve technical complexities that for the most part are neglected here. It should be mentioned that part of the growth of this literature can be attributed to computational innovations. New numerical algorithms and faster computers to implement them have made possible the investigation of these often complex equilibrium models.

The survey is organized as follows. Section 1 reviews a simple two-period model that illustrates the potential of uninsurable idiosyncratic shocks to produce a low risk-free rate and a high equity premium. Section 2 describes a model that is used in several recent papers to show that incompleteness has little impact on asset prices when transitory idiosyncratic shocks are imbedded into a standard infinite horizon model. A number of related papers are also discussed. Section 3 reviews some recent results on how the permanence of uninsurable shocks and their correlation with aggregate shocks affects asset prices in infinite horizon models. Transactions costs can be the source of incompleteness, or exacerbate the effects of incompleteness. Section 4 describes several papers that examine how transactions costs affect asset prices. Section 5 summarizes the empirical evidence on the practical importance of market incompleteness. Section 6 concludes.

I. *A Two Period Asset Pricing Model*

The idea that market incompleteness might resolve the equity premium and risk-free rate puzzles was first suggested by Mehra and Prescott (1985), and illustrated in a two period model by Mankiw (1986).[1] This section presents a simplified version of Mankiw's model that shows why uninsurable income shocks can result in a high equity premium and a low risk-free rate.[2]

Consider an economy with two groups of agents who differ only in their labor income realizations in the second period (time 1). At time 0 all agents have equal labor and dividend income: $Y_0+\delta_0/2$. At time 1 stochastic income is composed of an aggregate stock dividend, δ, and individual labor income, Y^i. Markets are incomplete in that agents cannot write contracts at time 0 that are contingent on the realization of time 1 labor income, so that idiosyncratic shocks cannot be insured.

The asset market consists of a stock that entitles the owner to a share of the time 1 dividend, and a bond market in which risk-free claims to time 1 income can be traded. At the beginning of time 0, each agent is assumed to own half of the stock and hold no debt. Thus bonds are in zero net supply. The stock price is denoted by p^s, and the bond price by p^b.

At time 0 each agent i chooses consumption, c_0^i, stock purchases, s^i, and bond purchases, b^i, to maximize

$$U(c_0^i)+\beta EU(c_1^i) \tag{1}$$

[1] Bewley (1984) shows that idiosyncratic risk depresses the risk-free rate. His framework is the basis for a number of recent papers including Clarida (1990), and Aiyagari and Gertler (1991).

[2] This section follows the example in Lucas and Heaton (1992). It differs from Mankiw (1986) in that Mankiw assumes a continuum of agents and a more general labor income process. Furthermore, he examines the proportional premium rather than the level of the premium.

subject to the budget constraints

$$c_0^i = Y_0^i + \frac{\delta_0}{2} - b^i p^b - s^i p^s$$
$$c_1^i = Y_1^i + \frac{\delta_1}{2} + b^i + s^i \delta_1 \qquad (2)$$

Market clearing requires

$$c_t^1 + c_t^2 = Y_t^a + \delta_t \quad t=0,1 \quad y_t^a = y_t^1 + y_t^2$$
$$s^1 + s^2 = 0 \qquad (3)$$
$$b^1 + b^2 = 0$$

The first order conditions for the agents' maximization problem imply that asset prices satisfy

$$p^b = \beta \frac{E[U'(c_1)]}{U'(c_0)} \qquad (4)$$

$$p^s = \beta \frac{E[U'(c_1)\delta]}{U'(c_0)}. \qquad (5)$$

These prices depend on the exogenous stochastic process governing dividends and labor income. Assume that at time 1, aggregate output is high with probability q, and low with probability $(1-q)$. The aggregate dividend is $\delta_1 \in \{0, \delta_H\}$, and aggregate labor income is $Y_1^a \in \{Y_L, Y_H\}$. If realized output is high, each agent receives $Y_H/2$. If realized output is low, one agent receives λY_L, while the other receives $(1-\lambda)Y_L; \lambda \geq 1/2$. This can be interpreted as the increased probability of becoming unemployed in a recession. At time 0 each agent faces the same distribution of future labor income. Notice that the case $\lambda = 1/2$ corresponds to the standard representative agent model.

Under these assumptions, the equity premium is higher than in the representative consumer case and the risk-free rate lower, as long as $U''' > 0$. This restriction on U''' is satisfied by the most common utility specifications, including constant relative risk aversion (CRRA) and constant absolute risk aversion (CARA). Note first that because agents have the same wealth and information at time 0, there are no transactions in the asset markets. As in the representative consumer model, consumption equals income in each period. Asset prices are found by substituting income into equations (4) and (5). The stock price is constant for all λ since the stock only pays a dividend in the good aggregate state, when labor income is riskless. It is easy to show that the convexity of the marginal utility function implies that the risk-free rate falls as λ increases. Intuitively, there is an increase in the precautionary demand for bonds due to the anticipated increase in time 1 consumption volatility. These two results imply that the premium, $E(\delta_1)/p^s - 1/p^b$, increases in λ. If $U'(0) = \infty$, the premium becomes arbitrarily large as the share of income, λ, received by the employed agent goes to one.[3]

[3] Rietz's (1988) demonstration that the equity premium puzzle can be resolved with the assumption of a small probability of extremely low (aggregate) consumption relies on a similar mechanism.

In Mankiw's model and the one here, the idiosyncratic shocks are assumed to be larger when aggregate output is low. Weil (1992) examines a similar model in which the idiosyncratic shocks are independent of the aggregate shocks. With constant relative risk aversion, he shows that the risk-free rate falls in this case as well, and the proportional equity premium increases.[4] However, in Weil's model idiosyncratic risk has two offsetting affects on the level of the premium. While idiosyncratic risk increases the required return on stocks relative to bonds,[5] it also decreases the required return on both stocks and bonds because it creates a precautionary demand for assets. When both rates fall, the difference between the two rates tends to fall. In practice, it is difficult to construct numerical examples using Weil's specification in which the risk-free rate is low and the equity premium is large. Thus, it appears that the impact of idiosyncratic risk on the equity premium is only significant when the idiosyncratic shock and aggregate shock are correlated. We will return to this idea in Section 3.

II. *Infinite Horizon Models with Transitory Idiosyncratic Shocks*

a. *Pricing Debt and Equity*

Assessing the quantitative importance of uninsurable income for asset prices requires a longer horizon model. Recently, several authors (D. Lucas (1991), Marcet and Singleton (1991), and Telmer (1991)) study a modified version of the R. Lucas (1978) asset pricing model which incorporates uninsurable income shocks, borrowing constraints in the debt market, and short sales constraints in the stock market. In contrast to the model of Section I, they find that predicted asset prices and consumption patterns resemble those produced by the representative agent model, even with fairly sizeable income shocks. This section gives an overview of these models and their empirical results.

i. *Model*

All three models share a similar theoretical structure, which can be summarized as follows (the notation is taken from Lucas (1991)). The economy has two large classes of agents that are distinguished by their labor income realizations. A type i agent, $i=1,2$, maximizes expected lifetime discounted utility

$$E\sum_{t=0}^{\infty}\beta U_i(c_{it})$$

by choosing consumption c_{it}, stock share holdings $s_{i,t+1}$ and bond share holdings $b_{i\,t+1}$, subject to a flow wealth constraint

$$(6)\qquad c_{it}+p_t^s s_{i,t+1}+p_t^b b_{i,t+1}B_t\leq s_{it}(p_t^s+d_t)+b_{it}B_{t-1}+x_{it}$$

and short sales constraints

[4] The proportional equity premium is defined as the ratio of the gross stock return to the gross bond return.

[5] Whether the proportional equity premium rises or falls with an increase in the riskiness of consumption depends on the utility function.

$$(7) \quad s_{it} \geq K_t^s \qquad t=0,1,2,\ldots$$

$$(8) \quad b_{it} \geq K_t^b \qquad t=0,1,2,\ldots$$

p_t^s and p_t^b are the real stock and bond prices at time t in terms of the time t consumption good. B_t is the face value of the privately issued bond at time t.

In the representative agent model no asset trading occurs because the initial allocation of risk cannot be improved upon. In Mankiw (1986) no trading occurs because the idiosyncratic shock is realized only in the final period. By contrast, in this model agents trade in the asset markets to partially offset their idiosyncratic income shocks. If agent 1 realizes a good labor income shock ($x_{1t} > x_{2t}$), some of the additional income can be saved by purchasing stocks or bonds. Similarly, agents can sell assets in response to bad shocks. The extent to which shocks can be offset by trading in this model depends on the persistence of the shocks, and on the severity of short sales or borrowing constraints, as discussed below.

Exogenous aggregate output, which equals aggregate consumption, consists of the aggregate dividend, d_t, and the sum of individuals' labor income $\sum_i x_{it}$. Both d_t and x_{it} are stochastic; the specifications for these processes are described below. Agents cannot write contracts contingent on future realizations of labor income, nor can they trade in more complex contingent claims such as options on the aggregate dividend.

The conditions for market clearing are

$$(9) \quad b_{1t} + b_{2t} = 0 \quad t=0,1,2,\ldots$$

$$(10) \quad s_{1t} + s_{2t} = 1 \quad t=0,1,2,\ldots$$

$$(11) \quad c_{1t} + c_{2t} = d_t + x_{1t} + x_{2t} \quad t=0,1,2,\ldots.$$

When the short sales constraints (7) and (8) are not binding, the first order necessary conditions from the agents' optimization problem imply that for all i and t,

$$(12) \quad p_t^b = \frac{\beta_i E[U_i'(c_{i,t+1})(p_{t+1}^s + d_{t+1}) | \Theta_t]}{U_i'(c_{it})}$$

and

$$(13) \quad p_t^b = \frac{\beta_i E[U_i'(c_{i,t+1}) | \Theta_t] B_t}{U_i'(c_{it})}$$

where $E[x|\Theta_t]$ is the expectation of x conditional on the common time t information set Θ_t.

If an agent is constrained by a short sales constraint, (12) is replaced by

$$(12') \quad s_{it} = K_t^s,$$

or (13) is replaced by

$$(13') \quad b_{it} = K_t^b$$

If a short sales constraint is binding for an agent, the market price exceeds the marginal rate of substitution for that agent. At least one type of agent will have a slack short sales constraint at all times, since there is no upper bound on the quantity of securities that can be purchased. Thus the market clearing prices are always well-defined by (12) and (13), using the marginal rate of substitution of the unconstrained agents.

Given the exogenous labor and dividend income process and a functional form for utility, finding an equilibrium requires solving (6), (9), (10), (11), (12) or (12′), and (13) or (13′) for p_t^s; p_t^b; c_{it}, $i=1,2$; s_{it}, $i=1,2$; and b_{it}, $i=1,2$ as a function of the state. The endogenous component of the state is the wealth of each agent, and the exogenous component includes the current dividend and the current labor incomes. In general, no closed form solution exists, so the model must be solved by numerical simulation. Because of the heterogeneity, even finding numerical solutions is problematic. Each paper employs a different solution method; the interested reader should refer to the original papers for a description of the algorithms.

ii. Empirical Specifications

To calibrate the model, one must specify a functional form for utility, the income process, and the level of borrowing constraints. The common assumption is that agents have CRRA utility, with an identical risk aversion coefficient for the two agents. In principle, however, the model could be used to examine the implications of differential risk aversion for portfolio composition and asset prices.

Lucas (1991) and Telmer (1991) assume that aggregate income is stationary in growth rates, and that it follows a first order markov chain. This aggregate income process is taken from Mehra and Prescott (1985), who chose these parameters to match the average, standard deviation, and first order serial correlation of the growth rate of aggregate consumption in the U.S. economy between 1889 and 1978. Following Mankiw (1986), the uninsurable component of income is evenly distributed between the two agents when aggregate output growth is high, but unevenly distributed when aggregate growth is low.[6] The high idiosyncratic income realization is assumed to be 1.5 to 1.8 times larger than the bad realization. The idiosyncratic shocks are transitory; the probability of receiving a bad shock at t is independent of the shock received at t-1. In contrast, Marcet and Singleton (1991) assume that aggregate income is stationary in levels rather than in growth rates, and that the volatility of aggregate income is about twice that of the other studies. They identify the income from the production of durable goods with one group of agents, and income from nondurables with the second group.

The severity of the assumed borrowing and short sales constraints also varies across specifications. I consider two cases: (a) trading only in stocks, with no short selling, and (b) trading only in bonds, with a ceiling on indebtedness at 60% of annual per capita income. The borrowing constraint seldom is binding. Telmer (1991) assumes that only bonds are traded, and also sets the borrowing constraint so that it rarely binds. Marcet and Singleton (1991) allow simultaneous trading in both markets. Because the shocks are large, the short sales and borrowing constraints bind fairly frequently.

[6] Lucas (1991) also considers the case in which the individual and aggregate shocks are uncorrelated.

iii. Results

In these models agents trade in the asset markets to partially offset their idiosyncratic income shocks. Short sales and borrowing constraints limit the extent to which consumption smoothing is feasible. When no asset trading is allowed, the assumed income processes are all sufficiently volatile to generate a high equity premium and a low risk-free rate. However, for the more moderate short sales and borrowing constraints described above, Telmer (1991) and I find that agents effectively trade back to the fully insured allocations. A single asset market (only stocks or only bonds) is sufficient for smoothing to take place. Since individual consumption is proportional to aggregate consumption, the implied asset prices are almost identical to those of the representative agent model. The specification of Marcet and Singleton (1991) produces more consumption variability and a higher equity premium. However, because their assumed aggregate income process is significantly more volatile, the marginal impact of market incompleteness is difficult to evaluate.

Table 1 presents summary statistics on realized stock returns, bond returns and consumption growth. Table 2 shows some typical simulation results from my specification.[7] The discount rate, β, is .95, the coefficient of relative risk aversion is 2.5, and the idiosyncratic shock is set so that income in the good state 1.2 times as large as income in the bad state. No idiosyncratic shocks occur when aggregate output is high. The first column of Table 2 illustrates that the simulated model produces a low risk-free rate and a large equity premium when agents experience idiosyncratic income shocks, but cannot borrow or trade

TABLE 1: SAMPLE STATISTICS ON ASSET RETURNS AND CONSUMPTION GROWTH, 1889–1978

$E(R_S)$	.0704
$\sigma(R_S)$	.1667
$E(R_B)$	.0104
$\sigma(R_B)$	.0549
$E(R_S-R_B)$	.0599
$\sigma(R_S-R_B)$	.1694
$E(C'/C)$	.0180
$\sigma(C'/C)$	.0365

TABLE 2: MODEL STATISTICS ON ASSET RETURNS AND CONSUMPTION GROWTH WITH IDIOSYNCRATIC SHOCKS

	Imcomplete Markets and no Trade	Complete Markets	Incomplete Markets and Trade
$E(R_S)$	.0976	.0971	.0962
$\sigma(R_S)$	.1002	.0469	.0481
$E(R_B)$	.0015	.0946	.0914
$\sigma(R_B)$	.0457	.0135	.0111
$E(R_S-R_B)$	.0991	.0025	.0048
$\sigma(R_S-R_B)$	.0807	.0452	.046
$E(C'/C)$	.0212	.0170	.0161
$\sigma(C'/C)$	.0807	.0360	.0363

[7] These are taken from Lucas (1991).

in stocks to offset the shocks. The second column reports simulation results under the assumption of complete markets (the Mehra and Prescott case). The third column shows that with idiosyncratic shocks and trading in the stock market, the equity premium shrinks and the risk-free rate rises to that of the complete markets case.

It appears that qualitatively similar results would obtain in economies with many agents. Clarida (1990) and Huggett (1991) consider related models without aggregate uncertainty, but with a continuum of agents who can trade in a single risk-free asset. Consistent with the above results, agents trade to partially offset the idiosyncratic shocks. The risk-free rate is depressed relative to the rate of time preference because idiosyncratic risk induces a precautionary demand for the bond.

These results are also related to those of an earlier paper by Scheinkman and Weiss (1986), who consider a two agent economy in which only one agent at a time is employed, and in which an agent's employment status follows a Poisson process. They show that if agents can trade in an asset market, a large part of the idiosyncratic income risk will be smoothed. However, they do not consider the role of exogenous aggregate uncertainty, nor do they focus on relative asset prices.

Finally, incomplete markets are also being explored in the international asset pricing and portfolio choice literature. One related result is by Cole and Obstfeld (1991), who construct examples in which perfect risk-sharing is achieved across countries, even if some risky assets cannot be traded.

b. The Term Structure of Interest Rates

The standard representative-agent model does not predict a number of the empirical regularities found in interest rate data such as the upward sloping term structure (Backus, Gregory, and Zin (1989)). Incomplete markets models may help to explain this data, although little work in this area has been done so far. The exception is Mehrling (1991), who considers an economy with idiosyncratic income risk and a complete set of aggregate state-contingent bonds. For tractability, he assumes that the uninsurable idiosyncratic income shocks occur at a much higher frequency than the aggregate shocks, so that the wealth distribution depends only on the current aggregate state. In this setting, it is shown that if aggregate risk is more important than idiosyncratic risk, then real interest rates are lower in booms than in contractions, and that the average term structure is upward sloping. Whether the model is empirically plausible remains to be tested.

III. *Permanent Idiosyncratic Shocks*

In the papers discussed in Section 2.a, the idiosyncratic shocks were assumed to be independently distributed over time, and consumption was effectively smoothed by trading in financial assets. Less consumption smoothing can be shown to occur, however, when idiosyncratic shocks have a permanent component. Examples of permanent shocks might include events such as suffering a serious injury, or winning a Nobel prize. Empirical evidence on the persistence of income shocks is discussed in Section 5 below.

To see why persistence matters, consider an infinite horizon economy in which agents have CARA utility. With CARA utility, consumption generally equals rW, where r is the risk-free rate, and W is wealth (appropriately defined). Thus the change in consumption is proportional to the change in wealth: $C_{t+1}-C_t=r(W_{t+1}-W_t)$. Let Y_t denote the innovation to uninsurable income, and W_t^b denote wealth at time t before the realization of Y_t. If uninsurable income shocks are i.i.d., then $W_t=W_t^b+Y_t$. Now assume instead that $Y_t=Y_{t-1}+\epsilon_t$, where ϵ_t is a mean zero uncorrelated shock. If uninsurable income were discounted at the risk-free rate, then $W_t=W_t^b+Y_t/r$. The change in wealth, and hence in consumption, due to a permanent shock is $1/r$ times greater than that due to a transitory shock. Thus higher persistence in uninsurable income shocks should lead to more volatile consumption realizations.

The above argument is only heuristic because with incomplete markets wealth is not well-defined. One would expect wealth to include the present discounted value of all future income, but by assumption future uninsured income has no market price.[8] However, Svensson and Werner (1991) show that with CARA utility, if stock returns and uninsurable income both follow Brownian motions, then agents assign a shadow present value to future uninsurable income. The shadow discount rate depends on the risk-free rate, and on the covariance of uninsurable income with the returns on traded assets. This nontradeable wealth is then treated symmetrically with financial wealth in terms of agents' consumption policies. Koo (1991) applies a similar idea to value uninsurable income with CRRA utility, but does not find a closed form solution.

In an equilibrium infinite horizon model with many agents and a finite number of risky stocks, Constantinides and Duffie (1991) show analytically that for certain income processes with persistent shocks, no trade occurs to buffer income shocks; consumption equals income each period. The paper is notable in that it derives one of the few aggregation results in this literature, and it provides closed form expressions for asset prices that can be easily interpreted. The rest of this section describes their findings.

In the case of CRRA utility,[9] Constantinides and Duffie consider a continuum of agents who maximize

$$\frac{1}{1-A}E\left(\sum_{t=0}^{\infty} e^{-\rho t}C_t^{1-A}\right)$$

The aggregate endowment is composed of labor income I, and aggregate dividend income D. The growth of aggregate consumption, $g_t=\ln(C_t/C_{t-1})$ is bounded above and below, where $C_t=I_t+D_t$. As in the models discussed above, individual labor income is risky and cannot be contracted upon. Agent a has labor income

$$I_{at}=(I_t+D_t)\exp\left[z_{at}+\gamma\log(I_0+D_0)-\gamma\log(I_t+D_t)-\alpha t\right]-D_t \tag{15}$$

where the idiosyncratic shock follows a random walk

$$z_{at}=z_{a,t-1}+\sigma\epsilon_{at}+(2\beta+2\gamma g_t)^{\frac{1}{2}}\eta_{at} \tag{16}$$

[8] More precisely, uninsured income has no market price unless it is spanned by the payoffs from traded assets. The models described in sections 2, 3, and 4 are constructed so that uninsurable income is not spanned. If uninsured income were spanned and in the absence of borrowing constraints, traded assets could be used to hedge against income shocks and markets would be effectively complete.

[9] A similar result is derived for CARA and no growth in the aggregate endowment.

Constant parameters include $A, \rho, \sigma, \beta, \gamma$, and α, where $\alpha \equiv \beta + \sigma^2/2$. $\{(\epsilon_{at}, \eta_{at}) : t = 1,2,...\}$ is a white noise process.

A no-trade equilibrium is shown to exist, with asset prices given by

$$P_t = E_t\left[\sum_{s=1}^{\infty} e^{-\hat{\rho}s}\left(\frac{C_{t+s}}{C_t}\right)^{-\hat{A}} d_{t+s}\right] \tag{17}$$

where d_t is the asset's dividend at time t, $\hat{\rho} = \rho - A\alpha - A^2\alpha$, and $\hat{A} = (1-\gamma-\gamma A)A$. Note that equation (17) gives the price of a one period risk-free bond when $d_{t+1}=1$ and $d_{t+s}=0$ for $s>1$.

The aggregate pricing operator, $e^{-\hat{\rho}}(C_{t+s}/C_t)^{-\hat{A}}$, depends only on the aggregate growth rate of consumption. It has the same form as the individual valuation operators, but with a different discount rate and a different coefficient of relative risk aversion. The aggregate discount rate reflects the increased precautionary demand for assets due to undiversifiable idiosyncratic risk. The discount rate falls with risk aversion, A, and with the magnitude of uncorrelated idiosyncratic risk, σ and β. Note that this increase in precautionary demand reduces the required return on both stocks and bonds, but only indirectly affects the equity premium.

The risk aversion parameter $\hat{A}$ determines how uninsurable shocks affect the relative returns on stocks and bonds. This in turn depends on γ, which controls the correlation between the idiosyncratic and aggregate shock. When the variance of idiosyncratic shocks is higher when aggregate output is low, effective risk aversion increases and the equity premium increases. This corresponds to Mankiw's (1986) case. When $\gamma=0$ (corresponding to Weil (1992)), the effective aggregate risk aversion coefficient is unaffected by idiosyncratic risk. Any change in the equity premium is due to the indirect influence of the discount rate.

Constantinides and Duffie observe that although the model might be successfully calibrated using cross-sectional data on uninsurable income variability, their aggregation result implies that the Euler equation tests of Hansen and Singleton (1983) reject the model. Put differently, although the model can potentially match the equity premium and the risk-free rate, the earlier Euler equation tests suggest that it fails for higher moments. However, their aggregation result depends on a very particular income growth process. One would expect that similar income processes would also induce a higher precautionary demand for assets and increase effective risk aversion, but that an aggregation result generally would not obtain.

IV. *Models with Transactions Costs*

In Section 2 we saw that with transitory idiosyncratic shocks, agents trade in financial assets to smooth consumption, and asset prices look like those with complete markets. One factor that could significantly alter these results is transactions costs. For instance, borrowing involves such expenses as credit checks, points, application fees, and verification of collateral value. Transacting in stocks entails brokerage fees, bid-ask spreads, and information costs. These costs tend to discourage trading, and impact upon asset prices.[10]

[10] See Amihud and Mendelson (1986) for an earlier exposition of these idea.

Aiyagari and Gertler (1991) study the affects of transactions costs in an economy with a continuum of agents with uninsurable idiosyncratic income risk, and no aggregate uncertainty.[11] As in the model of Section 2, agents trade to offset idiosyncratic shocks. Stocks and differentiated from bonds by the fact that they are in positive net supply, and that there is a proportional or fixed stock trading cost. Motivated by the low observed cost of trading U.S. Treasury bills, bond market transactions are assumed to be costless, or close to costless. Borrowing constratints limit the extent to which agents can substitute towards trading in the low-cost bond market. Aiyagari and Gertler fix the return differential between stocks and bonds at 3%, and choose a quantity of government bonds and a tax policy that supports an equilibrium. They find that although the model does a fairly good job explaining the relative transactions velocities of stocks and bonds, and the ratio of stocks to income, it cannot explain the large observed holdings of liquid assets by U.S. households.

Lucas and Heaton (1991) incorporate transactions costs and the possibility of persistent idiosyncratic shocks into the model described in Section 2. Transactions costs have both a direct and indirect affect on asset prices in this model. The direct affect is that the relative required rates of return on securities may be altered because trading costs shift the supply and demand schedules for assets. For instance, if the Treasury bill rate is identified with the lending rate, and if borrowers but not lenders pay a transactions cost in the bond market, then the equity premium increases with the transactions cost. This occurs because the demand for borrowed funds falls relative to the frictionless case. The indirect affect of transactions costs is that they discourage the use of asset markets to self-insure against transitory income shocks, so that consumption becomes more variable. This increased idiosyncratic volatility reduces the tolerance for aggregate uncertainty, and hence tends to increase the equity premium and lower the risk-free rate.

To calibrate the model, we estimate a statistical model of individual income that captures the size of the idiosyncratic shocks and the persistance of these shocks over time, based upon evidence from the Panel Study of Income Dynamics (PSID). The time series properties of aggregate income and dividends are also estimated using the National Income and Product Accounts. The calibrated model produces the following results. Agents strongly substitute towards trading in the lower-cost market, so that for transactions costs to affect prices, they must be simultaneously present in both markets. With a moderate wedge between the borrowing and lending rate or credit rationing, the model can generate a high equity premium, and a bond return that is close to the observed return on U.S. Treasuty securities. This occurs via the direct effect described above. The indirect effect, however, appears to be relatively unimportant.[12]

The impact of transactions costs on asset prices depends critically on how often agents choose to trade. For instance, Constantinides (1986) argued that transactions costs should have only a small effect on asset returns when individuals trade only to rebalance their portfolios. In such a situation, the main effect of transactions costs is to cause agents to delay their portfolio rebalancing in response to rtae of return shocks. As a result, asset returns

[11] Clarida (1990) uses a similar model without transactions costs to analyze international borrowing and lending patterns.

[12] For an analytic demonstration that the incidence of transactions costs affect asset prices in a simpler model, see Lucas and Heaton (1992).

are not much affetced. In the models of Aiyagari and Gertler (1991) and Lucas and Heaton (1991), agents trade frequently, and hence transactions costs have a significant affect on prices.

Consumption adjustments may be more costly than portfolio adjustments, especially for durable goods such as housing or autos. Grossman and Laroque (1990) show that such costs also affect the correlation structure of consumption and asset returns. They study a partial equilibtium, continuous-time asset pricing model in which ownership of a durable good provides a flow of consumption services. Because adjusting the stock of durables is costly, agents optimally follow an *s-S* policy under which consumption changes only after a sufficiently large change in wealth. Trafiing in asset markets is costless. The value of the risky stocks fluctuates with the aggregate endowment, but not with individual consumption. The CCAPM does not hold in this environment, although a two fund separation theorem still obtains.

Finally, recall that the rejection of the CCAPM can be attributed to too little volatility in the marginal rate of substitution of consumption. Luttmer (1991), applying the theoretical work of Jouini and Kallal (1991), shows that for a large class of models, even small transactions costs have the effect of widening the acceptable range of variation in the marginal rate of substitution.

V. *Related Empirical Evidence*

The most striking fact about portfolio composition in the U.S. is the high concentration of stock ownership. Using the 1984 Panel Study of Income Dynamics (PSID), one finds that obly 28% of the families surveyed own any stock. In the lower tail of wealth, 14% own neither stocks nor liquid assets.[13] This appears to be primafacie evidence against frictionless markets, since one can show that riks-averse agents with smooth preferences and positive wealth will hold at least a small portion of their wealth in stocks, as long as stocks have a higher mean return.[14]

The high concentration of stock ownership suggests that, at very least, asset pricing models should be tested conditional on consumption data only from those households that own stocks. Unfortunately, obtaining disaggregated consumption data to use in these tests is difficult. For the U.S., the one readily available proxy measure is food consumption. Mankiw and Zeldes (1990) compare the food consumption volatility of stockholders and nonstockholders, and calculate the correlation between consumption and asset returns for these two groups. They find that consumption volatility is higher for stockholders, and that consumption is more highly correlated with market returns. Conditioning on

[13] These statistics are from Mankiw and Zeldes (1990). It appears that these numbers do not significantly understate the scope of stock ownership. Some have suggested that the large stockholdings of private pension plans should be attributed to the individual beneficiaries of those plans. Haliassos and Bertaut (1991) point out for the majority of pension plans (those with defined benefits), the policyholders do not bear market risk. Furthermore, the overlap between stockholders and pensionholders is likely to be large. Mankiw and Zeldes (1990) note that approximately 1/6 of stock owners hold less than $100 of stock. These small holdings may not be actively managed, having been received as an inheritance or gift.

[14] See Haliassos and Bertaut (1991).

the consumption of stockholders, however, does not resolve the equity premium puzzle. Recalibrating the Mehra and Prescott (1985), model Mankiw and Zeldes find a risk aversion coefficient of 100 for all families in the PSID, and 35 for stockholders. This is well above the generally accepted range for these parameters.[15]

Many authors have used cross-sectional data to more directly examine how effectively households smooth consumption, and to look for evidence of borrowing or short sales constraints.[16] Zeldes (1989) tests for the presence of liquidity constraints, making use of the observation that the Lagrangian multiplier associated with the borrowing constraint of a constrained household will be positive, while that of an unconstrained household will be zero. Zeldes splits the sample between constrained and unconstrained households in the PSID, using other information to categorize each household. He concludes that there is evidence for borrowing constraints for constrained households. Using a different approach, Cochrane (1991) observes that if there is full insurance, then the correlation between consumption growth and a number of exogenous variables should be zero. He calculates the correlation between the food consumption growth rate and a set of exogenous variables that one might expect to affect consumption in the absence of complete insurance. Also using data from the PSID, he finds that full insurance is rejected for long illnesses and involuntary job loss, but not for spells of unemployment, loss of work due to strikes, or an involuntary move. These results seem loosely consistent with the idea that temporary shocks are smoothed, but that permanent shocks affect consumption. McCarthy (1991) strengthens Cochrane's results by performing a similar exercise while controlling for wealth, and considering the role of non-financial wealth such as housing. However, using data from the Consumer Expenditure Suvery and a broader measure of consumption, Mace (1991) reports more mixed results. Complete insurance cannot be rejected for a CARA utility specification, although it is rejected for CRRA utility.

As discussed in Section 3 above, with incomplete insurance the persistence of income shocks will influence the degree of consumption smoothing. Carroll (1991) decomposes income innovations into a permanent and a transitory component. Using data from the PSID, he fits the following specification:

$$\begin{aligned} YL_t &= P_t E_t \\ P_{t+1} &= G P_t N_{t+1} \end{aligned} \tag{18}$$

Here YL_t is non-capital income (labor income+transfers), P_t is "permanent" labor income, E_t, is an idiosyncratic shock, and N_{t+1} is a permanent shock. Both E_t and N_{t+1} are assumed to be lognormally distributed. The estimates of the variance of the transitory and permanent components are approximately equal; the standard deviation of each shock is approximately .1.[17] Whether this degree of persistence will have a significant affect on predicted asset prices in a model such as that of Constantinides and Duffie (1991) remains to be seen.

[15] Mehra and Prescott consider 10 an upper bound on risk aversion, and many authors suggest that a coefficient of 1 or 2 is more reasonable. For a dissenting opinion, however, see Kocherlakota (1990).

[16] Most of these studies were designed as tests of the permanent income or lifecycle savings hypotheses. To avoid straying too far afield, I only discuss a small fraction of this extensive and closely related literature.

[17] These estimates understate actual income variance because they are conditioned on households with no zero income events.

The biggest impediment to evaluating the importance of self-insurance by saving is the paucity of data. For instance, it would be informative to look directly at how much households use financial wealth to buffer income shocks. Unfortunately, household time series data on financial asset is unavailable. The financial data used in the studies discussed above rely on the single observation on financial data collected in the 1984 PSID survey.

One potentially important (but largely unobservable) factor that complicates these empirical investigations is taste shocks. The theoretical models discussed thus far have assumed nonstochastic preferences. However, taste shocks can make true income shocks difficult to identify. For instance, if the marginal utility of consumption increases due to a taste shock, individuals may respond by working longer hours. To the econometrician, the increase in earnings appears to be an income shock, but actually it is the result of smoothing the marginal utility of consumption. Conversely, if an individual becomes ill and requires expensive medication, the marginal utility of income may rise just when realized income falls. An interesting theoretical analysis of the implications of incomplete markets for risk-sharing with taste shocks can be found in Atkeson and Lucas (1991).

VI. *Concluding Remarks*

The main findings from this still evolving literature can be summarized as follows. Incomplete markets in the form of limited consumption insurance can have a significant impact on predicted asset prices. The presence of uninsurable idiosyncratic risk tends to depress the risk-free rate below the subjective rate of time preference. For most common utility specifications, it also tends to increase the predicted equity premium. However, the models we have considered seem unlikely to explain the higher moments of asset returns. For instance, none of the models predict the high variance of stock returns relative to bond returns. The quantitative predictions of these models are sensitive to the assumed persistence of the idiosyncratic shocks, to their correlation with aggregate shocks, and to the presence of transactions costs. Transitory idiosyncratic shocks appear to be unimportant when individuals can freely trade in financial securities, since the securities can be used to effectively self-insure. On the other hand, if idiosyncratic shocks are persistent, or if transactions costs limit trades, then examples can be constructed that match the average observed returns on stocks and bonds. Obtaining a better empirical understanding of the cross-sectional income process, the nature of borrowing constraints, and the size transactions costs seems necessary to conclusively evaluate the quantitative significance of these models.

There are many unresolved issues that remain for future research. For one, with incomplete markets the equilibrium is generally not unique (see He and Pearson, 1979). In most of the papers surveyed, the numerical solution method produced only one equilibrium, or the authors chose to focus on the equilibrium of economic interest. It would be useful to see whether qualitatively different equilibria exist. Perhaps more importantly, examples have been constructed in similar models where the price predictions are very sensitive to the assumed market structure. For instance, Ketterer and Marcet (1989) show that in an economy with two classes of agents with different risk tolerances, the introduction of an options market can significantly lower the equilibrium variability of stock returns.[18] Whether the results of the models considered in this survey are similarly sensitive has not been investigated. Finally, these models may provide a useful framework in which to study the implications of incompleteness for other aspects of financial markets such as trading volume and portfolio choice.

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[18] There are many recent theoretical papers in the related area of optimal financial innovation with incomplete markets. The interested reader should see Allen and Winton (1992) and the references therein.

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