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INFLATION TAX AND CAPITAL FLIGHT IN AN OVERLAPPING GENERATIONS MODEL

HIDENOBU OKUDA

Abstract

This essay analyzes smuggling as an intertemporal activity in a dynamic general equilibrium model where only distorting taxes are available to finance a given time-path of government spending. I use a small country overlapping generations framework and assume that the government can raise revenue by creating currency and/or by confiscating illegally held foreign currency obtained through illegal trade. In this model, smuggling arises entirely by way of attempts by people to hold foreign currency and, thereby, avoid the inflation tax on holdings of domestic currency. The government manipulates the growth rate of money supply (thereby manipulating the inflation tax) and the degree to which it confiscates illegal holdings of foreign currency. With risk-neutral preferences, it does not matter, in terms of the Pareto criterion, from which of the two sources the government obtains its revenue. However, with risk-averse preferences, the welfare comparison of the two revenue sources depends on how the size of total savings of a representative agent changes when holding foreign currency illegally is introduced as a portfolio choice.

I. *Introduction*

In many less developed countries (LDCs), creating money is an important source of government revenue. The resulting inflation creates a potential demand for foreign currencies which seem to be among the few available inflation-tax-free assets (see pp-67 and Box 3.4 in pp-68 in World Development Report (1988)). Consequently, in many such economies with high inflation rates, the authorities tightly control the trading of foreign currencies. This, in turn, leads to incentives for illegal trade in such currencies—illegal markets for foreign currencies accomplished through illegal trade in commodities.

Since the pioneering work by Bhagwati and Hansen (1973), "smuggling activities," both in commodity and financial trade, have been analyzed by many economists mainly in the static trade-theoretic framework. However, although several studies have pointed out the importance of the connection between inflation and illegal trades (for example, Sheikh (1976), Blejer (1978), Pitt (1984), and DeMacedo (1985)), illegal trade activities have not been formally investigated in the context of an optimal taxation problem. Previlous wefare studies of smuggling have assumed that government revenue from price distorting (such as tariffs) are redistributed among agents in the form of negative lump-sum taxes. This

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is equivalent to assuming that the government has available as revenue sources both price distorting taxes and lump-sum taxes. Provided the real cost of smuggling is negligibly small, it is obvious then that social welfare is maximized when all price distortionary taxes are evaded via smuggling activities. That is, since smuggling is equivalent to tariff evasion, it is always beneficial when the real costs of smuggling are small enough.

This paper, in contrast to previous work, analyzes smuggling as an intertemporal activity in a dynamic general equilibrium model where only distorting taxes are available to finance a given time path of real government spending. I assume that the government can raise revenue by creating currency and/or by confiscating illegally traded goods. The government manipulates the growth rate of money supply (thereby manipulating the inflation tax) and the degree to which it confiscates goods involved in illegal trade. Thus smuggling is viewed in the context of portfolio choice under uncertainty in the presence of a risk-free asset (domestic currency) and a risky asset (foreign currency).

In this paper, two regimes are considered: the government can either raise revenue from the taxation of domestic currency alone or, in addition, from the confiscation of illegally traded goods. With risk-neutral preferences, it does not matter, in terms of the Pareto criterion, from which of the two sources the government obtains its revenue. However, with risk-averse preferences, the welfare comparison of the two regimes depends on how the size of total savings of a representative agent changes when holding foreign currency illegally is introduced as a portfolio choice. If the level of total savings, which consists of savings in the form of domestic and foreign currencies, decreases after smuggling is introduced, then the non-smuggling regime strictly Pareto dominates the smuggling regime, ex ante. But, if the total savings increases after smuggling is introduced, then there are cases where the smuggling regime strictly Pareto dominates the non-smuggling regime. In such cases, obtaining positive government revenue from random confiscation of illegally traded goods can be better in terms of the Pareto criterion than setting it to be zero.

II. *Model*

II.1. The Agents

Consider a small open economy consisting of overlapping generations of two-period lived agents and a government. The model contains one exportable and one importable good at each date $t\geqq 1$. I assume that both goods are non-storable and the exportable good cannot be consumed. At each date $t\geqq 1$, a continuum of identical people of total measure one (called generation t) is born and lives through the next date $t+1$. Each agent of generation t is endowed with $w>0$ units of date t exportable good when young, and nothing when old. In order to focus on the intertemporal distortions in consumption allocations, I assume that there is no production in this economy. A representative agent's preferences is represented by a von Neumann-Morgenstern ulitity function $U:R_+{}^2\rightarrow R$, such that

$$U[c_t(t),c_t(t+1)]=u_1[c_t(t)]+u_2[c_t(t+1)]$$

where $c_t(s)$ is consumption of the date s importable good by generation t. I assume that $u_j[\cdot]\in C^2$, $Du_j[\cdot]>0$, $D^2u_1[\cdot]<0$, and $D^2u_2[\cdot]\leqq 0$ $(j=1,2)$. This implies that U is a concave (strictly concave if $D^2u_2[\cdot]<0$) function. In addition, I assume that all generations are identical in all respects. For the current old (generation 0), I assume that each such agent is endowed with $H(0)>0$ units of unbacked domestic currency and maximizes his current consumption.

II.2. Market Structure

At each date $t\geqq 1$, young agents can export their endowments, either legally to receive their income in the form of domestic currency or illegally to receive their income in the form of foreign currency. At each date $t\geqq 1$ both young and old agents can purchase from abroad the date t importable good, either legally in exchange for domestic currency or illegally in exchange for foreign currency. The probability of successful illegal import is controlled costlessly by the government and given by a positive constant $\alpha\in[0,1]$. When illegal import fails, the commodity is confiscated by the government. I assume that when agents purchase the importable good illegally, the agent must pay the bill in advance regardless of the attempted trade, *i.e.* the agent bears the full risk of the trade.

Finally I assume that, in this small open economy, the prices in foreign currency of exportable and importable goods are taken as given equal to unity for all date $t\geqq 1$.

II.3. The Government

The economy also contains a government which spends $G(t)>0$ units of importable good in per capita terms at each date $t\geqq 1$. The government finances its spending by creating money and/or confiscating the illegally imported good. It increases the money supply according to the scheme:

$$H(t)=n\cdot H(t-1)$$

where $H(t)$ is the per capita money supply at date t and $n>1$ is a constant growth rate of money supply for all date $t\geqq 1$. The per capita government budget constraint at date $t\geqq 1$ is

$$(1)\quad G(t)=\{H(t)-H(t-1)\}p(t)+X(t)$$

where $H(t)-H(t-1)$ is the per capita amount of domestic fiat currency newly created by the government at date t, $p(t)$ is the date t price of domestic currency in terms of the date t good, and $X(t)$ is the per capita amount of date t illegal import which is confiscated by the government. The first and second terms of the right hand side of equation (1) represent the date t government revenue from money creation and confiscation of illegally imported good, respectively. It should be noted that there is no aggregate randomness in $X(t)$, since the risk of being confiscated is shared among a measure of agents. I assume that the government prohibits agents from carrying over foreign currency from date t to date $t+1$.

III. *Agents' Maximization Problem*

The representative agent of generation $t \geqq 1$ maximizes his expected utility $E_t\{U[c_t(t), c_t(t+1)]\}$, subject to his budget constraints at date t

$$c_t(t)+p(t)h_t(t)+q(t)f_t(t) \leqq w,$$

and at date $t+1$

$$c_t(t+1) \leqq p(t+1)h_t(t)+q(t+1)f_t(t) \quad \text{with probability } \alpha$$
$$c_t(t+1) \leqq p(t+1)h_t(t) \quad \text{with probability } 1-\alpha,$$

where $h_t(t)$ and $f_t(t)$ are the units of domestic and foreign currencies carried over by the representative agent of generation t from date t to date $t+1$, respectively, and $q(t)$ is the date t price of foreign currency in terms of the date t good and is an exogenously given constant to this small economy. Thus $p(t)$, $q(t)$, w and α are taken parametrically by the agent in his maximization problem.

Let $s_h(t)$ and $s_f(t)$ denote the real balances of domestic and foreign currencies demanded by the representative agent of generation $t \geqq 1$, respectively (*i.e.* $s_h(t)=p(t)h_t(t)$ and $s_f(t)=q(t)f_t(t)$). Then his maximization problem can be written as

$$\text{(2)} \quad \underset{\{s_h, s_f\}}{\text{Max.}} \quad V[s_h(t), s_f(t)]$$

subject to $s_h(t)+s_f(t) \leqq w$, $s_h(t) \geqq 0$, and $s_f(t) \geqq 0$.

where the objective function, $V: R^2{}_+ \to R$, is defined by

$$V[s_h(t), s_h(t)] = u_1[w-s_h(t)-s_f(t)]+\alpha u_2[r_h(t)s_h(t)+r_f(t)s_f(t)]+(1-\alpha)u_2[r_h(t)s_h(t)].$$

and $r_h(t)=p(t+1)/p(t)$ and $r_f(t)=q(t+1)/q(t)$ are the real gross rates of return on home and foreign currencies.

It should be noticed that the objective function $V[s_h(t), s_h(t)]$ is concave (strictly concave, if $D^2u_2[\cdot]<0$) since the agent's utility function $U[c_t(t), c_t(t+1)]$ is concave (strictly concave, if $D^2u_2[\cdot]<0$). Since V is continuous and the constraint set for $(s_h(t), s_f(t))$ is closed and bounded, the problem given in (2) has a nonnegative optimal solution. If $D^2u_2[\cdot]<0$, it has a unique nonnegative optimal solution, which is given by a pair of functions: $s_h(t)^* = s_h[r_h(t), r_f(t), w, \alpha] \geqq 0$ and $s_f(t)^* = s_f[r_h(t), r_f(t), w, \alpha] \geqq 0$. If $D^2u_2[\cdot]=0$, the optimal solution is indeterminate because the Jaacobian matrix is singulr.[1]

1. The Jacobian matrix J is

$$\begin{pmatrix} \partial^2 V/\partial s_h^2 & \partial^2 V/\partial s_h \partial s_f \\ \partial^2 V/\partial s_f \partial s_h & \partial^2 V/\partial s_f^2 \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}.$$

Its determinant is given by det.$J=(J_{11}J_{22}-J_{12}J_{21})$ where

$$J_{11}=D^2u_1[\cdot]+a(r_h)^2D^2u_2[r_hs_h+r_fs_f]+(1-a)(r_h)^2D^2u_2[r_hs_h],$$
$$J_{12}=D^2u_1[\cdot]+ar_hD^2u_2[r_hs_h+r_fs_f],$$
$$J_{21}=D^2u_1[\cdot]+ar_hD^2u_2[r_hs_h+r_fs_f],$$
$$J_{22}=D^2u_1[\cdot]+a(r_f)^2D^2u_2[r_hs_h+r_fs_f] \text{ with } u_1[\cdot]=u_1[w-s_h-s_f].$$

Therefore, since $D^2u_f[\cdot] \leqq 0$ $(j=1,2)$,

$$\text{det.} \quad J=a(r_h-1)^2D^2u_1[\cdot]D^2u_2[r_hs_h+r_fs_f]+(1-a)(r_h)^2D^2u_1[\cdot]D^2u_2[r_hs_h]$$
$$+a(1-a)(r_h)^2D^2u_2[r_hs_h+r_fs_f]D^2u_2[r_hs_h] \geqq 0.$$

IV. *Equilibrium*

IV-1. Definitions

At each date $t \geqq 1$, all currencies (*i.e.* domestic and foreign currencies) and all goods (*i.e.* exportable and importable goods) are traded. However, since the demand for the exportable good and the supply of the importable good are perfectly elastic in this small open economy, the commodity markets are always cleared. Therefore, by Walras' law, all markets are in balance, if the excess demand for domestic currency by young agents is zero. At each date $t \geqq 1$, the market clearing condition for domestic currency is in per capita terms represented by,

(3) $h(t) - H(t) = 0.$

I define the stationary equilibria with and without illegal trade (smuggling) as follows.

Definition 1: A stationary *smuggling equilibrium* (SE) consists of a set of sequences for non-negative prices, $\{p(t), q(t)\}$, with constant rates of return, $p(t)/p(t-1) = \hat{r}_h$, and non-negative constant allocations, $\{s_h(t), s_f(t)\} = (\hat{s}_h, \hat{s}_f)$ with $\hat{s}_f > 0$, such that, for given $\alpha = \hat{\alpha}$, $n = \hat{n}$, and $q(t)/q(t-1) = \hat{r}_f$.
(i) $(\hat{s}_h, \hat{s}_f)$ solves the maximization problem (2) for all dates $t \geqq 1$;
(ii) $h(t) - H(t) = 0$ for all dates $t \geqq 1$.

Definition 2: A stationary *non-smuggling equilibrium* (NSE) consists of a set of sequences for non-negative prices $\{p(t), q(t)\}$ with constant rates of returns, $p(t)/p(t-1) = \tilde{r}_h$ and $q(t)/q(t-1) = \tilde{r}_f$, and constant resource allocations $\{s_h(t), s_f(t)\} = (\tilde{s}_h, \tilde{s}_f)$ with $\tilde{s}_f = 0$, such that, for given $\alpha = \tilde{\alpha}$, $n = n$, and $q(t)/q(t-1) = \tilde{r}_f$,
(i) $(\tilde{s}_h, \tilde{s}_f)$ solves the maximization problem (2) for all dates $t \geqq 1$;
(ii) $h(t) - H(t) = 0$ for all datIs $t \geqq 1$.

In the following sections, all variables in a stationary smuggling equilibrium and in a stationary non-smuggling equilibrium are denoted by ($\wedge$) and ($\sim$), respectively.

V. *Welfare Comparison*

Using the expected utility of a representative agent, we compare what happens if the control of the government on the illegal holding of foreign currency is so tight (say $\alpha = 0$) that the representative agent does not want to buy foreign currency at all (resulting in a NSE with $s_h(t) > 0$ and $s_f(t) = 0$) with what happens if the control of the government on the illegal holding of foreign currency is so loose that an agent wants to buy some positive amount of foreign currency (the case of a SE with $s_h(t) > 0$ and $s_f(t) > 0$), assuming first risk-neutral and then risk-averse preferences on the part of the agents.

V.1. The Case of Risk Neutrality

I begin the discussion with the case in which each agent has risk neutral preferences, *i.e.* $D^2u_2[\cdot]=0$.

Proposition 1: Suppose that $u_2[\cdot]$ is a linear function. Then there is a SE with $\hat{s}_h$, $\hat{s}_f$, and $\hat{G}$, if and only if there exists a NSE with $\tilde{s}_h$, $\tilde{s}_f$, and $\tilde{G}$ such that the expected utilities are the same, *i.e.* $V[\hat{s}_h,\hat{s}_f]=V[\tilde{s}_h,\tilde{s}_f]$, and the level of government revenue is the same, *i.e.* $\hat{G}=\tilde{G}$.

Proof of Proposition 1: Suppose that there is a SE with $\hat{s}_h$ and $\tilde{s}_f$ for given $\hat{\alpha}$. Then by choosing $n=1/\hat{\alpha}$, we can immediately construct a NSE with $s_h(t)=\hat{s}_h+\hat{s}_f$ since $u_2[\cdot]$ is linear.

Suppose that there is a NSE with $\tilde{s}_h$ for given $\tilde{n}$. By choosing $\alpha=1/\tilde{n}$, we can immediately construct a SE with $s_h(t)+s_f(t)=\tilde{s}_h$ since $u_2[\cdot]$ is linear.

Since $u_2[\cdot]$ is linear, the expected utility is, for $\hat{s}_h+\hat{s}_f=\tilde{s}_h$,

$$\begin{aligned}V[\hat{s}_h,\hat{s}_f]&=u_1[w-\hat{s}_h-\hat{s}_f]+\alpha u_2[r_h\hat{s}_h+r_f\hat{s}_f]+(1-\alpha)u_2[r_h\hat{s}_h]\\&=u_1[w-\hat{s}_h-\hat{s}_f]+u_2[r_h\hat{s}_h+r_f\hat{s}_f]\\&=u_1[w-\tilde{s}]+u_2[r_h\tilde{s}_h]=V[\tilde{s}_h].\end{aligned}$$

Since $\hat{r}_h=\hat{r}_f=\alpha$ in a SE and $\tilde{r}_h=1/n$ in a NSE, the government revenue for $\alpha=1/n$ is given by

$$\begin{aligned}\hat{G}&=N\{(1-\hat{r}_h)\hat{s}_h+(1-\hat{\alpha})\hat{s}_f\}\\&=N\{(1-\hat{\alpha})(\hat{s}_h+\hat{s}_f)\}\\&=N(1-\tilde{r}_h)\tilde{s}_h=\tilde{G}.\quad \text{Q.E.D.}\end{aligned}$$

If the expected rate of return on home currency is equal to the probability of success in illegal trade, *i.e.* $r_h=\alpha$, for each individual with risk neutral preferences, holding his savings in the form of home currency to purchase his second period consumption good legally is a perfect substitute to illegally holding his savings in the form of foreign currency to purchase his second period consumption good. On the other hand, when $r_h=\alpha$, the inflation tax ratio, $1-r_h$, is equal to the expected confiscation ratio (*i.e.* the probability of failure in illegal trade), $1-\alpha$, and government revenue in per capita terms, $(1-r_h)s_h(t)+(1-\alpha)s_f(t)$, is unchanged as long as the size of the total tax base, $s_h(t)+s_f(t)$, is unchanged. Therefore, regardless of his portfolio selection, for $r_h=\alpha$, the optimal total amount of savings of each individual, $s_h(t)+s_f(t)$, is constant and hence the government revenue is also unchanged.

V.2. The Case of Risk Aversion: Non-increasing total savings

Next I consider the more interesting case in which each agent has risk averse preferences, *i.e.* $D^2u_2[\cdot]<0$. This implies that (2) has a unique optimal solution $\{s_h(t)^*,s_f(t)^*\}$

since the objective function V is strictly concave. There are two sub-cases here depending on the particular preferences: first I consider the economies with utility functions for which the total savings is not greater in a SE than in a NSE (*i.e.* $\hat{s}_f+\hat{s}_h \leqq_h \tilde{s}$}. Then I consider the case where the total savings is greater in a SE than in a NSE (*i.r.* $\hat{s}_f+\hat{s}_h>\tilde{s}_h$).

Proposition 2: If a SE and a NSE are two stationary equilibria which satisfy $\hat{G}=\tilde{G}$ and $\hat{s}_f+\hat{s}_h\leqq\tilde{s}_h$, then the expected utility associated with the SE is less than that of the NSE, *i.e.* $V[\hat{s}_h,\hat{s}_f]<V[\tilde{s}_h,\tilde{s}_f]$.

Proof of Proposition 2: Since $\hat{G}=\tilde{G}$, the following is true:

$$
\begin{aligned}
(\tilde{G}-\hat{G})/N &= (1-1/\tilde{n})\tilde{s}_h-(\{1-1/\hat{n})\hat{s}_h+(1-\hat{\alpha})\hat{s}_f\} \\
&= (1/\hat{n})\hat{s}_h+\hat{\alpha}\hat{s}_f-(1/\tilde{n})\hat{s}_h+d=0
\end{aligned}
$$

where $d=\tilde{s}_h-\hat{s}_f-\hat{s}_h\geqq 0$.

Since $u_2[\cdot]$ is strictly concave, the expected utility of a representative agent in a SE is

$$
\begin{aligned}
&V[\hat{s}_h,\hat{s}_f] \\
&=u_1[w-\hat{s}_h-\hat{s}_f]+\hat{\alpha}u_2[\hat{s}_h/\hat{n}+\hat{s}_f]+(1-\hat{\alpha})u_2[\hat{s}_h/\hat{n}] \\
&<u_1[w-\hat{s}_h-\hat{s}_f]+u_2[\hat{\alpha}(\hat{s}_h/\hat{n}+\hat{s}_f)+(1-\hat{\alpha})(\hat{s}_h/\hat{n})] \\
&=u_1[w-\hat{s}_h-\hat{s}_f]+u_2[\hat{s}_h/\hat{n}+\hat{\alpha}\hat{s}_f] \\
&=u_1[w-(\tilde{s}_h-d)]+u_2[\tilde{s}_h/\tilde{n}-d] \\
&\leqq u_1[w-(\tilde{s}_h-d)]+u_2[(\tilde{s}_h-d)/\tilde{n}], \text{ since } \tilde{n}>1. \\
&\leqq u_1[w-\tilde{s}_h]+u_2[\tilde{s}_h/\tilde{n}], \text{ by revealed preference theory,} \\
&=V[\tilde{s}_h,\tilde{s}_f]. \quad \text{Q.E.D.}
\end{aligned}
$$

To finance the given time path of the real government spending $\hat{G}=\tilde{G}$, the policy parameters $(\hat{n},\tilde{n},\hat{\alpha})$ must satisfy $(1-1/\tilde{n})\tilde{s}_h=\{(1-1/\hat{n})\hat{s}_h+(1-\hat{\alpha})\hat{s}_f\}$. This implies that, if $\hat{s}_h+\hat{s}_f\leqq\tilde{s}_h$, the weighted average of the expected rate of return on total savings in a SE is not higher than that in a NSE.

$$\hat{\alpha}\{\hat{s}_f/(\hat{s}_f+\hat{s}_h)\}+(1/\hat{n})\{\hat{s}_h/(\hat{s}_f+\hat{s}_h)\}\leqq 1/\tilde{n}.$$

the above holding with equality for $\hat{s}_f+\hat{s}_h=\tilde{s}_h$. Therefore, it is obvious that the expected utility of each agent in a SE is lower than that in a NSE, since he is risk averse and his portfolio always contains a risky asset in the SE.

For example, if $U[c_t(t),c_t(t+1)]=\ln c_t(t)+\ln c_t(t+1)$, then $\hat{s}_f+\hat{s}_h=\tilde{s}_h=w/2$ for all $n>1$ and $\alpha\in[0,1]$. Obviously, $\hat{s}_f+\hat{s}_h\leqq\tilde{s}_h$ and $V[\hat{s}_h,\hat{s}_f]<V[\tilde{s}_h,\tilde{s}_f]$.

V.3. The Case of Risk Aversion: Increasing total savings

Now I consider the economies in which $\hat{s}_f+\hat{s}_h>\tilde{s}_h$ is possible. For these economies I have the following proposition.

Proposition 3: For some economies, there exists a SE such that for all NSE where the same revenue is obtained, *i.e.* $\hat{G}=\tilde{G}$, the expected utility in the SE is greater than that in any NSE, *i.e.* $V[\hat{s}_h,\hat{s}_f]>V[\tilde{s}_h,\tilde{s}_f]$.

Proof of Proposition 3: I prove this proposition by constructing a class of examples with the aid of a diagram (Figure 1) in which $c_t(t)$ and $c_t(t+1)$ are measured along the axes. The per capita social consumption frontier and the after tax individual consumption frontier are represented by ww' and gg', respectively. Here, for convenience, I show in advance the first order necessa)y conditions for the interior solutions.

From the maximization problem (2), given n and α, the first order necessary conditions for the interior solutions in a SE are

(4a) $Du_2[r_h\hat{s}_h+\hat{s}_f]/Du_1[w-\hat{s}_h-\hat{s}_f]=1/\alpha$

(4b) $Du_2[r_h\hat{s}_h]/Du_1[w-\hat{s}_h-\hat{s}_f]=(n-1)/(1-\alpha)$.

and for given n and α, the first order necessary condition for the interior NSE solution is

(4c) $Du_2[r_h\tilde{s}_h]/Du_1[w-\tilde{s}_h]=n$.

Let $(w,g)\in R^2_{++}$ with $g<w$. Assume that an indifference curve is tangent to wE at $E=(E_2,E_1)$ on gg'. Let $(\tilde{s}_h,\tilde{s}_f)=(w-E_1,0)$. Let $n=\tilde{s}_h/E_2$. Since the first order necessary condition (4c) is satisfied at E and the objective function V is strictly concave, a NSE occurs at E with $\tilde{s}_h$. Below, I will show that there are preferernces such that E is the best NSE with $\tilde{G}=w-g$.

Now choose $(\hat{s}_h,\hat{s}_f,\alpha)$ so that

(i) $\hat{s}_f>0$

(iii) $0<1/\alpha<n$

(iii) $(1-1/n)\tilde{s}_h=(1-1/n)\hat{s}_h+(1-\alpha)\hat{s}_f$

(iv) $(w-\hat{s}_h-\hat{s}_f)/(\hat{s}_h/n)>E_1/E_2$

(v) $(\hat{s}_h+\hat{s}_f)/(\hat{s}_h/n+\hat{s}_f)>1/\alpha$.

This can always be done since by choosing $\hat{s}_h+\hat{s}_f$ close to $\tilde{s}_h$ and $1/\alpha$ close to unity, (iv) and (v) are implied by (i)–(iii). Here (iii) implies that $\hat{G}=\tilde{G}$ and (iv) implies that the ratio of the second preiod consumption to the first period consumption in NSE is higher than that in SE when smuggliing fails. In Figure 1, C and C' represent the consumption baskets when smuggling succeeds and fails, respectively, where $C=(\hat{s}_h/n+\hat{s}_f,\ w-\hat{s}_h-\hat{s}_f)$ and $C'=(\hat{s}_h/n,\ w-\hat{s}_h-\hat{s}_f)$. It should be noticed that C is outside gg' from (iii) and that the ray from (0,0) to C' is above the ray from (0,0) to (E_2,E_1) from (iv).

Next fill out indifference curves so that

(vi) (4a) and (4b) are satisfied at C and C'.

(vii) all indifference curves are strictly concave to (0,0).

Conditions (vi), (vii) and (ii) imply that $(\hat{s}_h,\hat{s}_f,1/\alpha)$ and n is a SE. Since $\hat{s}_h=0$ is never an individual optimum, and since $1/\alpha<n$ is sufficient to imply that $\hat{s}_f>0$ in equilibrium, it follows from the convacity of V that (4a) and (4b) are sufficient for an optimum. The strict concavity of V and revealed preference imply that $V[\hat{s}_h,\hat{s}_f]>V[\tilde{s}_h,\tilde{s}_f]$.

It remains to show that (vi) and (vii) are consistent when E being the best NSE. The latter is equivalent to the condition that the offer curve for all returns higher than $1/n$ be above gg'. It should be noted that the tangencies assumed at C, C' and E are consistent even with homothetic preferences. And even with homothetic indifference curves, as I now show, there is no conflict between the tangency conditions and the offer curve condition that is equivalent to E being the best NSE.

For homothetic preferences, from condition (v) the offer curve goes through a point $X=(X_2,X_1)$ on ray Oc which is above C and therefore outside gg' (see Figure 2). I will show that well behaved homothetic preferences are consistent with the offer curve being the straight line connecting E and X for all returns $r\in[1/n,\alpha]$.

To do that we construct a marginal rate of substitution function f which assigns a marginal rate of substitution to each $z\in[E_2/E_1,X_2/W_1]$. Let $z\in[E_2/E_1,X_2/X_1]$ and consider the intersection of a ray through the origin with slope $1/z$ and the line connecting E and X. Next consider the line that connects that intersection with the point $(0,w)$. Let $f[z]$ be defined to be the negative of the slope of that line. It follows that f is continuous and strictly decreasing. Moreover, by construction, the offer curve implied by f is the straight line connecting E and X. (Obviously we could have connected E and X by a "curve" outside gg' and followed a similar construction procedure to get a continuous and strictly decreasing function f.) Q.E.D.

The restriction on the policy parameters $(\hat{n},\tilde{n},\hat{\alpha})$ under the fixed government spending, $\hat{G}=\tilde{G}=\{(1-1/\hat{n})\hat{s}_h+(1-\hat{\alpha})\hat{s}_f\}=(1-1/\tilde{n})\tilde{s}_h$, implies that the weighted average of the expected rate of return on total savings in the SE can be higher than in the NSE, $\alpha\{\hat{s}_f/(\hat{s}_f+\hat{s}_h)\}+(1/n)(\hat{s}_h/(\hat{s}_f+\hat{s}_h)\}>1/\tilde{n}$, if $\hat{s}_f+\hat{s}_h>\tilde{s}_h$. Thus, the less risk-averse a agent's preferences, the more likely it is that his expected utility in the SE will be higher than in the NSE. The welfare effect of illegal trade depends on the elasticity of total savings with respect to the tight-

FIGURE 1.

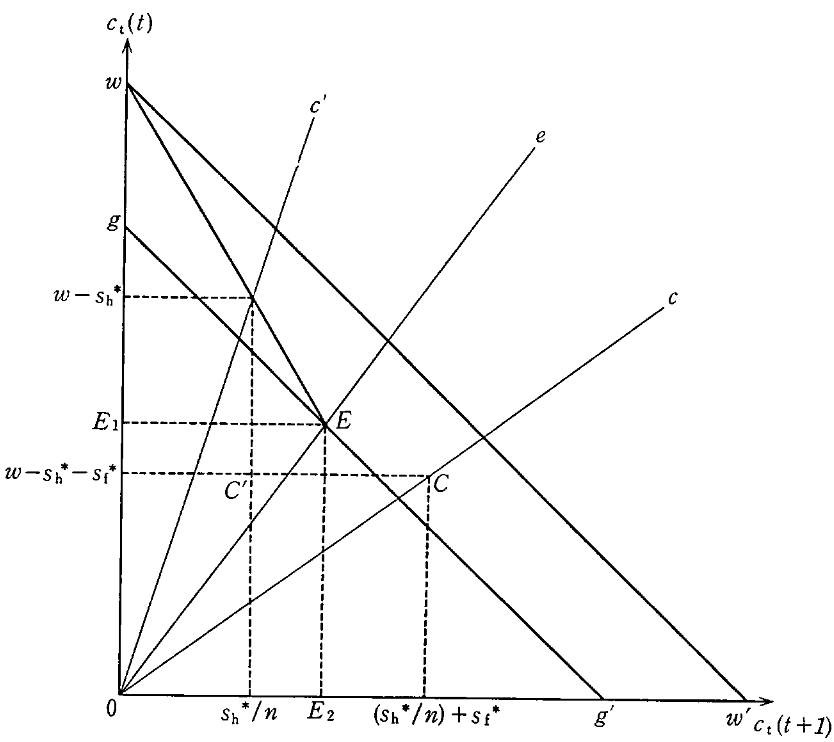
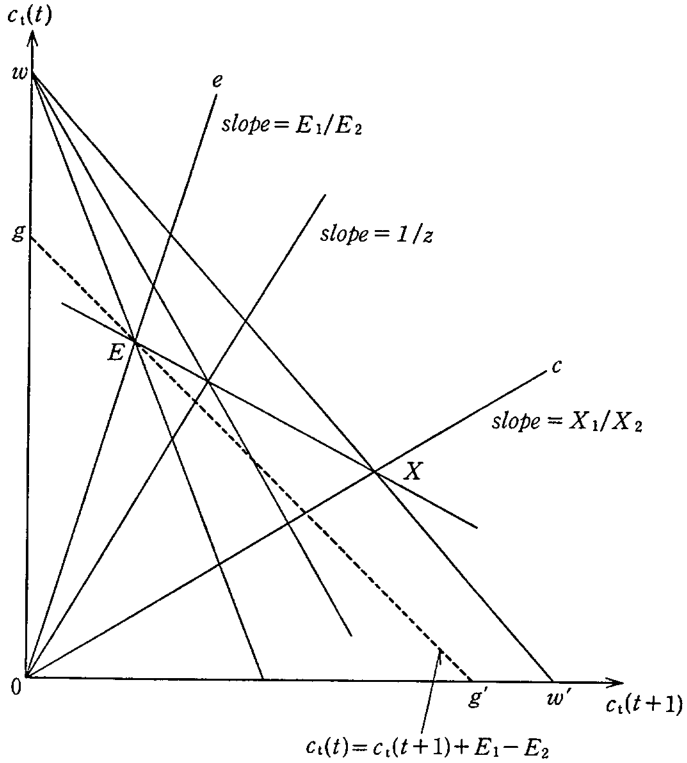


FIGURE 2.



ness of government control α. If the total savings increases as government control on smuggling is relaxed (*i.e.* α increases), illegal trade, which is a form of inflation tax evasion, may increase welfare vis-a-vis the non-smuggling situation.

V.5. An Example

I will display Proposition 3 for the following example: $U[c_t(t), c_t(t+1)] = c_t(t) + \{c_t(t+1)\}^{1/2}$ with given $1/2 < r < 1$. For these specification, $\tilde{s}_h = r/4$ where $r = 1/n$. It follows that $\tilde{G} = (r - r^2)/4$ with $d\tilde{G}/dr < 0$ for $r > 1/2$. Therefore corresponding to each $r \in [1/2, 1)$ is a NSE which has the property that there is no better NSE yielding the same government revenue. It is sufficient for the existence of a SE with higher utility and the same government revenue that the following equations be satisfied for a given $r \in [1/2, 1)$.

(5) $\hat{s}_h = r/4 - \{(1-\alpha)/(1-r)\}\hat{s}_f$

(4a′) $\hat{s}_f = \alpha^2/4 - r\hat{s}_h$

(4b′) $\hat{s}_h = \{r(1-\alpha)^2\}/\{4(1-r)^2\}$.

Here (5) represents the condition for $\hat{G} = \tilde{G}$ and (4a′) and (4b′) are the first order necessary conditions (4a) (4b), respectively. By substituting the right hand side of (4b′) into (4a′) and (5), I have

$$\hat{s}_f=\{(\alpha-r)(\alpha+r-2\alpha r)\}/\{4(1-r)^2\}$$

$$\hat{s}_f=\{r(1-r)(\alpha-r)(2-\alpha-r)\}/\{4(1-\alpha)\}.$$

Then from the equality of the right hand sides, I get

$$(6)\quad (1-\alpha)(\alpha+r-2\alpha r)=r(1-r)^3(2-\alpha-r).$$

Letting $L(\alpha)$ and $R(\alpha)$ denote the left-hand side and the right-hand side of (6), respectively, and setting $\alpha=r$, I have

$$L(r)=2r(1-r)^2>2r(1-r)^4=R(r)$$

and for $\alpha=1$

$$L(1)=0<r(1-r)^4=R(1).$$

Since $L(\alpha)$ and $R(\alpha)$ are continuous functions, there exists at least one $\alpha\in(r_h,1)$ that satisfies (6). For this α, $\hat{s}_h$ and $\hat{s}_f$ are uniquely determined without conflict to the constraint $\hat{s}_h+\hat{s}_h<w$ for some large enough w.

VI. *Concluding Remarks*

In order to focus on the essential link between smuggling activities as an intertemporal activity and inflation tax in a dynamic general equilibrium model, so far, I have assumed that the real cost of smuggling for private individuals and the law enforcement cost for the government are both zero. Changing these assumptions would change the results in the following way. A positive real cost of smuggling to the agents would reduce the expected rate of return on smuggling activities, r_f. The higher the real cost bedomes, the more likely it is that social welfare is maximized by financing a given government spending only through non-random inflation tax. On the other hand, positive law enforcement costs would make it less efficient to reduce smuggling activities and to finance government spending by the inflation tax only. The higher the enforcement costs become, the more likely it is that social welfare is maximized by obtaining government revenue not only from inflation tax but also from confiscation of illegal imports.

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