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THE CAUSES AND DEGREE OF EMPLOYMENT INTERNALIZATION*

KAZUHIRO ARAI

I. Introduction

Since the publication of the work of Doeringer and Piore (1971), the concept of the internal labor market has been one of the most important topics in the field of labor economics. Many people have used the concept to discuss economic issues, but it is surprising that few have showed exactly under what conditions it arises. Most people are surely aware of some of the fundamental causes that bring about internal labor markets, but again few have described the mechanism through which they arise. In other words, what has not been much discussed is why some firms have internal labor markets and others do not. The remarkable theories of Coase (1937), Oi (1962), and Azariadis (1975) and the persuasive discussion by Aoki (1984, Part I) lead us to believe that the internal labor market reduces a variety of transaction costs for the firm. Why then do not all firms have internal labor markets in the real world? More specifically, why does the degree of internalization vary from firm to firm? Williamson’s (1985, Chap. 10) explanation based on the differences in the degree of human asset specificity is useful in answering this question, but it is very sketchy.

This paper considers a model that produces a spectrum of different types of employment as equilibria. On one extreme is lifetime employment where the internalization of labor input is complete, and on the other extreme is spot-market employment where it is nil. In this paper we formulate the model as a simple game played between firm and worker, and pay particular attention to the firm’s dismissal policy and the worker’s incentives to choose to invest in firm-specific as opposed to general human capital.\footnote{Some authors have analyzed the problem of labor turnover in relation to specific and general capital. However, they have assumed either that the absolute amounts of the two kinds of capital are fixed [Mortensen (1978)], that the ratio of their amounts is fixed [Pencavel (1972)], or that only specific training can be undertaken on the job [Donaldson and Eaton (1976) and Hashimoto (1979)]. Contrary to these assumptions, there is very serious interaction between dismissal probability and investment in specific or general capital. On the one hand, if much specific capital has been accumulated, the dismissal probability tends to}

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\footnote{1 See Becker (1975) for the definitions of firm-specific and general human capital. In this paper the two kinds of human capital are referred to as specific and general capital for short.}
be low as some of these authors show. On the other hand, if the dismissal probability is high, the worker seeks to invest more in general rather than in specific capital. Therefore it is necessary to build a model in which these factors are determined simultaneously.

We endogenize in the game the firm's dismissal policy (probability) and the worker's decision to allocate his limited resources between specific and general capital. If the probability of dismissal is high, the worker will spend most of his time and/or effort on the investment in general rather than on specific capital. (Equivalently, if the firm has a high probability of dismissal but requests accumulation of specific capital, it cannot attract the worker.) If the worker is not likely to be dismissed, he is probably more willing to invest in specific capital. The firm, however, might suffer a loss in this case when the product price becomes low. We will see in this paper what employment policy the firm will choose and how much of the two kinds of human capital will actually be accumulated in this situation.

The equilibria of the model are classified into the following: Lifetime Employment Equilibrium (LEE), where the worker is never dismissed and accumulates some amount of specific capital; Conditional Employment Equilibrium (CEE), where the probabilities of dismissal and retention are both positive, but where there is some accumulation of specific capital; and Spot-market Employment Equilibrium (SEE), where the worker accumulates only general capital. We see all of these types of employment rather widely in the real world. Japan and the United States are typical examples where lifetime employment is not exceptional [see Hall (1982)]. The main feature of this paper is that we demonstrate exactly under what conditions each of these types of equilibrium arises.

The equilibrium type of employment that will arise depends on many factors rather than simply the degree of specificity of human capital. Even in our simple model, it depends additionally on the physical productivity of specific capital relative to that of general capital within the training firm, the market wage, the distribution of the product price, the worker's propensity to quit, the degree of his risk aversion, and so on. The equilibrium reflects the employment system that provides the worker with incentives to accumulate the most desirable combination of specific and general capital in each different condition. In other words, the degree of employment internalization is determined in accordance with the amount of incentives the firm provides with the worker to invest his time and/or effort in specific as opposed to general capital. An especially interesting feature of our model is that we can distinguish between the condition for LEE and that for CEE. Though both types of employment encourage investment in specific capital, the former does so by guaranteeing complete job security, while the latter does so by offering the possibility of high wages.

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2 The fact that the probability of dismissal affects the worker's incentives to accumulate human capital can be seen in the following example. Suppose he is not a native English speaker and that the firm requests him to learn a foreign language because it will increase his productivity when the firm transfers him abroad. If the dismissal probability is low, he may be willing to learn even a minor language. If it is high, he will be more willing to learn English. Becker et al. (1977) point out this causal relation in the economics of marriage and divorce. If the probability of divorce is high, people tend to have fewer children. An argument of this type applies to any specific capital that requires the worker's time and/or effort for accumulation. In general, we can regard the completion of any training (either formal or informal) or work that will later bring about a difference in the usefulness of this experience between inside and outside the firm as accumulation of specific capital. Thus in this paper we treat very common or even daily problems for workers and firms. See Becker (1975) and Doeringer and Piore (1971) for various examples of specific capital.
We will consider the framework of the basic model in Section II. In Section III we will obtain the equilibrium of the game and examine its optimality. Then we will extend the basic model by generalizing the distribution of the product price and the worker's quitting behavior (Section IV) and by introducing risk aversion on the part of the worker (Section V). The last section will provide some remarks.

II. Framework of the Basic Model

We consider the simplest possible two-period model of a firm-worker game. In the first period human capital investment is undertaken, while in the second period production is carried out using the human capital accumulated in the first period. At the beginning of the first period the two parties strike an employment contract. The contract represents the firm's strategy and has essentially two components. One is the wage for the second period and the other is the level of job security, which will be discussed in detail later. Taking the firm's strategy into consideration, the worker determines the allocation of his total resources, such as time and effort, toward either specific or general capital formation.

When the employment contract is offered, the price of the product in the second period is uncertain. However, since this uncertainty diminishes gradually, we assume that the price will be known for certain at the end of the first period. As soon as price information is obtained, the previously offered contract comes into effect. If the firm dismisses the worker, he must work for a new firm using the general and specific capital accumulated in the first period. If not, he can continue to work for the same firm with the same capital portfolio. Of course, he is free to quit at the end of the first period and work for a new firm in the second period.

Suppose the worker has a fixed amount of resources that can be invested in human capital in the first period. These resources are composed of either time or effort or both, but to simplify the exposition let a fixed amount \( c > 0 \) of his time represent the total resources available for investment. Choosing suitable units, let one unit of his time produce one unit of specific capital or one unit of general capital. The amount of specific capital he accumulates is denoted by \( x \) and that of general capital by \( y \), where \( x \geq 0 \), \( y \geq 0 \), and \( x + y = c \). The variable \( x \) represents the worker's only strategy in our game. We assume that...
there is some difference in productivity between the two types of human capital. More specifically, let \( a \) units of general capital be equivalent in productivity to one unit of specific capital within the firm. If \( a \leq 1 \), there is little to analyze in our framework. Thus we assume \( a > 1 \) in the following unless we explicitly assume the contrary.

Again choosing a suitable unit, let one unit of general capital have a capacity for producing one unit of output in the firm in the second period. Thus the total physical product of the worker is equal to \( ax + y \), if the two parties do not sever their relationship. The price of the product in the second period is denoted by \( p \). We assume in the basic model that \( p \) can take on only two values, \( p_1 \) and \( p_2 \), where \( 0 < p_1 < p_2 \). The probability that \( p = p_i \) is \( \pi_i > 0 \) for \( i = 1, 2 \), where \( \pi_1 + \pi_2 = 1 \). The worker’s value product can be written as \( p(ax + y) \). The worker will receive a fixed wage \( w \) in the second period if the two parties maintain their relationship. This \( w \) is one of the two components in the firm’s strategy.

If the worker is dismissed after price information is obtained, he must work in the second period for a new firm using the human capital portfolio \((x, y)\) he accumulated in the first period. The specific capital now is not as productive in the new firm as in the old one. Let \( \beta \) units of general capital be equivalent in productivity to one unit of specific capital, where \( 0 \leq \beta < a \), and \( \beta < 1 \). \( \beta \) is a measure of the specificity of specific capital. The smaller the value of \( \beta \), the higher the specificity. If \( \beta = 0 \), it is completely specific. We assume that the wage the worker will receive in the new firm is equal to \( r(px + y) \), where \( r > 0 \) is fixed. This assumption approximates the case where there is a large market for general capital. It also simplifies the worker’s decision to quit. He will quit if and only if the initial firm offers a lower wage than the highest possible wage he can earn elsewhere.

We will now consider again the initial firm-worker relationship and see how the firm makes the determination to dismiss or retain the worker. When specific capital exists in an uncertain world, a firm does not necessarily dismiss a worker even if the value product becomes less than the wage. To describe this we introduce a measure of job security. We assume that the firm dismisses the worker if and only if \( p(ax + y) + t < w \), i.e., iff the worker’s wage exceeds his value product by more than \( t \). Equivalently, the firm retains the worker iff \( w - p(ax + y) \leq t \), i.e., iff the difference between wage and value product lies within the limit of \( t \). The value of \( t \) measures the job security offered by the firm, and \textit{ceteris paribus} the larger the value of \( t \), the less likely is the occurrence of dismissal. If \( t > 0 \), it is possible that the firm will retain the worker even when his value product is less than his wage. This is an example of labor hoarding. If \( t < 0 \), the firm might dismiss the worker even when his value product is larger than his wage. This variable \( t \) is the second component in the firm’s strategy, and we call it the security limit of employment since employment is guaranteed as long as the difference between wage and value product lies within this limit. Using the firm’s dismissal-retention rule, we can write the probabilities of dismissal and retention as functions of the combination \((w, t, x)\) of the strategies of the two parties. Mak-

\[ \tag{6} \text{Making } p \text{ continuous would make manipulation of the model difficult.} \]

\[ \tag{7} \text{Strictly speaking, the condition that } \beta < 1 \text{ is unnecessary. But the case where } 1 \leq \beta < a \text{ is not especially interesting. We can easily extend the basic model to include this case and show } x = c \text{ in the equilibrium. We note that specific capital is characterized by } (a, \beta) \text{ and that specificity is measured by } \beta \text{ when } \beta < 1. \text{ Some might consider a function of } a \text{ and } \beta \text{ such as } a/\beta \text{ as a measure of specificity, but it is not a proper measure. It is the value of } \beta \text{ and not that of such a function that affects the worker’s incentives to invest in specific capital.} \]

\[ \tag{8} \text{We think that the randomness of wages is much smaller than that of product prices in the real world. Without this assumption, the computation would become much more complex.} \]
ing use of the constraint $x+y=c$, we have: probability of dismissal $= \text{Prob} \left\{ p((\alpha-1)x+c)+t < w \right\}$, and probability of retention $= \text{Prob} \left\{ p((\alpha-1)x+c)+t \geq w \right\}$.

Given $(w, t)$, we can write the worker's evaluation $U(x)$ of the firm's strategy as a function of his choice of $x$. Suppose the worker is interested in maximizing the expected wage he will receive (This implies that he is risk neutral). Then the evaluation function represents simply the expected wage he will receive if he chooses $x$ in reaction to $(w, t)$. Allowing for the above dismissal-retention rule, for $0 \leq x \leq c$ we have:

\begin{enumerate}
\item[(1a)] $U(x)=w, \quad \text{if} \quad \frac{w-t}{p_1(\alpha-1)} - \frac{c}{\alpha-1} \leq x$;
\item[(1b)] $U(x)=\pi_1 r(c-(1-\beta)x)+\pi_2 w, \quad \text{if} \quad \frac{w-t}{p_2(\alpha-1)} - \frac{c}{\alpha-1} \leq x < \frac{w-t}{p_1(\alpha-1)} - \frac{c}{\alpha-1}$;
\item[(1c)] $U(x)=r(c-(1-\beta)x), \quad \text{if} \quad x < \frac{w-t}{p_2(\alpha-1)} - \frac{c}{\alpha-1}$.
\end{enumerate}

If $x$ is in the region of (1b) for example, the probability of dismissal is $\pi_1$ and that of retention is $\pi_2$. Since the worker will work either for the initial firm or for a new one in the second period, taking account of his evaluation function enables us to write his expected wage of this game as

\[ S(x) = \max\{U(x), r(c-(1-\beta)x)\}, \quad 0 \leq x \leq c. \]

The second component in the braces represents the wage the worker will receive if he quits with $x$ units of specific capital. Given $(w, t)$ by the firm, the worker chooses $x$ so as to maximize $S(x)$ subject to $0 \leq x \leq c$. Since he is always free to choose $x=0$, $S(x) \geq rc$.

Next we consider the payoff to the firm. We assume that if the firm dismisses the worker, it can instantaneously hire a new worker with $c$ units of general capital for the market wage rate. As we see later, we need this assumption only for the sake of formality. Let $v_i$ denote the profit the firm gains if it dismisses the initial worker when $p=p_i$. Then

\[ v_i = \begin{cases} (p_i-r)c & \text{if} \quad r < p_i, \\ 0 & \text{if} \quad p_i \leq r, \quad (i=1, 2). \end{cases} \]

Using (3), we can write the firm's expected profit as follows:

\begin{enumerate}
\item[(4a)] $V(w, t)=E(p)\ ((\alpha-1)x+c)-w$,
\item[(4b)] $V(w, t)=\pi_1 v_1 + \pi_2[p_2((\alpha-1)x+c)-w]$,
\item[(4c)] $V(w, t)=\pi_1 v_1 + \pi_2 v_2$,
\end{enumerate}

where $E(p)=\pi_1 p_1 + \pi_2 p_2$. The region of $(w, t, x)$ for each of the above three equations is the same as the corresponding region in (1). In order for (4a) and (4b) to hold, it is necessary by (2) that $U(x) \geq rc$, otherwise the firm's payoff is equal to that in (4c). In the region for (4c), the above condition is automatically satisfied with an equality. The firm chooses $(w, t)$ so as to maximize $V(w, t)$ subject to $U(x) \geq rc$ in anticipation of the worker's reaction to the chosen strategy.

We would like to add a note here. There are cases where the worker chooses $x=0$ and the firm dismisses him formally as in (1c). However, we can assume the firm rehires him
for \( rc \) as his wage, if it intends to employ a worker with only general capital after it dismisses the old one. The reason is that the firm is equally served by a new worker or the old one when both have only general capital, and the old worker is completely indifferent to working for either a new firm or the old one because the wage is equal to \( rc \) in both. This can occur because the two parties have no special attachments to each other due to the worker’s lack of specific capital investment.

### III. The Equilibrium of the Basic Model

In this section we obtain the equilibrium of the basic model. We define it as the combination of strategies \((w, t, x)\) that maximizes the firm’s payoff. Though the computation is slightly complex, the basic method for obtaining the equilibrium is simple. We first characterize each type of equilibrium by using simple linear programming. Then we examine which payoff of the firm is actually the largest by comparing the payoffs obtained in the characterization. We show explicitly the conditions in which LEE, CEE, or SEE arises. We also consider briefly the optimality of the equilibrium.

First, we would like to characterize LEE. (1a) implies that in this case the worker chooses \( x \) such that \( x \geq (w-t)/p_{1}(\alpha-1)-c/(\alpha-1) \) and the firm does not dismiss him even when the product price is the lowest. Since his wage is independent of the chosen level of \( x \) in this region, we assume for simplicity that he chooses \( x=(w-t)/p_{1}(\alpha-1)-c/(\alpha-1) \). We can easily show that the results will not change even if we assume that he chooses any \( x \) level in the above region. Since \( 0<x<rc \), \( t<w-p_{1}c \) and \( t \geq w-\alpha p_{1}c \). As mentioned below (4), it is necessary that \( w \geq rc \). Substituting the above \( x \) level into (4a), we can write the firm’s expected profit in LEE as \( V(w, t)=(E(p)/p_{1})w-E(p)t/p_{1} \). The firm maximizes this \( V(w, t) \) subject to \( t<w-p_{1}c \), \( t \leq w-\alpha p_{1}c \), and \( w \geq rc \). The equilibrium is obviously \((w, t, x)=(rc, (r-c(pl)c, c))\) and the firm’s payoff \( V^{L} \) in this equilibrium is given by

\[
5) \quad V^{L}=(\alpha E(p)-r)c
\]

Next we characterize SEE. Any of the firm’s strategies that induce the worker to choose \( x=0 \) leads to SEE. There are a continuum of such strategies.\(^9\) The firm’s payoff \( V^{S} \) in this equilibrium is the same as (4c), i.e.,

\[
6) \quad V^{S}=\pi_{1}v_{1}+\pi_{2}v_{2}
\]

According to (1b), CEE can arise only when the worker chooses \( x=(w-t)/p_{2}(\alpha-1)-c/(\alpha-1) \). Since \( 0<x<rc \) in CEE, \( t<w-p_{2}c \) and \( t \geq w-\alpha p_{2}c \). Substituting the above \( x \) level into (1b), we can compute the condition for \( U(x) \geq rc \). It is expressed as

\[
7) \quad t \geq \left[ 1-\frac{\pi_{2}}{\pi_{1}} \frac{\alpha-1}{1-\beta} \frac{p_{2}}{p_{1}} \right] w + \frac{\pi_{2} \alpha + \pi_{1} \beta - 1}{\pi_{1}(1-\beta)} p_{2} c.
\]

Substituting again the above \( x \) level into (4b), we can write the firm’s expected profit as \( V(w, t)=\pi_{1}v_{1}-\pi_{2}t \). In CEE the firm maximizes this \( V(w, t) \) subject to the three inequality conditions we have just obtained. A simple computation shows that CEE is \((w, t, x)=(1-\)

\(^9\) Two examples are \((w, t)=(3p_{2}ac, p_{2}ac)\) and \((w, t)=(2p_{4}ac, 0)\).
\( \pi_1 \beta )r / p_2, (1 - \pi_1 \beta )r / p_2 - \alpha p_2 c, c \). The firm's maximized expected profit \( V^c \) in this equilibrium is given by

\[
V^c = \pi_1 v_1 + (\alpha \pi_2 p_2 - (1 - \pi_1 \beta )r)c,
\]

and all of these hold when\(^{10}\)

\[
\alpha \geq \frac{\pi_1}{\pi_2} - \frac{1 - \beta}{p_2} r + 1.
\]

So far we have characterized the three types of equilibrium. We would now like to investigate which one will actually arise. We can do this by comparing the firm's payoffs in (5), (6), and (8) to see which of them is actually the largest. We summarize the relative magnitude of the three payoffs in Table 1, which shows, for example, that when \( p_1 \leq r < p_2 \), \( V_L \geq V^c \) iff \( \alpha \geq \beta r/p_1 \). The two blanks in the table imply that the corresponding relationships hold unconditionally.\(^{11}\)

Using Table 1, we can show which payoff is the largest in \((r, \alpha)\) planes. The combinations of the values of the exogenous parameters that give rise to different types of equilibrium can be classified as areas in \((r, \alpha)\) planes with the values of parameters other than \( r \) and \( \alpha \) determining the boundaries of the areas. The classified equilibria are depicted in Figures 1 and 2. Figure 1 applies to the case where \( 0 \leq \beta < p_1/E(p) \), while Figure 2 applies to the case where \( p_1/E(p) \leq \beta < 1 \). The Appendix explains how we derive these figures. In Figure 1 we have LEE in the area above ABCD and SEE in the area below it. In Figure 2 we have LEE in the area above FGHJ, CEE in the area encompassed by IHJK, and SEE in the area below FGHJK. On the boundaries, either of the adjacent equilibria can arise.

We now discuss the optimality of the equilibrium. We can show that the equilibrium we have obtained maximizes the joint wealth of the initial two parties. Instead of providing the details, we here sketch the method of proof. Assume both parties behave cooperatively so as to maximize the sum of their payoffs. We consider the following three exhaustive cases of employment relations and the corresponding joint wealth. The first is the case

| Table 1. |
| --- | --- | --- |
| \( V_L \geq V^c \) | \( V_L \geq V_S \) | \( V^c \geq V_S \) |
| \( r < p_1 \) | \( \alpha \geq \beta r/p_1 \) | \( \alpha \geq \pi_1 (1 - \beta )r / \pi_2 p_2 + 1 \) |
| \( p_1 \leq r < p_2 \) | \( \alpha \geq (\pi_1 r + \pi_2 p_2) / E(p) \) | \( \alpha \geq \pi_1 (1 - \beta )r / \pi_2 p_2 + 1 \) |
| \( p_2 \leq r \) | \( \alpha \geq \beta r/p_1 \) | \( \alpha \geq r / E(p) \) | \( \alpha \geq (1 - \pi_1 \beta )r / \pi_2 p_2 \) |

\(^{10}\) The equilibrium values of \( w \) and \( x \) are not unique when an equality holds in (9). In particular, \( x \) can be less than \( c \). We ignore this in the following discussion, because it is not very important (The reader will see that this type of CEE arises only in some cases where the two parties are indifferent to CEE and SEE).

\(^{11}\) We adopt simplification of notation here. The statement that \( A \geq B \) iff \( C \geq D \) implies that \( A > B \) iff \( C > D \), that \( A = B \) iff \( C = D \), and that \( A < B \) iff \( C < D \). We actually have strict inequalities for the relative magnitude of the payoffs corresponding to the two blanks. The table can be obtained easily. If \( p_1 \leq r \), \( V_L \geq V^c \) iff \( \alpha \geq \beta r/p_1 \). All the others can be proved similarly.
where the two parties will not terminate their relationship. This is the same as the relationship between the two parties in LEE if $x>0$. The joint wealth in this case is

$$W^L(x) = E(p) ((\alpha-1)x + c).$$

The second is the case where they will definitely separate formally. Since obviously $x=0$, this corresponds to SEE and the maximized joint wealth is
The third is the case where the two parties will separate only when the product price is low, i.e., \( p = p_1 \), because the benefit derived from maintaining the relationship is low. This corresponds to CEE if \( x > 0 \) and the joint wealth is

\[
W^s = \pi_1 v_1 + \pi_2 v_2 + rc .
\]

We compare the joint wealth in (10) through (12) and examine which is actually the largest (Note maximizing (10) and (12) leads to either \( x = c \) or \( x = 0 \)). We can easily prove that when \( \alpha > 1 \) the relative magnitude among \( W^L(c) \), \( W^C(c) \), and \( W^S \) is exactly the same as that among \( V^L \), \( V^C \), and \( V^S \), i.e., that we have exactly the same relationships as those in Table I with \( V^L \), \( V^C \), and \( V^S \) replaced by \( W^L(c) \), \( W^C(c) \), and \( W^S \) respectively. When \( \alpha \leq 1 \), it is easy to see that \( W^S \) is the largest. These facts imply that the equilibrium is joint-wealth maximizing.

We see in Figures 1 and 2 that the type of equilibrium that will arise depends on many parameters rather than simply on the specificity of human capital. All the exogenous parameters but \( c \) in our model contribute to the determination of the types of equilibrium. We notice first that when the specificity of human capital is high (\( \beta \) is small), we have either LEE or SEE and no CEE. High specificity makes conditional employment costly to the firm, because the wage must be high enough to compensate for the low wage the worker will receive if he is dismissed. We would like to emphasize that SEE can arise even if the specificity is high and specific capital is more productive than general capital (see the area between BC and \( \alpha = 1 \) in Figure 1). In this case specific capital is surely valuable to the firm when the product price is high (\( p = p_2 \)) but not valuable enough when it is low, so lifetime employment is costly.

The figures imply that LEE tends to arise when \( \alpha \) is large, or \( r \) is low relative to the product price. If specific capital is productive within the training firm, it encourages investment in specific capital by guaranteeing perfect job security. We would like to emphasize that \( \alpha \) is not a measure of specificity, but that it is a very important factor for employment internalization. When \( r \) is low the loss that may arise when \( p = p_1 \) is likely to be small. The realization of LEE depends on more parameters. It is not difficult to show from the figures that ceteris paribus LEE is more likely to arise as \( p_1 \), \( p_2 \), or \( \pi_2 \) is larger, or \( \beta \) is smaller. We note that labor hoarding can arise in LEE. The equilibrium security limit of employment is positive in the areas encompassed by EBCD in Figure 1 and by LGHI in Figure 2 (see the discussion above (5)). In this case the firm will retain the worker when \( p = p_1 \), even though the value product is actually less than the wage. It chooses this strategy in order for the worker to accumulate the productive specific capital, which brings about large profits when the product price is high (\( p = p_2 \)).

As noted above, CEE arises only when the specificity of human capital is low (\( \beta \) is large). It is not difficult to see further that ceteris paribus CEE is more likely to arise as \( p_1 \) is smaller, or \( p_2 \) or \( \pi_2 \) is larger. The conditions for the product prices seem especially interesting. If \( p_1 \) is small (relative to \( r \)), then the advantage of dismissal at \( p = p_1 \) is large. If \( p_2 \) is large (relative to \( r \)), then the advantage of retention at \( p = p_2 \) is large. If \( \pi_2 \) is large, the wage cost in CEE is small (see the discussion above (8)). Though we have assumed that the firm can hire a new worker with \( c \) units of general capital when it dismisses the old one, it does not
actually hire a new one in CEE. We can see this from the fact that CEE arises only when \( p_1 \leq r \). If retaining a productive worker is not profitable, hiring a less productive worker is much less profitable.

SEE arises in the areas where neither LEE nor CEE arises. SEE is a type of employment similar to that in the traditional general equilibrium model, where there is no (productive) specific capital (see Debreu (1959)). Since no specific capital is accumulated in SEE, the two parties have no special attachment for the other and separation does not cause any external diseconomies. The labor input in one period does not have to include the same people as those in the previous period and the firm loses nothing in the general equilibrium model. Continuation of employment has no significance in this case. In this sense, the areas where \( \alpha < 1 \) and \( r < p_1 \) in Figures 1 and 2 are noteworthy. A type of employment that might be called lifetime employment arises here, since the firm might rehire the initial worker after formal dismissal and the latter might work for the former again in the second period. This type of employment, however, differs from LEE, because no attachment exists between the two parties.

The fact that the equilibrium is joint-wealth maximizing has some important implications. Since the worker’s wage is kept to the lowest level, no other system can increase the firm’s payoff. This implies that the two components \((w, t)\) of the firm’s strategy alone can bring the highest possible expected profit. It implies in particular that a fixed wage maximizes the firm’s payoff.\(^{12}\) It might be of interest to formulate the problem we have considered in terms of a cooperative game, since it involves only two parties and information is likely to be shared. An important implication of our analysis is that even such a formulation is likely to result in the equilibrium types of employment we have obtained above.

In summary, it is instructive to note that the structure of employment is stratified in a certain fashion. As we have already emphasized the effect of \( \beta \) on realized types of equilibrium, we now turn our attention to other parameters. If we assume that all the parameters but \( \alpha \) are fixed, then as \( \alpha \) becomes smaller, the equilibrium type of employment changes in the order of LEE without labor hoarding, LEE with labor hoarding, CEE, and SEE, though some of these may be absent for certain parameter values. If we assume next that all the parameters but \( r \) are fixed, then as \( r \) increases, we get the equilibrium type of employment in the same order as above, though again some may be absent.

### IV. Some Generalizations

Among the results we obtained in the previous section, the one on the nonexistence of CEE for low levels of \( \beta \) seems of special interest. We would like to show here that even if

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\(^{12}\) This observation eases the treatment of the first period wage the initial firm might offer. Let \( w_1 \) denote it. If we assume that the first period is simply a training period, then it will be reasonable to assume \( w_1 = 0 \) for simplicity. If we assume that human capital is accumulated through on-the-job training or learning by doing, the firm has only to set \( w_1 = r_1 \), where \( r_1 \) is the wage the worker would receive outside the firm. The wage and the security limit of employment we have obtained so far for the second period can be combined with this \( w_1 \) and there is no change in the outcomes we get, if the value product in the first period is not less than \( w_1 \). Even if it is less than \( w_1 \), we need only a minor change which results from the condition that the sum of the (expected) profits in the two periods must be nonnegative. (We have assumed for simplicity that the discount factor is unity and that the ‘normal’ level of profit is zero. These can be easily modified.)
the product price has a very general probability distribution, CEE does not exist when human
capital is highly specific.

Assume now that the product price is a continuous random variable which is distributed
on the interval $0 < p \leq \bar{p} < +\infty$ with probability distribution function $F(p)$ and density
function $f(p)$. Since the procedure of proof is very similar to that in the previous sections,
we will only sketch the outline. The maximized expected profit in LEE is

$$V_L = (\alpha E(p) - r)c,$$

where $E(p)$ denotes the expected value of the product price. Formally, (13) is exactly the
same as (5). The expected profit in SEE can be written as

$$V_S = \int_{p}^{\bar{p}} v(p)f(p)dp,$$

where $v(p) = (p - r)c$ if $p > r$ and $v(p) = 0$ if $p \leq r$. (14) is a generalization of (6).

In CEE there must exist $p'(p < p' < \bar{p})$ such that the firm will dismiss the worker if $p < p'$
and will retain him if $p \geq p'$. The maximized expected profit in CEE is

$$V_C = \int_{p}^{p'} v(p)f(p)dp + \alpha \int_{p'}^{\bar{p}} pf(p)dp - (1 - F(p'))c,$$

and this holds when

$$\alpha \geq F(p')(1 - \beta) \int_{p'}^{\bar{p}} pf(p)dp + 1.$$ 

Note the similarity between (15) and (8), and that between (16) and (9).

Now we compare $V_C$ with $V_L$. Note first that $p' \leq r$, since otherwise the firm would
dismiss the worker in CEE even when the profit was positive, i.e., $r < p < p'$. Then

$$V_C - V_L = \left[ F(p') \beta r - \alpha \int_{p}^{p'} pf(p)dp \right] c.$$

We consider an extreme case where $\beta = 0$, i.e., human capital is completely specific. In this
case it is obvious that $V_C < V_L$, and we never have CEE. We have either LEE or SEE. The
boundary similar to ABCD in Figure 1 can be obtained by comparing $V_L$ with $V_S$. It is

$$\alpha = 1 \text{ if } r < \bar{p},$$

if $p \leq r < \bar{p}$, and $\alpha = r/E(p)$ if $\bar{p} \leq r$. It is easy to show that this boundary is continuous, increasing, and convex. The condition that $\beta = 0$ is sufficient for the nonexistence of CEE. We conjecture from our discussion that it is unlikely that CEE exists for small positive values
of $\beta$.

Next we consider another generalization of the basic model. Up to this point we have
assumed that the worker's decision to quit is based only on wages. In the real world, how-
ever, there are cases where workers quit for reasons unrelated to wages. One example is the case of women, who often quit their initial firms after marriage. In some economies workers tend to quit more often for non-wage reasons than in others. We assume here that the worker quits for non-wage as well as wage reasons. The easiest way to approach this problem is to introduce a fixed quitting probability as in Salop and Salop (1976). Suppose the worker quits at the end of the first period for non-wage reasons with probability $0 \leq q < 1$ and independently of the product price. If he does quit, he has to work for a new firm with the human capital portfolio $(x, y)$ he accumulated in the first period, while the initial firm has to hire a new worker with $c$ units of general capital, provided the profit is positive.

The procedure for the derivation of equilibrium is similar to that in the previous section. Without going into detail, we would like to point out only the major results of this generalized model. We can show that each of the three types of equilibrium can arise again depending on the relative magnitude of the parameters. However, the wages in LEE and CEE are now increasing functions of $q$. The existence of CEE depends again on the magnitude of $\beta$. This time, however, CEE exists only if $\beta \geq p_1/(E(p) - q\pi_2(p_2 - p_1))$. Figure 3 classifies the equilibria when $\beta$ is large enough for CEE to exist. If $q=0$, we obtain the results shown in Figure 2. We omit a figure corresponding to Figure 1. We see in Figure 3 that as $q$ increases, the existence of LEE and CEE becomes less likely, while the likelihood of SEE arising becomes greater. When the worker’s probability of quitting is high, the firm has to offer a sufficiently high wage to encourage investment in specific capital. Since this is costly, SEE is more likely to arise. In an economy where workers tend to quit often for non-wage reasons, the types of employment that involve specific capital are less likely to arise. Moreover, those who are likely to quit easily for non-wage reasons (e.g., women) tend to be discouraged to invest in specific capital. Typically, they remain outside of lifetime employment systems.\(^{13}\)

\(^{13}\) Career women tend to choose occupations that require formal qualifications rather than those that require firm-specific ability. Women have traditionally been denied access to lifetime employment systems in Japan.
V. Risk Aversion

We have so far assumed that both parties are risk neutral. We would like to assume in this section that the worker is risk averse and see the effects of this on equilibrium. We assume again that the firm is risk neutral. Let \( u \) denote the utility function of the worker, where \( u' > 0 \) and \( u'' < 0 \). From the viewpoint of the worker, there is no uncertainty in either LEE or SEE. So we have only to examine CEE in detail.

Rewriting (1) by using \( u \) implies that the boundary of the condition that the expected utility in CEE is no smaller than \( u(rc) \) becomes

\[
\pi_1 u \left[ r \left( \frac{d-\beta}{\alpha-1} - \frac{1-\beta}{\alpha-1} \frac{w-t}{p_2} \right) \right] + \pi_2 u(w) = u(rc).
\]

Applying Jensen's inequality to the left-hand side of (19), we have\(^{14}\)

\[
\pi_1 r \left( \frac{d-\beta}{\alpha-1} - \frac{1-\beta}{\alpha-1} \frac{w-t}{p_2} \right) + \pi_2 w > rc.
\]

This is similar to (7). In fact if the strict inequality in (20) was replaced with a weak one, they would be exactly the same. This situation is depicted on the \((w, t)\) plane in Fig. 4. \( R_0 \) is equivalent to the boundary of (7). (20) implies that the value of \( t \) that satisfies (19) is

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\(^{14}\) It is easy to show that the two arguments of \( u \) in the left hand side of (19) are not equal for \( w \geq rc \) and \( w - \alpha p_2 c \leq t < w - p_2 c \).
larger than that on the boundary of (7) for a given value of \( w \) in the region under consideration. Thus (19) lies above \( R_0 \). The more risk averse in terms of absolute risk aversion, the higher the position of (19) relative to \( R_0 \). In Figure 4 the utility function for \( R_2 \) is more risk averse than that for \( R_1 \). We note that (19) is convex. Differentiating (19) with respect to \( w \), we have

\[
\frac{dt}{dw} = 1 - (\alpha - 1) \pi_2 p_2 u'(w) / [(1 - \beta) \pi_1 r u'[u^{-1}((u(rc)) - \pi_2 u(w))] / \pi_2],
\]

which is increasing in \( w \).

Since the firm maximizes \( \pi_1 v_1 - \pi_2 t \) (see the discussion below (7)) subject to \( t < w - \alpha p_2 c, \ t \geq w - \alpha p_2 c \), and (19) in CEE, the more risk averse the worker, the smaller the expected profit. Since the expected profits in LEE and SEE are the same as (5) and (6) respectively, this implies that the more risk averse the worker, the less likely it is for CEE to arise. An interesting feature of this CEE is that the amount of specific capital accumulated may be less than \( c \) because of the convexity of (19). In the basic model \( x \) was equal to \( c \) in CEE, but this time the firm may not encourage the worker to invest all his resources in specific capital because his risk aversion may make the wage cost very high. We can show explicitly in relatively simple examples that \( x \) in CEE is actually less than \( c \) in some cases.

When the worker is risk averse, the expected profit is smaller and the amount of specific capital accumulated may be smaller in CEE than when he is risk neutral. Even the expected profits in some LEE and SEE are smaller, because they would actually be CEE when the worker was risk neutral. Thus risk aversion on the part of the worker affects the allocation of resources through the firm’s dismissal-retention decision and the worker’s investment behavior.

There is an employment system that Pareto-improves this situation. Consider a severance pay system in which the firm pays a fixed amount of money \( s \) in case of dismissal to the worker if he has accumulated a certain amount of specific capital. The expected utility that corresponds to (1b) can now be written as

\[
U(x) = \pi_1 u[r(c - (1 - \beta)x) + s] + \pi_2 u(w),
\]

and the worker chooses \( x = (w - t)/p_2(\alpha - 1) - c/(\alpha - 1) \) in CEE. Substituting this level of \( x \) in the firm’s expected profit \( \pi_1 (v_1 - s) + \pi_2 [(\alpha - 1)x + c - w] \), which corresponds to (4b), we have

\[
V(w, t, s) = \pi_1 (v_1 - s) - \pi_2 t.
\]

Now let \( w = rc, \ t = rc - \alpha p_2 c, \) and \( s = (1 - \beta)rc \). Then \( x = c \), the worker’s expected utility is equal to \( u(rc) \) in (22), and the firm’s expected profit is equal to

\[
\pi_1 v_1 + (\alpha \pi_2 p_2 - (1 - \pi_1 \beta)r)c
\]

in (23). We note that (24) is exactly the same as (8). Since the expected profits in LEE and SEE are again the same as (5) and (6) respectively, this implies that we have the equilibria exactly the same as those depicted in Figures 1 and 2. As discussed before, the equilibria

\[15\] A utility function that is more risk averse than \( u \) can be given by \( k(u) \), where \( k' > 0 \) and \( k'' < 0 \). For \( k(u) \) write the condition similar to (19) and apply Jensen’s inequality.
are joint wealth maximizing for risk neutral parties. Since the worker is risk averse in the new equilibria we have just obtained, they are *a fortiori* efficient and no other system can increase the firm's expected profit. Therefore, even if the worker is risk averse, by choosing a suitable amount for the severance pay, the firm can increase its expected profit to the level that could be achieved when he was risk neutral.

**VI. Concluding Remarks**

In this paper we have endogenized both dismissal probability and investment in specific as opposed to general capital, and derived three types of employment equilibrium that differ in the degrees of internalization of labor input. In the basic model we have given the exact conditions in which each type of equilibrium arises. We have shown that the degree to which labor input is internalized depends on several factors rather than simply the degree of specificity of human capital and that it is determined in accordance with the amount of incentives the firm provides with the worker to invest his time and/or effort in specific as opposed to general capital. Each employment equilibrium is optimal in the sense that it maximizes the joint-wealth of the initial worker and firm.

We have examined some extensions of the basic model. We have shown that the equilibrium types of employment are similar to those in the basic model even if the distribution of the product price is general. If the probability that the worker quits for non-wage reasons is high, the degree of internalization becomes low. The more risk averse the worker, the more likely it is for internalization to be an extreme, either perfect or no internalization. However, if severance pay is introduced, the equilibrium with risk aversion on the part of the worker will be exactly the same as that of the basic model and, is, therefore, efficient.

The results of our model help us in understanding the reality. I would like to remark especially that they are consistent with the historical facts that brought about internal labor markets in Japan in the early period of this century. One important fact was that since different firms in the modern (oligopolistic) sector introduced different types of new technology, their workers' skills became firm-specific. Systematic empirical analyses based on the results of this paper will be of special interest, if they allow for technological and product market differences in various industries.

**APPENDIX**

By using Table 1, we would like to prove the results depicted in Figures 1 and 2. Figure 1 applies to the case where $0 \leq \beta < p_1/E(p)$ and Figure 2 to the case where $p_1/E(p) \leq \beta < 1$.

First we examine the case where $0 \leq \beta < p_1/E(p)$. An important feature of this case is that $\alpha = (1 - \pi_1) r/ \pi_3 p_3$ lies above $\alpha = \beta r/p_1$. Suppose that $r < p_1$. Then $\nu^L$ is larger than both $\nu^C$ and $\nu^S$. So we have LEE in this case. Next suppose that $p_1 \leq r < p_2$. For $\alpha > \beta r/p_1$, we have $\nu^L > \nu^C$. Since $\alpha = \beta r/p_1$ lies below $\alpha = \pi_1 (1 - \beta) r/\pi_3 p_3 + 1$, $\nu^C$ does not exist for $\alpha \leq \beta r/p_1$ according to the discussion related to (9). Thus we do not have CEE in this case.

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16 See e.g. Levine and Kawada (1980).
We have LEE for $\alpha > \frac{(\pi_1 r + \pi_2 p_2)}{E(p)}$ and SEE for $\alpha < \frac{(\pi_1 r + \pi_2 p_2)}{E(p)}$. If $\alpha = \frac{(\pi_1 r + \pi_2 p_2)}{E(p)}$, the two parties are indifferent to LEE and SEE. Finally suppose that $p_3 \leq r$. For $\alpha > \frac{\beta r}{p_1}$, we have $V_L > V_C$. Since $\alpha = \frac{\beta r}{p_1}$ lies below $\alpha = \frac{(1 - \pi_1 \beta) r}{\pi_2 p_2}$, we have $V^S > V_C$ for $\alpha \leq \frac{\beta r}{p_1}$. Thus we do not have CEE in this case either. We have LEE for $\alpha > \frac{r}{E(p)}$ and SEE for $\alpha < \frac{r}{E(p)}$. If $\alpha = \frac{r}{E(p)}$ the two parties are indifferent toward LEE and SEE. So far we have implicitly assumed that $\alpha > 1$. It is obvious that if $\alpha \leq 1$, the firm induces the worker to choose $x = 0$. Thus SEE arises if $\alpha \leq 1$.

Next we examine the case where $p_1 / E(p) \leq \beta < 1$. An important feature of this case is that $\alpha = \frac{\beta r}{p_1}$ lies above $\alpha = \frac{(1 - \pi_1 \beta) r}{\pi_2 p_2}$. As in the previous paragraph we first assume that $\alpha > 1$. Suppose that $r < p_1$. Then we have LEE as before. Next suppose that $p_3 \leq r < p_2$. We note that in this case $\alpha = \frac{\beta r}{p_1}$, $\alpha = \frac{(\pi_1 r + \pi_2 p_2)}{E(p)}$ and $\alpha = \frac{(1 - \beta) r}{\pi_2 p_2 + 1}$ intersect at $r = \frac{\pi_2 p_1}{\pi_2 p_2} \left( \beta E(p) - \pi_1 p_1 \right)$. Consider the subcase where $p_3 \leq r < \frac{\pi_2 p_1}{\pi_2 p_2} \left( \beta E(p) - \pi_1 p_1 \right)$, then $\alpha = \frac{\beta r}{p_1}$, we have $V_L > V_C$. Since $\alpha = \frac{\beta r}{p_1}$ lies below $\alpha = \frac{(1 - \pi_1 \beta) r}{\pi_2 p_2 + 1}$, $V_C$ does not exist for $\alpha \leq \frac{\beta r}{p_1}$. Thus we do not have CEE in this subcase. We have LEE for $\alpha > \frac{(\pi_1 r + \pi_2 p_2)}{E(p)}$ and SEE for $\alpha < \frac{(\pi_1 r + \pi_2 p_2)}{E(p)}$. As before $\alpha = \frac{(\pi_1 r + \pi_2 p_2)}{E(p)}$ is the boundary for these two equilibria. Consider now the subcase where $\pi_2 p_1 / (\beta E(p) - \pi_1 p_1) \leq r < p_2$. If $\alpha > \frac{\beta r}{p_1}$, $\alpha > \frac{(\pi_1 r + \pi_2 p_2)}{E(p)}$ and we have $V_L > V^S$ and $V^C > V_S$. Thus the equilibrium is LEE if $\alpha > \frac{\beta r}{p_1}$. If $\pi_1 (1 - \beta) r / \pi_2 p_2 + 1 < \alpha < \frac{\beta r}{p_1}$, $V^C > V_S$ and $V^C > V_S$, which implies that the equilibrium is CEE. If $\alpha < \frac{(1 - \pi_1 \beta) r}{\pi_2 p_2 + 1}$, $V_C$ does not exist and $\alpha < (\pi_1 r + \pi_2 p_2) / E(p)$ implies that the equilibrium is SEE. On the boundaries of $\alpha = \frac{\beta r}{p_1}$ and $\alpha = \pi_1 (1 - \beta) r / \pi_2 p_2 + 1$, the parties are indifferent to the adjacent equilibria. Finally suppose that $p_2 \leq r$. If $\alpha > \frac{\beta r}{p_1}$, $V_L > V^C$ and $V^L > V_S$. Thus we have LEE. If $(1 - \pi_1 \beta) r / \pi_2 p_2 < \alpha < \frac{\beta r}{p_1}$, $V^C > V_S$ and $V^C > V_S$. Thus the equilibrium is CEE. If $\alpha < (1 - \pi_1 \beta) r / \pi_2 p_2$, we obviously have SEE. On the boundary of $\alpha = \frac{\beta r}{p_1}$, the parties are indifferent to LEE and CEE, while on the boundary of $\alpha = (1 - \pi_1 \beta) r / \pi_2 p_2$, they are indifferent to CEE and SEE. If $\alpha \leq 1$, we obviously have SEE as before.

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