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THE COST OF LIVING AND THE SENIORITY-BASED WAGE SYSTEM IN JAPAN*

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I. *Introduction*

The seniority-based wage system, or the positive correlation between wages and age (or length of service), has been one of the most exciting research topics for both theoretical and empirical labor economists. The first serious research on this matter was undertaken by advocates of the human capital theory, such as Mincer (1958), Oi (1962), and Becker (1964). Institutionalists also showed interest, but some used the concept of specificity of human capital [Doeringer and Piore (1971)]. In the 1970s, models of the human capital approach incorporated quitting behavior and layoffs, and became rather sophisticated. They include models developed by Parsons (1972), Mortensen (1978), and Hashimoto (1979), some parts of which can be interpreted as theories on this matter. There are a few additional theories on this subject. One is based on self-selection or sorting [Salop and Salop (1976) and Guasch and Weiss (1980)]. Another is based on the cheating behavior of workers [Lazear (1979)].

In Japan there have primarily been two different traditional theories regarding this subject. One is the theory by Ujihara (1966) and Koike (1966). It insists, in essence, that wages increase with age or length of service because workers acquire more skills and knowledge. Though historical factors are also considered in this theory, it is quite similar to the human capital approach. The other theory has been developed by Funabashi (1961, 67). This second theory, probably unique to Japanese economics, insists that older workers receive higher wages because their costs of living are higher.

The purpose of this paper is to examine this second Japanese theory, or the cost-of-living approach, from a viewpoint slightly different from that proposed by Funabashi. Though the cost-of-living approach has strong support among some Japanese labor economists, the concept behind it appears very strange to modern economists. A naive question is why would firms employ older workers for high wages when they could employ younger workers for low wages. Since the traditional approach does not explicitly consider the effect of competeition in labor markets on wage determination, it cannot provide a satisfactory answer to

* The basic concept of this paper was conceived earlier (Arai (1980)) and an extension of the work was published in Japanese (Arai (1984)). Recent controversies as to the empirical validity of several standard theoretical models on upward sloping age-wage profiles have compelled me to publish in English with some additions and all necessary proofs, which were mostly absent in the Japanese version. The author is grateful to Mr. Ronald M. Siani for his proofreading of this paper. This research was partially supported by the Grant in Aid for Scientific Research of the Ministry of Education.

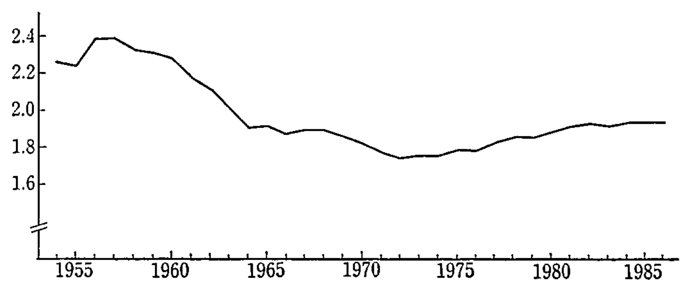
this question. Another question is why should firms have to take into account the living costs of workers at different ages when workers behave rationally and make their own consumption plans. In the discussion of this paper, we will answer these questions and show that under certain circumstances the wage contract which allows for costs of living at different ages is superior to the contract that guarantees wages which are equal to productivity.

The basic concept of our model is that wages are not equal to productivity because there are intergenerational transfers within firms in wage determination. This has been developed from an observation of extremely low real rates of interest as compared with those predicted by the economic principle. Because of this fact, workers rely primarily upon the intergenerational transfer of incomes within firms rather than upon bank savings when making consumption plans. Thus, the wage contract in our model specifies the wage levels at different ages for the worker. This idea is an application of the exact consumption loan model developed by Samuelson (1958).

There are two major advantages in our model. First, our theory reflects important aspects of the wage determination practice in Japan. When they determine wages, most Japanese firms seriously take into consideration the living costs of their workers [Funabashi (1967) pp. 69–71 and Shimada (1980)]. Ono (1987) showed in his elaborate empirical research that age or the living cost, as a function of age, is the most important factor in wage determination. It is commonly held among Japanese labor economists that age-wage profiles in Japan appear very similar to profiles of age and living costs of model households. Though many models can explain the discrepancy between wages and productivity, they cannot explain why the above two different types of profiles coincide.

The second advantage is that our theory can provide testable hypotheses and forecasts of changes in the degree to which age or seniority affects wage levels. Figure 1 shows the ratio of the average wage of 40–49 year-old male workers to that of 20–24 year-old male workers during the past few decades. Clearly, the ratio tended to decrease in the late 1950s and 60s, and began to increase in the early 70s. Our model can explain this phenomenon by connecting the ratio with some economically important variables such as the growth rates of population and productivity, but no other models have this feature. Furthermore, most Japanese labor economists are intuitively aware that there is some relationship between the ratio and the rate of economic growth and/or the growth rate of population. Therefore, our model provides a formulation of this intuition. Our theory differs from that of Reder (1955), who developed his theory of the change in wage differentials in terms of a somewhat

FIG. 1



different situation.

We contended above that there are intergenerational transfers in wage determination when workers face relatively low interest rates. There is undeniable evidence and reasons for the fact that very low interest rates prevailed during the past several decades in Japan. We will see in the last section just how low these interest rates were in the past. The system of interest rates was essentially controlled by the government, and their levels were kept low in order to reduce the interest costs of national loans and to encourage investment by large corporations [see Koizumi (1965) and Suzuki (1974)]. This is the well-known 'easy money' policy.

In this paper we will consider a simple imaginary economy where there are no banks, and where workers rely on wage contracts to achieve their most desirable consumption plans. In Section II we will discuss the assumptions and framework of our model. In Section III we will consider the optimal wage contract and its properties. Section IV provides a few remarks.

II. *Assumptions and Framework of the Model*

We consider the simplest possible situation to show only the essence and to avoid complication. Each worker participates in production for two periods. In his first period he is a young worker, while in his second period he is an older worker. At the beginning of his first period he makes a wage contract with the firm that will employ him. The wage contract specifies the wage he will receive in each period. At the end of his second period he will retire. We consider periods from minus infinity to plus infinity. In each period there are two different types of workers, i.e., young and older workers. We assume that all workers have identical preferences. As we see later, we can assume that workers' abilities increase at a constant rate, but all the workers in the same period are assumed to have the same abilities.

Consider a representative worker. He makes his lifetime consumption plan when he seeks employment, i.e., at the beginning of his first period. Let (c_1, c_2) denote his consumption plan in which c_1 is his (family's) consumption in his first period and c_2 is that in his second period. His consumption plan is based solely on the wages he will receive. All consumption plans and wages are measured in real terms. Let $u(c_1, c_2)$ denote his utility function. It is increasing both in c_1 and in c_2, and is strictly quasi-concave. Further, it is assumed to be homothetic. This last assumption might seem rather restrictive, but it is adopted to avoid complications such as different generations striking qualitatively different wage contracts. It will become clear later that this assumption is necessary to simplify our analysis. Essentially, it implies that the pattern of his consumption plan, or the planned ratio of c_1 to c_2, is invariant to changes in his lifetime income. This is less unnatural than the same assumption for ordinary goods.

As the consumption plan considered here covers almost the entire span of the worker's life, it might be allowable to make slightly uncommon assumptions about his utility function. Usually the term 'consumer' implies an individual, a household, or a larger group with a common purpose [see Debreu (1959)]. It seems implicit here that the 'members' of a consumer are invariant over time. But in our situation the consumer-worker's family will become larger, i.e., he will get married and have children as he becomes older. We would like to capture this effect on 'his' utility function. More specifically, we adopt either or both of the following two assumptions in our analysis.

Assumption 1: The worker tends to place higher value on consumption in his second period.

Assumption 2: The substitutability of the worker's utility function between consumption in his first period and that in his second period is relatively small.

We shall examine the backgrounds of Assumption 1 first. When a utility function is written as $u(c_1, c_2)=v(c_1)+\delta v(c_2)$, where $v'>0$, $v''<0$, and $\delta>0$, we usually assume that δ is less than unity. But Assumption 1 states that δ is larger than unity. A reason for this is that a unit consumption in his second period, when he has a spouse and children to support, provides him higher utility than that in his first period, when he is single or his family is small. Another reason is that $u(c_1, c_2)$ here can be regarded as a kind of 'social' welfare function. The word 'social' refers here to the worker's family. Roughly, δ larger than unity implies a larger number of family members in his second period than in his first period. A third possible reason is that a unit consumption in his second period is more valuable when he sees whether or not his life is a comfortable one or a success.

Since the second reason given above might appear slightly unusual, let us investigate its meaning in more detail. We shall consider examples to develop this concept further. Suppose the number of members of the worker's family in his i-th period is n_i $(i=1, 2)$, where $n_1<n_2$. Suppose further that all the members of his family have identical and invariant preferences in each period and that they are expressed by $\log c$. We construct the following additive social welfare function:

$$\tilde{u}(c_1, c_2)=n_1\log(c_1/n_1)+n_2\log(c_2/n_2),$$

where, for simplicity, we have assumed that the 'discount factor' is unity and that each member consumes an equal portion of the worker's planned consumption for his family. Simple calculation shows

$$\tilde{u}(c_1, c_2)=n_1[\log c_1+(n_2/n_1)\log c_2]-(n_1\log n_1+n_2\log n_2).$$

Since n_1 is positive and the second term of the right-hand side is constant, this social welfare function is equivalent to $u(c_1, c_2)=v(c_1)+\delta v(c_2)$, where $v(c)=\log c$ and $\delta=n_2/n_1>1$. Incidentally, $\tilde{u}(c_1, c_2)$ satisfies all the assumptions required above. In particular, it is homothetic.

We provide one more example. Suppose that the preferences of the family members are given by c^α, where $0<\alpha<1$, and construct a similar social welfare function:

$$\bar{\tilde{u}}(c_1, c_2)=n_1(c_1/n_1)^\alpha+n_2(c_2/n_2)^\alpha.$$

By calculation, this can be reduced to

$$\bar{\tilde{u}}(c_1, c_2)=n_1^{1-\alpha}[c_1^\alpha+(n_2/n_1)^{1-\alpha}c_2^\alpha].$$

By the same reasoning this is equivalent to $u(c_1, c_2)=v(c_1)+\delta v(c_2)$, where $v(c)=c^\alpha$ and $\delta=$

$(n_2/n_1)^{1-\alpha}>1$. It is easy to show that $u(c_1, c_2)$ satisfies all the assumptions required above. These two examples show that a large class of relatively normal utility functions $v(c)$ might fit our requirements.

Next we examine the meaning of Assumption 2. This assumption states that the worker tends to secure a certain standard of living in each period subject to his budget constraint. Thus it implies, for instance, that very large consumption in his first period and very small consumption in his second period do not provide him with a very large measure of satisfaction. This assumption may be used later together with another assumption that the worker can consume only a portion of his planned consumption in his second period because his family increases in size in his second period. In order to model this we introduce the parameter $\theta>1$ which measures the worker's cost of living in his second period. Though the consumption plan for his family is (c_1, c_2), the portion that contributes to the worker's own satisfaction is now $(c_1, c_2/\theta)$. Assumption 2 then implies that the worker has to choose (c_1, c_2) for his family to secure a certain standard of living $(c_1, c_2/\theta)$ for himself.

A few comments on the use of parameter θ follow. First, the parameter is determined outside the model, and it reflects several social factors. In particular, it is a function of the number of children per family, the level and private costs of their education, availability of social security services, and so on. If θ is large, the worker's cost of living in his second period is large and the portion which contributes directly to his own consumption is small. Second, it is only for the sake of simplicity that we have used the cost-of-living parameter solely in the worker's second period. We can introduce another parameter and divide c_1 by it, but since only the relative magnitude of the costs of living in the two periods is important, we simply assume that the cost-of-living parameter for the first period is unity. Finally, under Assumption 2 with the cost-of-living parameter, the worker is assumed to choose his family's consumption plan (c_1, c_2) so as to maximize his own utility function $u(c_1, c_2/\theta)$ subject to his budget constraint. Thus, the other members' utility functions do not appear in this maximization problem. This is slightly different from the method discussed previously in relation to Assumption 1 (recall the argument of 'social' welfare functions). In this cost-of-living approach we assume that the worker makes his wage contract and a consumption plan for his family not in consideration of the other members' utility functions but of the cost-of-living parameter θ.

As we have already remarked, we will use either or both of the above two assumptions in the following discussion. When both are used simultaneously, we adopt interpretations which are not inconsistent. That is, though we can use the two assumptions simultaneously, we must interpret $u(\cdot, \cdot)$ either as a 'social' welfare function or as the worker's own utility function. This will become clearer when the two assumptions are actually used.

In the discussion of intertemporal consumption the interest rate plays an important role, but for the reason mentioned in the previous section we consider a simple imaginary economy where there are no banks. This corresponds to the situation in the real economy where the real interest rate the worker can make use of is relatively low. It will become clear in the next section that when the real rate of interest available to the worker is low, it is more advantageous for him to use a wage contract for his consumption plan than to use a savings account. Our hypothesis is that wage contracts have a savings function. If we accept this we can better explain some important phenomena. However, we must be careful since consumer-workers actually use savings accounts in the real economy. From the viewpoint of our highly simplified model, this is because consumers have actually different preferences, and there is some uncertainty in the wages and expenditures. Since our model does not consider these two factors, it must be interpreted to provide a rough approximation of reality.

We next consider firms. As in the case of workers, we adopt the simplest assumption that the number of firms is given and they are identical in every respect. In particular, all firms produce the same product with the same technology. There is no uncertainty either in production or in the markets. Though the markets are assumed to be competitive, it implies that workers have no incentives to quit and firms have no incentives to dismiss them.

As we mentioned at the beginning of this section, we consider periods from minus infinity to plus infinity. Corresponding to the periods, there are generations of workers from minus infinity to plus infinity. The t-th generation of workers enter the labor market and strike wage contracts at the beginning of the t-th period. They live for two periods as workers and retire or die at the end of the $(t+1)$-th period. We assume that the marginal productivity of the worker grows at the constant rate of $(\gamma-1)$ each period and that the population of workers grows at the constant rate of $(\xi-1)$ each period.

Since all firms are assumed to be identical, we have only to analyze a representative firm. In each period each firm has an equal share of young and older workers of the total population of young and older workers. In particular, the number of workers in the representative firm grows at the rate of $(\xi-1)$. Thus, the changes in productivity and population in the firm can be summarized as in Table 1. The table contains data for three periods and the corresponding four generations. Each element of the table is the total productivity (population times marginal productivity) of the corresponding generation in the corresponding period. Without loss of generality the (marginal) productivity in period 1 is assumed to be one and the number of older workers (workers of generation zero) is assumed to be one unit. Therefore, the number of workers of generation 1 is equal to ξ, that of generation 2 is equal to ξ^2, and so on. The productivity in period 2 is equal to γ, that in period 3 is equal to γ^2, and so on. In general, the total productivity of older workers and that of young workers in period t are $\xi^{t-1}\gamma^{t-1}$ and $\xi^t\gamma^{t-1}$, respectively. Since the total number of workers in period 1 is $(1+\xi)$, that in period 2 is $\xi(1+\xi)$, and so on, the population is really growing at the rate of $(\xi-1)$. On the other hand, the total productivity in period 1 is equal to $(1+\xi)$, that in period 2 is equal to $\xi\gamma(1+\xi)$, and so on. Thus the total product can be regarded as growing at the rate of $(\xi\gamma-1)$. This reasoning can also be applied to the entire economy.

We would like to remark on the assumption of constant rates of growth of population and productivity. Though this assumption is adopted to simplify our analysis, the two rates, especially the latter, are not constant in the real economy. Therefore, when interpret-

TABLE 1.

Generation	Period 1	2	3
0	1		
1	ξ	$\xi\gamma$	
2		$\xi^2\gamma$	$\xi^2\gamma^2$
3			$\xi^3\gamma^2$

ing our results for the real economy, we must assume that some adjustments were, or will be, made in accordance with changing trends in the real economy.

III. *The Optimal Wage Contracts*

In the discussion related to Table 1 we referred to the marginal productivity of the worker in each period. According to the basic economic principle it is equal to his wage when competition prevails in the labor market. However, if the real rate of interest is low, there is a type of wage contract that is superior to the contract which guarantees that the wage is equal to the worker's productivity in each period. We consider wage contracts which involve intergenerational transfers of incomes from young to older workers within each firm. If the total wages a firm pays in each period are equal to the total productivity of the workers of the firm in that period, and if each worker is equally better off after the transfers, then all parties in the firm will unanimously agree to wage contracts with intergenerational transfers. Thus if wage contracts involve intergenerational transfers, the wage the worker receives in each period is not equal to his productivity in that period. We are interested in finding the optimal intergenerational transfers. To find this we first have to see what intergenerational transfers are possible. This will require deriving a type of budget constraint for each worker.

In the following discussion we consider a representative firm and representative workers in it. Let w_t^o denote the wage the older worker receives after a transfer in period t. Similarly, let w_t^y denote the wage the young worker receives after a transfer in period t. We introduce here the concept of the *ratio of seniority wages* defined as $s \equiv w_t^o/w_t^y$. This ratio measures the degree to which age or seniority affects wages. If it is large, the degree is large. Otherwise, the degree is small. In our analysis we are not so much interested in absolute levels of wages. Our main interests are the level of the ratio of seniority wages and how it is affected by economically important exogenous variables. The wage contract is essentially to determine the ratio such that the total wages paid are equal to the total productivity within the firm in each period. In our analysis of stationary states the worker who makes a wage contract with s gives a transfer corresponding to s when he is young and receives a transfer corresponding to s when he is older.

Let us refer to Table 1 to see what wage contracts with intergenerational transfers are possible. Consider the wages in period 1. According to the above discussion, w_1^o and w_1^y must satisfy the following two conditions:

(1) $w_1^o/w_1^y = s, \quad w_1^o + \xi w_1^y = 1 + \xi.$

The first expression of (1) provides the condition for the ratio of the older worker's wage to the young worker's. The left-hand side of the second expression shows the firm's total wage payment, while the right-hand side shows the total productivity of the workers. Similarly, in period 2 we have

(2) $w_2^o/w_2^y = s, \quad \xi w_2^o + \xi^2 w_2^y = \xi\gamma + \xi^2\gamma.$

Solving (1) for w_1^o and w_1^y, we have

(3) $w_1^o = (1+\xi)s/(\xi+s),$

(4) $w_1^y = (1+\xi)/(\xi+s).$

Similarly, from (2), we have

$$(5)\quad w_2{}^o=(1+\xi)s\gamma/(\xi+s),$$

$$(6)\quad w_2{}^y=(1+\xi)\gamma/(\xi+s).$$

We eliminate s by using (4) and (5). Then we have

$$(7)\quad \xi\gamma w_1{}^y+w_2{}^o=(1+\xi)\gamma,\quad (w_1{}^y\leq 1).$$

Equation (7) shows all possible wage contracts ($w_1{}^y$, $w_2{}^o$) available to the worker of generation 1.

More generally, we have the following conditions in period t:

$$(8)\quad w_t{}^o/w_t{}^y=s,\quad \xi^{t-1}w_t{}^o+\xi^t w_t{}^y=\xi^{t-1}\gamma^{t-1}+\xi^t\gamma^{t-1}.$$

Obviously, the second equation of (8) can be rewritten as

$$(9)\quad w_t{}^o+\xi w_t{}^y=(1+\xi)\gamma^{t-1}.$$

Solving (8) for $w_t{}^o$ and $w_t{}^y$, we have

$$(10)\quad w_t{}^o=(1+\xi)s\gamma^{t-1}/(\xi+s),\quad w_t{}^y=(1+\xi)\gamma^{t-1}/(\xi+s).$$

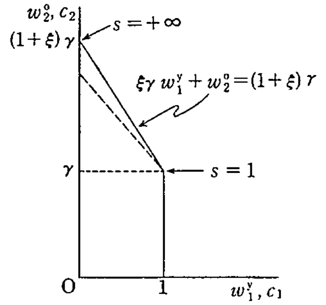
From (10) we can derive an equation similar to (7) for the worker of generation t:

$$(11)\quad \xi\gamma w_t{}^y+w_{t+1}{}^o=(1+\xi)\gamma^t,\quad (w_t{}^y\leq\gamma^{t-1}).$$

In the imaginary economy without banks, (7) can be regarded as a type of budget constraint for the worker of generation 1. Figure 2 depicts equation (7). In (7), or in Figure 2, the worker is assumed to be unable to borrow in order to augment his consumption in his first period. This is not an unnatural assumption, because borrowing possibilities are limited in the real economy, and because, as we shall see, he does not have to borrow when either Assumption 1 or Assumption 2 holds and the exogenous variables take on realistic values.

Equation (7), or Figure 2, shows that (the absolute value of) the slope of the budget constraint is equal to $\xi\gamma$, which is simply the product of one plus the population growth rate and one plus the productivity growth rate. The downward sloping dotted line is the budget constraint corresponding to the (real) interest rate in the real economy, its slope being, of course, one plus the rate of interest. If the interest rate is low, $\xi\gamma$ is larger than one plus the

FIG. 2



interest rate in the real economy. This implies that the budget constraint (7) is above that corresponding to the interest rate and, therefore, that the worker has a more preferable opportunity set. They in turn imply that we have only to consider an imaginary economy without banks. More precisely, the condition for the above argument is that $\xi\gamma=1+(\xi-1)+(\gamma-1)+(\xi-1)(\gamma-1)>1+r$ or that

(12) $(\xi-1)+(\gamma-1)+(\xi-1)(\gamma-1)>r,$

where r is the rate of interest. We will show later that this condition held true in most of the years examined in the real economy.

We note that the point $(1, \gamma)$ in Figure 2 corresponds to $s=1$. This can be seen from (4) and (5). Similarly, $(0, (1+\xi)\gamma)$ corresponds to $s=+\infty$. It can be plainly seen that the value of s increases from one to plus infinity along the budget constraint. Thus the closer the optimal contract point is to $(0, (1+\xi)\gamma)$, the larger s is. A large s means that the wage contract involves large intergenerational transfers and, therefore, the degree to which age or seniority affects wages is large. Figure 2 depicts only the budget constraint for the worker of generation 1. But (11) shows that the budget constraint for each generation expands at the rate of $(\gamma-1)$ preserving similarity. More precisely, the budget constraint for the t-th generation has a kink at (γ^{t-1}, γ^t) and its slope is equal to $\xi\gamma$.

We are now in a position to dicuss the optimal wage contracts and their properties. In the following discussion we will use either Assumption 1 or Assumption 2, or both. The above argument implies that the worker's utility function is maximized in his opportunity set if he concludes a wage contract equal to his optimal consumption plan. Thus finding the optimal wage contract is equivalent to finding the optimal consumption plan. We note that in our analysis of stationary states the optimal wage contract expressed by s is the same for each generation. This follows from the similarity preserving expansion of budget constraints and from the homotheticity of the utility function. More precisely, (10) implies that $w_{t+1}{}^o/w_t{}^y=s\gamma$ and, therefore, that the same s corresponds to the same ratio of wages for workers of various generations when γ is constant. This implies that to find the optimal value of s we have only to confine our analysis to generation 1.

Consider a representative worker of generation 1. His optimal consumption plan can be obtained by solving the following problem:

(13) $\max u(c_1, c_2)$

subject to

(7a) $\xi\gamma c_1+c_2=(1+\xi)\gamma, \quad (c_1\leq 1),$

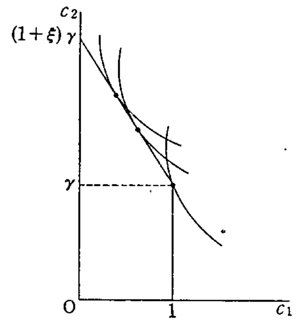
where we have used c_1 and c_2 instead of $w_1{}^y$ and $w_2{}^o$ for the budget constraint (7). The first-order condition for the above problem is obviously

(14) $u_1(c_1, c_2)/u_2(c_1, c_2)=\xi\gamma,$

where u_i $(i=1, 2)$ denotes the partial derivative of u with respect to c_i. The assumption of strict quasi-concavity of u guarantees that the second-order condition will hold. This situation is depicted in Figure 3.

Let us see the implications of Assumption 1. The condition that the worker places relatively high value on consumption in his second period is equivalent to the condition that

FIG. 3



the marginal rate of substitution on a given ray from the origin is relatively small. Thus, as the figure shows, the higher the value placed on consumption in his second period, the larger the optimal ratio of seniority wages or s. If we use the utility function $u(c_1, c_2)=v(c_1)+\delta v(c_2)$ and interpret it as a social welfare function as in the previous section, then the above result implies that the larger the family size in his second period relative to that in his first period (i.e., the larger δ), the larger the optimal ratio of seniority wages. When the worker's valuation of consumption in his second period is not high, the optimal consumption plan corresponds to the corner solution $(1, \gamma)$ in the figure and the worker receives wages equal to the productivity in the two periods, in which case (14) does not necessarily hold.

We next examine the effect of the population growth rate on the optimal ratio of seniority wages. Figure 2 shows that as ξ increases when γ is held constant, the budget constraint rotates upward with $(1, \gamma)$ as its axis. Since the slope $\xi\gamma$ of the budget constraint increases, the marginal rate of substitution at the optimal point increases by (14). But this implies that c_2/c_1 also increases, because the utility function is assumed to be homothetic and strictly quasi-concave. On the other hand, (4) and (5) imply $c_2/c_1=w_2^o/w_1^y=\gamma s$. If c_2/c_1 increases when γ is held constant, s must also increase. Therefore, a large population growth rate leads to a large optimal value of the ratio of seniority wages.

How then does the rate of productivity growth affect the optimal ratio of seniority wages? This problem cannot be analyzed by the same method described above. Since c_2/c_1 is equal to γs, we cannot immediately know how the change in c_2/c_1 due to a change in γ affects s. We therefore adopt a different method and rewrite (7) (or (7a)) as

(15) $\xi c_1+(1/\gamma)c_2=1+\xi, \quad (c_1\leq 1),$

where c_1 and c_2 are used for w_1^y and w_2^o, respectively, and ξ is held constant. In (15) the coefficients of c_1 and c_2 can be interpreted as the corresponding prices of the consumption in the two periods, while $(1+\xi)$ on the right-hand side can be interpreted as the worker's lifetime income. According to this interpretation, an increase in γ is equivalent to a decrease in the price of the consumption in his second period. Therefore, our problem is reduced to a study of the substitution and income effects based on the Slutsky equation. To judge the eventual effect of γ on the optimal value of s, we use (4), which does not depend on γ. If $w_1^y=c_1$ increases, then s decreases, and vice versa.

For ease of notation we adopt symbols more suitable to the present context and let $p_1=\xi$, $p_2=1/\gamma$, and $y=1+\xi$. Then (15) can be rewritten as

$$p_1c_1+p_2c_2=y, \quad (c_1\leq 1).$$

We would like to know the effect of p_2 on c_1. The Slutsky equation for the cross effect is given by

$$(16) \quad \frac{\partial c_1}{\partial p_2}=\left(\frac{\partial c_1}{\partial p_2}\right)_{u=\mathrm{const}}-c_2\left(\frac{\partial c_1}{\partial y}\right),$$

where the first term on the right-hand side is the substitution effect and the second term is the income effect. It is obvious that under our assumptions the first term is positive and the second term is negative, but we cannot immediately determine the sign of $\partial c_1/\partial p_2$. So let us adopt Assumption 2 and assume that the substitution effect is relatively small. Then $\partial c_1/\partial p_2$ is likely to be negative. This implies that if $p_2=1/\gamma$ decreases, c_1 increases. From the argument above, a large γ leads to a small s.

In the above discussion we interpreted Assumption 2 as implying a small substitution effect in (16) or a negative cross effect. However, is there any way to connect the assumption directly with the utility function? We can show that this interpretation is equivalent to the assumption that the elasticity of substitution σ of $u(c_1, c_2)$ is less than unity in our situation (The proof is listed in the Appendix). Therefore, if the elasticity of substitution of the worker's utility function is less than unity, a large γ leads to a small s. If it is larger than unity, equivalently, if the cross effect is positive, then we arrive at an opposite conclusion. Since Assumption 2 implies that the worker tries to secure a certain standard of living in each period subject to his budget constraint, he can do this with small intergenerational transfers when the rate of productivity growth is high, i.e., when the productivity in his second period is much larger than that in his first period. On the other hand, if Assumption 2 does not hold ($\partial c_1/\partial p_2>0$ or $\sigma>1$), the worker does not act in this way and chooses instead a large c_2 or a large s because the price of c_2 is low when γ is large. Roughly, the growth rate of productivity is equal to the growth rate of the GNP per capita. Thus, if Assumption 2 holds, the ratio of seniority wages is small when this growth rate is high.

Finally, we consider a different specification mentioned previously. The worker is now assumed to choose his family's consumption plan (c_1, c_2) so as to maximize his own utility function $u(c_1, c_2/\theta)$ subject to his budget constraint, where θ is a measure of the cost of living in his second period. We would like to know how θ affects the optimal ratio of seniority wages. To address this question we use the same approach as that adopted above. Using the same symbols, we can formulate the problem as

$$(17) \quad \max_{c_1, c_2} u(c_1, c_2/\theta) \quad \text{subject to} \quad p_1c_1+p_2c_2=y, \quad (c_1\leq 1).$$

This can be solved easily by introducing new notations. We let $c_1'=c_1$ and $c_2'=c_2/\theta$. Then $c_2=\theta c_2'$. Using these notations, (17) can be rewritten as

$$(18) \quad \max_{c_1', c_2'} u(c_1', c_2') \quad \text{subject to} \quad p_1c_1'+\theta p_2c_2'=y, \quad (c_1'\leq 1).$$

What is new in this problem is that we have θ in the coefficient of c_2'. Letting $p_2'=\theta p_2$, we can interpret a large value of θ as a high price of c_2'. Thus, it may be really proper to say that θ is a measure of the cost of living in the worker's second period. To determine the effect of θ on the optimal s, we repeat the same reasoning as above. If θ increases, p_2' increases. Thus, $\partial c_1'/\partial p_2' < 0$, when $\sigma < 1$ or when Assumption 2 is valid. Since $c_1' = c_1$, a decrease in c_1' implies a larger s by (4). When $\sigma > 1$, or when Assumption 2 is not valid, we get an opposite result. This result shows that if the substitutability of consumption is low (or $\sigma < 1$), such factors as a smaller family size, a more liberal security system, and less costly education of children all act to decrease the optimal ratio of seniority wages.

Summarizing the results in this section, we have the following proposition:

Proposition: In the framework of our model the optimal ratio of seniority wages is larger, (a) the larger the growth rate of population, (b) the higher the value the worker places on consumption in his second period (because of larger family size, etc.), (c) the lower the growth rate of productivity, and (d) the larger the cost of living in his second period (because of larger family size, more costly education for his children, a less liberal social security system, etc.). For (c) and (d) the elasticity of substitution of the utility function is assumed to be less than unity. If it is larger than unity, we get opposite results.

IV. *Concluding Remarks*

In this paper we have discussed wage contracts with intergenerational transfers and derived some testable hypotheses. In order for this theory to be valid, the real rate of interest must be sufficiently low. We would like to show here that this has been the case in the past few decades. The left-hand side of (12) can be interpreted as equal to the growth rate of the real GNP, while the right-hand side reflects the real rate of interest. We compare these two from 1952 to 1986 by using *Keizai Yoran* by the Economic Planning Agency. According to it, the average growth rate of the real GNP is equal to 7.2%. The average nominal rate of interest for one-year deposits and the average growth rate of the consumer price index are 5.8% and 5.4%, respectively, so the average real rate of interest is equal to 0.4%. Therefore, the average real growth rate of the GNP is higher than the average real rate of interest by 6.8%. In fact, (12) was satisfied in all the years examined with the exception of 1958, 1983, and 1986. Slightly higher interest rates were introduced during the middle of the period under observation, but the differences are negligible. Since workers actuily have access to risky investment with higher expected rates of return, the above nominal rate of interest might be an underestimate. But it is doubtful that a difference as large as 6.8% would be reduced to almost zero, even if a more properly defined rate of interest (adjusted for risk) was used.

Next we would like to view the validity of our theory in terms of the history of Japanese labor markets. We would like to examine very roughly the effects of the growth rates of population and productivity. More detailed examination is left for future study. Since our analysis has been based on stationary states, we need to identify long-range historical trends. Figure 1 shows two trends: one is a decline in the ratio of seniority wages between the late 1950s and early 70s, and the other is a slight increase since the early 70s. The former trend is probably due to the effect of the high growth rate of productivity (we assume $\sigma < 1$), which dominated that of the relatively high rate of population growth. The latter trend is probably due to the opposite. Ono (1973, Figure 7–8) shows that the ratio of seniority wages generally increased between the mid-1920s and the mid-50s. According to our theory, this is due to the high growth rate of population and to the fact that, increasingly, more firms adopted seniority-based wage systems during this period.

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APPENDIX

Theorem: Assume $u(c_1, c_2)$ is homothetic as well as strictly quasi-concave and increasing in c_1 and c_2. Then the cross effects are negative if and only if the elasticity of substitution of $u(c_1, c_2)$ is less than unity.

Proof: To solve the consumer's problem, max $u(c_1, c_2)$ subject to $p_1c_1+p_2c_2=y$, we use the Lagrangian $u(c_1, c_2)+\lambda(y-p_1c_1-p_2c_2)$. Then the cross effects are

$$\partial c_i/\partial p_j=D_{ji}\lambda/D+c_jD_{3i}/D,$$

where $i, j=1, 2,$

$$D=\begin{vmatrix} u_{11} & u_{12} & -p_1 \\ u_{21} & u_{22} & -p_2 \\ -p_1 & -p_2 & 0 \end{vmatrix},$$

and D_{ji} is the cofactor of the element in the j-th row and the i-th column of D.

Using $p_1=u_1/\lambda$ and $p_2=u_2/\lambda$, we get

$$D_{12}\lambda/D=D_{21}\lambda/D=u_1u_2/\lambda D,$$

$$c_1D_{32}/D=-c_1(u_1u_{21}-u_2u_{11})/\lambda D,$$

$$c_2D_{31}/D=-c_2(u_2u_{12}-u_1u_{22})/\lambda D.$$

Therefore,

$$\text{(A-1)}\quad \partial c_1/\partial p_2=D_{21}\lambda/D+c_2D_{31}/D=(u_1u_2-c_2u_2u_{12}+c_2u_1u_{22})/\lambda D,$$

$$\text{(A-2)}\quad \partial c_2/\partial p_1=D_{12}\lambda/D+c_1D_{32}/D=(u_1u_2-c_1u_1u_{21}+c_1u_2u_{11})/\lambda D.$$

It can be shown that $c_jD_{3i}/D=-c_j(\partial c_i/\partial y)$. Since u is homothetic, at the consumer's equilibrium point (c_1, c_2) we have

$$\frac{c_2(y, p_1, p_2)}{c_1(y, p_1, p_2)}=k(p_2/p_1),$$

where k is a function of p_2/p_1 only. Then

$$\text{(A-3)}\quad c_2(y, p_1, p_2)=k(p_2/p_1)c_1(y, p_1, p_2).$$

Differentiate both sides of (A-3) with respect to y, then

$$\text{(A-4)} \quad \frac{\partial c_2}{\partial y} = k \cdot \left(\frac{\partial c_1}{\partial y} \right).$$

(A-3) and (A-4) imply

$$\frac{\partial c_2}{\partial y} \cdot \frac{1}{c_2} = \frac{\partial c_1}{\partial y} \cdot \frac{1}{c_1},$$

or that $c_1(\partial c_2/\partial y) = c_2(\partial c_1/\partial y)$. Thus, under our assumptions, not only the substitution effects, but also the income effects in (A-1) and (A-2) are equal, so

$$\text{(A-5)} \quad \frac{\partial c_1}{\partial p_2} = \frac{\partial c_2}{\partial p_1}.$$

Now the elasticity of substitution of u can be given by

$$\sigma = \frac{u_1 u_2 (u_1 c_1 + u_2 c_2)}{c_1 c_2 (2u_{12} u_1 u_2 - u_1{}^2 u_{22} - u_2{}^2 u_{11})},$$

where the denominator is positive. Consider the condition $\sigma < 1$. Simple calculation shows that this condition is equivalent to

$$\text{(A-6)} \quad u_1 c_1 (u_1 u_2 - c_2 u_2 u_{12} + c_2 u_1 u_{22}) + u_2 c_2 (u_1 u_2 - c_1 u_1 u_{21} + c_1 u_2 u_{11}) < 0.$$

Applying (A-1) and (A-2) to the left-hand side of (A-6), we have

$$\begin{aligned} \text{LHS of (A-6)} &= u_1 c_1 \lambda D(\partial c_1/\partial p_2) + u_2 c_2 \lambda D(\partial c_2/\partial p_1) \\ &= \lambda D(u_1 c_1 + u_2 c_2)(\partial c_1/\partial p_2) \qquad \text{by (A-5).} \end{aligned}$$

Since the assumptions imply that $\lambda D(u_1 c_1 + u_2 c_2) > 0$, we have proved the theorem.

REFERENCES

Arai, K. (1980), "On the Seniority-Based Wage System in Japan," Discussion Paer No. 724, Krannert Graduate School of Management, Purdue University.

Arai, K. (1984), "*Nenko chinginsei no seikatsu hosho kasetsu saiko* [The Cost-of-living Approach to the Seniority-based Wage System Revisited]," *Hitotsubashi Ronso* 92, 728–743.

Becker, G.S. (1964), *Human Capital: A Theoretical and Empirical Analysis, with Special Reference to Education*, National Bureau of Economic Research.

Debreu, G. (1959), *Theory of Value*, New York, Wiley.

Doeringer, P.B. and M.L. Piore (1971), *Internal Labor Markets and Manpower Analysis*, D.C. Heath and Co.

Economic Planning Agency, *Keizai Yoran* [The Statistical Abstract of the Japanese Economy].

Funabashi, N. (1961), "*Kigyonai chingin kozo* [The Internal Wage Structure]," in M. Shinohara and N. Funabashi ed., *Nihongata chingin kozo no kenkyu* [Studies in the Japanese Wage Structure], Rodo Hogaku Kenkyujo.

Funabashi, N. (1967), "*Kigyonai Chingin Kozo* [The Internal Wage Structure], in N. Funabashi ed., *Nippon no Chingin* [Wages in Japan], Nihon Hyoron-sha.

Guasch, J.L. and A. Weiss (1980), "Wages as Sorting Mechanisms in Competitive Markets with Asymmetric Information: A Theory of Testing," *Review of Economic Studies* 47, 653–65.

Hashimoto, M. (1979), "Bonus Payments, On-the-job Training, and Lifetime Employment in Japan," *Journal of Political Economy* 87, 986–1104.

Koike, K. (1966), *Chingin* [Wages], Diamond-sha.

Koizumi, A. (1965), "*Kinri Suijun to Kinri Taikei* [The Levels of Interest Rates and the System of Interest Rates], in H. Kawaguchi and I. Kawai ed. *Kinyu-koza* [Lectures on Monetary Theory] 5.

Lazear, E.P. (1979), "Why Is There Mandatory Retirement?" *Journal of Political Economy* 87, 1261–1284.

Mincer, J. (1958), "Investment in Human Capital and Personal Distribution of Income," *Journal of Political Economy* 66, 281–302.

Mortensen, D.T. (1978), "Specific Capital and Labor Turnover," *Bell Journal of Economics* 9, 572–586.

Oi, W.Y. (1962), "Labor as a Quasi-Fixed Factor," *Journal of Political Economy* 70, 538–555.

Ono, A. (1973), *Sengo Nippon no Chingin Kettei* [The Wage Determination in Postwar Japan], Toyo Keizai Shimpo-sha.

Ono, A. (1987), "Two Competing Hypotheses for the Nenko Wage System—Skill or Living Cost Compensation?—," *Hitotsubashi Journal of Economics* 28, 1–25.

Parsons, D.O. (1972), "Specific Human Capital: An Application to Quit Rates and Layoff Rates," *Journal of Political Economy* 80, 1120–1143.

Reder, M. (1955), "The Theory of Occupational Wage Differentials," *American Economic Review* 45, 833–852.

Salop, J.K. and S.C. Salop (1976), "Self-Selection and Turnover in the Labor Market," *Quarterly Journal of Economics* 90, 619–628.