THE YEN AND THE DOLLAR:
THEIR PURCHASING POWER PARITY
IN EFFICIENT MARKETS

ALI M. FATEMI

I. Introduction

In perfectly competitive markets the “law of one price” should hold. When applied to an international setting, the law requires that each currency have the same purchasing power in its home country as it would in a second country if converted to the currency of that country at the equilibrium exchange rate. This concept is referred to as the purchasing power parity (PPP) theory, conception of which is credited to Cassel (1918). The theory, however, has been subject to different interpretations. The broadest interpretation [Cassel (1922)] maintains that exchange rates are determined by the relative purchasing power of each currency as well as other nonmonetary relevant variables. Other interpretations are more specific, in that they maintain exchange rates are exactly determined by changes in the relative purchasing power of each currency. These various interpretations have been empirically tested [e.g. Balassa (1964), Folks and Stansell (1975), Gailliot (1970), Hodgson and Phelps (1975), Isard (1977), Thomas (1973)] with varying degrees of sophistication. Results have indicated that price level changes directly affect exchange rate changes. However, some findings [e.g. Folks and Stansell, Hodgson and Phelps, and Thomas] have indicated that price level changes precede exchange rate changes, implying lack of efficiency in foreign exchange markets.

This implication has been most significant for the Japanese yen markets. Hodgson and Phelps, examining the data over the period April 1919–April 1925, reported a long lag of almost “eighteen” months between changes in price levels and changes in the yen-dollar exchange rate.

It has been only recently that the conflict between these inefficiency implications and the efficient markets literature has come under scrutiny [Rogalski and Vinso (1977), Roll (1979)]. Roll in a path-breaking work has applied the efficiency concept to the PPP theory and formulated a simple theory of PPP from an efficient markets perspective. According to this theory the current exchange rate is the best predictor of the future exchange-adjusted difference in price levels between two countries. Using data for twenty three countries and twenty years Roll found that his efficient markets version of PPP holds well for most pairs of countries. Contrary to previous results Roll finds that “the current exchange rate is an excellent predictor of the subsequent month’s exchange-adjusted inflation rate differ-

\[^1\] See Officer (1976) for a comprehensive review of the literature on PPP.
ence between two countries, and no other past information used here adds to the predictive power." (p. 174)

This study discusses and evaluates some of the econometric problems of the classical version of PPP as used by Hodgson and Phelps. Additionally, the validity of the efficient markets version of the PPP theory for the U.S. dollar against the Japanese yen is examined. The period covered by the study is the seven year period April 1973—March 1980 of the current floating exchange rate system.

II. The Classical Model of PPP

Hodgson and Phelps' model is based on the law of one price which requires a good to have the same price (in terms of purchasing power) in two different countries. Let \( iUSP \) denote the U.S. price of good \( i \) in dollars and \( iJP \) denote its Japanese price in yens. The law of one price would require

\[
(EXCH)_t \cdot iUSP_t = iJP_t, \quad \text{where} \quad EXCH = \text{the exchange rate of yens per dollar.}
\]

This equilibrium is in effect for every period and all goods. If we now define \( i \) as an index of goods, the composition of which is different than index \( j \), and denote the relative price of index \( j \) per index \( i \) in yens by \( k \left( \frac{jJP}{iJP} \right) \), Equation (1) can be rewritten as

\[
\left( \frac{1}{EXCH} \right)_t = k_t \cdot \frac{iUSP_t}{iJP_t}
\]

Equation (2) can be recast in the following form

\[
ln \left( \frac{1}{EXCH} \right)_t = ln k_t + ln \left( \frac{iUSP_t}{iJP_t} \right)
\]

Hodgson and Phelps estimated an unconstrained generalized-least-squares analog of (3) as

\[
ln \left( \frac{1}{EXCH} \right)_t = \alpha_0 + \alpha_1 ln \left( \frac{USPI_t}{JPI_t} \right) + \epsilon_t, \quad \text{where}
\]

\( USPI = \text{U.S. Price Index} \) and \( JPI = \text{Japanese Price Index} \).

It is obvious that if \( k_t \) and \( \left( \frac{USPI}{JPI} \right)_t \) are correlated the estimated slope will be different than unity. Recalling that \( k_t \) is the relative price of index \( j \) per index \( i \) in yens and that \( i \) and \( j \) are the U.S. and the Japanese price indexes respectively, the covariance term in (4) may indeed be nonzero which will result in \( \alpha_1 \) estimates different than unity.

Furthermore, as the estimate of \( \alpha_0 \) is a function of \( \hat{\alpha}_1 \), biased estimates of \( \alpha_1 \) would tend to reduce the standard error of the estimated \( \alpha_0 \), and thus increase its observed level of significance. Finally exclusion of the time-related variable \( ln k_t \) in the estimated form (4) would cause autocorrelation of residuals.

All these econometric problems are, of course, created by the existence of relative price \( (k) \) variations. Therefore, a poor fit of Equation (4) can not be used as evidence against the PPP.
The model of Equation (4) is an instantaneous adjustment model. However, Hodgson and Phelps argue that if there is a time lag between changes in the price levels and subsequent changes in the exchange rate, a distributed lag model will be more appropriate. A Koych-type distributed lag model of (4) would be

\[ \ln\left( \frac{1}{EXCH} \right)_t = \alpha_0 + \alpha_1 \ln\left( \frac{USPl_t}{JPl_t} \right) + \alpha_2 \ln\left( \frac{1}{EXCH} \right)_{t-1} + \epsilon_t. \]  

(5)

Their estimated version of (5) for the Japanese yen is:²

\[ \ln\left( \frac{1}{EXCH} \right)_t = \alpha_0 + .056 \ln\left( \frac{USPl_t}{JPl_t} \right) + .948 \left( \frac{1}{EXCH} \right)_{t-1} + \epsilon_t \]

Using the estimated coefficients \( \hat{\alpha}_1 \) and \( \hat{\alpha}_2 \) they report an average lag of 17.87 months, which is, of course, suggestive of (1) a high degree of inefficiency and (2) the violation of the PPP principle.³

The latter, i.e., the violation of PPP, is suggested despite the fact that in estimating the model Hodgson and Phelps used wholesale price indexes as proxies for each country’s price index which according to Officer (1976) should bias their results in favor of the PPP. In this study each country’s price index is proxied by its consumer price index to minimize the bias. The classical version of PPP (i.e. Equations 4 and 5) is then estimated and the estimates are examined for the presence of the hypothesized econometric problems. Furthermore, the model is reestimated using wholesale price indexes and the bias is empirically examined.

III. The Efficient Markets Version of PPP

Roll (1979) has formulated a simple theory of PPP from an efficient markets perspective. According to this model PPP holds on an expected value basis, i.e.,

\[ E\{\ln(EXCH)_t + \ln\left( \frac{USPl_t}{USPl_{t-1}} \right) - \ln\left( \frac{JPl_t}{JPl_{t-1}} \right) | \phi_t\} = \ln(EXCH)_{t-1} \]

(6)

where

\[ E = \text{expected value operator} \]

\[ \ln\left( \frac{PI_t}{PI_{t-1}} \right) = \text{continuously compounded inflation rate} \]

\[ \phi_t = \text{information set available at time } t, \text{ and the quantity in braces may be referred to as the exchange-adjusted differential inflation rate.} \]

Accordingly, the preceding period’s exchange rate contains all information relevant for predicting the exchange-adjusted differential inflation rate.⁴

Equation (6) is stated in expected value terms. However, if the foreign exchange markets are efficient, deviations in expectations should be uncorrelated over time. This can be seen by subtracting \( \ln(EXCH)_{t-1} \) from both sides of Equation (6).

³ The reported average lag of 17.87 months is apparently obtained by using the un-rounded estimate of \( \hat{\alpha}_2 \) and not the rounded one reported.
⁴ Equation (9) is Roll’s Equation (6.9), p. 140.
Equation (8) may be empirically tested as
\[ X_t = \hat{b}_0 + \hat{b}_1 \ln(\text{EXCH})_{t-1} + \varepsilon_t, \quad (8) \]

where \( X \) is the exchange adjusted differential inflation rate. To test whether lagged values of the dependent variable are relevant in predicting the current period's exchange adjusted differential inflation rate the following equation is also fitted.\(^5\)
\[ X_t = \hat{b}_0 + \hat{b}_1 \ln(\text{EXCH})_{t-1} + \hat{b}_2 X_{t-1} + \ldots + \hat{b}_7 X_{t-7} + \varepsilon_t \quad (9) \]

According to the efficient markets version of the PPP we would expect \( b_1 = 1.0 \) and \( b_0 = b_2 = \ldots = b_7 = 0. \)

Equation (7) was empirically tested as
\[ Y_t = \hat{\gamma}_0 + \hat{\gamma}_1 Y_{t-1} + \hat{\gamma}_2 Y_{t-2} + \ldots + \hat{\gamma}_8 Y_{t-8} + \varepsilon_t \quad (10) \]

where \( Y \) is deviations in expectations
\[ = \ln(\text{EXCH})_t - \ln(\text{EXCH})_{t-1} + \ln \left( \frac{USPI_t}{USPI_{t-1}} \right) - \ln \left( \frac{JPI_t}{JPI_{t-1}} \right). \]

The efficient markets version of PPP would require \( \gamma_1 = \gamma_2 = \ldots = \gamma_8 = 0. \)\(^6\)

Notice that Equations (8), (9) and (10) are free of the econometric problems associated with the existence of relative price variations. Therefore, a good fit of these equations is all that is needed to support PPP. A poor fit of these equations can be used as evidence against PPP.

Because of the inherent bias of the wholesale price indexes in favor of the PPP, consumer price index are used to estimate the efficient markets version. The data used to estimate both models (the classical, and the efficient markets) are monthly observations on exchange rates and price indexes for the seven-year period April 1973—March 1980 of the contemporary floating exchange rate system. All data were obtained from IMF statistics tape.

IV. Empirical Results

A. The Classical Version of PPP

The unconstrained generalized-least-squares estimate of Equation (4) with CPI's used as proxies for price levels is
\[ \ln \left( \frac{1}{\text{EXCH}} \right)_t = -5.626 + 0.119 \ln \left( \frac{USPI_t}{JPI_t} \right) + \varepsilon_t, \]
\[ (-43.28) \quad (0.46) \]
\[ F=0.21, \quad R^2=0.002, \quad d=0.06. \]

The estimated slope is significantly different than unity \((t = -3.37)\) and identical to zero. The estimated intercept is highly significant and variations in relative prices between the

\(^5\) Equation (12) is Roll's Equation (6.9\(^*\)), p. 145.

\(^6\) Equation (13) is Roll's Equation (6.11), p. 146.
U.S. and Japan explain virtually none (but 0.2 percent) of the variations in the exchange rate between the dollar and the yen. Furthermore, the $F$-statistic (a measure of the strength of association) indicates that the variability of the exchange rate explained by the model is a small fraction (0.21) of the variability not explained. Finally, the very small value of Durbin-Watson $d$-statistic indicates presence of severe positive autocorrelation.

Estimates significantly improve in favor of PPP when WPI's are used instead of CPI's. The estimated equation is

$$\ln\left(\frac{1}{EXCH}\right)_t = -6.369 + 1.489 \ln\left(\frac{USPI_t}{JPI_t}\right) + \varepsilon_t$$

\((14.51)\)

\(F=210, R^2=0.71, d=0.22.\)

Notice, however, that the estimated slope, although significantly different than zero, is still significantly different than unity \((t=4.77)\). Also, notice that the estimated intercept is highly significant and that severe positive autocorrelation is still present.

The estimated form of Equation (5), with CPI's used as proxies for price levels is

$$\ln\left(\frac{1}{EXCH}\right)_t = -0.138 - 0.066 \ln\left(\frac{USPI_t}{JPI_t}\right) + 0.969 \ln\left(\frac{1}{EXCH}\right)_{t-1} + \varepsilon_t$$

\((-0.9)\) \((-1.02)\) \((37.06)\)

\(F=691, R^2=0.94, h=0.59\)

The estimated $\alpha_1$, although insignificant, is negative, and the computed average lag is 31.25 months. However, as judged by the Durbin's $h$-statistic, the distributed lag model of (5) does not suffer from autocorrelation problems present in (4). This holds regardless of whether CPI's or WPI's are used. The estimated form of (5) with WPI's is

$$\ln\left(\frac{1}{EXCH}\right)_t = -0.688 + 0.155 \ln\left(\frac{USPI_t}{JPI_t}\right) + 0.891 \ln\left(\frac{1}{EXCH}\right)_{t-1} + \varepsilon_t$$

\((-2.15)\) \((1.75)\) \((17.79)\)

\(F=709, R^2=0.94, h=1.19.\)

The bias of WPI in favor of PPP shows up in the estimate of $\alpha_1$ being of the expected sign (yet insignificant) and a computed average lag of 8.17 months.

B. The Efficient Markets Version

The estimated form of Equation (8) is

$$X_t = 0.032 + 0.943 \ln(EXCH)_{t-1} + \varepsilon_t$$

\((2.17)\) \((35.92)\)

\(F=1290, R^2=p.94, d=1.82.\)

Ninety-four percent of the variations in the exchange-adjusted differential rate of inflation is explained by the variations in the previous period's exchange rate. The $F$-statistic indicates that the variance explained is significantly greater than the variance not explained by the model and the Durbin-Watson $d$-statistic indicates that serial correlation does not pose any significant problem. The estimated intercept, although small in magnitude is significantly different than zero. The estimated slope is close to unity but significantly smaller than 1.0 \((t=-2.19)\). Nonetheless, this does not, in itself, imply that the markets
for the dollar/yen are inefficient. Inefficiency may be concluded if lagged values of exchange-adjusted differential inflation rate added to the predictive performance. The estimated form of Equation (9) shows that this clearly is not the case.

\[ X_t = 0.035 + 1.113 \ln(EXCH)_{t-1} - 0.172X_{t-1} + 0.065X_{t-2} + 0.111X_{t-3} - 0.131X_{t-4} \]
\[ \begin{align*}
(2.1) \\
(2.6) \\
(-0.4) \\
(0.4) \\
(0.7) \\
(-0.8)
\end{align*} \]

\[ -0.007X_{t-5} - 0.043X_{t-6} + \varepsilon_t \]
\[ \begin{align*}
(-0.1) \\
(-0.4)
\end{align*} \]

\[ F=183, \quad R^2=0.94, \quad h=0.59 \]

Inclusion of lagged values of the dependent variable does not add to the predictive performance. The only effect, however, is to make the estimated \( b_t \) identical to 1.0. This evidence indicates that the current dollar/yen (or yen/dollar) exchange rate contains all information relevant to predicting next period's exchange-adjusted differential inflation rate between the U.S. and Japan, i.e., the foreign exchange markets for these currencies are efficient.

Further evidence supporting efficiency of the dollar/yen exchange markets is provided by the estimate of Equation (10):

\[ Y_t = -0.0 + 0.007Y_{t-1} + 0.05Y_{t-2} + 0.106Y_{t-3} + 0.046Y_{t-4} + 0.01Y_{t-5} + 0.046Y_{t-6} \]
\[ \begin{align*}
(-0.0) \\
(0.1) \\
(0.4) \\
(0.9) \\
(0.4) \\
(0.1) \\
(0.3)
\end{align*} \]

\[ F=0.25, \quad R^2=0.02, \quad h=0.41 \]

These results indicate that none of the six lagged values of deviations in expectations provide a significant contribution in predicting next period's deviation from expectation. In other words, deviations in expectations are uncorrelated over time, reinforcing the conclusion that the dollar/yen markets are efficient.

V. Summary and Conclusions

The classical version of PPP was estimated for the seven-year period April 1973–March 1980 of the contemporary floating exchange rate system. The results show that the use of the classical formulation as a test vehicle may lead to reinforced conclusions that there exist long average lags between changes in price levels and changes in the dollar/yen exchange rates and as such a high degree inefficiency is present in the foreign exchange markets. However, it was shown that econometric problems caused by the existence of relative price variations create bias in the estimates of the classical model and may lead to erroneous conclusions on both the validity of PPP and the efficiency of the foreign exchange markets.

Results of the efficient markets version tests support the hypothesis that the current dollar/yen exchange rate contains all relevant information for predicting next-period's exchange-adjusted inflation rate differences between the U.S. and Japan. It was found that lagged values of exchange-adjusted differential inflation rate do not add to the predictive performance of this rate. Finally, it was found that as the efficient markets theory requires, the observed deviations in PPP are intertemporally independent.
REFERENCES


Cassel, Gustav (1922), Money and Foreign Exchange After 1914, New York, Macmillan Co.


