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OPTIMAL RATIONAL EXPECTATIONS

By TAKEAKI KARIYA*

Using Muth's (1961) model, this paper examines the postulate of rational expectation hypothesis, and it is shown that expectation formation is not optimal relative to profit or sales maximization. Alternatively, under full information, the postulates of optimal expectation formation are proposed relative to the economic variables an economic agent wishes to finally forecast. These expectation formations share with that of rational expectation a property that expectation be formed in a model with full information.

I. Summary

In this paper, we shall use the term “forecast” rather than “expectation” for predicting a value of a variable in advance. The term “expectation or expected” is used exclusively for the (mathematical) expectation or expected value of a random variable. Hence in this paper, rational expectation, adaptive expectation, etc., are called rational forecast, adaptive forecast, etc., respectively.

The rational forecast postulated by Muth (1961) is so defined that the expected equilibrium of an economic model an economic agent knows as a part of his information may be attained in advance. That is, under the postulate that subjective forecast be equal to objective expectation in the market, it is determined as an endogenous variable which together with other endogenous variable gives the expected equilibrium of an economic model expected at the end of \( t-1 \), where period \( t \) is the period for forecast. [See, e.g., Muth (1961), Barro (1976) or Shizuki and Muto (1981)]. This implies that the forecast thus formed is a function of all the variables that are predetermined up to the end of \( t-1 \), and that it depends heavily on the model. Consequently, if a model includes policy variables, the rational forecast is affected by the realized values of these policy variables up to the end of \( t-1 \) and the expected values of these variables for the period \( t \). Due to this fact, rational forecast is often applied to policy evaluation in macroeconomic models.

However, the rationality of the rational forecast formation has not well been discussed. We here do this based on Muth’s (1961) model, and show that the behavior of an economic agent who forms the rational forecast with the information assumed is not optimal (rational) relative to his profit or sales. In assuming that the agent knows the model with the values of parameters, he eventually knows that his expected profit or sales evaluated with the expected equilibrium price is a function of his forecast. Hence maximizing it with respect to the forecast yields the optimal forecast for him, which is called the subjective optional rational forecast in this paper. But, among the agents, these optimal forecasts are in general different

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unless the technical levels in production are same. If the information on the technical levels is assumed to be available to the suppliers in the market, there may be a minimax-type optimal forecast, which is common to them and better than the rational forecast (section III). In section II, the optimal forecast which maximizes the aggregate expected profit (or the total expected market profit) is explicitly derived, and there the forecast is considered that of a representative agent. Naturally, the maximized aggregate expected profit under the optimal forecast is greater than the aggregate expected profit under the rational forecast. It is noted that in Muth's model, the forecast of price level at the end of $t-1$ determines the planned supply for the period $t$ via a supply function, which is derived by maximizing profit with the forecasted price. In this sense, forecasting price is regarded as determining the behavior of supply or production for period $t$, and hence the optimal forecasts proposed here seem more natural as such a behavior than the rational forecast, so long as the agents are assumed to behave rationally with the information given. In section III, some remarks are made. It is pointed out that Muth's derivation of an adaptive forecast through the rational forecast is not effective as it stands. In section IV, considering the situation Baumol (1959) described, we derive the alternative optimal forecast which maximizes the expected sales (revenue) of a representative producer, and this forecast is shown to minimize the variance (risk) of the sales (revenue). In this argument, it is assumed that the supply function of each producer is derived through profit maximization based on his forecast. This is because the market (aggregate) supply function in Muth's model is considered thus derived. And when he supplies with the supply function, he again knows that his sales evaluated with the expected equilibrium price in the market is a function of his forecast. Although he also knows that his expected profit is a function of his forecast, here he is supposed to choose the optimal forecast which maximizes his expected sales rather than the optimal forecast which maximizes his expected profit.

The utilization of the information in the rational forecast hypothesis is rather limited. Although the agents know the (true) model, they only use it to get the expected equilibrium price which holds only when they compete perfectly without knowing the model. And given the expected equilibrium price, they are supposed to maximize their profit, as is described in the case of perfect competition. However, for any given forecast, the expected market equilibrium price is obtained as a function of the forecast, since the model is known to them. That is to say, in the rational forecast hypothesis, what is utilized is the information on the expected equilibrium price under perfect competition, but as is shown in this paper, given the information on the model, the situation described under perfect competition no longer holds. And the suppliers can control the expected equilibrium price by suitably setting their price forecasts.

In this paper, assuming the same information as in the rational forecast hypothesis, we derive the alternative forecasts through optimization. But this simply implies that the assumption produces the better forecasts. What is more important will be the plausibility of the assumption. Hence, once a similar set of assumptions are made in macro-economic context, the arguments here will be applicable to derive alternative forecasts. There the final variable (objective function) to be optimized may vary according to agents. For example, laborers may need to forecast price level to forecast real wage rate in the next period before a contract is made. Then there will be an optimal forecast which maximizes the expected real wage rate, and a corresponding level of supply of labor.
II. Optimal Rational Forecast

Muth's demand-supply market model with explicit constant term is expressed as

\[
\begin{align*}
S_t &= gx + S_0 + u_t, \quad g > 0 \quad \text{(supply)} \\
D_t &= -bp_t + D_0, \quad D_0 > 0, b > 0 \quad \text{(demand)} \\
S_t &= D_t
\end{align*}
\]

where \( S_t \) and \( D_t \) are respectively supply and demand of a commodity in the market in period \( t \), \( x \) is the price to be forecasted at the end of \( t-1 \) and \( P_t \) is the price realized in the market in period \( t \). Here \( u_t \) is a disturbance (shock or innovation) in period \( t \) to supply caused by ill weather or some random effects and it is considered a random variable with

\[
E(u_t) = 0 \quad \text{and} \quad \text{Var}(u_t) = E(u_t^2) = \sigma^2 > \infty
\]

where \( E(\cdot) \) denotes conditional expectation taken at the end of \( t-1 \) given the information up to then. To examine the rationality or optimality of rational price forecast, let us assume that there are \( n \) producers or suppliers in the market and that each producer knows the model (2.1), the values of parameters \( g, b, D_0, S_0 \), and the fact (2.2) as well. What he cannot know at the end of \( t-1 \) is the value of the shock \( u_t \) and so the values of \( S_t = D_t \) and \( p_t \). Without knowing the value of \( u_t \), at the end of \( t-1 \), he has to forecast the price \( p_t \) or give \( x \) to decide how much he plans to supply in the next period. The supply function of each producer is given through profit maximization. That is, forecasting \( p_t \) by \( x \), the \( i-th \) producer maximizes his profit \( \Pi_{it} = 1 \)

\[
\Pi_{it} = px - ct(st(t))
\]

so that he obtains his supply function

\[
x = c_i'(s_t(x)) \quad \text{or} \quad s_t(x) = h_i(x) \quad \text{for} \quad x \geq z_i(> 0),
\]

where \( c_i(\cdot) \) is the cost function, \( c_i'(\cdot) \) is the derivative, and \( h_i \) is the inverse function of \( c_i' \) on the set \( x \geq z_i \). The usual assumptions are made upon the cost functions. Further, we assume that there exists no random factor in \( s_t(x) \) and

\[
\sum_{t=1}^{n} s_t(x) = E(S_t) = gx + S_0.
\]

Under this situation, if \( x \) is given at the end of \( t-1 \), the \( i-th \) producer plans to supply the amount \( s_t(x) \). Although he does not know the value of \( u_t \), he knows the model (2.1) and so the model expected at the end of \( t-1 \);

\[
\begin{align*}
E(S_t) &= gx + S_0 \quad \text{(expected supply function)} \\
E(D_t) &= -bp_t + D_0 \quad \text{(expected demand function)} \\
E(S_t) &= E(D_t) \quad \text{(expected equilibrium condition)}
\end{align*}
\]

In this expected model, if \( x \) is given, (2.6) can be solved for \( E(S_t) = E(D_t) \) and \( E(p_t) \). That is, each producer can evaluate the expected market price

\[
E(p_t) = \frac{-(gx + S_0 - D_0)}{b}
\]

for each forecast level \( x \), where \( x \leq (D_0 - S_0)/b \) from \( \bar{p}_i(x) \geq 0 \). Therefore, the \( i-th \) producer who supplies \( s_t(x) \) based on the forecasted price \( x \) eventually knows that he can expect to receive the profit

\[
\Pi_{it}(x) = \bar{p}_i(x)s_t(x) - ct(s_t(x)) = E[p_t^*s_t(x) - ct(s_t(x))]
\]

where \( p_t^* = \bar{p}_i(x) - u_t/b \) is the equilibrium price of the model (2.1) or the equilibrium market price in period \( t \). This expected profit is not the forecasted profit \( x s_t(x) - ct(s_t(x)) \) obtained by maximizing (2.3), unless \( \bar{p}_i(x) = x \). This provides a justification from the viewpoint of
subjective equilibrium for the rational forecast hypothesis. In fact, in the rational forecast hypothesis, it is required to set \( x = E(p_t) = p_t(x) \) so that \( x \) is made an endogenous variable of the expected model (2.6) at the end of \( t-1 \). It is noted that \( x \) is not made an endogenous variable of the actual model (2.1) in the market in period \( t \). Solving \( p_t(x) = x \) by (2.7) yields (2.9) \( x^* = (D_0 - S_0)/(b + g) \).

If the producer forecasts \( x \) by \( x^* \), he plans to supply \( s_t(x^*) \) and expects to receive the maximized profit \( x^* s_t(x^*) - c_t(s_t(x^*)) \) since in (2.4) \( x^* = c_t'(s_t(x^*)) \).

However, this rational forecast formation is not optimal in the sense that the attainable maximum of his expected profit function in (2.8) is not obtained. In (2.8), the expected profit function \( \Pi_t(x) \) is a function of his forecasted price \( x \). This is because his forecast can affect the expected equilibrium price through the supply \( s_t(x) \) and (2.5). This is implied by the assumption that each producer knows the model (2.1) with (2.2). Hence, we here define what we call the subjective optimal rational forecast of the \( i \)-th producer by the forecast \( x^i \) which maximizes the expected profit function \( \Pi_t(x) \) in (2.8). Clearly \( x^i \) needs to satisfy \( \Pi_t(x^i) = 0 \) and \( \Pi_t(x^i) < 0 \) for the maximization. Further \( x^i \) must lie in the domain \( A_i = \{ z_i \leq x \leq (D_0 - S_0)/b \} \). Although from (2.4)

(2.10) \( c_t'(s_t(x)) = x \) for \( x \in A_i \) \( (i = 1, \ldots, n) \),

the \( x^i \) satisfying \( \Pi_t(x^i) = 0 \) depends on the cost function \( c_t \) and so it is not common for all \( i \).

In this sense, although each producer can compute the optimal rational forecast, he may not use it unless he knows that all the cost functions or all the technical levels of \( n \) producers are almost same. On the other hand, compared to the rational forecast \( x^* \) in (2.9), he also knows (2.11) \( \Pi_t(x^i) \geq \Pi_t(x^i) \) \( (i = 1, \ldots, n) \), where \( x^i \in A_i = \cap \cap A_i \) may be assumed (if not, \( x^\ast \) is not common either). Hence he has an incentive to use it. But whether he in fact uses it or not will depend on whether he has the information on the cost functions of the other producers, the difference of the profits in (2.11) and his relative market share.

Alternatively, as is often done in macroeconomics, let us consider the behavior of a "representative" producer in the market. We assume that he has the supply function \( \tilde{S}_t(x)/n \) and the cost function \( C(x)/n \), where

(2.12) \( \tilde{S}_t(x) = E(S_t) = g_x + S_0 \)

and \( C(x) = S_0 + c_t(s_t(x)) \).

Then his expected profit function is given by \( \Pi_t(x)/n \), where

(2.13) \( \Pi_t(x) = \tilde{p}_t(x) \tilde{S}_t(x) - C(x) \).

As a matter of fact, \( \Pi_t(x) \) is the aggregate expected profit function obtained by aggregating \( \Pi_t(x) \) in (2.8) with respect to \( i \). To maximize \( \Pi_t(x) \), differentiating it yields (2.14) \( \Pi_t'(x) = p_t'(x)S_t(x) + p_t(x)S_t'(x) - \Sigma c_t'(s_t(x))s_t'(x) \)

\[ = -g[g_x + S_0 + g_x + S_0 - D_0 + bx]/b, \]

where the relations (2.12), (2.5), (2.7) and (2.10) are used (note \( C'(x) = xS_t'(x) \) from (2.10) and (2.5)). Therefore

(2.15) \( x^0 = (D_0 - 2S_0)/(2g + b) \), where \( D_0 \geq S_0 \) is assumed, satisfies \( \Pi_t'(x^0) = 0 \). Assuming \( x^0 \in A \), the second order condition is given by (2.16) \( \Pi_t''(x^0) = -(g^2/b) - \Sigma c_t''(s_t(x^0))(s_t'(x^0))^2 < 0 \), which holds if \( c_t''(s_t(x^0)) > 0 \). In the below, we shall call \( x^0 \) the optimal rational forecast of a representative producer or the optimal rational forecast in the market. That is, \( x^0 \) maximizes the aggregate expected profit and from the assumption \( x^* \in A \), we have

(2.17) \( \Pi_t(x^0) > \Pi_t(x^\ast) \).
Also in this sense, the rational forecast $x^*$ is not optimal. Next, let us see the relationship between the aggregate expected profit function $\Pi_t(x)$ and the \textit{expected market profit function} defined by

\begin{equation}
\Pi_t(x) = E(p_t^* S_t^* - C(x)),
\end{equation}

where $p_t^* = \bar{p}_t(x) - u_t/b$ and $\bar{S}_t^* = \bar{S}_t(x) + u_t$ are respectively the equilibrium price and equilibrium supply of the model (2.1). It is easy to see that

\begin{equation}
\Pi_t(x) = \Pi_t(x) - \sigma^2/b.
\end{equation}

Hence, maximizing the aggregate expected profit $\Pi_t(x)$ is equivalent to maximizing the expected market profit $\Pi_t^*(x)$, and $\Pi_t^*(x^0) > 0$ if and only if $\Pi_t(x^0) > \sigma^2/b$. Therefore, if the variance (level of uncertainty) $\sigma^2$ of $u_t$ is greater than the maximized forecasted profit $\Pi_t(x^0)$ times $b$, the representative producer expects the negative profit and so may stop production. It is noted that from (2.17), the negative expected profit under $x^0$ implies the negative expected profit under $x^*$. This point is not discussed in Muth (1961). Finally, we shall consider the variance of the market profit under a forecast $x$;

\begin{equation}
\text{Var}(p_t^* S_t^* - C(x)) = \text{Var}(p_t^* S_t^*) = E(p_t^* S_t^*)^2 - \{E(p_t^* S_t^*)\}^2
\end{equation}

\begin{equation}
= \left[ \Gamma - \sigma^2 + (2g + 2S_0 - D_0)^2 \sigma^2 \right] / b^2 = V(x), \text{ say},
\end{equation}

where we assumed $E(u_t^3) = 0$ and $E(u_t^4) < \infty$. This variance is minimized by

\begin{equation}
x^* = (D_0 - 2S_0) / 2g,
\end{equation}

but the sign of $V(x^0) - V(x^*)$ depends on the values of $g, b, S_0$ and $D_0$.

What we observed in this section is as follows. The rational forecast $x^*$ is not optimal in the sense that it maximizes neither the expected profit function of each producer nor the aggregate expected profit function (nor the expected market profit function). In general, unless all the cost functions of $n$ producers are same, no common subjective optimal rational forecast exists. It is noted that if all the cost functions are same, the common subjective optimal forecast is equal to the optimal rational forecast $x^0$ in (2.15) of a representative producer. The forecast $x^0$ maximizes the aggregate expected profit function in (2.13) (or the expected market profit function in (2.18)), the $n$-th of which may be regarded as the expected profit function of a representative producer. These observations are deduced from the assumption that each producer knows the model (1.1). In the rational forecast hypothesis, although the same assumption is made, it seems that producers are supposed to behave as if they did now know the model (1.1) except the expected equilibrium price $x^*$, and given $x^*$, they regard it as their demand function and act as if it were the only possible forecast. However, as has been seen in (2.8), the assumption necessarily implies that the expected profit function of each producer is a function of each forecast. Hence if each producer is assumed to behave rationally, it does not seem natural that he only uses the information on the expected equilibrium price $x^*$ from the sufficient information assumed, and acts as if it were given.

\section{Maximin Forecast and Some Remarks}

In section II, it has been shown that unless all the cost functions of the $n$ producers are same, in general there exists no common subjective optimal rational forecast. However, if the producers are assumed to know all the cost functions, they can evaluate the following minimax-type forecast. Let
(3.1) \( X_t = \{ x \in A_t : \Pi_t(x) > \max_j \Pi_{jt}(x^*) \} \)

where \( \Pi_t(x) \) is the expected profit of the \( i \)-th producer in (2.8), \( A_t = \{ z_t \leq x \leq (D_0 - S_0)/b \} \) as before and the rational forecast \( x^* \) is given by (2.9). We assume \( A = \cap_t A_t \neq \emptyset \) and \( x^* \in A \). And assume \( X = \cap_t X_t \neq \emptyset \). Then the maximin forecast \( x^m \) satisfying

(3.2) \( \max \min_{x \in X} \Pi_{jt}(x) = \min_j \Pi_{jt}(x^m) \)

exists if \( \Pi_{jt}(x) \)'s are continuous (since \( X \) is compact), and from the definition

(3.3) \( \Pi_{jt}(x^m) \geq \Pi_{jt}(x^*) \) for \( j = 1, \ldots, n \).

And unless \( x^m = x^* \), at least one of the inequalities in (4.3) will be in general strict, even though \( x^m \) might not be unique. Hence the maximin forecast \( x^m \) is better than the rational forecast \( x^* \), provided the assumption made here and effective. And it is common to all the producers.

Some technical remarks follow below.

First, in the argument of the subjective optimal forecast, we tacitly assumed that there exists no random factor in \( s_t(x) \). This will be natural when the model (2.1) is the one for an industrial product. However, if the model represents the one for an agricultural product, \( s_t(x) \) may suffer from a random effect, say \( u_t \), mostly through weather. Then, the profit of the \( i \)-th producer becomes

(3.4) \( \eta_t = p_t^* [s_t(x) + u_t] - c_t(s_t(x) + u_t) \)

where \( p_t^* = p_t(x) - u_t/b \) as before. Since \( \Sigma t-1 u_t = u_t \) in this case, the expected value of this profit \( \eta(x) = E(\eta) \) is not equal to (2.8). Maximizing this expected profit in principle gives an alternative optimal forecast which fits the situation under consideration. But, we may suppose that the correlation between the aggregate disturbance \( u_t \) and the individual \( u_t \) is small when \( u \) is relative large, and that \( c_t(s_t(x) + u_t) \) is approximately equal to \( c_t(s_t(x)) + c'_t(s_t(x)u_t) \) when the variance of \( u_t \) is relatively small. In such a situation, the expected profit function is approximately equal to the one in (2.8). Second, if the model (2.1) is a model for an industrial product, the disturbance term \( u_t \) is rather hard to interpret. It may be regarded as an aggregation error or an approximation error in the specification of the model, but it is often regarded as an innovation realized in the period \( t \). If we regard it as an uncertainty factor prevailing in the market, it will be more natural to introduce it into the demand function.

Third, even if a disturbance term, say \( v_t \), is introduced into the demand function in (2.1) in addition to \( u_t \) in the supply function, the results obtained above remain effective except (2.20) and (2.21). To see this, we suppose

(3.5) \( E(v_t) = 0 \) and \( Var(v_t) = \tau^2 \)

and \( u_t \) and \( v_t \) are independent. Further, set \( D'_0 = D_0 + v_t \). Then conditional on \( v_t \), the expressions \( \Pi_t(x) \) in (2.13) and \( \Pi_t*(x) \) in (2.18) hold as they are if \( D_0 \) is replaced by \( D'_0 \), and they are linear in \( v_t \). Hence taking expectation with respect to \( v_t \) yields \( \Pi_t(x) \) and \( \Pi_t*(x) \) exactly. On the other hand, the variance of \( p_t^* S_t - C(x) \) in this case is evaluated as

(3.6) \( Var(p_t^* S_t - C(x)) = [\alpha^2 + (2g + 2S_0 - D_0)^2 \sigma^2 + S_t(x)^2 \sigma^2 + \sigma^2 \tau^2]/b^2 \)

Minimizing this with respect to \( x \) gives

(3.7) \( x^* = [D_0 - 2S_0\sigma^2 - S_0\tau^2]/(2\sigma^2 + \tau^2) \).

Fourth, suppose that \( u_t \) in (2.1) is of the form

(3.8) \( u_t = \Sigma \sigma_j c_j w_{t-j} (j < \infty) \),

where \( w_{t-j} \) are mutually independent, \( E(w_{t-j}) = 0 \) and \( Var(w_{t-j}) = \delta^2 (j = 0, 1, \ldots) \). Since \( w_{t-j} \)'s \((j \geq 1)\) are realized at the end of \( t-1 \), set \( S'_0 = S_0 + \Sigma \sigma_j c_j w_{t-j} \). Then the result obtained above
is effective with $S_0$ replaced by $S_0'$ as it stands. But in this case, $S_0'$ can take any value so that $\Pi_\ast(x) > 0$ must be checked for the realized value of $S_0'$. This fourth remark is applicable to the result in section II.

In relation to the fourth remark, we comment on Muth's (1961) derivation of an adaptive price forecast via a rational price forecast. That is, he showed that when $u_t = \sum_{j=0}^{\infty} c_j w_{t-j}$ with $c_j = 1 (j \geq 0)$, the rational price forecast is under certain assumptions expressed as a geometrically weighted moving average of past prices. However, when $c_j = 1 (j \geq 0)$, the series $\sum_{j=0}^{\infty} w_{t-j}$ does not converge under the assumption that $w_{t-j}$'s are independently and identically distributed [see, e.g., Chung (1974) for the Three Series Theorem]. Hence the claim he made does not hold as it is.

IV. Optimal Rational Forecast Relative to Sales

In monopolistic competition where the number $n$ of producers are relatively small, producers may be more interested in sales than profit [see Baumol (1959) for such situations]. If we accept this hypothesis, another alternative optimal forecast is obtained. However, a crucial assumption for deriving it is that the market (aggregate) supply function in the model (2.1) is derived under the procedure described in (2.3)–(2.5). That is, the $i$-th producer first gives a forecast $x$ and maximizes the profit (2.3) so that he obtains his supply function $s_i(x)$ in (2.4). And he knows the relation between his supply function and the aggregate supply function in (2.5), and so he knows that under a forecast $x$, the expected equilibrium price $p_t(x)$ is given by (2.7) and that his expected sales (revenue) is evaluated as $r_t(x) = p_t(x)s_i(x)$. In other words, the market supply function is assumed to be the aggregate of $s_i(x)$'s obtained through profit maximization under a forecast $x$, and each producer is assumed to know the fact. Then, when he supplies $s_i(x)$ based on his forecast $x$, he eventually expects to receive $r_t(x)$, which is a function of his forecast $x$. Although he also knows his expected profit $\bar{\Pi}_t(x)$ in (2.8), here he is supposed to maximize the expected sales $r_t$ rather than $\bar{\Pi}_t(x)$. Baumol imposes on this behavior the condition that a certain level of profit must be guaranteed. But we ignore it.

The optimal forecast which maximizes $r_t(x)$ again depends on the supply function $s_t(x)$ of the cost function $c_t$, and so unless the information on the technical levels of other producers is available, he may not use it. Here once more we adopt the macroeconomic argument in which a representative producer is considered. Then assuming the representative producer's supply function is given by $S_i(x)/n$, his expected revenue is evaluated as $R_t(x)/n = \bar{p}_t(x)S_i(x)/n$, where $S_i(x)$ is given by (2.12). Alternatively, if the relative shares of $n$ producers, say $a_i$'s, are assumed to be known to the producers, the $i$-th producer's supply function is regarded as $a_i S_i(x)$ so that his revenue is given by $a_i R_t(x)$. In either case, the optimal rational forecast maximizing the expected revenue is given by the forecast $x^\ast$ in (2.21), which there minimizes the variance of $p_t^\ast S_t^\ast$. Since as is easily shown,

\[(4.1) \quad R_t^\ast(x) = E(p_t^\ast S_t^\ast) = R_t(x) - \sigma^2/b,\]

maximizing the expected revenue is equivalent to maximizing the expected market sales $E(p_t^\ast S_t^\ast)$ and simultaneously, the forecast minimizes the variance of the market sales. This means that the maximized expected revenue can be expected with most certainty or minimum risk. Therefore, the optimal rational forecast $x^\ast$ relative to sales is better than any other
forecast in terms of the expected revenue (expected market sales) $R_t(x)$ (or $R_t^*(x)$) and risk $V_t(x)$ in (2.20). Consequently, under the assumptions we made, the representative producer (or the $i$-th producer when the shares $a_i$'s are known) plans to supply $\tilde{S}_t(x')/n$ (or $a_i\tilde{S}_t(x')$ resp.) in the next period, and expects to receive the sales $R_t^*(x')/n$ (or $a_iR_t^*(x')$ resp.) with minimum risk, we make two remarks. First, in order for $R_t^*(x')$ in (4.1) to be positive,

$$\frac{(D_0-2S_0)^2}{4}+(D_0-S_0)S_0<\sigma^2$$

must hold. If this does not hold, the producers will stop production. Second, if a disturbance term, say $v_t$, is introduced into the demand function in (2.1), then as has been shown in section 3, $x^*$ in (2.21) no longer minimizes the variance of $p_t^*S_t^*-C(x)$ in (3.6) and so the variance of $p_t^*S_t^*$. But it still maximizes the expected revenue $R_t(x)$ or the expected market sales $R_t^*(x)$.

V. A Concluding Remark

From the arguments above, the scope of the availability of information turns out to be very important in describing the rational behaviors of economic agents. And if we assume that a true model is known to the agents as in the rational forecast hypothesis, the rational forecast proposed by Muth (1961) has been shown to be not the one which is derived under the rational behaviors. This will tempt us to say that assuming too much information produces conclusions which are often irrelevant to the real world, even if they are logically consistent within the assumptions. [See the chapter 2 of Tobin (1980) on this matter].

REFERENCES