I. Introduction and Outline of the Paper

The purpose of this paper is to investigate two ways in which the composition of demand affects the distribution of household income. First, the spending of one household results, via production and remuneration of factors of production, in income for another household. This process can usefully be examined when households are grouped together according to one or more socio-economic criteria. Second, the composition of investment, government spending and foreign trade determines the product mix in the economy and thus the flows of income to households.

The model presented in this paper captures both of these channels. It consists of an input-output model in which household expenditures are endogenous and in which the household sector has been disaggregated. The model provides an integrated explanation of the growth and distribution of household incomes from the demand side. Although the model is fairly conventional, we believe the interpretation is not. Specifically, two ways are shown to break down observed total income distribution. One aims at indicating the effect each final demand component has on the distribution of income, but without formally decomposing an inequality summary measure. The other indicates lower bound inequality levels that are compatible with some structural characteristics of the economy captured by the model, both in short-run equilibrium and in a long-run steady state situation. The comparison of these two levels with observed levels of inequality yields insights into distributinal trends not derivable from other methods of analysis and has important implications for the selection of redistributive policy measures.

For the empirical part of the paper, we present an application of the model to Japan over the years 1959-69, a period during which economic growth was steady and uninterrupted by exogenous shocks.

The plan of the paper is as follows. The next section contains the formal presentation of the model. Section three describes the data base and estimation methodology. The empirical results and a further analysis and interpretation of the model are in section four. Finally, section five contains some concluding remarks.

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† This paper is based on my doctoral dissertation. As such I am indebted to the members of my dissertation committee at the University of Hawaii. Some additional research was undertaken at the Development Research Center of the World Bank. For this I am grateful for valuable input from Jan Bisschop and Graham Pyatt. Of course, I remain solely responsible for any remaining errors. The views expressed herein are mine only and do not reflect those of the institutions I was and am affiliated with.
II. An Equilibrium Model of Income Distribution

For the purposes outlined in the previous section, the model in this paper includes the following features. First, the existence of socio-economic groups in society, each with a different consumption and saving behavior, is recognized. Second, the level and sectoral composition of final demand, broken down in its components, viz. consumption, investment, government spending, and net exports, is incorporated as an explanatory variable. Third, the model is able to generate both the level and distribution of household income. Finally, feedback effects from the level and distribution of income (itself generated by a given amount and pattern of spending), back to spending, then back to income, etc. are included. In other words, the existence of a multiplier mechanism of spending on household income is incorporated into the model.

The last requirement necessitates the use either of a dynamic model with feedback loops or of a static equilibrium model. The latter includes feedback effects but without accounting for the actual time lapse involved. Since our main interest is to include the impact of indirect effects, rather than to estimate their time pattern, and since the data available for the empirical application did not permit estimation of the complex lag structure between spending and production, production and income, income and spending, we opted for a static equilibrium model.

Formally, the model is derived from the general equilibrium condition that aggregate demand equals aggregate supply for each sector.

Aggregate supply is the sum of domestic output and imports. Aggregate demand is broken down in three components: intermediate demand, consumption (both of these are endogenous to the model) and exogenous final demand. We can thus write

\[ X + M = AX + C + FD \]  

where \( X \) is a \( j \times 1 \) vector of domestic output (\( j \) is the number of input-output sectors),

\( M \) is a \( j \times 1 \) vector of imports

\( A \) is a \( j \times j \) matrix of technical coefficients

\( C \) is a \( j \times 1 \) vector of household consumption

\( FD \) is a \( j \times 1 \) vector of exogenous final demand

The ability to distinguish between different socio-economic groups in society is one of the model requirements. Relevant criteria for defining such groups are income level, occupation or industrial affiliation of the household head, family composition, age of the household head, etc. Each socio-economic group is assumed to have a different pattern of consumption, saving, and paying taxes. We can therefore write

\[ C = cE \]  

where \( c \) is a \( j \times i \) matrix of consumption coefficients

(i is the number of socio-economic groups)

\( E \) is a \( i \times 1 \) vector of total household spending.

\(^1\) Actually \( FD \) represents the sum of \( n \) column vectors, where \( n \) is the number of final demand components distinguished.
Combining (1) and (2) yields, after re-writing,
\[ X - AX - cE = FD - M \]  
(3)

Next, we consider the income generation process, which we write as
\[ Y = VX + \bar{Y} \]  
(4)

where \( V \) is an \( i \times j \) matrix where each element \( v_{ij} \) represents the share of value added generated in sector \( j \) that is received by socio-economic group \( i \).

\( \bar{Y} \) is an \( i \times 1 \) vector of exogenous income (e.g. transfer payments).

Equation (4) can be re-written as
\[ -VX + Y = \bar{Y} \]  
(5)

We close the model by noting that
\[ E = eY \]  
(6)

where \( e \) is an \( i \times i \) diagonal matrix where the non-zero elements are the ratios of household expenditures to income for each socio-economic group.

We can combine (3), (5) and (6) to yield
\[
\begin{pmatrix}
I - A & -ce \\
-V & I
\end{pmatrix}
\begin{pmatrix}
X \\
Y
\end{pmatrix}
= 
\begin{pmatrix}
FD - M \\
\bar{Y}
\end{pmatrix}
\]
(7)

where \( I \) is the identity matrix.

Writing \( c' = ce \), solving and inverting (7), we obtain
\[
\begin{pmatrix}
X \\
Y
\end{pmatrix}
= 
\begin{pmatrix}
B(I - c'VB)^{-1} & Bc'(I - VBC')^{-1} \\
(I - VBC')^{-1}VB & (I - VBC')^{-1}
\end{pmatrix}
\begin{pmatrix}
FD - M \\
\bar{Y}
\end{pmatrix}
\]
(8)

where \( B = (I - A)^{-1} \).

Expression (8) describes the generation of the distribution of income over socio-economic groups in a manner fully consistent with the structure of production. Regarding distribution, expression (8) is flexible in that \( Y \) can contain distribution over socio-economic groups defined along any one criterion or set of criteria. The only restriction is that the rows of \( V \) and the columns of \( c' \) must be defined along these same criteria (in order to effectuate model closure).

For purpose of the empirical application in this paper, the criterion used was income, as this was the only dimension along which the available data permitted estimation of \( V \) and \( c' \). For this application we thus assume that income groups are relevant socio-economic groups with distinct patterns of consumption, saving, and paying taxes.

Expression (8) was estimated on Japanese data for the years 1959 and 1969.\(^2\) Economic growth in Japan was steady and uninterrupted between 1959 and 1969. Consequently, a comparison of these 2 years would allow the model to clearly perform its function which is to capture the effects of growth related changes in final demand on income distribution. 1974 on the other hand, was an "unusual" year for Japan, as it was suffering from the oil crisis, in the grip of a severe inflationary wage-price spiral, and in the middle of its first severe post-war recession. (See Patrick & Rosovsky, 1976, p. 14).


\(^3\) Data availability at the time of writing permitted model estimation for 1959, 1964, 1969 and 1974. Economic growth in Japan was steady and uninterrupted between 1959 and 1969. Consequently, a comparison of these 2 years would allow the model to clearly perform its function which is to capture the effects of growth related changes in final demand on income distribution. 1974 on the other hand, was an "unusual" year for Japan, as it was suffering from the oil crisis, in the grip of a severe inflationary wage-price spiral, and in the middle of its first severe post-war recession. (See Patrick & Rosovsky, 1976, p. 14).
section will briefly discuss the data base and the methodology used to obtain matrices $V$, $B$, and $c'$.

III. Data Base and Estimation Methodology

(a) *The Consumption Coefficients*

The data source for the estimation of the consumption coefficients (Engel coefficients) is the National Survey of Family Income and Expenditure (NFIE) published every five years since 1959 by the Bureau of Statistics, Office of the Prime Minister, Japan. This publication contains tabulations of average monthly receipts and disbursements per household by income group cross-classified by occupation of the household head, and so permits the estimation of marginal propensities to consume (for each spending category) within each income group, since multiple observations are available within each group. The major advantage of the NFIE over other publications is its large sample size, which makes the cross-classification possible while at the same time keeping sufficient observations in each category.

The major drawback of the survey is that the survey period extends only from September to November. We therefore compared the estimates of the Japanese income distribution made by Wada (1975) on the basis of several surveys with annual observation periods with the income distribution derived from the NFIE data, and we found that the correspondence is quite close. It would seem that the NFIE data have some bias toward equality (about .02 Gini-points), suggesting that seasonal payments (essentially bonus payments) are distributed a bit more unequally than regular monthly incomes (Grootaert, 1978).

We also compared, for selected income groups, the spending patterns reported in NFIE with those reported in the Annual Report of the Family Income and Expenditure Survey (FIES). The latter distinguishes between the average spending computed over the period January-November and the spending pattern in December, when the bonus is received by most households. In general the NFIE spending percentages differ only slightly from the January-November FIES estimates and fall between the latter and the December pattern of FIES. In the majority of cases they are close to the percentage that obtains from a weighted average of the January-November and the December pattern (details are in Grootaert (1978), Chapter 3). We may conclude that the 3-month survey period of NFIE does not result in any major income group specific biases, and that, therefore, NFIE qualifies as a data source for our model.

The consumption package purchased by an income group differs from that purchased by another group in two basic ways: the pattern of consumption differs across income groups, and the total level of expenditure differs. In order to estimate the consumption coefficients for our model, we need a set of sectoral consumption functions (Engel Functions) that capture both of these aspects. In addition, the functions must fulfill the additivity condition, i.e., the sum of all Engel coefficients needs to sum up to one minus the propensities to save and to pay taxes within each income group.

Of the different functional forms that meet these conditions and that we fit to the data, we selected, on the basis of conventional statistical criteria, the following form:

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*The data permit us to make this comparison only for urban workers' households.*
\[ C_j = a_j + b_j Y + c_j Y^2 + \sum_i d_{ij} D_i Y + \sum_i e_{ij} D_i Y^2 + \sum_k f_{kj} D_k Y + \sum_k g_{kj} D_k Y^2 \]  

(9)

where \( C_j \) = average monthly spending on item \( j \)
\( Y \) = average monthly income
\( D \) = dummy variable (e.g., \( D_2 = 1 \) when \( i = 2 \); \( D_2 = 0 \) otherwise)
\( j \) = subscript for spending categories (\( j = 1, 2, \ldots, 12 \))
\( i \) = subscript for income groups (\( i = 1, 2, \ldots, 5 \))^5
\( k \) = subscript for occupations (\( k = 1, 2, \ldots, 10 \))

Expression (9) is a quadratic Engel function with dummy variables allowing for slope and curvature shifts both across income groups and across occupation groups. The inclusion of occupational dummy variables guarantees that the estimated coefficients of the income variables pick up the effect of income only, i.e., excluding any differences due to occupation. The estimates of equations (9) were the basis for constructing the matrix of income group specific Engel coefficients, the latter being computed at the average income level of each income group.

(b) *The Value Added Coefficients*

The computation of the matrix describing the distribution of households’ share of value added over income groups required the combination of several data sources. Information on the share of sectoral value added taken up by wages and profits was found in the 1960-65-70 Link Input-Output Tables. The profit component was broken down into two parts, corporate profits and income of unincorporated enterprises, on the basis of the National Income Accounts. Income of unincorporated enterprises was assumed to go to households completely. We relied upon the results of the Corporation Enterprise Survey to split corporate profits into distributed and retained earnings.

Wages, income of unincorporated enterprises, and distributed corporate profits make up the share of value added going to households. The distribution of that share over income groups was made on the basis of NFIE and FIES tabulations of household income by income group and by sectoral affiliation of the household head. This information is only available for twelve major sectors and determined the level of aggregation of our study.

(c) *The Input-Output Matrix*

The data base for the Leontief-inverse in the model is taken from the 1960-65-70 Link Input-Output Tables. These tables are fully consistent and comparable with one another over time.

IV. *Empirical Results*

As indicated above, the model was estimated for 1959 and 1969. Solution equation (8) will be analyzed for each of those two years, in three steps. First, we shall focus on demand linkages between income groups based on their expenditures. Second, we shall investigate sectoral differences in the generation of household income, and, finally, we shall find out the impact of each final demand component on income distribution.

^5 NFIE distinguishes 16 income groups in 1959 and 9 groups in 1969. To make overtime comparison possible the results are presented using income quintiles, hence \( i = 1, 2, \ldots, 5 \).
The dominant feature of solution equation (8) is the submatrix \((I - VBc')^{-1}\), since it appears in each part of the solution matrix. This submatrix is of dimension \(i \times i\) with elements \(z_{k,l}\), where \(k = 1,2,\ldots, i\) and \(l = 1,2,\ldots, i\). Each element represents the income received by income group \(k\) as a result of the spending by group \(l\) out of a yen’s worth of its income. For example, in 1959 (see Table 1 (a)), a yen earned by someone in the third quintile (or received as a transfer) will eventually generate \(0.172949\) yen of income for the second quintile. The elements of submatrix \((I - VBc')^{-1}\) are multipliers, representing a three-step propagation process.

The first step, represented by \(c'\), is the consumption spending out of earned or received income. The second step, captured by the Leontief-inverse matrix, “translates” the spending into industrial output. The third step, \(V\), tells how much income is generated for each income group by the production of this output. This newly generated income results in new spending, and the process repeats itself until an equilibrium position is reached (the latter is guaranteed by the inversion procedure). Submatrix \((I - VBc')^{-1}\) thus portrays how each income group is linked to every other income group through the processes of consumption, production, and income generation. We will refer to this matrix as “matrix of inter-income-group linkages” or, briefly, “linkages matrix.”

Table 1(a) presents the linkages matrix for 1959. Looking at the main diagonal, we observe that the within-quintile multipliers increase as one moves to higher quintiles. This means that the spending from a yen earned by the “poor” (or given to them as transfer income) would eventually generate more income for the poorer quintiles than for the richer quintiles.

### Table 1. Linkages Matrix, 1959, 1969

#### (a) 1959

<table>
<thead>
<tr>
<th>Receiving Quintile</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.147104</td>
<td>.150793</td>
<td>.142964</td>
<td>.118925</td>
<td>.083129</td>
<td>1.642913</td>
</tr>
<tr>
<td>II</td>
<td>.182554</td>
<td>1.179211</td>
<td>.172949</td>
<td>.151254</td>
<td>.113318</td>
<td>1.799284</td>
</tr>
<tr>
<td>III</td>
<td>.207779</td>
<td>.196121</td>
<td>1.193753</td>
<td>.177572</td>
<td>.141620</td>
<td>1.916843</td>
</tr>
<tr>
<td>IV</td>
<td>.274649</td>
<td>.257007</td>
<td>.258794</td>
<td>1.243527</td>
<td>.201791</td>
<td>2.235768</td>
</tr>
<tr>
<td>V</td>
<td>.502478</td>
<td>.466563</td>
<td>.456943</td>
<td>.440655</td>
<td>1.366360</td>
<td>3.232999</td>
</tr>
<tr>
<td>Total</td>
<td>2.314563</td>
<td>2.249693</td>
<td>2.225403</td>
<td>2.131933</td>
<td>1.906217</td>
<td>10.827807</td>
</tr>
</tbody>
</table>

#### (b) 1969

<table>
<thead>
<tr>
<th>Receiving Quintile</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.091853</td>
<td>.081553</td>
<td>.074269</td>
<td>.067670</td>
<td>.044192</td>
<td>1.359535</td>
</tr>
<tr>
<td>II</td>
<td>.152028</td>
<td>1.133187</td>
<td>.121339</td>
<td>.111393</td>
<td>.071718</td>
<td>1.589663</td>
</tr>
<tr>
<td>III</td>
<td>.201928</td>
<td>.177373</td>
<td>1.161745</td>
<td>.147728</td>
<td>.095369</td>
<td>1.784142</td>
</tr>
<tr>
<td>IV</td>
<td>.263469</td>
<td>.233351</td>
<td>.213252</td>
<td>1.194310</td>
<td>.126270</td>
<td>2.030650</td>
</tr>
<tr>
<td>V</td>
<td>.423460</td>
<td>.378647</td>
<td>.348614</td>
<td>.315848</td>
<td>1.207425</td>
<td>2.673994</td>
</tr>
<tr>
<td>Total</td>
<td>2.132736</td>
<td>2.004109</td>
<td>1.919218</td>
<td>1.836948</td>
<td>1.549793</td>
<td>9.437984</td>
</tr>
</tbody>
</table>

6 It can be shown that \(B(I - c'VB)^{-1} = B(I + c'(I - VBc')^{-1}VB)\).

7 In addition to the conditions applying to the Leontief-inverse, the convergence of the propagation process is guaranteed if and only if for each income group the Engel coefficients add up to less than one.
come) generates less income for the poor themselves than the spending from a yen earned by (or given to) the “rich” generates for the rich. This result is in spite of the fact that the total income generated by a yen originally earned by the poor exceeds that originating from a yen earned by the rich. This can be seen by looking at the column totals which are total income multipliers.

The same conclusions can be drawn from the 1969 linkages matrix (Table 1(b)). The major difference is that all multipliers are smaller than in 1959. The decrease is not equal though for all income groups. It is quite small, relatively, for the middle income groups (quintiles III and IV) and especially large for the poorest quintile. It seems, therefore, that in terms of deriving income from the spending of other income groups, the middle class has fortified its relative position between 1959 and 1969.

The row totals of the linkages matrix show the total income received by each quintile. In other words, if each quintile earns (and spends out of) one yen, quintile I, for example, would in 1959 receive 1.642913 yen or 15.17% of the total 10.827807 yen of income generated.

A useful way of interpreting the row totals is the following: suppose that the marginal income (re)distribution were perfectly equal (i.e. any additional incomes are distributed 20% to each quintile), then the row totals represent the resulting equilibrium distribution of that marginal income. It is, in other words, the minimum level of inequality feasible, given the three structural elements of the Japanese economy captured by our model, viz. the consumption and saving behavior of households, the input-output technology, and the generation and distribution of households' share of value added.

The inequality of the actually generated income distribution will be above this minimum due to the existing (exogenous) inequality of income distribution and because the composition of exogenous final demand will not usually be so that inequality of distribution is minimized.

The linkages matrix provides the basis for breaking down the actually observed level of inequality into a component resulting from the three above mentioned structural elements—we shall refer to this component as “endogenous inequality”—and a component resulting from exogenous elements. This distinction has important policy implications. A transfer program, for example, which does not change the structure of taxes, will affect the exogenous level of inequality but not the endogenous one. If the latter is on a level fairly close to the former, then the equilibrium effect of a transfer program will be minimal with respect to distribution, and policy action should be focussed on structural changes. If, on the other hand, the level of endogenous inequality is far below the exogenous one (as in Japan), there is more scope for exogenous marginal redistribution.

The 1959 linkages matrix for Japan corresponds with a Gini-coefficient (G) of .1336 and a Kuznets' measure of inequality (K) of .1313, quite a bit below actual inequality of $G = .2907$ and $K = .2747$. In 1969, endogenous inequality was even a bit lower ($G = .1301$,

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8 This is due mainly to lower expenditure-income ratios and changes in the input-output structure. This conclusion is derived from a series of simulation exercises that were performed with the model. These are not reported on in this paper but details can be found in Chapter 5 of Grootaert (1978).

9 It is assumed that there are no continuous redistribution programs which give, at the margin, the lower income quintiles each more than 20% of income. If there was such a program, inequality of income distribution could fall below the level predicted by the linkages matrix.
$K = 0.1231$) while observed inequality rose a bit, viz. to $G = 0.2972$ and $K = 0.2798$.10

The linkages matrix can also be used to explore the steady state income distribution that is consistent with the three structural components captured by our model. Formally, this can be represented by the eigenvalue problem

$$(I - VBc')^{-1} y = \lambda y$$

where $y$ is an eigenvector of the linkages matrix and $\lambda$ its associated eigenvalue.

For $y$ to represent an income distribution vector, specifically, a vector of population quintile income shares, the sum of its elements must equal one, i.e. for $y = [y_i]$, $\sum_{i=1}^{5} y_i = 1$, and each element of $y$ must be non-negative, i.e. $y_i \geq 0$ for each $i$. If we call the vector that meets these conditions $s$, then (10) can be re-written as

$$(I - VBc')^{-1} s = \gamma s$$

where $\gamma$ is the new associated eigenvalue. Note that (11) eliminates the possibility of the trivial solution of a zero eigenvector. This implies that $(I - VBc')^{-1}$ must be non-singular if (11) is to have a solution.

The linkages matrix is obviously square and will usually be non-negative. A negative element would mean that an increase in the income of a socio-economic group would reduce the income of another group. Since the elements of $V$ and $B$ cannot be negative if they are to be economically meaningful, this could only occur if elements of $c'$ are negative, i.e. if some groups have negative marginal propensities to consume for certain items. This is possible (in fact, it was the case for a few of the coefficients estimated from the 1969 data) but it is not likely to result in negative elements in $(I - VBc')^{-1}$ due to interactions with the other components. Whether the linkages matrix is indecomposable will depend on the level of aggregation. At quintile level it is bound to be. It will be decomposable if there is at least one group in society whose spending feeds back in no way, neither via production nor value added generation, into the income of another group. This could occur if a large number of highly disaggregated groups are considered, e.g. broken down regionally in a situation where there is little inter-regional trade. The normal case, however, will not present a decomposable matrix.

If, then, the linkages matrix $(I - VBc')^{-1}$ is square, non-negative and indecomposable, the Frobenius-Perron theorem applies which ensures that the matrix has a dominant root (a unique largest eigenvalue) with which is associated one and only one strictly positive eigenvector. Therefore, for most economic applications and at normal levels of aggregation, equation (11) will have a unique solution vector that is economically meaningful and represents a steady state income distribution. The steady state interpretation can perhaps better be seen from the fact that $s$ can also be obtained non-algebraically, viz. by an iteration process as follows. Let $s_1$ be the initial distribution vector, then

$$d_2 = Ls_1$$

where $L = (I - VBc')^{-1}$. The elements of the resulting vector $d_2$ will not sum to one. If we therefore normalize $d_2$, the resulting distribution vector $s_2$ is the basis for the second
iteration:

\[ d_3 = Ls_2 \text{ where } s_2 = \left( \frac{d_{2,i}}{\sum d_{2,i}} \right) \]

This process can be continued until \( d_n = Ls_{n-1} \text{ where } s_{n-1} = \left( \frac{d_{n-1,i}}{\sum d_{n-1,i}} \right) \).

For \( n \to \infty \), \( s_n = s_{n-1} \), where \( s_n = \left( \frac{d_{n,i}}{\sum d_{n,i}} \right) \).

This result is independent of \( s_1 \), and \( s_n \) (which is equal to \( s \) in equation (11)) represents therefore the distribution that would maintain itself indefinitely if no structural changes would occur.\(^{11}\)

It is worth noting that \( s \) can also be obtained from an eigenvalue equation involving \( VBc' \) rather than \((I-VBc')^{-1}\). This can be seen by pre-multiplying (11) by \((I-VBc')\) which yields

\[
\begin{align*}
    s &= (I-VBc')s \\
    s &= \gamma s - VBc' \gamma s \\
    (1-\gamma)s &= -VBc' \gamma s \\
    \left( \frac{(\gamma - 1)}{\gamma} \right) s &= VBc' s
\end{align*}
\]

In other words, \( VBc' \) has a different dominant root but the same associated eigenvector. If \( VBc' \) meets the conditions specified for the Frobenius-Perron theorem to hold, the vector will be unique and strictly positive.

It should also be clear that \( s \) can be obtained equally from the full structural matrix of equation (7), and, alternatively from its inverse in equation (8). For the structural matrix, the eigenvalue problem is:

\[
\begin{pmatrix}
    I - A & -c' \\
    -V & I
\end{pmatrix}
\begin{pmatrix}
    x \\
    y
\end{pmatrix}
= \begin{pmatrix}
    \lambda_1 \\
    \lambda_2
\end{pmatrix}
\begin{pmatrix}
    x \\
    y
\end{pmatrix}
\]

Here, constraints are to be placed separately on \( y \) and \( x \) so that all their elements are positive and sum to one.

The eigenvector associated with the 1959 linkages matrix and representing the steady state distribution corresponds with a \( G = .2574 \) and \( K = .2249 \) which is a bit below actual inequality (\( G = .2907 \), \( K = .2747 \)) but well above the minimum endogenous inequality (\( G = .1336 \), \( K = .1313 \)). The latter, it will be recalled, we interpreted as the distribution which would result from an exogenously implemented (re)distribution that is perfectly equal at

\(^{11}\) The normalization of the successive vectors \( d \) to \( s \) is not required in practice; one normalization after the \( n \)th iteration will produce the same result.

\(^{12}\) Although the steady state distribution will be arrived at from any initial distribution, the time needed to reach it may be affected by the starting point.
the margin. The level of endogenous inequality cannot sustain itself but would deteriorate to the level of inequality indicated by the eigenvector. The policy implication is that, in the long run, equality can only be brought down below the level implicit in the linkages matrix eigenvector by continuous exogenous redistributive efforts. In 1969, the eigenvector corresponded with $G = .2778$ and $K = .2287$, which, again, is a little below actual but well above minimum inequality.

Consideration of the steady state level of inequality and the gaps between it and actual and endogenous inequality has important policy implications. Lowering steady state levels requires policy action geared towards structural changes such as redistribution of assets, changes in technology, etc., while income redistribution, altering the composition of investment or government spending, etc. may be used to reduce the gap between actual and steady state levels. If funds are redistributed perfectly equally at the margin, then the endogenous distribution will prevail initially, but the long-run effect will be to worsen that distribution again, and the more so the wider the gap between endogenous and steady state inequality.

We would argue that an international comparison of inequality levels could meaningfully be done by comparing linkages matrices and their implied levels of endogenous and steady state inequality. This would yield more insights than merely comparing total observed inequality. Countries can indeed have similar observed levels but widely varying steady state and/or endogenous levels, and vice versa. Obviously this has important implications for the correct understanding of the processes underlying cross-country distributional patterns. It would be interesting to explore, for example, whether the Kuznets curve is also observed for endogenous and/or steady state inequality, and whether the size of the gaps between the different levels of inequality is related to the level of development.

(b) Sectoral Differences in the Generation of Household Income

The next step in our analysis is to see how each sector in the economy affects income generation and distribution. For that purpose, we look at submatrix $(I - V B c')^{-1} V B$ of solution equation (8). It is the product of the linkages matrix with the matrix $V B$, and describes the effects of a one yen increase of final demand in each sector on the incomes of each quintile. This matrix holds the key to explaining how shifts in the sectoral composition of final demand affect income distribution. Since it gives directly the equilibrium implications for household income and its distribution, this matrix would be useful for project analysis in developing countries.\(^{13}\)

The matrix $(I - V B c')^{-1} V B$ for 1959 is presented in Table 2. The activities with the highest multipliers are agriculture, government spending and services. A one yen increase of final demand for agricultural output, for example, will generate 1.746893 yen of household income. Agriculture and services are sectors with a large percentage of unincorporated enterprises, which, in part, explains the high multipliers. In the case of government spending it is the large percentage of value added going to wages that accounts for the high multiplier.

\(^{13}\) It is worth pointing out the difference here between the final-demand-to-income link in this model and the same link in other studies of income distribution (e.g. Cline, 1972; Chinn, 1972; Morley and Williamson, 1974; Figueroa, 1975). The latter translate final demand into sectoral output via an input-output table. The sectoral output vector is then premultiplied with labor coefficients to obtain the effects on employment and then household income. In other words, they consider the matrix $V B$ and do not incorporate the feedback effects described by the linkages matrix.
Activities particularly favorable for the poorest income group are agriculture and construction. Particularly low first quintile multipliers are found in the finance-insurance-real estate sector and in the utilities sector. The richest quintile derives the most income from final demand for services, wholesale and retail trade, and agriculture. The activity generating most income for the middle groups is government spending. In terms of the summary measures G and K, the highest levels of inequality are generated by final demand in finance, insurance and real estate, and in services. The lowest levels of inequality stem from final demand in agriculture and construction.

In 1969 (see Table 3) the same three sectors have the highest multipliers. Wholesale and retail trade and services joined agriculture and construction as activities with high multipliers for the lowest quintile. Inequality rose in almost all sectors, but especially in agriculture, government spending and utilities. Construction and mining are now the sectors resulting in the lowest levels of inequality.

(c) Final Demand and Income Distribution

We are now in a position to investigate the contribution of each final demand component.

\[ \text{Final Demand and Income Distribution} \]

\[ \text{We are now in a position to investigate the contribution of each final demand component} \]
### TABLE 3. INCOME GENERATION PER SECTOR, 1969

<table>
<thead>
<tr>
<th>Quintile</th>
<th>AGR</th>
<th>MIN</th>
<th>MFG</th>
<th>CON</th>
<th>EGW</th>
<th>WRET</th>
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<td>.164345</td>
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<td>.255673</td>
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<tr>
<td>V</td>
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<td>1.043631</td>
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### TABLE 4. INCOME GENERATED BY FINAL DEMAND COMPONENT, 1959

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<th>Inv</th>
<th>X</th>
<th>X_s</th>
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</table>

(a) 10^4 yen

(b) Percentages

(c) Multipliers per Unit of Final Demand

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<th>G</th>
<th>I</th>
<th>Inv</th>
<th>X</th>
<th>X_s</th>
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### Table 5. Income Generated by Final Demand Component, 1969

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<th>I</th>
<th>Inv</th>
<th>X</th>
<th>X₅</th>
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<td>(c) Multipliers per Unit of Final Demand</td>
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*Code:* see Table 4.

### Table 6. Relative Importance of Final Demand Components as Income Source, 1959, 1969

#### 1959

<table>
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<tr>
<th>Quintile</th>
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<th>G</th>
<th>I</th>
<th>Inv</th>
<th>X</th>
<th>X₅</th>
<th>Total</th>
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#### 1969

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<th>I</th>
<th>Inv</th>
<th>X</th>
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</table>

*Code:* see Table 4.
to the process of income generation and distribution. The relevant results are summarized in Tables 4-6. These tables highlight the advantage of using a model which breaks down final demand into its components and which explicitly recognizes the sectoral composition of each component. These two features indeed make possible the computation of the amount of household income, and its distribution, generated by each final demand component separately. This yields more information on the factors that contribute to an overall observed distribution than could be obtained from any formal decomposition of inequality summary measures.

In 1959, the most important final demand component in absolute terms is investment. It is also the component which generates the most equal income distribution. This is predominantly due to the importance of the construction sector in investment, a sector which generates a very low level of inequality. The second most important component is government spending which channels income mostly into the middle quintiles.

It is interesting to note that the level of inequality associated with exports is higher than that which would result had imports been produced domestically. Again, the explanation is given by the sectoral composition: major export sectors are manufacturing and transportation and communication, which both have higher inequality associated with them than agriculture and mining, the major import sectors.

From a policy point of view, it is important to know not only the distributional impact of spending, but also how much income is generated per unit of spending. For that purpose we computed multipliers per unit of final demand. A unit of government spending generates more household income than any other final demand component. Second place is shared by investment and consumption outside households.

Comparing government spending with investment, we notice (from Table 4(b)) that the richest quintile receives about 35% of the total income generated by either. On a per unit basis, however, government spending is more favorable to the richest quintile than is investment. From Table 4(b) we also notice that, percentage-wise, investment is more favorable for the poorest income group. The multipliers of Table 4(c) fortify that statement by pointing out that on a per unit basis, investment spending results in more income for the poorest quintile, and this is in spite of the overall higher multiplier of government spending.

Altogether, we notice the very special role of investment. It generates the most equal income distribution, and at the same time has the second highest household income multiplier per unit of spending. Investment is also the largest component of exogenous final demand. As such, we may conclude that investment spending was not only a major force behind the level (and growth) of household incomes, but, at the same time, a major reason for the fairly low level of inequality in the distribution of household incomes in 1959.

In 1969, investment still generated by far the most household income. Its distribution, however, had become less equal as the share of the poorest quintile was reduced by almost 4 percentage points in favor of the middle quintiles. This phenomenon was typical, in fact, for all final demand components. As a result, the level of inequality associated with each final demand component went up. This can be traced back, to a large extent, to the altered sectoral composition of each component. For example, in 1969, the share of the construction sector in investment fell compared to 1959, while the shares of the manufacturing and wholesale and retail sectors rose. The sectoral composition of government spending
also shifted between 1959 and 1969 towards more spending in the services, utilities and finance-insurance-real estate sectors, which are all characterized by fairly high inequality of the incomes they generate. Note that income from exports was again, as in 1959, distributed less equally than income from imports, had they been produced domestically.

The multipliers per unit of final demand are lower in 1969 than 1959. This is in line with the changes observed in the linkages matrix. Contrary to 1959, consumption outside the household has the highest multiplier. This is due to a large shift to services in that component. Government spending comes in second. Investment is only third, but still has the highest multiplier for the poorest quintile. In addition, looking at Table 6, the relative importance of investment as an income source increased for all quintiles, while government spending and consumption outside the household became less important.

We believe, therefore, that the increased importance of investment as a source of income, combined with a less equal distribution of that income, is a major explanatory factor in the observed increase in income inequality in Japan between 1959 and 1969. However, had government spending kept its 1959 relative importance, the total income distribution would have been more unequal, since the increase in inequality was greater in the incomes derived from government spending than in those derived from investment.

V. Conclusions

The model discussed in this paper belongs to the literature that investigates the relationship between the size distribution of income and various facets of the growth performance of an economy. The facet given emphasis here is the demand side, specifically the impact of the composition of final demand on the level and distribution of household income and the linkages, based on household expenditures, that exist between different socio-economic groups in society. Such linkages are seen as a key aspect in the joint explanation of growth and distribution.

The model presented is a conventional input-output model augmented with a distribution matrix and closed for the household sector which is disaggregated according to one or more socio-economic criteria. The closure of the model effectuates the inclusion of indirect (or feedback) effects from household spending onto production, income generation, and back onto spending. The model yields the equilibrium level and distribution of household income in a manner fully consistent with the production structure of the economy.

Exogenous final demand is broken down into its components and full account is taken of the sectoral composition of each component. This makes possible the derivation of the amount of household income and its distribution generated by each final demand component separately. This form of “distribution-accounting” yields more information about the factors contributing to an observed overall distribution than conventional decomposition of inequality summary measures.

As a result of the explicit recognition of socio-economic groups, the solution of the model contains a sub-matrix which conveniently summarizes the demand linkages between socio-economic groups (it was therefore labelled “linkages matrix”) and which yields useful interpretations regarding different politically relevant levels of inequality.

Specifically, it was shown that one can derive from the linkages matrix a lower bound
level of inequality which corresponds to the minimum inequality feasible in equilibrium, consistent with certain structural features of the economy. This level, which was labelled "endogenous inequality" can, in the long run, only be obtained by continuous exogenous redistributive efforts. The comparison of endogenous with actual inequality provides useful indications about the appropriate policy tools for affecting the existing distribution.

It was also shown that the linkages matrix implies a steady state level of inequality which would maintain itself indefinitely if no structural changes occur. We argued that an international comparison of actual, endogenous and steady state levels of inequality would yield valuable information about the causes and underlying mechanisms of distributional patterns and be much more relevant than the customary comparisons of actual levels only.

The empirical analysis in the paper used data from Japan pertaining to the years 1959-1969, a period selected because of the steady and uninterrupted growth experienced by Japan.

The analysis showed that there were interesting differences in the amounts of income generated as a result of spending by the poor vs. the rich. The web of linkages between income groups changed over the period considered to favor the middle income classes.

It was found that during the period under consideration the actual level of inequality was not much above its steady state level. The endogenous level, however, was substantially below the steady state level.

It was further demonstrated that final demand in different sectors was associated with widely different levels of inequality of the incomes generated. Construction, for example, was found to be particularly favorable for the lower income groups, while the government sector generated income flows mostly to middle income groups.

Of all exogenous final demand components, investment generated the most equal income distribution, because of the importance of the construction sector for investment. Since investment was the largest component of exogenous final demand, it played a major role in explaining the low level of inequality in the distribution of household incomes in 1959. The increased importance of investment as a source of income in 1969, combined with a less equal distribution of that income, explained in part the observed increase in income inequality between 1959 and 1969. At the same time, it helped to check that increase, which indeed would have been larger had government spending (the second largest component of exogenous final demand) kept its 1959 relative importance, since the increase in inequality was greater in the incomes derived from government spending than in those derived from investment.

Finally, a word needs to be said about the limitations of the model used. These limitations are by implication suggestions for future research. First, the model is not able to trace the effects of changes in relative prices. This could be remedied by the incorporation of, for example, a linear expenditure system which would allow for price effects on expenditure patterns. Second, the value added shares are fixed and given, which assumes an exogenously determined and fixed distribution of ownership of resources. Adding a labor market model would, in part, solve this problem. It seems unlikely to be feasible to construct a model of capital goods distribution in the present state of data availability. Third, capacity constraints could be acknowledged by adding an investment matrix to the model. Fourth, the study of the linkages between socio-economic groups could usefully be done, data permitting, along other dimensions than income, e.g. occupation. Finally, in the
spirit of semi-input-output models, the endogenous variable in the model (the level and distribution of household income) could be made into a pre-specified target and the model then run in reverse to determine the required levels of certain policy instruments such as exports, income transfers, etc. that are compatible with the target.

We believe that the methodology and the empirical application and findings of this paper present some new insights into the process of income distribution which could not have been derived from distributional studies using alternative approaches, and which might warrant the undertaking of similar studies for other countries.

REFERENCES


DATA SOURCES


------------------, Statistical Yearbook, various issues.