ON THE INTERTEMPORAL NON-NEUTRALITY OF TAXES ON LAND: A DYNAMIC MARKET CLEARING MODEL

By YUKIO NOGUCHI*

I. *Introduction*

Foresters have long recognized that the classical proposition that a tax on land value is neutral does not hold in an intertemporal setting.[1] Their view on the intertemporal effect of taxes on land can be summarized by the following propositions.

(i) The imposition or an increase in the rate of a perfectly administered *ad valorem* property tax has the same effect as an increase in the market rate of interest: It discourages land uses with long gestation periods.

(ii) An income tax on yield is neutral with respect to the intertemporal allocation of land.

This view can be found already in the Fairchild Report on Forest Taxation (1935), which used the above propositions to justify a preferential property tax treatment for forests. Although there has been controversy as to whether such a treatment can be justified (e.g. Trestrail [1969], Klemperer [1974]), the validities of the propositions themselves have been reconfirmed each time and have been used as the basis of the arguments.

Recently, Skouras (1978) and Bentick (1979) examined the intertemporal effects of taxes on land in the context of urban development. Using similar models, they concluded that the above propositions are valid in this context too.[2] This paper examines the same problem (the intertemporal allocation of land for urban uses) and argues that although the non-neutrality of a property tax holds under fairly general conditions the validities of the above propositions hinge upon a strong assumption concerning demand elasticities. It also demonstrates that if the assumption is removed the conclusion becomes different. In particular it is shown that:

(i) It is a tax on the value of vacant land that has the same effect as an increase in the market rate of interest.

(ii) An income tax on rentals has the same effect as a flatter user demand curve and is in general non-neutral with respect to the intertemporal allocation of land.

* Associate Professor (*Jokyōju*) of Public Finance.

[1] The problem discussed here is the *intertemporal* allocation of land, i.e., the choice between putting land to immediate uses (or uses with short gestation periods) and waiting for future uses (uses with long gestation periods). It is usually assumed that both the market rate of interest and rentals are exogenously given and are unaffected by taxes. In this respect the problem must be distinguished from the "capitalization issue" such as the one discussed by Neutz (1970), Mieszkowski (1972), Hamilton (1976) and Blake (1979). The necessity to make such a distinction has been correctly pointed out by Bentick (1979, p. 860).

[2] Skouras (1978) does not prove the second proposition directly. Instead he shows that a tax on vacant land has the same effect as a property tax (except for its effect on the price of land). As is argued later this is equivalent to the second proposition.

(iii) The effect of a property tax is a combination of the above two effects and is different from an increase in the market rate of interest.

(iv) If elasticities take extreme values the above taxes become neutral.

Before presenting our model it is necessary to review the present theory in order to identify the point which we wish to modify in this paper. The theory is essentially a comparison of two kinds of demand prices (bid prices); the user's price and the speculator's price.[3] The former is defined as the present value of rentals obtainable from the present use. Thus if R_0 is the rental and r is the discount rate, the user's price is $P_0^u = R_0/r$. The latter is the discounted value of the future user's price. Thus if development is expected to take place in period T which raises the rental to R_T, the speculator's price in period 0 is $P_0^s = R_T/r(1+r)^T$. It is argued that if P_0^s exceeds P_0^u then land will be held by speculators and will be kept idle until T. It is further argued that if a property tax of rate b is imposed, the user's price and the speculator's price fall to $R_0/(r+b)$ and $R_T/(r+b)\ (1+r+b)^T$, respectively, so that there could be cases in which the result of the above comparison is altered in favor of the present use. An income tax on rentals is regarded as neutral because it reduces the user's price and the speculator's price by the same proportion.

The most remarkable feature of the above theory is the fact that the price movement is analyzed without considering market clearing conditions. The trick is in the (implicit) assumption that the user demand in period T is perfectly elastic.[4] Namely, it is assumed that an infinite amount of land can be allocated to the "highest and best use" so that the demand curve in period T is horizontal at the price R_T/r.

Once the assumption is removed, the problem becomes fairly complicated. If the demand curve is allowed to be downward-sloping as in the usual microeconomic theory it is necessary to distinguish the demand prices represented by the present values of rentals from the market price. But the market price at T depends upon the amount of speculative holding accumulated before T because its decumulation exerts a downward pressure on the price. The problem then becomes difficult because the speculative demand before T depends upon the expectation of *the market price* at T. Thus in general the present and the future are interrelated; *the present cannot be determined unilaterally from the fixed future.*

In spite of the difficulties in the analysis, generalization of the model is definitely worth undertaking, because, as is well known, elasticity of demand is the key factor in the analysis of the neutrality of taxes. Fortunately, the recent development in economic theory provides us with a method to cope with the difficulty mentioned above. By adopting the rational expectation hypothesis, we can solve a problem in which expected price appears as an endogenous variable. Thus the model presented in Section II is essentially an application of the capital asset pricing models under ration expectation, especially the one developed by Black (1972, 1976).

The property of the solution in the absence of taxes is analyzed in Section III. The main conclusions of the analysis are: (1) The adjustment of the price to an anticipated

[3] For the detail of the model, see Skouras (1978) and Bentick (1979). Klemperer (1974) presents a similar model in the context of rural land use. Although each author presents his model in a slightly different way, the essence of the theory is not destroyed by the summary given here.

[4] Furthermore, it is assumed that vacant land and other assets are perfect substitutes so that infinite amount of vacant land will be demanded if the market price is lower than P_0^s, i.e., that the speculative demand curve too is horizontal. As is shown later the speculative demand is a decreasing function of the current market price if future land price is not known with complete certainty and if speculators are risk averse.

change in the user demand takes place both before and after the time of the change. (2) Higher interest rate has the effect of *bringing the adjustment into more distant futures.* (3) Flatter user demand curve or greater risk aversion of speculators has the effect of *concentrating the adjustment around the time of the change.* Although conclusion (1) is the usual one in the dynamic asset pricing models, conclusion (2) and (3) have not been recognized in the literature.[5] They become the basis for the analysis of the effects of taxes which is done in Section IV.

II. *A Market Clearing Model of Land Price Movement under Rational Expectation*

The basic factor characterizing our theory as well as the theory reviewed in the introduction is the assumption that the cost of converting land uses is sufficiently high. This assumption is necessary because, as has been correctly pointed out by Skouras (1978), if conversion cost is low, land would always be in the present uses and there would be no holding of vacant land. For analytical convenience we assume that once land is put to a use by means of building a facility on it, it cannot be shifted to a different one as long as the facility remains. We further assume that the expected lives of facilities are sufficiently long. These assumptions imply that if it is decided to put land to a use which is available in the future, it is necessary to keep land vacant until then.

The stock of vacant land is reduced if it is fixed to a specific use by means of building a facility on it. We call this "user demand" and denote by D_t the amount of user demand (in terms of flow) in period t. On the other hand, the stock of vacant land is increased if facilities built on land become obsolete. We call this "supply of vacant land" and denote by S_t the amount of supply (in terms of flow) in period t. Because the change in the stock must be equal to the net flow, we have the following market clearing condition.[6]

$$H_t - H_{t-1} = S_t - D_t \qquad (1)$$

where H_t is the stock of vacant land in period t. Some situations are formalized (rather artificially) by supposing that the same person appears on both sides of the market at the same time. For example, if vacant land is put to a use by the owner, we suppose that the owner as a holder of the vacant land sells it to himself as a user.

Because of the assumptions stated earlier, the possibility of converting land use is not taken into account in the user demand decision. Thus given the streams of rentals and the discount rate, D_t depends only upon p_t, the price of vacant land in period t.[7] Because less profitable uses are justified under a lower price, D_t is a decreasing function of p_t. The supply of vacant land in period t is determined by the pattern of land uses determined in the past and by the physical condition of obsolescence. Here we assume that it is described by a random variable independent of p_t. Then, "net user demand" $D_t - S_t$ is represented by

[5] Although Black (1976) points out that the rate of price increase is dependent on the elasticity of supply, he does not describe the effect in this way.

[6] This is the usual market clearing condition in the dynamic asset pricing models. See, for example, Muth (1961), Black (1976).

[7] Strictly speaking, there are other factors affecting the user demand. For example, the existing pattern of land uses affects the user demand because it affects the rental. We neglect these factors by assuming that the rentals are exogenously given as in the theory reviewed in the introduction.

a decreasing function of p_t. For analytical convenience we assume a linear function:

$$D_t - S_t = \delta_t - \eta p_t \qquad (2)$$

where η is a positive parameter assumed to be a constant over time, and δ_t is a time dependent random variable (expressing the random part of the user demand as well as the supply of vacant land).

The demand to hold vacant land (called "speculative demand") is derived from asset holders' portfolio selection decisions. Vacant land is regarded as one of the assets and each asset holder is supposed to make a portfolio decision at the beginning of each period with the aim of maximizing the utility of his wealth at the end of the period. Thus unlike the decision determining the user demand, this is a repetitive decision. Assuming (i) the existence of a safe asset whose rate of return is fixed at i, (ii) the possibility of unlimited borrowings at rate i, (iii) the absence of taxes on land, and (iv) the existence of "intermediate uses" of vacant land (uses with no facilities built on land such as an open parking lot or a lumberyard), the speculative demand can be described by an increasing function of

$$\pi_t = E_t p_{t+1} - p_t - i p_t + q \qquad (3)$$

where $E_t p_{t+1}$ is the expectation of the price of land in period $(t+1)$ formed in period t, and q is the return per unit of land from the intermediate use (assumed to be a constant over time). It can be shown that if it is further assumed that speculators' absolute risk aversion is a constant and that the future price is normally distributed, the speculative demand is represented by a linear function:[8]

$$H_t = h\pi_t \qquad (4)$$

where h is a parameter which is inversely proportional to the (average) degree of risk aversion of speculators.

Substituting (2), (3) and (4) into (1) and rearranging, we obtain

$$E_t p_{t+1} - \alpha p_t - E_{t-1} p_t + \beta p_{t-1} + \delta_t / h = 0 \qquad (5)$$

where

$$\alpha = 1 + i + \eta / h$$
$$\beta = 1 + i$$

If the expectation formation mechanism and the movement of δ_t are specified, the movement of p_t is completely determined by equation (5).

Here we assume that expectation is rational in the sense of Muth (1962), i.e., $E_t p_{t+1}$ is the mathematical expectation of p_{t+1} conditional upon available information at t. As has been done by Black (1972, 1976) we decompose δ_t into two parts:

$$\delta_t = u_t + v_t$$

The first term represents the unpredictable part, i.e., at the time of speculators' decisions at t only u_{t-j}s $(j=0, 1, 2, \ldots)$ are known. We assume that u_ts are serially uncorrelated random variables with zero mean and a constant variance. The second term represents the predictable part, i.e., at t all v_{t-j}s $(-\infty < j < \infty)$ are known. Thus the information set at t is $I_t = \{u_t, u_{t-1}, \ldots : \ldots v_{t+1}, v_t, v_{t-1}, \ldots\}$.

Under the above setting the solution of (5) is found by the same procedure as Muth's. Namely, we express p_t as a linear combination of u_{t-j}s $(j=0, 1, 2, \ldots)$ and v_{t-j}s $(-\infty < j < \infty)$, substitute into (5) noting that $E_t p_{t+1} = E(p_{t+1} / I_t)$, then determine the coefficients with the restriction that they be finite in the limits $j = -\infty$ and ∞.[9] Then we obtain the

[8] For example, see Black (1976, p. 768).

[9] Sargent and Wallace (1975) justify this restriction as ruling out speculative bubbles. Black (1976) calls this "inevitable collapse of tulipomania."

following solution.

$$p_t=\frac{A}{h}\sum_{j=0}^{\infty}\lambda_1{}^j u_{t-j}+\frac{B}{h}\sum_{j=0}^{\infty}\lambda_1{}^j v_{t-j}+\frac{B}{h}\sum_{j=1}^{\infty}\left(\frac{1}{\lambda_2}\right)^j v_{t+j} \tag{6}$$

where

$$A=\frac{1}{\alpha-\lambda_1}=\frac{2}{\alpha-1+\sqrt{C}}=\frac{1-\alpha+\sqrt{C}}{2(\alpha-\beta)}>0$$

$$B=\frac{\lambda_2}{(1+\alpha)\lambda_2-2\beta}=\frac{1}{\sqrt{C}}>0$$

$$C=(\alpha+1)^2-4\beta=(\alpha-1)^2+4(\alpha-\beta)>0$$

and where λ_1 and λ_2 $(0<\lambda_1<1<\beta<\alpha<\lambda_2)$ are the roots of the characteristic equation

$$\lambda^2-(\alpha+1)\lambda+\beta=0$$

The above solution, which may be regarded as a special case of Black's solution (19), states that the price is described by the weighted averages of the past and future parameters of the net user demand, the weights being decreased geometrically for periods distant from t. Substituting (6) into (3), we find

$$H_t=-(\beta-\lambda_1)\sum_{j=0}^{\infty}\lambda_1{}^j(Au_{t-j}+Bv_{t-j})+(\lambda_2-\beta)B\sum_{j=1}^{\infty}\left(\frac{1}{\lambda_2}\right)^j v_{t+j}+qh \tag{7}$$

The stationary solution is found by letting

$$v_t=\delta_0, \qquad u_t=0$$

for all t. The result is

$$p_0=\delta_0/\eta \tag{6a}$$

$$H_0=h(q-i\delta_0/\eta) \tag{7a}$$

In what follows all variables represent the deviations from the stationary solution.[10]

III. *Response to an Anticipated Increase in the User Demand*

In this section we examine the properties of the solution for the case in which an anticipated change occurs in the user demand. The change is due to such factors as development of land or a sudden increase in the population. Let us suppose that the user's price is anticipated to rise by a unit at T, namely

$$v_t=0, \qquad t<T$$

$$v_t=\eta, \qquad T\leq t$$

Substituting the above into (6) and (7) we obtain the following results.

$$p_t=B\frac{\eta}{h}\frac{\lambda_2{}^{t+1-T}}{\lambda_2-1}+\frac{\eta}{h}U_t, \qquad t<T \tag{6b}$$

$$p_t=1-B\frac{\eta}{h}\frac{\lambda_1{}^{t+1-T}}{1-\lambda_1}+\frac{\eta}{h}U_t, \qquad t\geq T \tag{6c}$$

$$H_t=B\eta\frac{\lambda_2-\beta}{\lambda_2-1}\lambda_2{}^{t+1-T}-(\beta-\lambda_1)\eta U_t, \qquad t<T \tag{7b}$$

$$H_t=-ih+B\eta\frac{\beta-\lambda_1}{1-\lambda_1}\lambda_1{}^{t+1-T}-(\beta-\lambda_1)\eta U_t, \qquad t\geq T \tag{7c}$$

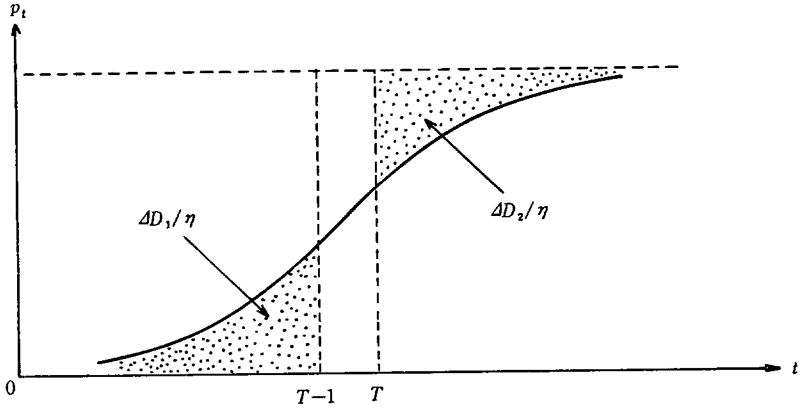
where

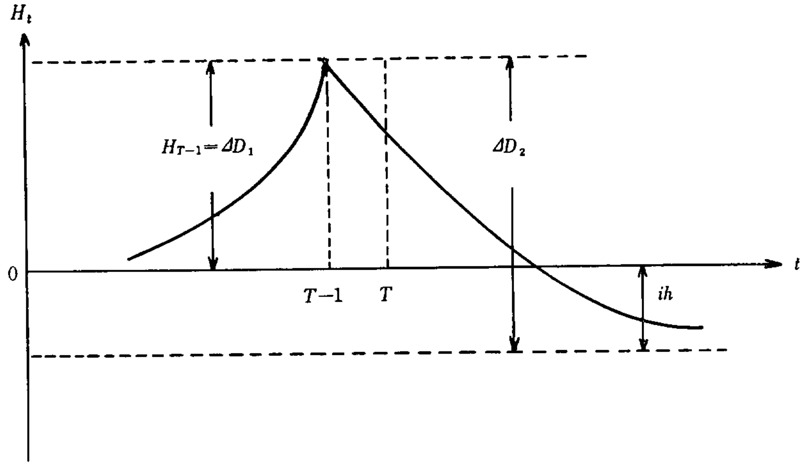
$$U_t=A\sum_{j=0}^{\infty}\lambda_1{}^j u_{t-j}=Au_t+\lambda_1 U_{t-1}$$

The last terms of the equations represent the effects of the unpredictable disturbance.

[10] We assume that q is greater than $i\delta_0/\eta$ so that the stationary holding of vacant land is positive.

FIG. 1. THE MOVEMENTS OF THE PRICE AND THE SPECULATIVE HOLDING OF VACANT LAND





Since they are not important in the following discussions, they are omitted for simplicity.[11] The movements of the price and the speculative holding described by other terms of the

[11] These terms are expected to decrease in absolute values in the next period since $E(U_{t+1}/I_t)=\lambda_1 U_t$. Reflecting this property, the speculative holding becomes smaller if current price is higher due to the unpredictable disturbance (i.e., the last term of H_t is negative if U_t is positive), and *vice versa*. If we consider expected price at t conditional upon available information at $t=0$, the last terms of the price equations become

$$\frac{\eta}{h}E(U_t/I_0)=\frac{\eta}{h}\lambda_1{}^t U_0$$

which may be neglected if t is sufficiently large.

above equations are depicted in Fig. 1.[12] The speculative demand begins to increase as soon as the future change in the user demand is anticipated. Because this exerts an upward pressure on the price, the price rises and the user demand is reduced.[13] The total amount of the reduction in the user demand before T is

$$\varDelta D_1=\eta\sum_{t=-\infty}^{T-1}B\frac{\eta}{h}\frac{\lambda_2^{t+1-T}}{\lambda_2-1}=B\frac{\eta^2}{h}\frac{\lambda_2}{(\lambda_2-1)^2} \tag{8}$$

It can be easily verified that this is equal to the stock of the speculative holding at $(T-1)$ derived from (7b):

$$H_{T-1}=B\eta\frac{\lambda_2-\beta}{\lambda_2-1} \tag{9}$$

Unlike what has been supposed in the theory reviewed in the introduction, the price does not rise to its new stationary level immediately after T. This is a natural result since the speculative holding is decumulated after T and this eases the demand and supply condition. Namely, although the price ultimately rises to the level high enough to offset the exogenous increase in the user demand, the process is smoothened by (gradual) decumulations of the speculative holding. Thus the adjustment to the exogenous change takes place not only before T but also after T.

Because of the gradual change in the price, user demand after T is not reduced immediately to the long-run level. The total amount of the difference is

$$\varDelta D_2=\eta\sum_{t=T}^{\infty}\frac{B\eta}{h}\frac{\lambda_1}{1-\lambda_1}\lambda_1^{t-T}=\frac{B\eta^2}{h}\frac{\lambda_1}{(1-\lambda_1)^2} \tag{10}$$

It can be easily verified that this is equal to the reduction in the speculative holding after $T-1$:

$$H_{T-1}-H_{\infty}=B\eta\frac{\beta-\lambda_1}{1-\lambda_1} \tag{11}$$

where $H_{\infty}=-ih$ represents the (new) stationary level of the speculative holding (cf. Fig. 1).

It can also be verified that

$$\varDelta D_1+ih=\varDelta D_2 \tag{12}$$

The above identity states that the total amount of the user demand which is permitted to exceed the long run level after T $(\varDelta D_2)$ is greater than the total reduction in the user demand before T $(\varDelta D_1)$, and that the difference is equal to the reduction in the long run level of the speculative holding (ih). Although obvious, this relationship has not been recognized in the literature and plays an important role in the following discussion.

Let us now examine the effect of the change in the parameter i, the rate of return on the safe asset. It is shown in the Appendix that before T the stock of the speculative holding is smaller, the rate of price increase is higher and the price level is lower for a higher i. This is to be expected since a higher i makes the speculative holding less profitable for an unchanged rate of price increase.

[12] In drawing the Figure we note the following fact.

i. Equations (6b) and (6c) yield the same value for $t=T-1$. They yield the same value also for $t=T$.

ii. $H_{T-1}>H_T$ because using the relationship

$$B\eta\left(\frac{\lambda_2-\beta}{\lambda_2-1}-\frac{\beta-\lambda_1}{1-\lambda_1}\right)=-ih$$

We have
$$H_T=H_{T-1}-B\eta(\beta-\lambda_1)$$

[13] Note that the rate of increase of price (λ_2) is in general greater than i. Namely, the price before T cannot be calucalted as a discounted value of the future price.

What is not obvious is the fact that after T the rate of price increase is lower and the price level is lower for a higher i. Namely, the approach of the price to the new stationary level is *carried forward into more distant futures*. This might seem curious because for a higher i less vacant land is accumulated before T and the decumulation after T would be more rapid. The answer is found in the identity (12). Although the speculative holding accumulated before T (ΔD_1) is smaller for a higher i, the reduction in the holding after T (ΔD_2) is greater due to an even greater reduction in the long-run level (ih).

Another parameter affecting the above process is η/h. It is shown in the Appendix that before T the rate of price increase is higher for a greater η/h. This is the same result as the one for a higher i and is to be expected since both a smaller h and a higher i imply less speculative demand for an unchange rate of price increase.[14] What is not obvious is the fact that after T the rate of price increase is higher for a greater η/h. Therefore a greater η/h has the effect of *concentrating the adjustment process around T*. It is important to recognize that this is completely different from the result for a higher i. The reason for the difference is once again found in (12). Unlike in the case of i, a greater η/h reduces or keeps unchanged the second term of the left hand side of (12). As a result, the amount that ΔD_2 can exceed ΔD_1 is reduced or kept unchanged. In the extreme case in which η/h tends to infinity (this occurs either when the user demand is perfectly elastic or when the speculators are infinitely risk averse), we have $p_t=0$ for $t<T$ and $p_t=1$ for $t\leq T$, i.e., the change occurs discontinuously at T.[15]

IV. *Effects of Taxes on Land*

In this section we discuss the effects of taxes using the result of the previous section. First we consider a tax on the value of *vacant* land. Referring to (3) it is apparent that the imposition of the tax (or a rise in the tax rate) is equivalent to a rise in the parameter i. Thus the analysis in the previous section tells us that the tax has the effect of *spreading the adjustment process into more distant futures*. The price and the stock of the speculative holding at any period is made lower as a result of the tax.[16] In this sense, the tax has a distorting effect on the intertemporal allocation of land. It must be remarked, however, that the distorting effect depends upon the elasticities of demands for land. It can be shown that the tax becomes neutral (except for its effect on the stationary solution) when the parameter η/h tends to infinity.[17] This is a result that can be readily conjectured from what has been

[14] As far as the deviation from the stationary solution is concerned, a greater η is equivalent to a smaller h.

[15] This can be confirmed as follows. Rewrite (5) in the form

$$(E_t p_{t+1}-\beta p_t)-(E_{t-1}p_t-\beta p_{t-1})=\frac{\eta}{h}\left[\frac{\delta_t}{\eta}-p_t\right]$$

If η/h tends to infinity, the market price is determined only by the user's demand and we have $p_t=\delta_t/\eta$. If η/h tends to zero, the market price is determined by setting the left-hand side equal to zero, i.e., it is determined only by the speculator's demand. In this case we have either $E_t p_{t+1}/p_t=1+i$ for all t (expected rate of price increase is i) or $p_t=0$ for all t. Thus in terms of our model, the theory reviewed in the introduction assumes (for the case when land is held vacant until T) that η/h is zero before T and tends to infinity after T.

[16] The rate of price increase (λ_2) is raised by the tax. From (A2) in the Appendix, the derivative of λ_2 with respect to the tax rate is $(\lambda_2-1)/(2\lambda_2-\alpha-1)$ which is less than unity for positive η/h.

[17] In the theory reviewed in the introduction a tax on vacant land is non-neutral in spite of the fact that the user demand is perfectly elastic ($\eta=\infty$). This does not contradict our result because as was remarked in footnote 15 the assumption concerning the parameter η/h is different from ours.

mentioned at the end of the previous section. The formal proof that the derivative of the price with respect to the tax rate tends to zero in this case is given in the Appendix.

Next, consider an income tax on rentals. Assuming a constant tax rate, user's price is reduced at every level by the same proportion.[18] In terms of equation (2) this is represented by an increase in the parameter η. Thus the analysis in the previous section tells that the tax has the effect of *concentrating the adjustment process around T.* In this sense, the tax is non-neutral.

In terms of the market clearing condition (1), the process before T is described as follows. An income tax increases the right-hand side while keeping the left-hand side unchanged for an unchanged rate of price increase. In order to restore equilibrium, the speculative holding must increase more rapidly, which is made possibly by an increased rate of price change.

The above analysis conflicts with Bentick (1979), who has emphasized that an income tax is neutral *because tax payments are synchronized with rental receipts.* Although it is true that in his model an income tax is neutral, the reason must be found in the *assumed elasticities of demands.* It is clear that the above mechanism does not work in his model because both the user demand and the speculative demand are perfectly elastic. Even in our model the tax becomes neutral in the extreme case when η/h tends to infinity (for a proof, see Appendix).

Next, consider a property tax, namely a tax which is imposed on the value of all land. As far as the vacant land is concerned the tax is equivalent to a tax on vacant land. Under the assumption in Section II the supply of vacant land is unaffected by the tax, while the user demand for vacant land is in general reduced by the tax. Because of the assumed impossibility of changing the land use, the value of land which is fixed to a user demand is independent of the price of vacant land and is given by the present value of rentals. Thus the effect of the tax can be described by reductions in the user's prices as in the case of an income tax.[19] Therefore a property tax can be regarded as a combination of a tax on vacant land and an income tax (with an appropriately adjusted rate). In general, the tax is non-neutral because both taxes are non-neutral *in different ways.* However, the tax is neutral in the extreme case when η/h tends to infinity. Note that the effect of a property tax is different from that of a tax on vacant land in the sense mentioned above. This conclusion is different from that of Skouras (1978), who argued that the two taxes have the same effect on the intertemporal allocation of land.

V. *Concluding Remarks*

The present theory to analyze the intertemporal effects of land taxes is too restrictive in its assumption on demand elasticities. We have examined in this paper how the conclusion is revised if the assumption is removed. Our main conclusion can be summarized as follows. A tax on vacant land has the same effect as an increase in the interest rate: it

[18] The tax also reduces q in equation (3). However, this has no effect on the result except for the stationary level of the speculative holding.

[19] Let b be the tax rate and r be the discount rate. Then as is shown in the theory reviewed in the introduction the user price is reduced by the factor $b/(r+b)$.

spreads the adjustment to an exogenous change into more distant futures. An income tax on rentals has the same effect as a flatter user demand curve: it concentrates the adjustment around the time of the change. A property tax can be regarded as a combination of the above taxes. Thus its effect is different from that of an increase in the interest rate. These taxes are in general non-neutral but become neutral when demand elasticities take extreme values. This last conclusion has an important policy implication because it suggests the possibility that taxes on land can still be regarded as ideal (or nearly ideal) taxes as classical writers believed.

APPENDIX

1. Note that

$$\frac{\partial \lambda_j}{\partial \alpha}=\frac{\lambda_j}{2\lambda_j-(\alpha+1)},\quad \frac{\partial \lambda_j}{\partial \beta}=-\frac{1}{2\lambda_j-(\alpha+1)}\quad (j=1,\ 2) \tag{A1}$$

Thus

$$\frac{\partial \lambda_j}{\partial i}=\frac{\partial \lambda_j}{\partial \alpha}\frac{\partial \alpha}{\partial i}+\frac{\partial \lambda_j}{\partial \beta}\frac{\partial \beta}{\partial i}=\frac{\lambda_j-1}{2\lambda_j-(\alpha+1)}>0 \tag{A2}$$

We also have

$$\frac{\partial C}{\partial \alpha}=2(\alpha+1),\quad \frac{\partial C}{\partial \beta}=-4 \tag{A3}$$

Thus

$$\frac{\partial C}{\partial i}=\frac{\partial C}{\partial \alpha}\frac{\partial \alpha}{\partial i}=2(\alpha-1)>0 \tag{A4}$$

This implies that

$$\frac{\partial B}{\partial i}=-\frac{\alpha-1}{C^{3/2}}<0 \tag{A5}$$

(A2) and (A5) imply that $p_{T-1}=\frac{\eta}{h}\frac{B}{\lambda_2-1}$ is lower for a higher i. Because the rate of increase of price before T (λ_2) is higher for a higher i, the price before T is always lower for a higher i. This in tern implies that the reduction in the user demand caused by the price increase is smaller for a higher i. Consequently, H_t ($t<T$) is smaller for a higher i. Furthermore, the price after T is always lower for a higher i because the rate of price increase is lower.

2. Using (A1) we have

$$\frac{\partial \lambda_j}{\partial(\eta/h)}=\frac{\partial \lambda_j}{\partial \alpha}\frac{\partial \alpha}{\partial(\eta/h)}=\frac{\lambda_j}{2\lambda_j-(\alpha+1)} \tag{A6}$$

This is negative for λ_1 and positive for λ_2. Using (A3) we have

$$\frac{\partial C}{\partial(\eta/h)}=\frac{\partial C}{\partial \alpha}\frac{\partial \alpha}{\partial(\eta/h)}=2(\alpha+1)>0 \tag{A7}$$

Thus

$$\frac{\partial B}{\partial(\eta/h)}=-\frac{\alpha+1}{C^{3/2}}<0 \tag{A8}$$

3. If we let $\eta/h\to\infty$, we have

$$\lambda_1\to 0,\quad \lambda_2\to\infty,\quad C\to\infty,\quad B\to 0$$

Note that

$$\lim_{\eta/h\to\infty}\frac{\eta}{h}B=\frac{1}{\lim\limits_{\eta/h\to\infty}\frac{\partial}{\partial(\eta/h)}\sqrt{C}}=\frac{1}{\lim\limits_{\eta/h\to\infty}\frac{\alpha+1}{\sqrt{C}}}=1 \tag{A9}$$

Therefore $p_t\to 0$ for $t<T$, and $p_t\to 1$ for $t\geq T$.

If we let $\eta/h\to 0$, we have

$$\lambda_1\to 1,\quad \lambda_2\to 1+i,\quad C\to i^2,\quad B\to 1/i$$

Thus $p_t\to 0$ for $t<T$. We also have

$$\lim_{\eta/h\to 0}\frac{\eta/h}{1-\lambda_1}=\frac{1}{-\lim\limits_{\eta/h\to 0}\frac{\partial\lambda_1}{\partial(\eta/h)}}=i \tag{A10}$$

Thus $p_t\to 0$ for $t\geq T$.

4. Because a rise in the rate of tax on vacant land is equivalent to a rise in i, the derivative of the price with respect to the tax rate is represented by dp_t/di. From (5d), (5e) and using (A2) and (A3), we have

$$\frac{\partial p_t}{\partial i}\Big/p_t=-\frac{\alpha-1}{C}+\frac{t+1-T}{\sqrt{C}}\frac{\lambda_2-1}{\lambda_2}-\frac{1}{\sqrt{C}},\quad t<T \tag{A11}$$

$$\frac{\partial p_t}{\partial i}\Big/(p_t-1)=-\frac{\alpha-1}{C}-\frac{t+1-T}{\sqrt{C}}\frac{\lambda_1-1}{\lambda_1}+\frac{1}{\sqrt{C}},\quad t\geq T \tag{A12}$$

If we let $\eta/h\to\infty$ then the first and the third terms tend to zero. For λ_2 the second term also tends to zero so that we have $\partial p_t/\partial i\to 0$. For λ_1, the second term converges to a finite number because

$$\frac{\lambda_1-1}{\sqrt{C}\lambda_1}=\frac{1}{\sqrt{C}}-\frac{1}{\beta}\frac{\lambda_2}{\sqrt{C}}\to-\frac{1}{\beta} \tag{A13}$$

But since $p_t\to 1$ we have $\partial p_t/\partial i\to 0$.

5. Because a rise in the rate of income tax is equivalent to a rise in η, the derivative of the price with respect to the tax rate is represented by $\partial p_t/\partial\eta$. Using (A6) and (A7) we have

$$\frac{\partial p_t}{\partial\eta}\Big/p_t=\frac{1}{\eta}-\frac{\alpha+1}{hC}+\frac{t+1-T}{h\sqrt{C}}-\frac{\lambda_2}{\lambda_2-1}\frac{1}{h\sqrt{C}},\quad t<T \tag{A14}$$

$$\frac{\partial p_t}{\partial\eta}\Big/(p_t-1)=\frac{1}{\eta}-\frac{\alpha+1}{hC}-\frac{t+1-T}{h\sqrt{C}}-\frac{\lambda_1}{1-\lambda_1}\frac{1}{h\sqrt{C}},\quad t\geq T \tag{A15}$$

If we let $h\to 0$ the right hand sides of the above equations tends to finite numbers in view of (A9). Since $p_t\to 0$ for $t<T$ and $p_t\to 1$ for $t>T$, we have $\partial p_t/\partial\eta\to 0$.

References

[1] Bentick, B.L., "Improving the Allocation of Land Between Speculators and Users: Taxation and Paper Land," *The Economic Record*, Vol. 48, No. 121, (March 1972), pp. 18–41.

[2] ——, "The Allocation of Land Between Speculators and Users Under a Land Ownership Tax: A Reply," *The Economic Record*, Vol. 50, No. 131, (September 1974), pp. 451–52.

[3] ——, "The Impact of Taxation and Valuation Practices on the Timing and Efficiency of Land Use," *Journal of Political Economy*, Vol. 87, No. 4, (August 1979), pp. 859–74.

[4] ——, "Property and Income Taxation in Rural Industries with Different Rotation Periods", *National Tax Journal*, Vol. 33, No. 2, (June 1980), pp. 219–25.

[5] Black, S.W., "The Use of Rational Expectations in Models of Speculation," *The Review of Economics and Statistics*, Vol. 54, No. 2, (May 1972), pp. 161–5.

[6] Black, S.W., "Rational Response to Shocks in a Dynamic Model of Capital Asset Pricing," *American Economic Review*, Vol. 66, No. 5, (December 1976), pp. 767–79.

[7] Blake, D.R., "Property Tax Incidence: An Alternative View," *Land Economics*, Vol. 55, No. 4, (November 1979), pp. 521–31.

[8] Fairchild, F.F. and Associates, *Forest Taxation in the United States*, U.S. Department of Agriculture, Misc. Publication, No. 218, Washington: U.S. Government Printing Office, 1935.

[9] Hamilton, B.W., "Capitalization of Intrajurisdictional Differences in Local Tax Prices," *The American Economic Review*, Vol. 66, No. 5, (December 1976), pp. 743–53.

[10] Klemperer, W.D., "Forests and the Property Tax—A Re-examination," *National Tax Journal*, Vol. 27, No. 4, (December 1974), pp. 645–51.

[11] Markusen, J.R. and Scheffman, D.T., "The Timing of Residential Land Development: A General Equilibrium Approach," *Journal of Urban Economics*, Vol. 5, No. 4, (October 1978), pp. 411–24.

[12] Mieszkowski, P., "The Property Tax: An Excise Tax or a Profit Tax?," *Journal of Public Economics*, Vol. 1, No. 1, (April 1972), pp. 73–96.

[13] Muth, J.F., "Rational Expectations and the Theory of Price Movements," *Econometrica*, Vol. 29, No. 3, (July 1961), pp. 315–35.

[14] Neutz, M., "The Price of Land for Urban Development," *Economic Record*, Vol. 46, No. 115, (September 1970), pp. 313–28.

[15] Sargent, T.J. and Wallace, N., "Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply," *Journal of Political Economy*, Vol. 83, (1975), pp. 241–54.

[16] Skouras, A., "The Allocation of Land between Speculators and Users under a Land Ownership Tax: A Comment," *The Economic Record*, Vol. 50, No. 131, (September 1974), pp. 449–50.

[17] Skouras, A., "The Non-Neutrality of Land Taxation," *Public Finance*, Vol. XXXIII, No. 1-2, (1978).

[18] Trestrail, R., "Forests and the Property Tax—Unsound Accepted Theory," *National Tax Journal*, Vol. 22, No. 3, (November 1969), pp. 347–56.