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A DYNAMIC MODEL
OF INCREMENTAL BUDGETING

By YUKIO NOGUCHI*

Abstract

Previous literature on incremental budgeting has emphasized the stability of budgetary decisions, thus year-to-year changes in the growth rates of expenditures have not been adequately explained in the models. This paper emphasizes dynamic properties of budgetary decisions and tries to explain such changes in the model. The model relates year-specific (or transitory) components of the growth rates of expenditures and tax reduction to the history of the "easiness" of budget preparation which is represented by "preliminary surplus."

It is shown that estimated values of parameters yield plausible interpretation and that the model reproduces actual data fairly well. Based on the empirical results, it is argued that from the point of view of dynamic properties, budget items can be classified into four categories; countercyclical items, stable items, sluggish items and buffer items.

1. Introduction

Behavioral budgetary theory has undergone many refinements since the pioneering work of Wildavsky (1964) and the first empirical study by Davis, Dempter and Wildavsky (1966: hereafter referred to as DDW). We begin by offering a very brief and casual review of the development of the theory, in order to clarify the purpose of this paper.1

The basic idea of the DDW model is that since budget preparation is a stable process, any decision (e.g., request) is expressed as a stable linear function of the previous decision (e.g., previous year's appropriation), at least in the short run. It can be argued that although budget figures do not exhibit radical fluctuations, fixedness of parameters is extreme, in the sense that year to year changes in the budget preparation policy is not explicitly taken into account. We may then argue that error terms in their regressions should be the object of study.

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† Earlier version of this paper was presented at the Zushi Conference of the Tokyo Center for Economic Research, April 1, 1979. I am indebted to the stimulating discussions with Eisuke Sakakibara (Saitama University) and Taizo Yakushiji (Saitama University) for getting the key idea in the model building. I am also grateful to Richard Samuels (MIT), Mikiro Otsuki (Tohoku University), Masahiko Aoki (Kyoto University) and Yasuko Niimura (Economic Planning Agency) for constructive comments.

1 For a more systematic survey of the literature in this field, see; Hoole (1976, chapter I). For a survey of literatures on budgetary decision in general, see; Crecine (1969, chapter II).
Crecine extended the DDW model in his computer simulation model of municipal budgeting (1969). There, the budgetary process is decomposed into several sub-processes, and "preliminary figures" of budget calculated in DDW fashion are modified step by step in order to balance the budget. In this model, expenditure allocation rule does change from year to year in accordance with the sign (and magnitudes) of the preliminary figure of budget surplus. The model is not completely satisfactory, however, since the specification of allocation rules and the choice of some parameter values seem arbitrary, although these are common problems in Cyert-March type computer simulation process models.

Wanat (1974) pointed out the necessity of distinguishing intentional policy decisions represented by "programmatic" increases in expenditure from the stable routine decision represented by "mandatory" increases. From the point of view of the present discussion, however, Wanat's argument is insufficient in that it establishes no quantitative relationship between the budget figures of the current year (e.g., request) and that of the previous year (e.g., previous year's appropriation).

This paper is an attempt to extend the frontier of research a little further. While incorporating the various properties of budgetary decision identified in the previous literature (such as Wildavsky's "fragmented incremental" property, Crecine's "internal-bureaucratic" property, Wanat's distinction between mandatory and programmatic increase), it tries to explain the fluctuations in the growth rate in the model. The key idea of the model building is to "let the unobservable be unobserved in the model," thereby avoiding the arbitrariness mentioned in regard to the Crecine's model. This idea is due to Sakakibara and Yakushiji (1979), or more classically, to Friedman (1957: In particular, the introductory statements to chapter III).

Before presenting the model, we shall take a brief look at the Japanese budget figures to see why the previous models are insufficient.

Figure 1 illustrates the growth rate of tax revenue and that of total expenditure (All budget figures treated in this paper are those of the initial budget: not those of the settlement.)

**FIG. 1. GROWTH RATES OF EXPENDITURE AND TAX**

![Graph showing growth rates of expenditure and tax](image-url)
The difference between the two concepts is especially significant with respect to tax revenue, since tax revenue in the initial budget is of a forecast nature. Observe, first, that the growth rate of expenditure exhibits a gradual change, forming a flattened U-shaped curve (the bottom being in the mid 1960s). If we are interested in this change, then DDW type model would be insufficient, since the fixed parameter models would be incapable of explaining such changes (even if expenditure is broken down into several items and price movement is explicitly incorporated). Second, the above change in the growth rate is not directly related to the short run fluctuations in tax revenue. This observation suggests that Crecine type balancing-the-budget model would also be insufficient to explain the Japanese data.

One possible explanation of the above phenomenon may be found in the long run change in fiscal conditions. Actually, the fiscal year 1966 was the first year after the World War II in which the long term bond to finance the budget deficit was issued, i.e., this year was the turning point from the surplus-budgeting periods to the deficit-budgeting periods (In the years before 1965, the “carried-over surplus” was significant. This surplus is, roughly speaking, the difference between the expenditure and the tax revenue in the settlement base). The fact that the growth rate of expenditure was lower than that of tax revenue in the fiscal years 1967 through 1971 may be explained as the “tightening” of the budget preparation policy caused by this “fiscal crisis.” In the fiscal years 1972 through 1975, the growth rate of expenditure caught up to that of tax revenue, indicating an “organizational forgetting” of the past crisis (Tax reduction was also significant in this period, although this is not observable in the Figure, since tax revenue in the Figure refers to the tax revenue after tax reduction). This observation suggests the existence of some dynamic mechanism, more specifically, some kind of organizational learning process in budgeting. The purpose of the model presented in the next section is to capture this dynamic property.

2. The Model

Unlike what has been done in the previous literature, we shall not distinguish in this paper different participants in the budgetary process and consider a (hypothetical) single decision-making organ called “the budget authority.”2 The boundary of the organ is left unspecified at this stage, so that the “budget authority” may be interpreted as the whole of the politico-bureaucratic complex.

Following Wanat’s distinction of mandatory and programmatic increases of expenditure, we shall here distinguish “secular” and “year-specific” (or transitory) components of increments. Unlike Wanat, however, we do not decide in advance the precise meaning to be attached to the “secular” component, other than saying that it is a time invariant. We thus leave the line to be drawn between the two components to be determined by the data. This is precisely the approach Friedman adopted in his permanent income theory of con-

---

2 One reason to adopt this approach is just to make the model simple. Thus it is in principle quite easy to extend the model to multi-actor settings. The other reason is a specific one associated with the empirical study on Japanese data. In Japan, the Diet usually passes the Government's Budget with no amendment, so that, as far as the budget figure is concerned, there is no distinction comparable to the President's Budget and the Congressional Appropriation of the US Federal Budget. All negotiations in the budgetary process are “below the surface” and virtually no data are available in this respect.
We assume that these two components are additive in terms of (exponential) growth rates (i.e., additive in logarithms), rather than of absolute values. (This formulation seems to be more faithful to the actual thinkings of decision makers.) Incidentally, such a formulation has become quite customary in the recent macroeconomic literature.

Thus letting \( g_{i,t}, h_i \) and \( r_{i,t} \) represent the growth rate of the \( i \)-th expenditure item in the fiscal year \( t \), its secular component and its year-specific component respectively, we write

\[
(1) \quad g_{i,t} = h_i + r_{i,t}.
\]

We assume, for the sake of simplicity, that

\[
(2) \quad h_i = h_0 \quad \text{(a constant)}
\]

for every \( i \).

Similarly, we assume that tax reduction can be decomposed into secular and year-specific components. Thus the growth rate of tax after tax reduction, \( x_t \), is

\[
(3) \quad x_t = x^*_t - (x_0 + q_t)
\]

where \( x^*_t \) is the growth rate of tax revenue before tax reduction, \( x_0 \) is a constant representing secular component of tax reduction and \( q_t \) is a variable representing year-specific tax reduction.

Our task, then, is to construct a model which explains the behaviors of year-specific components. To do this, we begin by specifying the forecasting behavior of the budget authority in the budget preparation process.

Let \( y^*_t \) be the budget authority’s forecast of the growth rate of nominal GNP in the calendar year \( t \). Assume that the forecast is made according to an adaptive mechanism:

\[
(4) \quad y^*_t - y^*_{t-1} = (1 - \mu)(y_{t-1} - y^*_{t-1}), \quad 0 \leq \mu \leq 1
\]

where \( y_{t-1} \) is the actual growth rate of nominal GNP in the calendar year \((t-1)\). The above equation can be transformed into a geometrically declining distributed lag equation:

\[
(5) \quad y^*_t = (1 - \mu)\sum_{k=1}^{\infty} \mu^k y_{t-k}.
\]

Assume further that the budget authority’s tax revenue forecasting function is a simple constant elasticity function:

\[
(6) \quad x^*_t = \delta y^*_t, \quad \delta > 0.
\]
We define the "preliminary surplus" $d_t$ as the difference between the forecasted growth rate of tax revenue before tax reduction and the growth rate of secular components:

$$d_t = x_t - a$$

where $a = h_0 + x_0$. Thus $d_t$ represents, if positive, the margin (in terms of growth rate) that can be allocated to year-specific increase of expenditure, and/or year-specific tax reduction, and/or reduction of budget deficit or accumulation of budget surplus\(^8\) (If $d_t$ is negative, the above statements should be appropriately modified).

In the sense just mentioned, $d_t$ is a measure that represents the relative "easiness" or the "degree of freedom" of budget preparation in the fiscal year $t$. Note that if $x_t$ in equation (7) is replaced with the growth rate of tax revenue after tax reduction, then $d_t$ would not be a proper measure of the easiness of budgeting.

The sign and the magnitude of the preliminary surplus would obviously affect the behavior of year-specific components. By reasons given in the introduction, we hypothesize that in general they are influenced also by the past values of the preliminary surplus. Thus consider the "atmosphere specific to the $i$-th item" $s_{i,t}$ which is defined by

$$s_{i,t} = \lambda_i s_{i,t-1} + (1-\lambda_i) d_t, \quad 0 \leq \lambda_i \leq 1.$$  

Then

$$s_{i,t} = (1-\lambda_i) \sum_{j=0}^{\infty} \lambda_i^j d_{t-j}.$$  

Decision rule of the year-specific increase of expenditure is now presented as

$$r_{i,t} = \gamma_i s_{i,t} + \varepsilon_{i,t}$$  

where $\gamma_i$ is a constant\(^9\) and $\varepsilon_{i,t}$ is a serially uncorrelated random variable with zero mean and a constant variance.

The decision rule adopted here assumes, first, that the budgetary process is primarily and "internal bureaucratic process" in the sense of Crecine (1969), i.e., pressures from outside the organization enters into policy making process primarily in regard to tax and other revenue considerations. This assumption prescribes the boundary of the "budget authority" in an implicit manner.

Second, the decision rule reflects the presumption that each budget item is determined by and large independently without paying due attention to the overall adjustment.\(^10\) This is what Wildavsky (1964) called the "fragmented" nature of budgeting, and may be interpreted as an example of "quasi-resolution of conflict" of organizational behavior in the terminology of Cyert and March (1963).

Note that equation (10) tries to capture the dynamic properties of budgeting by making the allocation of the preliminary surplus subject to the history of budgeting. More specifically:

(i) Positive preliminary surplus is allocated to the year-specific increase of expenditure

More specifically:

(i) If year-specific growth rate is $d_t$ for every expenditure item and no year-specific tax reduction is made, then deficit ratio is maintained at the same level as the previous year, since $(x_t - x_0)/(h_0 + d_t) = 1$.

(ii) If there is no year-specific increase in expenditures and the tax laws are revised so as to make $q_t = d_t$, then deficit ratio is maintained at the same level as the previous year since $(x_t - x_0 - d_t)/h_0 = 1$.

\(^8\) The parameter $\lambda_i$ should in general be a positive number. This may not be true if the item is used as an instrument of countercyclical fiscal policy: See discussions in section 3 and footnote 17.

\(^9\) Since current year's preliminary surplus $d_t$ is included in $s_{i,t}$, some overall consideration is in general supposed to be made, i.e., the decision rule adopted here is not "strictly fragmented." But if the relative weight $(1-\lambda_i)$ is small, then the year-specific decision on the $i$-th item is made relatively independently from the total figure of the budget. See discussions of "sluggishness" in section 3.
(for which \( \gamma \) is positive) only if atmosphere is favorable, i.e., only if \( s_{i,t} \) is positive. Otherwise it is allocated to the reduction of budget deficit or to the accumulation of budget surplus.

(ii) Negative preliminary surplus is eliminated by reducing the growth rate of expenditure (for which \( \gamma \) is positive) below its secular rate only if \( s_{i,t} \) is negative. Otherwise, it is financed by increasing the bond issue or by reducing the carried-over surplus, while letting expenditures grow above their secular rates. This behavior may be interpreted in the Cyert-March framework as “problemistic search” or “organizational learning.”

The model presented here is analogous to the Friedman’s consumption theory in that the outcome is generated by a time varying variable which is unobservable. The analogy ceases there, however, since we have explained the transitory rather than the secular component of the dependent variable.

Equations (1) through (10) enable us to write down \( g_{i,t} \) in the form

\[
g_{i,t} = h_0 - a \gamma i + \delta \gamma i (1 - \mu)(1 - \lambda_i) \sum_{j=0}^{\infty} \sum_{k=1}^{\infty} \lambda^j \mu^k - 1 y_{t-j-k} + \varepsilon_{i,t}.
\]

We specify the decision rule of tax revision in an analogous way as year-specific expenditure decision. Thus the growth rate of tax after tax reduction is

\[
x_t = x_t - x_0 - s_{x,t} + \varepsilon_{x,t}
\]

where \( \gamma_x \) is a constant, \( s_{x,t} \) is the atmosphere specific to tax revision decision defined in the same way as equation (8), and \( \varepsilon_{x,t} \) is a disturbance having the same property as \( \varepsilon_{i,t} \). The reduced form for \( x_t \) is

\[
x_t = h_0 - a(1 - \gamma_x) + \delta (1 - \mu)(1 - \lambda_x) \sum_{j=0}^{\infty} \sum_{k=1}^{\infty} \lambda^j \mu^k - 1 y_{t-k} - \delta \gamma_x (1 - \mu)(1 - \lambda_x) \sum_{j=0}^{\infty} \sum_{k=1}^{\infty} \lambda^j \mu^k - 1 y_{t-j-k} + \varepsilon_{x,t}.
\]

So far we have modelled expenditure decision and tax reduction decision. This formulation implicitly assumes that budget deficit (or surplus) can be regarded as a residual item. However, this may not be an appropriate description of the real decision structure, i.e., it may be that a separate decision is made for deficit per se. In that case, some of the decision functions formulated above should be regarded as the mirror image of the deficit decision function.

3. Empirical Results

It is not practical to estimate equations (11) and (13) directly since the sample size is extremely limited due to the annual character of budget data. In order to retain necessary degrees of freedom, the number of regressors must be reduced. For this purpose, we apply the “Koyck’s transformation” twice, i.e., lag equation (11) by one period, multiply through \( \lambda_i \), subtract it from the original equation, then apply the similar procedure to the new equation. Then we obtain:

\[
g_{i,t} = (h_0 - a \gamma i)(1 - \lambda_i)(1 - \mu) + \delta \gamma i (1 - \lambda_i)(1 - \mu)y_{t-1} + (\lambda_i + \mu)g_{i,t-1} - \lambda_i \mu g_{i,t-2} + \varepsilon_{i,t} + (\lambda_i + \mu)g_{i,t-1} + \lambda_i \mu \varepsilon_{i,t-2} + \varepsilon_{i,t}.
\]

Similarly, we obtain:

\[
x_t = [h_0 - a(1 - \gamma_x)](1 - \lambda_x)(1 - \mu) + \delta [1 - \gamma_x (1 - \lambda_x)](1 - \mu)y_{t-1} - \delta \lambda_x (1 - \mu)y_{t-2} + (\lambda_x + \mu)x_{t-1} - \lambda_x \mu x_{t-2} + \varepsilon_{x,t}.
\]

\[\text{While total expenditure data for earlier years are available, expenditure data by category are available only for limited periods, due to the discontinuity in the classification of expenditures.}\]

\[\text{Koyck (1954).}\]
The above equations have been estimated by the budget data of Japan. Expenditure equation (14) has been estimated for each expenditure item listed in Table 1. The results are summarized as follows:

**Table 1. List of Variables**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>growth rate of tax revenue after tax reduction</td>
</tr>
<tr>
<td>( g_t )</td>
<td>growth rate of social security expenditure including government employees' pension</td>
</tr>
<tr>
<td>( g_e )</td>
<td>growth rate of education expenditure</td>
</tr>
<tr>
<td>( g_d )</td>
<td>growth rate of defense expenditure</td>
</tr>
<tr>
<td>( g_f )</td>
<td>growth rate of subsidy for rice production</td>
</tr>
<tr>
<td>( g_p )</td>
<td>growth rate of public works expenditure</td>
</tr>
</tbody>
</table>

(i) While some coefficients of \( g_{t-1} \) are significantly different from zero, no coefficients of \( g_{t-2} \) (and \( x_{t-2} \)) are significant at 90% level. This implies that the null hypothesis \( \mu = 0 \) is not rejected.

(ii) Some coefficients of \( y_{t-1} \) are significant, which implies that hypothesis \( \mu = 1 \) is rejected.

It follows that if the choice is between the two pivotal cases \( \mu = 0 \) and \( \mu = 1 \), we accept \( \mu = 0 \). Of course, this does not deny the possibility that \( \mu \) is some positive number less than 1. But since no coefficients of \( g_{t-2} \) (and \( x_{t-2} \)) is significant, there seems to be no way of getting reliable point estimate of \( \mu \).

Apart from the statistical arguments given above, one could argue as follows. Suppose that the budget authority believes that time series \( y_t \) is generated by the following moving average process (random walk):

\[
y_t = \sum_{j=0}^{\infty} \theta_j y_{t-j}
\]

where \( u_t \) is a serially independent random variable with zero mean and a constant variance. Then \( \mu = 0 \) gives the optimal forecast of \( y_t \), because \( E y_t = y_{t-1} \). Since it would not be unreasonable to suppose that the budget authority believes in (16), it would not be unreasonable to assume apriori that \( \mu = 0 \).

Therefore, it would be practical and reasonable to proceed on the assumption that \( \mu = 0 \) and to reestimate equations by dropping the \( g_{t-2} \) and \( x_{t-2} \) terms. In this simplified version, \( R^2 \) (coefficient of determination adjusted for the degree of freedom) rises (except

---

13 In this regression, OLS was used. There are, however, problems in using OLS. See discussions in the Appendix.
14 In estimating education expenditure equation, dummy variable \( D_1 \) (=1 fiscal years 1974 and 1975; = 0 otherwise) was used in order to remove the effect of inflation in those years. This adjustment was made only for this item in view of the fact that it consists largely of personnel expenditure and that it is personnel expenditure that suffers significant influence from inflation.
15 In estimating subsidy for rice production, dummy variable \( D_1 \) (=1 fiscal year 1968; =0 otherwise) was used in order to remove extraordinary growth rate (67.1%) in that year. This extraordinary growth rate is due to the spectacular rice harvest in the previous year.
16 Expenditure items listed in Table 1 are not exhaustive but cover almost 60% of the total expenditure. The major item not covered here is the grand-in-aid to local governments. This item has been dropped from the estimation because the magnitude of this expenditure is determined almost automatically as a fixed percentage of tax revenue.
17 For a rigorous discussion, see: Muth (1960). Incidentally, there is an algebraical slip in Muth's paper. In his equation (3.9) the last term should read \( \sigma_y^2 \sqrt{\frac{1}{4} + \frac{\sigma_x^2}{\sigma_y^2}} \).
for $g_z$ and $g_f$ equations, where $R^2$ falls by about 0.01). This seems to support the above assumption.

Some of the coefficients in this version are still insignificant. For example, the coefficients of terms $y_{t-2}$ and $x_{t-1}$ in the tax equation are not significant (t-values are 0.388 and -1.54), which implies that hypothesis $\lambda_x = 0$ is not rejected. In such cases, we have reestimated the equation by dropping the variable whose coefficient is insignificant, and have adopted the simplified version when $R^2$ rises.

The final results are summarized in Table 2. All coefficients are significant at 90% level, except for the constant term in $g_f$ equation.

### Table 2. Estimated Results

<table>
<thead>
<tr>
<th></th>
<th>$g_z$</th>
<th>$g_s$</th>
<th>$g_d$</th>
<th>$g_f$</th>
<th>$g_p$</th>
<th>$x$</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>const.</td>
<td>$y_{t-1}$</td>
<td>$g_{t-1}$</td>
<td>dummy</td>
<td>$\bar{R}^2$</td>
<td>d.w.</td>
</tr>
<tr>
<td>$g_z$</td>
<td>*</td>
<td>0.365</td>
<td>0.723</td>
<td>*</td>
<td>0.585</td>
<td>1.31</td>
</tr>
<tr>
<td></td>
<td>(2.08)</td>
<td></td>
<td>(5.41)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_s$</td>
<td>0.140</td>
<td>*</td>
<td>*</td>
<td>0.120</td>
<td>0.693</td>
<td>1.75</td>
</tr>
<tr>
<td></td>
<td>(18.23)</td>
<td></td>
<td></td>
<td>(5.70)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_d$</td>
<td>*</td>
<td>0.320</td>
<td>0.645</td>
<td>*</td>
<td>0.189</td>
<td>2.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.87)</td>
<td>(3.68)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_f$</td>
<td>-0.209</td>
<td>2.317</td>
<td>*</td>
<td>0.504</td>
<td>0.742</td>
<td>2.05</td>
</tr>
<tr>
<td></td>
<td>(-1.60)</td>
<td></td>
<td></td>
<td>(4.89)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_p$</td>
<td>0.384</td>
<td>-2.003</td>
<td>0.396</td>
<td>*</td>
<td>0.542</td>
<td>1.95</td>
</tr>
<tr>
<td></td>
<td>(5.38)</td>
<td></td>
<td>(4.12)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>-0.286</td>
<td>2.969</td>
<td>*</td>
<td>*</td>
<td>0.737</td>
<td>2.82</td>
</tr>
<tr>
<td></td>
<td>(-4.15)</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Notes: 1. Figures in parentheses are t-values.
2. $\bar{R}^2$: Coefficient of determination adjusted for the degree of freedom
3. d.w.: Durbin-Watson statistics
4. $s$: Standard error
5. Estimation Period
   $g_z$: 1966-1978
   other items: 1961-1978
6. * indicates that relevant variable is omitted.
7. Dummy variables are defined in footnote 14.

Before interpreting the results, it would be necessary to check whether the estimated values of parameters $h_0$, $x_0$ and $d$ are reasonable. The values of $A_i = h_0 - a \gamma_i$ and $B_i = d \gamma_i$, which are calculated from the estimated values of coefficients, are plotted in Figure 2 (In case of tax equation, $A_i = h_0 - a(1 - \gamma_x)$, $B_i = d(1 - \gamma_x)$).

The theory predicts that points ($A_i$, $B_i$) should be on a straight line:

\(B_i = \frac{\delta}{a}(h_0 - A_i)\)

Since the plotted points are not strictly on a straight line, $\hat{B}_i$ is regressed on $\hat{A}_i$. The result is

\(\hat{B}_i = 1.02 - 6.78 \hat{A}_i \quad (\bar{R}^2 = 0.994)\).

Therefore, $\hat{h}_0 = 0.15$. This seems to be a reasonable value. Unfortunately, separate estimates of $\delta$ and $x_0$ are not obtained from the above regression. However, we know the relationship:

\(\hat{\delta} h_0 / \hat{a} = 1.02\)
or

\[ \dot{x}_0 = \left( \frac{1}{1.02} \delta - 1 \right) h_0. \]

This relationship can be regarded as a plausible one. The reason is given below for the case of income tax, which is the most important tax in the Japanese tax system.

Income tax revenue in the fiscal year \( t \) is approximated by the formula:

\[ X_t = \alpha (Y_t - M_t) \]

where \( Y_t \) is income, \( M_t \) is minimum taxable income, and \( \alpha \) is a constant representing the marginal tax rate (Although the actual marginal rates are progressive, constant marginal rate is assumed here).

In postwar Japan, \( M_t \) has been raised almost every year in order not to increase the relative tax burden, while \( \alpha \) has not been changed frequently. Thus the growth rate of tax revenue can be written as

\[ \dot{x}_t = \frac{Y_t}{Y_t - M_t} y_t - \frac{M_t}{Y_t - M_t} m_t \]

where \( y_t \) and \( m_t \) are growth rates of \( Y_t \) and \( M_t \), respectively. The first term of the above
equation is the growth rate of tax revenue when $M_t$ is not changed. Namely it is $x_t^*$ in equation (6). Thus, $\delta = Y_t/(Y_t-M_t)$. Hence, $x_0$ in equation (3), which is represented by the second term of (21), is
\[ x_0 = (\delta - 1)m_t \]

The data indicate that $m_t$ is approximately equal to $h_0$. Then:
\[ x_0 = (\delta - 1)h_0. \]

This is almost the same as the equation (19) (It is usually believed that $\delta$ is around 1.2. If we use this value, $x_0$ is around 0.03).

Let us now examine for each item the estimated value of $\gamma_i$ which is the sensitivity of year-specific growth rate with respect to change in atmosphere. In the last column of Table 2, the values of $\gamma_i$ which are calculated from the coefficients of $y_{t-1}$ terms on the assumption $\delta = 1.20$ are shown.

We observe that the values are negative for public work expenditure and tax reduction. This implies that these items are used as instruments of countercyclical fiscal policy.\(^{17}\)

We also observe that the value is zero for education. This implies that education expenditure is a "stable item" in the sense that its growth rate is completely determined by the secular rate (except for the extraordinary years of inflation which are represented by the dummy variable). This seems to be a reasonable result, because the education system in Japan has long been established so that there is little room left for non-routine type budgetary decisions.

Let us next examine the estimated value of $\lambda_i$ (the coefficient of $g_{t-1}$) for those items whose sensitivities are positive. The results in Table 2 indicate that "memory length" is fairly long for social security and defense, while no "memory" exists for subsidy for rice production. For example, preliminary surplus of three years ago has non-negligible influence on year specific growth rate of social security $\left( (1 - \lambda_i)x_{t-3} = 0.105 \right)$, whereas preliminary surplus of even the previous year has no effect on year specific growth rate of subsidy for rice production. Namely, the former items are "sluggish items" and the latter item is a "flexible item."

Again, this is a very reasonable result. For, consider the case of social security. Increase in this expenditure is usually caused by institutional changes such as initiation of new programs or upgrading of payment levels. The budget authority is cautious in admitting them, since it would be extremely difficult to abolish the once initiated program or to downgrade the once upgraded payment level. Therefore, they are admitted only when fiscal condition is exceptionally favorable, i.e., only when preliminary surplus continues to be positive for long enough periods. Furthermore, there are numerous bureaucratic procedures before the final stage of legislation and budgeting is reached. It is therefore natural that social security exhibits considerable sluggishness. Similar argument can be made for defense expenditure.

\(^{17}\) It might be argued that if these items are instruments of countercyclical fiscal policy, then year-specific growth rates should be formulated as being dependent not on the atmospheres which are functions of preliminary surplus but directly on the forecasted growth rate of GNP, so that, although the regression forms may remain unchanged, the coefficients should be given different interpretations. Although this is a reasonable argument, our formulation is not rejected, at least on apriori grounds, since model formulation is a matter of choice rejected or supported by empirical data. The fact that $(A_i, B_i)$ for these items are nearly on the straight line drawn in Figure 2 implies that interpretation of coefficients based on our formulation is plausible, which in turn implies that our formulation is plausible too.
In contrast to this, year-specific increase in subsidy for rice production is achieved simply by increasing the amount of subsidy, which requires no preparation procedure comparable to the one mentioned above. It is therefore natural that this item is "flexible." Stated differently, subsidy for rice production is used as a "buffer" against fluctuation in tax revenue before tax reduction. For example, if the preliminary surplus is large but at the same time the long term fiscal condition is not favorable, the surplus is reduced by increasing the subsidy for rice production (It is of course not generally the case that preliminary surplus is completely eliminated by this procedure. The residual is ultimately eliminated by adjustments in bond issue or carried-over surplus.).

Note that "stability" defined before is conceptually different from "sluggishness" defined here; the former is the insignificance of year-specific growth rate, whereas the latter is the rigidity of year-specific growth rate. In other words, "stability" reflects the absence of major policy decision, whereas "sluggishness" reflects the cautiousness (or the inertia) of policy decision.

Finally, Figure 3 compares the actual growth rate with the predicted growth rate on "partial test" basis. Observe that in general the model prediction captures the behavior of the actual data to a considerable extent. In particular, the turning points in the growth rates are predicted by the model fairly well. Note that our model explains the rise in the growth rate of social security in the mid 1970s as an internal bureaucratic phenomenon. This is radically different from the general understanding that this increase was caused by public opinion.

4. Conclusion

It is true that the model presented in this paper is exceedingly simple and primitive, leaving wide possibilities for improvement in future research. Nevertheless, it seems to have captured the dynamic properties of budgeting to a considerable extent.

In particular, we have been able to categorize budget items into four groups according to their dynamic properties, namely, countercyclical items (\( \gamma_i < 0 \)), stable items (\( \gamma_i = 0 \)), sluggish items (\( \gamma_i > 0 \) and \( \lambda_i \) not small) and buffer items (\( \gamma_i > 0 \) and \( \lambda_i = 0 \)). It should perhaps be emphasized that this categorization is not a mere confirmation of the common sense. For, although the "sluggishness" of social security expenditure is widely recognized, it is not clearly distinguished from a different concept "stability." Furthermore, the existence of "buffer items" such as the subsidy for rice production has been scarcely recognized in the previous literature. It seems that previous work on incremental budgeting has placed too much emphasis on the stability of budget figures, thereby overlooking its dynamic properties. This paper has demonstrated the need for the study of such structures.

\[ \text{18 The possibilities for future research have already been mentioned. Especially, in footnotes 2 and 5.} \]
FIG. 3. MODEL PREDICTIONS

\( g_s \) (Social security)

\( g_e \) (Education)

\( g_d \) (Defense)
Fig. 3. (continued)

- $g_5$ (Public works)
- $g_f$ (Subsidy for rice production)
- $x$ (Tax revenue)
APPENDIX

Problems of Interdependency and Serial Correlation

In this paper, OLS has been used throughout. It is widely recognized, however, that OLS is not even a consistent estimator when some of the regressors are lagged dependent variables, because there is a contemporaneous dependence between one of the regressors and the disturbance, and the disturbance is autocorrelated.

Koyck (1954) proposed a two-step procedure to obtain consistent estimates of parameters under this circumstance. The idea, translated to our case, is as follows: First apply OLS obtaining $\hat{\lambda}$ and $\sum z_i^2$ (sum of squared residuals), then substitute

$$\frac{\sum g_{t-1}g_t + \hat{\lambda}^* \sum z_i^2}{1 + \hat{\lambda}^* \hat{\lambda}}$$

for $\sum g_{t-1}g_t$ in the normal equation, where $\hat{\lambda}^*$ is the corrected estimate of $\lambda$, obtained by solving a quadratic equation in $\hat{\lambda}^*$. (Klein (1958) has demonstrated that Koyck’s method can be regarded as a maximum likelihood method).

In order to evaluate the magnitude of the correction, let us suppose that $\hat{\lambda}^*$ is not far from $\hat{\lambda}$. Then by substituting $\hat{\lambda}$ for $\hat{\lambda}^*$ in the above equation, the correction may be evaluated as

$$A = \frac{\hat{\lambda}}{1 + \hat{\lambda}^2} \sum z_i^2$$

It is easy to see from the result in Table 2 that relative weight of the correction term (i.e., $A / \sum g_{t-1}g_t$) is almost negligible. Furthermore, as Griliches (1967) pointed out, Koyck’s procedure depends crucially on the original disturbances being uncorrelated. Moreover, the need for consistency is not so urgent in our case, since the sample size is extremely limited. For these reasons, Koyck’s method has not been undertaken in this paper. In this respect, we may rely upon Klein’s suggestion that for practical purposes we may start out with equations such as (14) and (15) estimating parameters by OLS on the assumption that errors are nonautocorrelated.

Finally, the Durbin-Watson statistic shown in Table 2 should be read with great caution, since it may not be a proper measure to test serial correlations in error in cases such as ours. Durbin (1970) proposed a different statistic which has a desirable asymptotic property to test serial correlation when regressor are lagged dependent variables. This was not computed here either, in view of the smallness of our sample size.
REFERENCES


