ON THE FACTOR-PRICE FRONTIER IN THE PURE THEORY OF INTERNATIONAL TRADE[1]

By MAKOTO IKEMA*

I. *Introduction*

The purpose of this paper is to demonstrate that the analysis of the effect of commodity price change on the prices of factors of production can be conducted effectively by a technique based upon the Factor-Price Frontier (FPF). Suprisingly enough the FPF has so far been paid little attention, if not ignored, in the pure theory of international trade. Instead the Edgeworth-Bowley box diagram and / or the so-called Samuelson-Johnson diagram have been usually and extensively used.[2] What has been said in terms of these diagrams on the commodity-prices / factor-prices relationships can, however, be derived and proved more easily and clearly in terms of the FPF.

In Section II the FPF will be derived diagramatically from the familiar isoquant. Sections III and IV illustrate the effectiveness of the FPF in dealing with the factor price changes relative to commodity price changes, applying it to proving a few fundamental theorems, i.e., the Stopler-Samuelson theorem in a standard two-commodity, two-factor case, and the Haberler-Jones theorem[3] in a two-commodity, three-factor case. Section V analyses in terms of the FPF the effects of factor price differentials.

II. *The Factor-Price Frontier*

A Factor-Price Frontier is here defined as a locus of all combinations of factor prices for a specified price of a commodity.[4] It is the counterpart of a unit-isoquant which is

* Assistant Professor (*Jokyōju*) of International Economics.

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[2] At textbook level, see, among others, Batra [2], and Caves and Jones [3]. See also Magee [14].

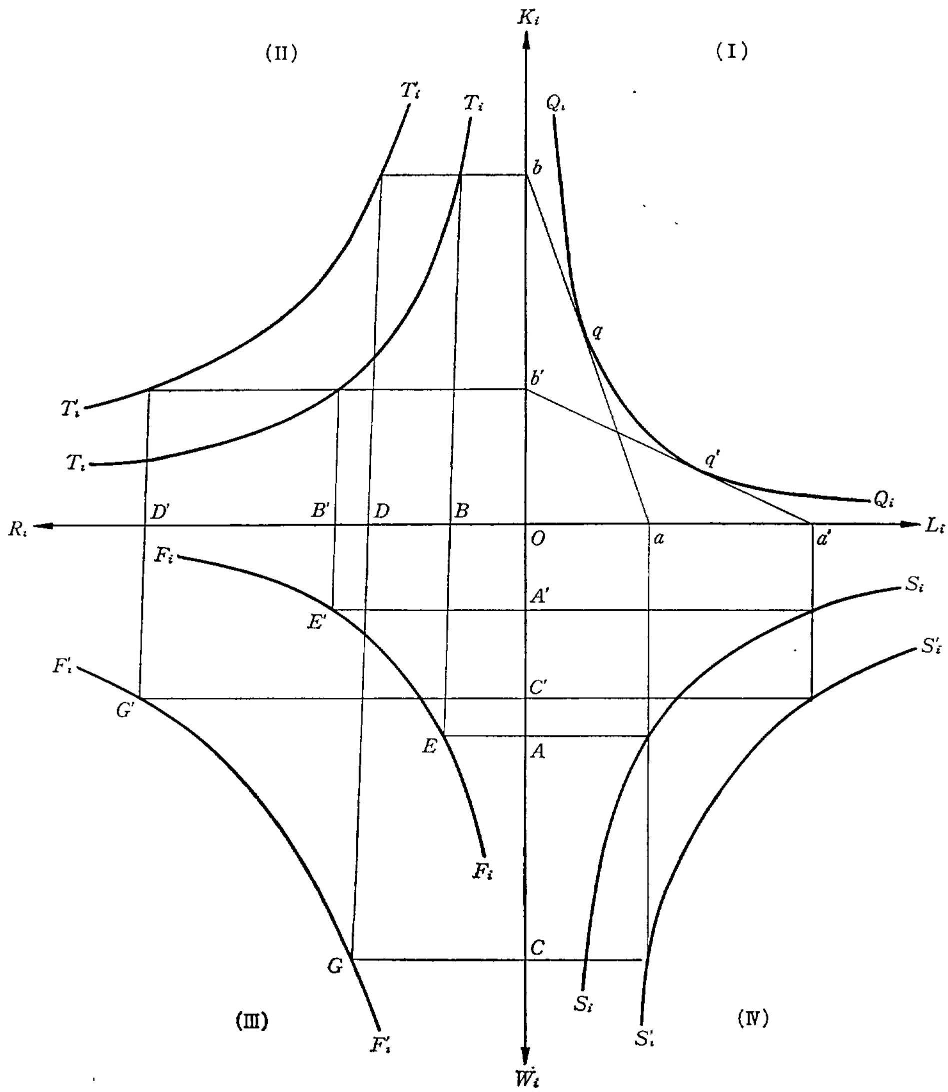
[3] This is my terminology.

[4] Samuelson [17] states "There is always a tradeoff between the wage and profit level .... A good name for this fundamental tradeoff relation would be the Factor-price Frontier." (pp. 195-6). Jones [9] defines a factor-price frontier as "a locus of all combinations of factor prices allowed by an economy's given technology. It is not an unambiguous concept. For example, if both factor prices are deflated by the price of the same commodity, the factor-price frontier is dependent only upon the production function of that sector." (p. 12). Since we are particularly interested in factor price changes relative to commodity price changes, we define the FPF as above in the text. In a sense, therefore, the term of *iso-commodity price* might be preferable. Hicks [6] calls it a "wage-equation" (p. 140).

itself the locus of all combinations of factors used to produce one unit of a commodity. In this section we derive diagramatically a FPF from a unit-isoquant.

Let commodity i be produced by using two factors of production, labour (L) and capital (K). The production function is assumed to be homogeneous of degree one with diminishing marginal product of each factor. The curve Q_iQ_i in Quadrant I of Figure 1 is the unit-isoquant for commodity i, labour and capital being measured respectively along the horizontal and the vertical axes.

FIG. 1



If the price of commodity i (P_i) is specified at the level of P_i' as well as the wage-rental ratio (W_i/R_i) as Ob/Oa (i.e., W_i'/R_i'), then maximization of profits is attained where the combination of factor inputs to produce one unit of commodity i is at point q on the unit-isoquant Q_iQ_i. Here the wage-rental ratio (or factor-price ratio) equals the marginal rate of substitution between two factors (i.e., the slope of the isoquant). The intersept on the horizontal axis (i.e., Oa) measures the commodity price in terms of wage rate (P_i'/W_i') and that on the vertical axis (Ob) is the commodity price in terms of rental on capital (P_i'/R_i').

Now if P_i and P_i/W_i are known, W_i must be determined because of the relation that $W_i=P_i/(P_i/W_i)$. This equation tells us that the relation between W_i and P_i/W_i is a rectangular hyperbola for any given P_i. The rectangular hyperbola SS in Quadrant IV is drawn for the case where P_i is specified as P_i'. When $Oa=P_i'/W_i'$, W_i' must be OA as shown on the vertical axis in Quadrant IV. Similarly the rectangular hyperbola TT in Quadrant II shows the relation that $R_i=P_i/(P_i/R_i)$ when $P_i=P_i'$, so that if $P_i'/R_i'=Ob$, then $R_i'=OB$. The combination of factor prices in Quadrants IV and II leads to a mapping in Quadrant III. Clearly, when commodity i is produced by using the technique as point q, the combination of W_i and R_i must be point E in Quadrant III.

Next suppose that the factor-price ratio changes to Ob'/Oa' (or W_i''/R_i'') which is smaller than Ob/Oa (or W_i'/R_i'), while the commodity price remains at the level of P_i'. In this case W_i'' and R_i'' are respectively OA' and OB', and the combination of these factor prices gives point E' in Quadrant III. By finding points such as E and E', the curve F_iF_i can be drawn. This is a FPF for commodity price P_i'. From its derivation the F_iF_i curve is the locus of all combinations of wage and rental rates for a commodity price specified at the level of P_i'.

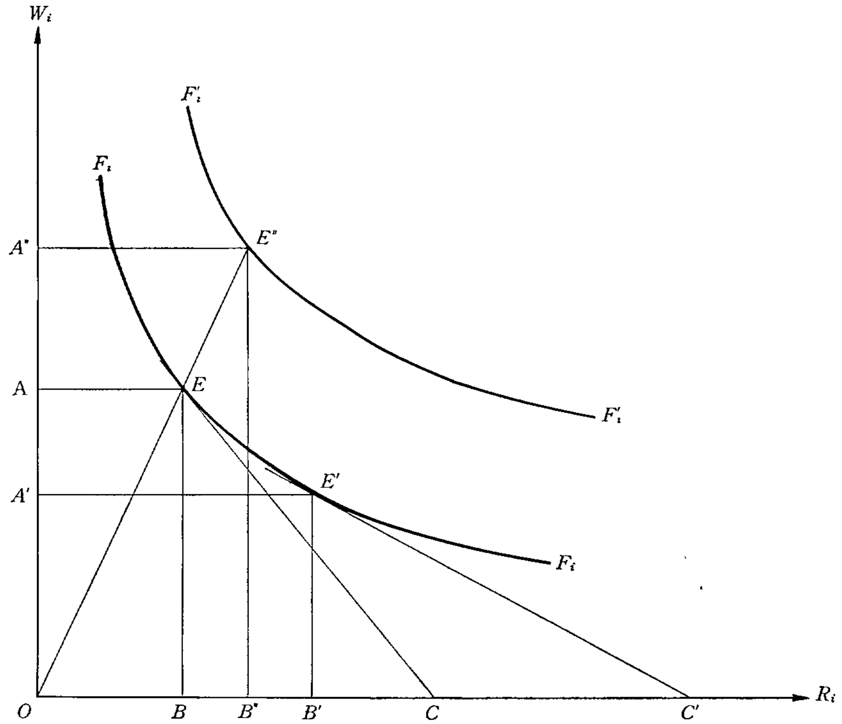
What will happen to the position of the FPF if commodity price P_i changes? Suppose that P_i is increased from P_i' to P_i'' which is, say, twice as high as P_i'. This changes the rectangular hyperbola from SS to $S'S'$ in Quadrant IV, and from TT to $T'T'$ in Quadrant II, shifting the FPF in Quadrant III from F_iF_i to $F_i'F_i'$. Since $P_i''=2P_i'$ by assumption, $OC=2OA$ and $OD=2OB$; hence $OG=2OE$. Similarly $OC'=2OA'$ and $OD'=2OB'$; therefore $OG'=2OE'$. That is to say, doubling the commodity price level leads to doubling the price of each factor. If both factor prices are increased by the factor h, then the price of commodity i increases by the same factor h, i.e., the FPF is homogeneous of degree one.[5] It is this property which gives the FPF an important role in the factor-prices / commodity-prices relationships.

Since our focus is on the FPF, Figure 2 reproduces Quadrant III in Figure 1, with some alteration in notation. In Figure 2 the wage rate W is measured vertically and the rental on capital R horizontally. It is noticiable that the factor price ratio now appears as the slope of the ray through the origin (e.g., BE/OB), while it was shown as the slope of the isoquant in Quadrant I of Figure 1 (e.g., Ob/Oa).

The slope of the FPF is, in absolute value, equal to the capital-labour ratio. For example, the slope of the tangent at point E on the F_iF_i curve, BE/BC, equals the ratio of capital to labour corresponding to the wage-rental ratio BE/OB. The proof is as follows. The price of a commodity is the sum of each factor price multiplied by its input coefficient.

[5] See Eq. (8) in The Mathematical Supplement to Section II.

FIG. 2



That is, $P_i = a_i W_i + b_i R_i$, where a_i and b_i are respectively labour and capital required to produce one unit of output, so that the ratio of b_i/a_i represents the capital-labour ratio. Remembering that $W_i da_i + R_i db_i = 0$ along the unit-isoquant, and that P_i is kept constant along the FPF, we have the relation that $a_i dW_i + b_i dR_i = 0$. Thus $dW_i/dR_i = -b_i/a_i$: The slope of the FPF (dW_i/dR_i) is equal in absolute value to the capital-labour ratio (b_i/a_i).[6] (Note that in the isoquant this ratio appears as the slope of the ray through the origin). Clearly, the capital-labour ratio decreases from BE/BC to $B'E'/B'C'$, as the wage-rental ratio decreases from BE/OB to $B'E'/OB'$.[7]

We now have the capital-labour ratio as well as the wage-rental ratio. From these two ratios we can derive the relative shares of factor incomes. The rental incomes relative to wage incomes amount to equal the capital-labour ratio divided by the wage-rental ratio. At point E the former is BE/BC and the latter BE/BC, so that the rental incomes relative to the wage incomes must be OB/BC.[8]

Finally, as already mentioned, the FPF is linearly homogeneous. This property implies that the commodity price can be measured as the distance from the origin. In terms

[6] See Eq. (9) in The Mathematical Supplement to Section II. Also refer to Samuelson [17] and Kemp [11].
[7] This relationship between factor-prices and factor-intensities is familiar from the Samuelson-Johnson diagram. Samuelson [16] and Johnson [7].
[8] See Eq. (11) in The Mathematical Supplement to Section II. Also refer to Samuelson [16] and Kemp [11].

of Figure 2, if we assume that the initial price level is OE, then the commodity price corresponding to the $F_i'F_i'$ relative to the initial price level must be OE' / OE; EE' / OE reprents the rate of change in the commodity price.

To sum up. The FPF contains the following six important properties: (1) Specified commodity price level, indicated by the distance from the origin, e.g., OE; (2) The level of factor prices, measured along the axes, e.g., OA or OB; (3) The factor price ratio shown as the slope of the ray through the origin, e.g., $OA/OB=BE/OB$; (4) The capital-labour ratio represented by the slope of tangent of the FPF, e.g., BE/BC; (5) The relative share of factor incomes, i.e., (4) / (3) or OB/BC which equals the ratio of rental incomes to wage incomes; and (6) The commodity price being homogeneous of degree one in each factor price, e.g., $EE''/OE=AA''/OA=BB''/OB$. All this information points, needless to say, to duality between the FPF and the isoquant.[9]

III. *The Stolper-Samuelson Theorem*

Up to this stage we have only concentrated on the FPF for commodity i. The properties of FPF described in the previous section must hold for any commodity. Consider that there are two commodities, 1 and 2, and commodity 1 uses labour intensively than commodity 2 for any given factor-price ratio common to both industries. As already proved, the slope of the FPF represents the capital-labour ratio in each industry. Thus it follows that the FPF for capital-intensive commodity 2 cuts that for labour-intensive commodity 1 from the above.[10]

Suppose now that these two commodities are the only ones produced and consumed in the economy, and that each factor moves freely among industries. Furthermore assume for simplicity that the economy is too small to influence commodity prices in international markets. Once the prices of commodities are given internationally, then the assumption of perfect factor markets ensures that factor prices are determined at the intersection of two FPFs as shown in Figure 3. Thus the wage rate must be OA and rental rate OB.[11] Here it is assumed that the slope of F_1F_1 and F_2F_2 at point E ensure full employment of each factor and incomplete specialization. That is to say, the slope of F_2F_2 at point E is greater than the overall capital-labour ratio while that of F_1F_1 at point E is less than the overall capital-labour ratio.

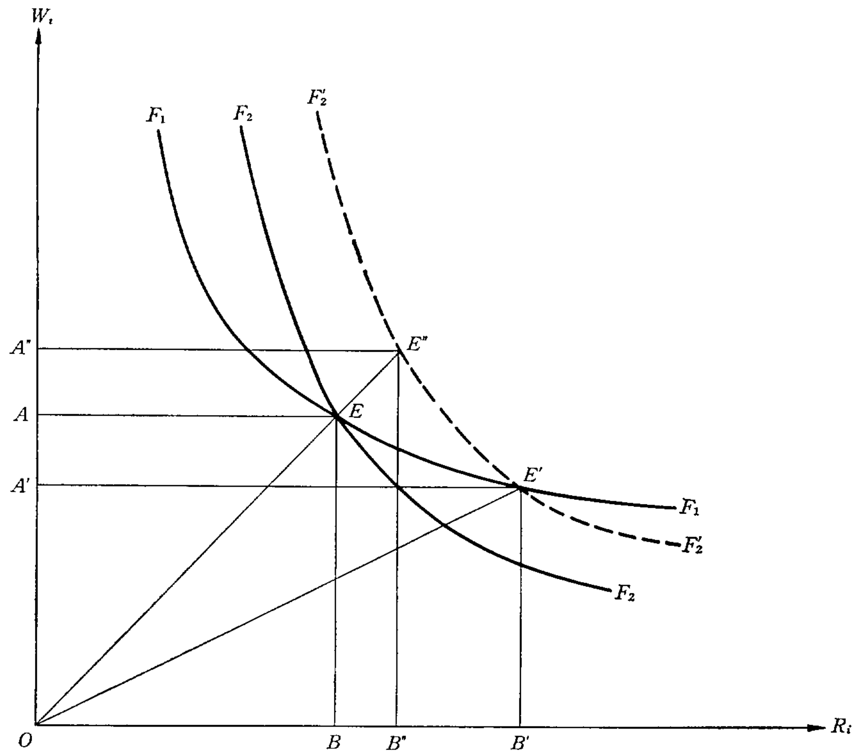
What will be the effect on factor prices if the price of capital-intensive commodity 2 assumed to be importables increases by EE'', say, due to a tariff imposed on it? Its FPF shifts accordingly from F_2F_2 to $F_2'F_2'$. This shift will instantaneously cause industry 2 to increase its wage rate to OA'' and rental to OB''. Since OA'' and OB'' are respectively greater than OA and OB, labour and capital should have moved from industry 1 to industry 2. With industry 2 being more capital-intensive than industry 1, relatively more capital

[9] To show duality between the Rybzynski theorem and the Stolper-Samuelson theorem, Jones [8] has produced the diagram measuring *commodity prices* along axes. The presentation here shows duality much clearer than Jones'.

[10] See also Kemp [11], pp. 14-5 and problems 2.4 and 2.5, pp. 21-2.

[11] Note that even when the price of commodity 2 and factor prices are measured in terms of commodity 1, this does not affect the shapes of FPFs, since the FPFs are linearly homogeneous. Therefore all prices in what follows can also interpreted in terms of commodity 1.

FIG. 3



than labour will flow into industry 2. Thus a new equlibrium will be reached at point E' where the F_1F_1 and $F_2'F_2'$ curves intersect.[12] A comparison of the initial point E with the new point E' makes it clear that wage rate decreases absolutely by AA' while the rental on capital increases absolutely by BB'. Hence the wage-rental ratio decreases from BE/OB to $B'E'/OB'$.[13]

What about the changes in *real* factor prices in terms of commodity 2? First, the rate of change in price of commodity 2 is EE''/OE, which is equal to AA''/OA, and the rate of change in wage rate is $-AA'/OA$. Thus the rate of change in wage rate in terms of commodity 2 is $(-AA'/OA)$ *minus* (AA''/OA), i.e., $-A'A''/OA$. Secondly, the rate of change of rental is BB'/OB. Since the rate of change in commodity 2's price EE''/OE also equals BB''/OB, the rate of change in rental in terms of commodity 2 should be BB''/OB $(=BB'/OB-BB''/OB)$: The rental on capital increases not only absolutely but also relatively to the price of commodity 2 which uses capital more intensively than commodity 1, as shown by Stolper and Samuelson.[14]

[12] Again at point E' it is assumed that the country can produce both commodities.
[13] This relationship is also familiar from the Samuelson-Johnson diagram. Note that this and Footnote 4 in Section II complete the Samuelson-Johnson diagram.
[14] Stopler and Samuelson [18].

It may be worthwhile pointing out that Figure 3 can also be used to show the effect on factor prices of Hicksian neutral technical change. If Hicksian neutral technical progress occurs in industry 2 its FPF will shift from F_2F_2 to $F_2'F_2'$, because at the *same* commodity price level the neutral technical progress enables each factor to be paid proportionally more than before. The initial equilibrium is E, and after the technical progress the equilibrium point should be E'. As a result, (1) wage rate decreases by AA'; (2) rental rate increases by BB', and hence (3) the wage-rental ratio decreases from BE/OB to $B'E'/OB'$. Generally speaking, when neutral technical progress occurs in an industry, the price of factor which is used intensively in that industry will increase while the price of the other factor used less intensively will decrease.[15]

Finally, though needless to say, the FPF can be utilized to show that the international trade of commodities, under certain additional assumptions, will result in the international equalization of factor prices not only in relative but also in absolute senses.

IV. *The Haberler-Jones Theorem*

In the previous section we dealt with the commodity-prices / factor-prices relationships in the standard two-by-two case. Five or eight years earlier than Stolper and Samuelson, however, Gotfried Haberler presented a proposition similar to the Stolper-Samuelson theorem.[14] It is concerned with the influences of international trade upon the relative prices of various specific and non-specific factors of production. The following conclusions were obtained: (1) When international exchange of commodities begins to take place, it will cause a rise in the price of those factors which are specific to exporting industries of a country; (2) it will cause a fall in the price of whatever factor are specific to those industries in which the country has a comparative disadvantage; (3) it will cause a rise in the prices of nonspecific factors, but this rise will be less than the rise under (1).

Thirty-five years later than Haberler, Ronald W. Jones reached, without any reference to Haberler, the same propositions as Haberler's, based on a "restricted" version of the three-factor, two-commodity case.[17] While Haberler did not accurately speak of real prices, Jones demonstrated rigorously that when the price of one commodity rises, with the other remaining constant,[18] then (a) the price of a factor specific to the former will increase more than the rise in the commodity price; (b) the price of a factor which is used for the production of two commodities will rise but less than the rise of the price of the commodity; and (c) the price of a factor used specifically to produce the commodity whose price remains unchanged will fall.[19]

The conclusions of Haberler and Jones can be called the *Haberler-Jones theorem*, which is in reality a Stolper-Samuelson theorem in the three-factor, two-commodity case. As shown above for the Stolper-Samuelson theorem, the FPF can be utilized to derive and prove

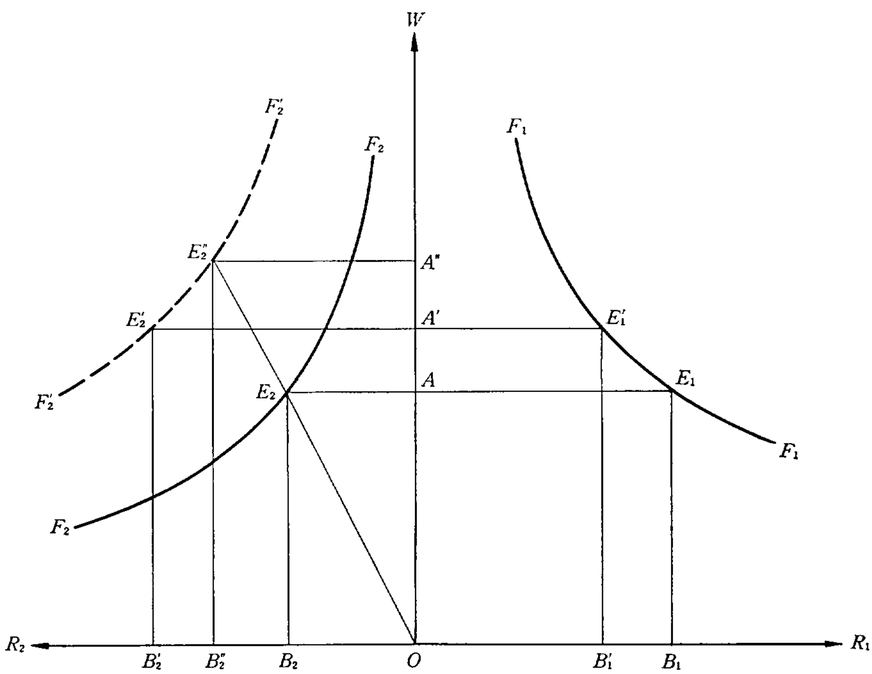
[15] These changes in factor prices are also in *real terms*, since commodity price are kept constant.
[16] Haberler [5], pp. 193-4.
[17] Jones [9]. Also see Amano [1]. A similar characteristic but more complicated model can be found in Gruen and Corden [4], and their model can also treated in terms of the FPF, though it is not attempted in this note.
[18] This is not Jones' assumption, but it is assumed here for simplicity.
[19] Jones [9], p. 9.

the Haberler-Jones theorem. A set of assumptions in the standard two-by-two model is retained, except that factor N (say, land) is used specifically to produce commodity 1, while factor K (say, capital) is specific to industry 2, and factor L (say, labour) is employed in both industries and can move freely between industries.

At the outset it should be noted that even in the three-by-two model under consideration, the FPF for each commodity is linearly homogeneous. In Quadrant I of Figure 4, wage rate is measured vertically and the return on N, R_1, is measured horizontally, so that for a specified price of commodity 1 the FPF can be drawn as F_1F_1. On the other hand, in Quadrant II return on K, R_2, is measured along the horizontal axis in order to give the F_2F_2 curve for a specified price of commodity 2. With these commodity prices, let initial equilibrium factor prices be OA for L, OB_1 for N and OB_2 for K. These commodity and factor prices are assumed to be capable of ensuring that all factors are fully employed.[20]

Suppose that the price of commodity 2 is increased from OE_2 to OE_2'', while that of commodity 1 remains the same. If the factor of production L could not allowed to move freely between the industries immediately after the commodity price has changed, then the

FIG. 4



[20] Given the total endowments of each factor of production, there is only one set of commodity and factor prices which ensures full employment of factors with perfect factor markets. This can be shown, though not attempted here, by measuring N and K along the vertical axis and L along the horizontal axis, and by taking into account that the slope of FPFs represent ratios of factors used in production.

return to factor L in industry 2 will increase to OA'' while that of industry 1 remains at OA. Since OA'' is greater than OA and factor L has been assumed to move freely from industry to industry, some of L must flow from industry 1 to industry 2. The final situation will be at points E_1' and E_2' where the price of L is the same in both industries and each factor of production is fully employed.

The comparison of factor prices before and after the rise in the price of commodity 2 shows that: (1) the price of mobile or non-specific factor L increases from OA to OA'; (2) the return to factor N, which is specific to industry 1 whose commodity price remains unchanged, falls from OB_1 to OB_1'; (3) the return to factor K used specifically in the production of commodity 2 whose price increases, raises from OB_2 to OB_2'; and (4) the rates of increases of prices of L and K are resepcetively OA'/OA and B_2B_2'/OB_2; the former is less than the latter. These results confirm the Haberler proposition of 1936, indicated above.

What has been concluded just above is, however, in nominal or money terms. Next consider the *relative* changes in prices. The rate of increase in the price of commodity 2 is $E_2''E_2/OE_2$ which equals AA''/OA or B_2B_2''/OB_2. The rate of change in the real price of factor L in terms of commodity 2 must be (AA'/OA) *minus* (AA''/OA), that is, $-A'A''/OA$: the real price of the mobile factor is decreased. The rate of change in rental on specific factor K in terms of commodity 2 is (B_2B_2'/OB_2) minus (E_2E_2'/OE_2), which equals $B_2''B_2'/OB_2$; the real price of specific factor K increases with a rise in the price of the commodity to which the factor is specific. These results are the same as those obtained mathematically by Jones.

V. *The Factor-Price Differentials*

The preceding sections assumed that the factor markets were competitive enough to bring about the price of each factor equal between industries. Now it can be shown that the FPF technique is also capable of dealing with the cases where the factor markets are distorted. As an illustration the FPF is here applied to analysis of the effects of *exogenous* factor-price differentials in a standard two-commodity, two-factor, small-open economy.[21] Apart from the introduction of factor-price differentials the assumptions made in Sections II and III are maintained.

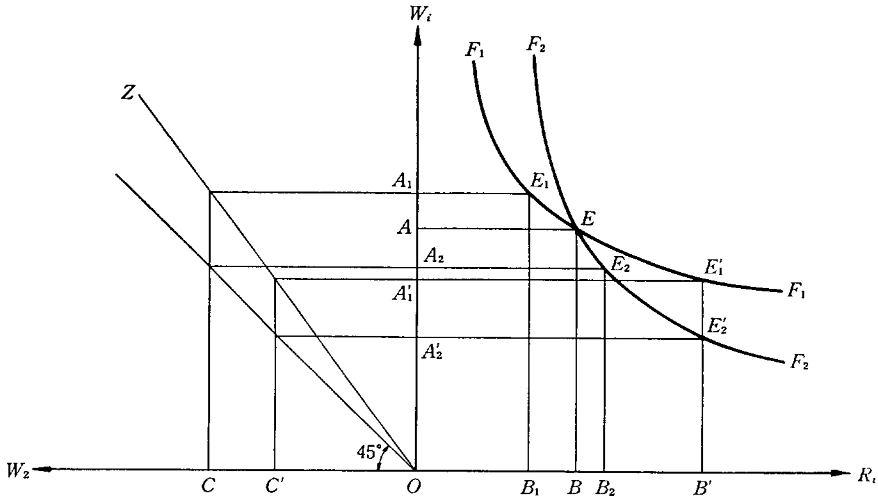
Quadrant I of Figure 5 is same as Figure 3. Without any distortion in factor markets, the equlibrium is achieved at point E, for the wage rate and the rental rate to be OA and OB respectively. Introduce now a wage differential such that $W_1/W_2=a>1$, where W_1 and W_2 are respectively the wage rates in industries 1 and 2, and a is a constant given exogenously and greater than one. The rental rate is assumed to be equal in both industries. The line OZ in Quadrant II shows the relation that $W_1=aW_2$.

To make our presentation somehow "realistic" or "vivid", assume that the labour force moves instantaneously from industry to industry, while capital is industry-specific in the "short" run.[22] The instantaneous effect of introduction of the wage differential is to reduce the employment of labour in industry 1 and to increase that in industry 2. This will lead

[21] Refer to Batra [2], Jones [10], and Magee [12], [13] and [14].

[22] An importance of the assumption made upon adjustment process is stressed by Neary [15].

FIG. 5



to an increase (or decrease) of capital-labour ratio in industry 1 (or 2). The wage rate in industry 1 will go up to, say, OA_1, while it will fall to, say, OA_2 in industry 2. As shown, of course, $OA_1/OA_2=a>1$.

But this situation can not be a "long" run equlibrium. For the rental rate in industry 1 (i.e., OB_1) is clearly lower than that in industry 2 (i.e., OB_2). Let now capital allow to move among industries, and capital will flow from industry 1 into industry 2. The final (long run) equilibrium is reached when the rental rates in the two industries become identical. This condition is satisfied where the rental rate is equal to OB' for the two industries, while the wage rates are OA_1' for industry 1 and OA_2' for industry 2.

Compare now the equlibrium situations with and without a wage differential. The wage rates for any of the two industries with the wage differential (i.e., either OA_1' or OA_2') are lower than wage rate in undistorted situation (i.e., OA). The rental rate is higher in distorted situation than in undistorted: $OB'>OB$. Conversely, it can be similarly shown that if a wage differential is in favour of industry 2 (or $W_1<W_2$), then the wage (or rental) rate will be higher (or lower) than what it would be without the wage differential.

What will be the effects of the introduction of a rental differential as well? Suppose that the factor-price differentials are such that $W_1/W_2=R_1/R_2=a$, where a is again a constant greater than one. Under this assumption it can be easily shown that the rental rate will be higher in the present situation than in the previous situation where $W_1=aW_2$, and the wage rate for each industry will be lower in the former than in the latter.[23] This might suggest that if there is a wage differential in favour of labour-intensive industry 1, then it is beneficial for the owners of capital to introduce as well a rental differential in favour of that industry. On the contrary if there are both wage and rental differentials in favour

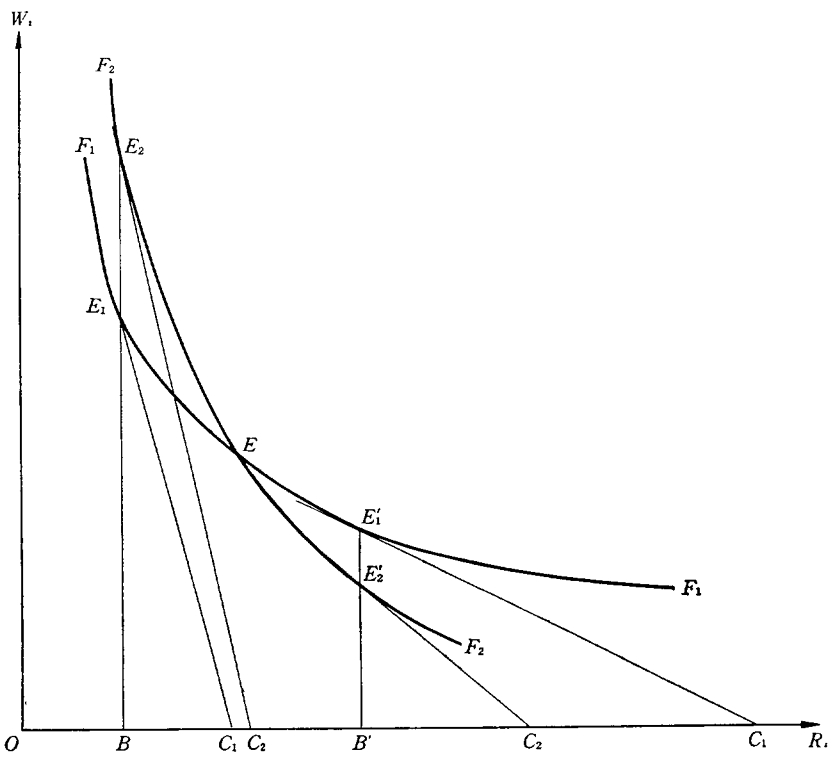
[23] Draw in Quadrant IV a rental differential line such as the wage differential line OZ in Quadrant II, and we can easily obtain the conclusion. Based upon the FPFs we can show explicitly the factor-price differentials, although it is in the Edgeworth-Bowley box diagram the wage-rental *ratio* differences that are dealt with explicitly.

of capital-intensive industry 2, then it would be advantageous for the owners of capital to remove the rental differential between industries.

Thus far the factor intensity has been used in the physical sense as the capital-labour ratio. It can be also defined, however, in the value sense, i.e., in terms of the rental incomes relative to the wage incomes in an industry. Since it has been known that the changes of outputs respond to the sign of the products of both definitions, it is important to identify the conditions under which the two definitions become different from each other.[24] On this matter we have the following theorem: (1) the necessary condition for the factor intensities to get reversed in the physical sense is that an industry pays the differential on its intensive factor; (2) the necessary condition for the reversal of factor intensities in the value sense is that an industry pays the differential on its non-intensive factor in the physical sense.[25] The reasoning behind the theorem is clear, but let us visualize it in the light of the FPF. For simplicity a wage differential alone is assumed.

Figure 6 shows the two FPFs intersecting at point E. If industry 2 pays the wage differential, the distorted equilibrium should be somewhere upward left to point E such as points E_1 and E_2. The factor intensities in the physical sense are BE_1/BC_1 for industry 1

FIG. 6



[24] See Jones [10].
[25] As a theorem it is stated in Batra [2].

and BE_2/BC_2 for industry 2: the latter is higher than the former. Clearly in the region left to point E the factor intensities in the physical sense cannot be reversed. Thus the probable region in which they may get reversed must be somewhere downward right to point E where industry 1 pays the wage differential. But even in this region the slope of the tangent to the point E_1' on the F_1F_1 curve (physical factor intensity in industry 1) is *always* less steeper than the slope of the tangent to the point E_2' on the F_2F_2 curve (that in industry 2). This is so, because, if we assume instead that the physical intensity at point E_1' is greater than that at point E_2', the physical intensity at point E should be also greater for industry 1 than for industry 2, which leads to a contradiction with the assumption made for the production functions.

Thus we can conclude that, though the necessary condition for the physical factor intensities to get reversed is for an industry to pay the differential on its intensive factor, this condition will *never be* satisfied *as long as* a single factor price differential is concerned with.[26] Furthermore under present assumption of the wage differential alone the physical factor intensity cannot get reversed for any commodity prices, since changes in commodity prices affect only the position of intersection of both FPFs but not their shapes. It should be immediately noted that if *each* industry pays the differential on its intensive factor then the physical factor intensities *may* be reversed. This case is represented by points E_1 and E_2'.

Next turn to the necessary condition for the reversal of value intensities. At the outset it is clear that if the (physical) capital-labour ratio is higher in one industry than the other, while the wage-rental ratio is lower in the former than in the latter, then the factor intensities in value sense cannot be reversed. This case is depicted at points E_1' and E_2': for the physical intensities $B'E_1'/B'C_1'<B'E_2'/B'C_2'$ and for the wage-rental ratios $E_1'B'/OB'>E_2'B'/OB'$; hence for the value intensities $OB'/B'C_1'<OB'/B'C_2'$, i.e., industry 2 has the greater value intensity than industry 1. On the contrary it is possible for the value intensities to get reversed if the wage-rental ratio is higher in the industry with a higher physical factor intensity. This possibility is shown as points E_1 and E_2: industry 1 is labour intensive in the physical sense because $BE_1/BC_1<BE_2/BC_2$, but it is capital intensive in the value sense because $OB/OC_1>OB/BC_2$.

Finally it should be noted that, if the production functions are assumed to be the Cobb-Douglass type, then the factor intensities in the value sense can never be reversed. This type of production function implies that the rental incomes relative to wage incomes in a industry is fixed. In terms of Figure 6 it means that OB/BC_1 and OB/BC_2 are both constant, and the latter is assumed to be always greater than former regardless of factor price differentials.

VI. *Conclusion*

From what has been said it follows clearly that FPF has a comparative advantage in analysis of commodity-prices / factor-prices relationships, just as an isoquant has a comparative advantage in dealing with commodity-outputs / facor-employments relationships. This note has applied the technique of the FPF to only some but basic theorems and problems

[26] See especially Magee [12].

in the pure theory of international trade. The information contained in the FPF has not been fully exploited in this note. In fact there may be scope for its application to analysis of other problems, not only in the field of international trade, but also in other fields.

The Mathematical Supplement to Section II

In order to make it more solid and rigorous that which has been said in Section II, it might be advisable to derive the FPF mathematically. First, the production function is assumed to be written as

(1) $Q_i=F_i(L_i, K_i)=L_i f_i(k_i)$,

where $f_i(k_i)=F_i(1, k_i)$ and $k_i=K_i/L_i$. And it is assumed that $f_i'=\partial f_i/\partial k_i>0$ and $f_i''=\partial^2 f_i'/\partial k_i^2<0$. Under the assumption of perfect markets the factor price is equal to its marginal value of product, i.e.,

(2) $W_i=P_i(f_i(k_i)-k_i f_i'(k_i))$,

and

(3) $R_i=P_i f_i'(k_i)$.

Divide Eq. (2) by Eq. (3) to obtain the wage-rental ratio (w_i) as

(4) $w_i=W_i/R_i=(f_i(k_i)/f_i'(k_i))-k_i$,

or

(5) $k_i=k_i(w_i)$,

i.e., the capital-labour ratio depends only upon the wage-rental ratio. From Eq. (4) we have

6) $dw_i/dk_i=-f_i f_i''/f_i'^2>0$,

which states that as the wage-rental ratio increases the capital-labour ratio will also increase.

Since the production is assumed to be linearly homogeneous, we have the following relationship:

(7) $Q_i=(\partial F_i/\partial L_i)L+(\partial F_i/\partial K_i)K_i$.

Multiplying both sides by P_i, and taking Eqs (2) and (3) into account, we have the following equation:

(8) $P_i(W_i+k_i R_i)/f_i(k_i)=G_i(W_i, R_i)$,

because k_i is a function of $w_i=W_i/R_i$. Eq. (8) gives the FPF for commodity i. That is to say, it gives the all possible combinations of W_i and R_i, which yield the specified level of commodity price P_i.

First, it can be proved that in Eq. (8) P_i is homogeneous of degree one with respect to W_i and R_i. Suppose that W_i and R_i increase by factor h. Then the wage-rental ratio w_i remains unchanged, so that from Eq. (4) or (5) the capital-labour ratio k_i and hence $f_i(k_i)$ also unchange. Thus in Eq. (8) P_i increases by the same factor h, when W_i and R_i increase by the factor h.

Secondly, differentiating Eq. (8) totally, putting P_i constant, and taking Eq. (4) into account, we obtain:

(9) $dW_i/dR_i=-k_i<0$.

In other words the slope of the FPF is negative and equals in absolute value capital-labour ratio. Furthermore, the slope of the FPF decreases in absolute value as R_i increases, because

(10) $d^2W_i/dR_i^2=(W_i/R_i)(\partial k_i/\partial w_i)>0.$

Finally the elasticity of W_i with respect to R_i equals in absolute value the relative share of factor incomes in industry i. By using Eq. (9) we have

(11) $|(W_i/R_i)(dR_i/dW_i)|=|(W_i/R_i)/(-k_i)|=(W_iL_i)/(R_iK_i).$

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