TYPOLOGY OF TRADE INTENSITY INDICES

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I. *Introduction*

In an article recently published in this journal, John E. Roemer presented four types of trade intensity indices, and developed an interesting piece of analysis.[1] It should be pointed out, however, that Kiyoshi Kojima had derived the same four indices, five in fact, in exactly the same manner as Roemer.[2] In addition, other authors have also utilized various indices of the kind to be discussed here. In most of these instances, the index used was devised by the author's inventiveness and ingenuity on an *ad hoc* basis without clear recognition of its logical connection with other studies, not to mention its theoretical foundation. In view of the multiplication of "new" inventions and discoveries in trade intensity indices of one type or another, it may be worthwhile to systematically present these indices. The approach adopted is the three-dimensional contingency-table analysis in statistics.[3]

In the next section, we shall offer an intuitive explanation of the index of geographic intensity of trade, this being one of the most frequently used, and hence, the most representative of all the indices to be considered. In section III, after pointing out the difficulties in applying the contingency-table analysis to a matrix of international trade flows, we shall derive eleven types of trade intensity indices from the analytical framework proffered. In section IV, we will briefly explore the meaning that each of these indices has, and give references to the existing studies which have made use of them. In section V, by way of an example, we will indicate the fact that there exists a certain relationship among these indices. Finally, we will summarize the analysis here in section VI.

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[1] John E. Roemer, "Extensions of the Concept of Trade Intensity," *Hitotsubashi Journal of Economics* 17, no. 1 (June 1976): 29–35. See also Roemer, *U.S.-Japanese Competition in International Markets: A Study of the Trade-Investment Cycle in Modern Capitalism* (Berkeley: University of California, Institute of International Studies, 1975), ch. 1.

[2] Kiyoshi Kojima, *Sekai keizai to Nihon bōeki* (World economy and Japan's foreign trade) (Tokyo: Keisō Shobō, 1962), ch. 7.

[3] The analysis here is based on the author's doctoral thesis: Kazutake Kunimoto, "The Contingency-Table Analysis of International Trade Flows: An Approach in Methodology" (Ph.D. dissertation, Yale University, 1975).

II. *The Index of Geographic Intensity of Trade*[4]

Let us denote by $Vi..$, $V.j.$, and $V...$ the values of country i's total exports, country j's total imports, and total world trade (exports or imports), respectively ($i, j=1, \ldots, n$). These values pertain to a given period of time, say, a year, and are expressed by a common currency, such as, the U.S. dollar.

Various factors contribute to the determination of a country's total exports and imports——the size of its economy, factor endowments, the heights of tariffs, and the distance from the major markets of the world, to name a few. Let us assume that the levels of total exports and imports of the countries in the world are determined according to their actual values as the result of the interactions of these and other factors.[5] Let us, then, imagine a hypothetical world as a frame of reference where there is no "geographic specialization" in international trade.

Put differently, the analysis here assumes that impediments (or inducements) to international trade, both tangible and intagible, can conceptually be divided into two categories: those which influence the levels of total exports and imports of the countries in the world and those which influence their geographical distribution. The hypothetical world of reference is then defined as an imaginary world in which the second category of trade impediments——factors which distort the direction of international trade flows——is absent. It should be noted that the levels of total exports and imports of the countries in the world are unchanged by the removal of these factors. The reason for this is that the second category of trade impediments is defined in such a way that it will not affect their values; it is the first category of trade impediments which is assumed to affect these values.

In the hypothetical world thus defined, it may be reasonably assumed that a country's total trade is distributed among countries according to the partner country's share in world trade. In other words, in the hypothetical world, country i's total *exports* ($Vi..$) will be allocated to country j in proportion to the latter's share in world *imports* ($V.j./V...$); and, similarly, country j's total *imports* ($V.j.$) will be allotted to country i proportionately to country i's share in world *exports* ($Vi../V...$). Symbolically, the hypothetical trade flow from country i to country j ($\overline{V}ij.$) is

$$(1) \quad \overline{V}ij.=\frac{Vi..V.j.}{V...}$$

or

$$1\text{-a)} \quad =Vi..\left(\frac{V.j.}{V...}\right)$$

or

$$(1\text{-b}) \quad =V.j.\left(\frac{Vi..}{V...}\right)$$

[4] For a more detailed analysis of the substance of this section, see Kunimoto, *op. cit.*, ch. I.

[5] This assumption is *not* crucial in the analysis to follow. Instead, the levels of total exports and imports may have been estimated. The advantages and limitations of utilizing actual trade values are carefully discussed in Edward E. Leamer and Robert M. Stern, *Quantitative International Economics* (Boston: Allyn and Bacon, 1970), pp. 157–168.

To recapitulate the argument stepwise, in the hypothetical world of reference set up above, country i first *exports* to the rest of the world where its total *exports* are distributed among the countries in the rest of the world according to their shares in world *imports*. Similarly, country j *imports* from the rest of the world, the distribution of which depends on the shares held by the countries of the rest of the world in world *exports*. As the use of the words "the rest of the world" signifies and as a country cannot trade internationally with itself, the hypothetical world of reference for country i as an exporter is composed of all the importers of the world except country i itself as an importer. The hypothetical world of reference for country j as an importer likewise comprises all the exporters of the world save country j as an exporter. This requires modification of the formula set out above. The hypothetical value of the trade flow from country i to country j is, now, from the standpoint of country i as an exporter,

$$\text{(1-a}'\text{)}\quad \bar{V}ij.\,(*)=Vi..\left(\frac{V.j.}{V...-V.i.}\right)$$

or, from the standpoint of country j as an importer,

$$\text{(1-b}'\text{)}\quad \bar{V}ij.(**)=V.j.\left(\frac{Vi..}{V...-Vj..}\right)$$

where $V.i.$ is the value of total imports of country i and $Vj..$ is the value of total exports of country j. (The asterisks * and ** in the parentheses indicate variations in the value of the same hypothetical trade flow based on different interpretations. The same rule will be applied hereafter whenever necessary.)

The actual trade flow from country i to country j ($Vij.$) will normally be different from the hypothetical value derived above because of the presence of the factors which were abstracted in order to set up the hypothetical world, i.e., the second category of trade impediments. When the deviation is expressed by their ratio, we obtain the index of geographic intensity of trade. If we use the hypothetical value before modification ($\bar{V}ij.$), the index of geographic intensity of trade from country i to country j ($Iij.$) is

$$\text{(2)}\quad Iij.=\frac{Vij.}{\bar{V}ij.}=\frac{Vij.}{Vi..}\Big/\frac{V.j.}{V...}$$

If we use instead $\bar{V}ij.(*)$ of (1-a′), the index of geographic *export* intensity of country i with country j [$Iij.(*)$] is

$$\text{(2-a}'\text{)}\quad Iij.(*)=\frac{Vij.}{\bar{V}ij.(*)}=\frac{Vij.}{Vi..}\Big/\frac{V.j.}{V...-V.i.}$$

while the index of geographic *import* intensity of country j with country i [$Iij.(**)$] is

$$\text{(2-b}'\text{)}\quad Iij.(**)=\frac{Vij.}{\bar{V}ij.(**)}=\frac{Vij.}{V.j.}\Big/\frac{Vi..}{V...-Vj..}$$

when $\bar{V}ij.(**)$ of (1-b′) is used.

The index shows unity when the actual trade flow from country i to country j is exactly the same as that found in the hypothetical world. If the trade flow is more intensive than expected, the value exceeds unity, whereas it is less than unity when the actual trade flow falls short of the expected value. The reason why such a deviation occurs is due to the presence of the factors which influence the direction of international trade flows among countries without affecting the levels of trade of the countries in the world. A discriminatory trade policy of a country, for example, affects not only the geographical distribution of its foreign trade but also the level of its total trade. These effects being considered separable, at least conceptually, the part that affects the direction of trade is removed in the hypothetical world. To give another example, the distance of a country from world markets would influence the level of its trade, while the *relative* distance from the country to its trade partners will affect its geographical distribution. The influence upon the index of discriminatory tariffs, relative distance, historical, cultural, and political affinities, the similarity or dissimilarity of commodity composition of trade, and so forth can be ascertained, for instance, by a regression analysis.[6]

In the analysis above, the hypothetical trade value was modified so as to be economically more meaningful. By this modification, however, for the same and identical trade flow from country i to country j we obtained two hypothetical values. $Vij.(*)$ of (1-a′) is the value of the hypothetical export of country i to country j viewed from the standpoint of country i as an exporter ($i=1, \ldots, n$), while $Vij.(**)$ of (1-b′) is the value of the hypothetical import of country j from country i viewed from the standpoint of country j as an importer ($j=1, \ldots, n$). Accordingly, the index of geographic export intensity given in (2-a′) can be used to compare various countries' export behavior, and the index of geographic import intensity (2-b′) to compare their import behavior, although in our formulation these two indices cannot be compared with each other.[7]

It may be pointed out that the modified indices $Iij.(*)$ and $Iij.(**)$ are formally the same as those derived by A.J. Brown.[8] While Brown's derivation was based on his own reasoning, our formulation has its theoretical foundation on the contingency-table analysis in statistics as we shall see presently. In this sense, our formulation is more general as compared with Brown's. Furthermore, in Brown's formulation, the geographic export and import intensity indices can be compared for the same country; but, within the limits of his fomulation, they cannot be compared internationally. That is to say, the geographic export and import intensity indices of country i with its trade partners can be calculated and compared. But they cannot be compared with the indices calculated for other countries.

We have thus formulated the geographic trade intensity indices in a different way from Brown's and advanced an alternative range for their applicability. The derivation of a hypothetical trade value which is consistent from the standpoints of both the exporting and importing countries is proposed by I. Richard Savage and Karl W. Deutsch.[9] Their formulation, too, is based on the contingency-table analysis, and, in the sense stated above, theoretically more satisfactory than the one presented here. However, it is much more complicated to calculate the hypothetical value under their formulation since it requires an iterative estimation procedure. On the other hand, the indices $Iij.(*)$ and $Iij.(**)$ proposed here are much simpler to calculate; and yet they take into consideration the fact that countries do not trade with themselves in the form of the modification of the denominators of the indices. For this reason, they are superior to the index $Iij.$ with no modification.

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[6] Cf. Hans Linnemann, *An Econometric Study of International Trade Flows* (Amsterdam: North-Holland Publishing Company, 1966), pp. 180–184; and Ippei Yamazawa, "Structural Changes in World Trade Flows," *Hitotsubashi Journal of Economics* 11, no. 2 (February 1971): 11–21.

[7] For more detail, see Kunimoto, *op. cit.*, pp. 56–62.

[8] A.J. Brown, *Applied Economics: Aspects of the World Economy in War and Peace* (London: George Allen & Unwin, 1947), pp. 212–226.

[9] I. Richard Savage and Karl W. Deutsch, "A Statistical Model of the Gross Analysis of Transaction Flows," *Econometrica* 28, no. 3 (July 1960): 551–572.

III. *The Analysis by a Three-Dimensional Contingency Table*

By classifying total world trade in a given year according to the exporting countries, the importing countries, and the commodities traded internationally, we can construct a three-dimensional matrix of international trade flows. If there are n countries in the world and m commodities are exchanged across their borders, this is an $(n\times n\times m)$ matrix. $Vijk$, being a representative element in the matrix, indicates the value of the trade flow from country i to country j in commodity k ($i=1, \ldots, n$; $j=1, \ldots, n$; $k=1, \ldots, m$). In the margins of the matrix, $Vij.(=\sum_{k=1}^{m} Vijk)$ denotes the value of the trade flow from country i to country j; $Vi.k(=\sum_{j=1}^{n} Vijk)$, the value of the export of country i in commodity k; $V.jk(=\sum_{i=1}^{n} Vijk)$, the value of the import of country j in commodity k; $Vi..(=\sum_{j=1}^{n}\sum_{k=1}^{m} Vijk)$, the value of the exports of country i; $V.j.(=\sum_{i=1}^{n}\sum_{k=1}^{m} Vijk)$, the value of the imports of country j; $V..k(=\sum_{i=1}^{n}\sum_{j=1}^{n} Vijk)$, the value of world trade in commodity k; and $V\ldots(=\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{m} Vijk)$, the value of total world trade, where all the values are expressed, as mentioned before, by a common currency.

We shall now apply the contingency-table analysis in statistics to the three-dimensional matrix of world trade thus constructed.[10] Before doing so, two problems must be considered. First, in the conventional analysis by a contingency table, the table to be analyzed does not have any *a priori* zero entry. On the other hand, in an international trade matrix, the elements which represent internal transactions of the countries in the world, i.e., $Viik$ and $Vii.(=\sum_{k=1}^{m} Viik)$ $(i=1,\ldots, n)$, are zero by definition. Secondly, in the conventional contingency-table analysis, the application is to be made to a frequency table, whereas in case of a trade matrix, the cell entry $Vijk$ indicates the value of trade, and *not* the number of transactions, from country i to country j in commodity k. The first problem was recognized in the traditional trade intensity analysis as well which was not based on the contingency-table analysis. Brown, for instance, modified the denominators of his indices in the manner we saw above. In what follows, we shall derive various types of trade intensity indices, first, ignoring the fact that countries do not trade with themselves internationally, and then give examples of partial modification to reflect this fact by referring to the existing

[10] Any textbook of statistics has at least a section or two on the contingency table. See, for example, Harald Cramér, *Mathematical Methods of Statistics* (Princeton, N.J.: Princeton University Press, 1946), pp. 441–452; G. Udny Yule and Maurice G. Kendall, *An Introduction to the Theory of Statistics*, 14th ed., revised and enlarged (New York: Hafner Publishing Company, 1950), chs. 1, 2, and 3; and Maurice G. Kendall and Alan Stuart, *The Advanced Theory of Statistics*, vol. 2, *Inference and Relationship*, 3rd ed. (London: Charles Griffin & Company, 1973), ch. 33. A series of articles by Leo A. Goodman and William H. Kruskal are also useful. Goodman and Kruskal, "Measures of Association for Cross Classifications," *Journal of the American Statistical Association* 49 (December 1954): 732–764; *idem*, "Measures of Association for Cross Classifications. II: Further Discussion and Reference," *ibid.*, 54 (March 1959): 123–163; and, *idem*, "Measures of Association for Cross Classifications. III: Approximate Sampling Theory," *ibid.*, 58 (June 1963): 310–364.

studies that have done so. As for the second problem, we will briefly touch on its significance in section V.[11]

VARIOUS INDEPENDENCE HYPOTHESES, HYPOTHETICAL TRADE VALUES, AND INTERNATIONAL TRADE INTENSITY INDICES

line	column A Independence Hypotheses	column B Hypothetical Trade Values	column C Trade Intensity Indices
	Ha: X, M, and C are mutually independent,		
1	$Pijk=Pi..P.j.P..k$	$\bar{V}ijk(a)=\frac{Vi..V.j.V..k}{V...V...}$	$Iijk(a)=\frac{Vijk}{\bar{V}ijk(a)}=\frac{V...V...Vijk}{Vi..V.j.V..k}$
	Hb: one classification is independent of the other two together;		
	$Hb1$: C is independent of X and M together,		
2	$Pijk=Pij.P..k$	$\bar{V}ijk(b1)=\frac{Vij.V..k}{V...}$	$Iijk(b1)=\frac{Vijk}{\bar{V}ijk(b1)}=\frac{Vijk}{Vij.}\Big/\frac{V..k}{V...}$
	$Hb2$: M is independent of X and C together,		
3	$Pijk=Pi.kP.j.$	$\bar{V}ijk(b2)=\frac{Vi.kV.j.}{V...}$	$Iijk(b2)=\frac{Vijk}{\bar{V}ijk(b2)}=\frac{Vijk}{Vi.k}\Big/\frac{V.j.}{V...}$
	$Hb3$: X is independent of M and C together,		
4	$Pijk=P.jkPi..$	$\bar{V}ijk(b3)=\frac{V.jkVi..}{V...}$	$Iijk(b3)=\frac{Vijk}{\bar{V}ijk(b3)}=\frac{Vijk}{V.jk}\Big/\frac{Vi..}{V...}$
	Hc: the margins of the two classifications are independent;		
	$Hc1$: X and M are independent in the margin,		
5	$Pij.=Pi..P.j.$	$\bar{V}ij.(c1)=\frac{Vi..V.j.}{V...}$	$Iij.(c1)=\frac{Vij.}{\bar{V}ij.(c1)}=\frac{Vij.}{Vi..}\Big/\frac{V.j.}{V...}$
	$Hc2$: X and C are independent in the margin,		
6	$Pi.k=Pi..P..k$	$\bar{V}i.k(c2)=\frac{Vi..V..k}{V...}$	$Ii.k(c2)=\frac{Vi.k}{\bar{V}i.k(c2)}=\frac{Vi.k}{Vi..}\Big/\frac{V..k}{V...}$
	$Hc3$: M and C are independent in the margin,		
7	$P.jk=P.j.P..k$	$\bar{V}.jk(c3)=\frac{V.j.V..k}{V...}$	$I.jk(c3)=\frac{V.jk}{\bar{V}.jk(c3)}=\frac{V.jk}{V.j.}\Big/\frac{V..k}{V...}$
	Hd: two classifications are independent in each of the third classification;		
	$Hd1$: X and M are independent in each of C,		
8	$Pijk=\frac{Pi.kP.jk}{P..k}$	$\bar{V}ijk(d1)=\frac{Vi.kV.jk}{V..k}$	$Iijk(d1)=\frac{Vijk}{\bar{V}ijk(d1)}=\frac{Vijk}{Vi.k}\Big/\frac{V.jk}{V..k}$
	$Hd2$: X and C are independent in each of M		
9	$Pijk=\frac{Pij.P.jk}{P.j.}$	$\bar{V}ijk(d2)=\frac{Vij.V.jk}{V.j.}$	$Iijk(d2)=\frac{Vijk}{\bar{V}ijk(d2)}=\frac{Vijk}{Vij.}\Big/\frac{V.jk}{V.j.}$
	$Hd3$: M and C are independent in each of X,		
10	$Pijk=\frac{Pij.Pi.k}{Pi..}$	$\bar{V}ijk(d3)=\frac{Vij.Vi.k}{Vi..}$	$Iijk(d3)=\frac{Vijk}{\bar{V}ijk(d3)}=\frac{Vijk}{Vij.}\Big/\frac{Vi.k}{Vi..}$
	He: there is no three-factor (or no second-order) interaction between X, M, and C,		
11	$Pijk=\frac{Pij.Pi.kP.jk}{P..kP.j.Pi..}$	$\bar{V}ijk(e)=V...\frac{Vij.Vi.kV.jk}{V..kV.j.Vi..}$	$Iijk(e)=\frac{Vijk}{\bar{V}ijk(e)}=\frac{Vijk}{V...\frac{Vij.Vi.kV.jk}{V..kV.j.Vi..}}$

[11] These problems are discussed in detail in Kunimoto, *op. cit.*

In column A of the table, we have presented five types of independence-no-interaction hypotheses——eleven variations in all——most conventional in the three-dimensional contingency-table analysis.[12] The following notations are used there:

Pijk is the probability (or the "theoretical tendency"[13]) that country *i* exports commodity *k* to country *j*;

$Pij.(=\sum_{k=1}^{m} Pijk)$ is the probability that country i exports to country j;

$Pi.k(=\sum_{j=1}^{n} Pijk)$ is the probability that country i exports commodity k;

$P.jk(=\sum_{i=1}^{n} Pijk)$ is the probability that country j imports commodity k;

$Pi..(=\sum_{j=1}^{n}\sum_{k=1}^{m} Pijk)$ is the probability that country i exports to the world;

$P.j.(=\sum_{i=1}^{n}\sum_{k=1}^{m} Pijk)$ is the probability that country j imports from the world; and

$P..k(=\sum_{i=1}^{n}\sum_{j=1}^{n} Pijk)$ is the probability that commodity k is traded internationally.

By definition, $\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{m} Pijk=1$.[14] In the table, the classifications of world trade by exporting countries, importing countries, and commodities traded are indicated by *X*, *M*, and *C*, respectively, for the sake of simplicity.

Take the hypothesis *Hc*1 as an example. This hypothesis assumes that the classification of world trade by exporting countries (*X*) is independent of the classification by importing countries (*M*). Under this hypothesis, the probability that country *i* exports to country *j* (*Pij.*) is the product of the probability that country *i* exports to the world (*Pi..*) and the probability that country *j* imports from the world (*P.j.*). That is,

$$Pij.=Pi..P.j.$$

(see line 5 of column A of the table). As we shall see in the next section, other hypotheses can be interpreted similarly.

In column B of the table, the hypothetical trade values under the respective independence hypotheses listed in column A are presented, and in column C, as the ratios of the actual and hypothetical values, the corresponding indices of trade intensities are presented, where the respective hypotheses are indicated in the parentheses.[15] Take the hypothesis *Hc*1 again. The hypothetical trade value $\overline{Vij}.(c1)$ under this hypothesis can be obtained by substituting $\overline{Vij}.(c1)\,/\,V...$ for $Pij.$, $Vi..\,/V...$ for $Pi..$, and $V.j.\,/V...$ for $P.j.$ in the relation given in line 5 of column A, namely,

[12] Cf., e.g., M.W. Birch, "Maximum Likelihood in Three-Way Contingency Tables," *Journal of the Royal Statistical Society*, series B (methodological) 25, no. 1 (1963): 220.

[13] Cf. Hayward R. Alker, Jr., "An IBM 709 Program for the Gorss Analysis of Transaction Flows," *Behavioral Science* 7, no. 4 (October 1962): 498; and Leo A. Goodman, "A Short Computer Program for the Analysis of Transaction Flows," *ibid.*, 9, no. 2 (April 1964): 180.

[14] The underlying structure of world trade (population) is assumed to be stable over some period of time, and, hence, the actual trade of any given year is a sample of that structure. Another sample will be obtained for another year. For the economic meaning of the probabilities shown in the text, see Kunimoto, *op. cit.*

[15] All the trade intensity indices in column C of the table take the value between zero and infinity around unity.

$$\overline{V}ij.(c1)=V\ldots\frac{Vi..}{V\ldots}\frac{V.j.}{V\ldots}=\frac{Vi..V.j.}{V\ldots}$$

(see line 5 of column B of the table).[16] It should be noticed that this is the same as *Vij.* of (1) we obtained in section II. The intensity index *Iij.*(*c*1) shown in line 5 of column C of the table is, therefore, the same as *Iij.* of (2). Thus, the index of geographic intensity of trade we derived previously is nothing but the trade intensity index under the hypothesis *Hc*1. The modification is needed, of course, so as to take into account the fact that a country does not trade with itself, as we did in the previous section.

IV. *Typology of Trade Intensity Indices*

We shall now inquire the meaning of the individual independence hypotheses, hypothetical trade values, and trade intensity indices given in the table. At the same time, from the analytical framework of the three-dimensional contingency table we shall examine some of the existing studies which have made use of these trade intensity indices. The studies dealt with here, of course, do not cover all the related studies. Moreover, much of the economic implications contained in these studies are not taken into account. For this reason, the articles cited below should also be consulted.

Before beginning our analysis, one work must be mentioned. The analysis by Kojima is essentially an extension of Brown's analysis referred to in section II.[17] As we noted at the outset, following Brown, Kojima proposed five types of indices of export intensities. Viewed from the standpoint of country *i* as an exporter, they are basically the same as *Iij.* (*c*l), *Ii.k* (*c*2), *Iijk* (*b*3), *Iijk* (*d*2), and *Iijk* (*d*1) in column C of the table.[18] Although he did not actually set out indices of import intensities, viewed from the standpoint of country *j* as an importer, they would be essentially the same as our *Iij.*(cl), *I.jk*(*c*3), *Iijk*(*b*2), *Iijk*(*d*3), and *Iijk*(*d*l) in the same column.[19] Kojima also sought to show the interre-

[16] Other hypothetical trade values in column B of the table can be obtained in a similar way. If we ignore the fact that *Viik*=0 (i=1, . . . , n), and assume that all the international transactions are of equal size, all the hypothetical values in column B except $\overline{V}ijk(e)$ are maximum-likelihood estimates. See Kunimoto, *op. cit.*, pp. 149–150.

[17] Kojima, *op. cit.*, ch. 7.

[18] These indices correspond to Kojima's Aj, Bi, Ci, D_{ij}... Dij, and Ej^{i}, respectively, except that in his formulation partial modifications were carried out on their denominators wherever pertinent—i.e., for Aj, Bi, Cij, and Ej^{i} (*ibid.*).

Roemer's four indices, αij, σi^{h}, αij^{h}, and σij^{h}, correspond to Kojima's Aj, Bi, Ej^{i}, and Dij, respectively (Roemer, "Extensions of the Concept of Trade Intensity," pp. 29–30). Apparently, he, too, derived these indices as an extension of Brown's analysis (see *ibid.*, p. 29, n. 1), and, hence, their use should have been confined to the analysis of a single country for the sake of consistency as Brown had explicitly warned (Brown, *op. cit.*). Unlike Kojima, however, Roemer ignored this limitation and sought to trace back actual trade values (*Vijk*'s) from his four indices *calculated for all the countries in the world.* In the Brown-type formulation, this limitation should have never been ignored unless there are only two countries in the world —Roemer assumed three—or all the export and import totals of the countries in the world are of equal size —something he did not assume (see Kunimoto, *op. cit.*, p. 37, n. 19). Even if he had used trade intensity indices of the Savage-Deutsch type formulation in a three-dimensional case, his attempt would probably have been futile, since the estimation of globally consistent hypothetical trade values itself requires an iterative procedure. Interesting as his intention may have been, the appropriateness of Roemer's analysis must be questioned.

[19] It is apparent that he had these indices, too, in mind (see Kojima, *op. cit.*, p. 308). In fact, he used some of them elsewhere as we shall see below.

lationships among the trade intensity indices he utilized. We will briefly refer to this problem in the next section.

In what follows, we shall proceed in the order of the hypotheses listed in the table. Note again that the classifications of world trade by exporting countries, importing countries, and commodities traded internationally are denoted by X, M, and C, respectively.

Ha: X, M, and C are mutually independent

The first hypothesis (*Ha*) assumes that the geographical export structure (X), the geographical import structure (M), and the commodity composition (C) of world trade are independent of each other. Under this hypothesis, the probability that country i exports commodity k to country j ($Pijk$) can be reduced to the product of the probability that country i exports to the world ($Pi..$), the probability that country j imports from the world ($P.j.$), and the probability that commodity k is traded in the world market ($P..k$):

$$Pijk = Pi..P.j.P..k$$

(see line 1 of column A of the table). In this situation, we obtain the hypothetical value of trade from country i to country j in commodity k [$\bar{V}ijk\,(a)$] as the product of (1) the value of total world trade ($V...$), (2) country i's export share in world trade ($Vi..\,/\,V...$), (3) country j's import share in world trade ($V.j.\,/\,V...$), and (4) the share of commodity k in world trade ($V..k\,/\,V...$). That is,

$$\bar{V}ijk\,(a) = V...\frac{Vi..}{V...}\,\frac{V.j.}{V...}\,\frac{V..k}{V...} = \frac{Vi..\,V.j.\,V..k}{V...\,V...}$$

(see line 1 of column B). Actually, the geographical export structure, the geographical import structure, and the commodity composition of world trade are not mutually independent; and, therefore, the observed trade value ($Vijk$) would normally be different from the hypothetical value derived above. When the deviation of the two values is expressed by their ratio, we obtain the trade intensity index under hypothesis *Ha*:

$$Iijk\,(a) = \frac{Vijk}{\bar{V}ijk\,(a)} = \frac{V...}{Vi..}\,\frac{V...}{V.j.}\,\frac{Vijk}{V..k}$$

(see line 1 of column C). If the index $Iijk(a)$ is greater or less than unity, it indicates that the three classifications of world trade are, in fact, not mutually independent.

This index was used by J.D.A. Cuddy for the projection of future world trade.[20]

Hb1: C is independent of X and M together

Under the hypothesis *Hb*1, the commodity composition of world trade is assumed to be independent of the geographical trade structure of world exports and imports. As is shown in line 2 of column A of the table, this hypothesis can be expressed as

$$Pijk = Pij.P..k$$

In this case, the hypothetical value of the trade flow from country i to country j in commodity k [$\bar{V}ijk\,(b1)$] is

$$\bar{V}ijk\,(b1) = \frac{Vij.\,V..k}{V...}$$

(see line 2 of column B); and any deviation of the actual from the hypothetical trade values would be indicated by the value of the index

[20] J.D.A. Cuddy, "A Note on Projections of International Trade Based on Coefficients of Trade Intensity," *Economic Journal* 83 (December 1973): 1222–1235, especially equation (3), p. 1224.

$$Iijk(b1)=\frac{Vijk}{\bar{V}ijk(b1)}=\frac{Vijk}{Vij.}\Big/\frac{V..k}{V...}$$

(line 2, column C) being greater or less than unity. In other words, the denominator of the index ($V..k\,/\,V...$) indicates the importance of commodity k in total world trade, while the numerator ($Vijk\,/\,Vij.$) indicates its importance in the trade from country i to country j. As the ratio of the two, the index $Iijk$ ($b1$) shows the extent to which commodity k is more (or less) intensively traded from country i to country j as compared with the world market.

The use of the index $Iijk(b1)$ was proposed by Ippei Yamazawa, but he did not carry out its actual calculation.[21]

Hb2: M is independent of X and C together

The third hypothesis $Hb2$ assumes that the geographical import structure of the world is independent of its geographical export structure and the commodity composition of world trade. Given this hypothetical situation as a frame of reference (i.e., the hypothetical world of reference under the hypothesis $Hb2$), the index

$$Iijk(b2)=\frac{Vijk}{\bar{V}ijk(b2)}=\frac{Vijk}{Vi.k}\Big/\frac{V.j.}{V...}$$

(see line 3 of column C of the table) indicates the deviation from this hypothesis of the trade flow from country i to country j in commodity k.

We have noted that $Iijk(b2)$ was implied in Kojima's analysis, although he did not explicitly set out the formula. Had he done so, it would have been

$$Iijk(b2^{**})=\frac{Vijk}{V.j.}\Big/\frac{Vi.k}{V...-Vj..}$$

viewed from the standpoint of country j as an importer. The index can be interpreted as indicating the degree of similarity (or dissimilarity) between country j's import demand structure by sources and commodities imported ($Vijk/V.j.$) and the export supply structure of the rest of the world [$Vi.k\,/\,(V...-Vj..)$].

Hb3: X is independent of M and C together

The hypothesis $Hb3$ assumes that the geographical export structure of the world is independent of its geographical import structure and the commodity composition of world trade (i.e., $Pijk=P.jkPi..$ in line 4 of column A of the table).

The hypothetical trade value under this hypothesis [$\bar{V}ijk$ ($b3$)] is

$$\bar{V}ijk(b3)=\frac{V.jkVi..}{V...}$$

(line 4 of column B); and the trade intensity index

$$Iijk(b3)=\frac{Vijk}{\bar{V}ijk(b3)}=\frac{Vijk}{V.jk}\Big/\frac{Vi..}{V...}$$

(line 4, column C) indicates the deviation of the trade flow from country i to country i in commodity k from the hypothesis $Hb3$.

The index $Iijk(b3)$ is essentially one of Kojima's five indices of export intensities. The actual formula he used was

[21] Ippei Yamazawa, "Intensity Analysis of World Trade Flow," *Hitotsubashi Journal of Economics* 10, no. 2 (February 1970): 61–90, especially equation (20), p. 70.

$$Iijk(b3^*)=\frac{Vijk}{Vi..}\Big/\frac{V.jk}{V...-V.i.}$$

viewed from the standpoint of country i as an exporter. The numerator of the index ($Vijk\,/\,Vi..$) can be interpreted as indicating country i's export supply structure by commodities exported and destinations, and the denominator $[V.jk\,/\,(V...-V.i.)]$ as the import demand structure of the rest of the world. More will be said about this index in the next section.

*Hc*1: *X and M are independent in the margin*

Under the hypothesis *Hc*1, the geographical export structure of the world is assumed to be independent of the geographical import structure. In this situation, we obtain the hypothetical trade value $\bar{V}ij.(c1)$ and the trade intensity index $Iij.(c1)$ in line 5 of the table. We have already discussed this case in the previous two sections.[22]

*Hc*2: *X and C are independent in the margin*

Much attention has been given to this case. The international trade matrix in which we are interested is the one where total world trade is classified according to the exporting countries and the commodities traded internationally. Since $Vi.i$ is the value of the export of country i in commodity i, it is not *a priori* zero. Thus, in contrast to the case of *Hc*1 above, the direct application of the contingency-table analysis is possible.

The hypothesis *Hc*2 assumes that the geographical export structure of the world is independent of the commodity composition of world trade. Any deviation from this hypothetical situation (i.e., the hypothetical world of reference under the hypothesis *Hc*2) will be indicated by the value of the index $Ii.k(c2)$ over or under unity.

Hisao Kanamori used the index $Ii.k(c2)$ under the name of the (export) specialization index (tokka keisū) and it has been extensively used by Japanese government economists.[23]

Another interesting example is offered by Bela Balassa who termed $Ii.k(c2)$ the index of the relative (export) share and used it as a basis of his investigation of "revealed" com-

[22] The index $Iij.(c1)$ and its variants have been extensively used. To give some of examples: Brown, *op. cit.*, pp. 212–226; Shinichi Ichimura, "Nihon keizai no kōzōteki fukinkō to sekai bōeki no kōzō (The structural disequilibrium of Japanese economy and the structure of world trade), *Keizai Bunseki*, no.16 (1954), pp. 1–20; Kojima, *Nihon bōeki to keizai hatten* (Japan's foreign trade and economic development) (Tokyo: Kunimoto Shobō, 1958), pp. 13–39; *idem*, *Sekai keizai to Nihon bōeki* (World economy and Japan's foreign trade), ch. 7; *idem*, "Japan's Trade Policy," *Economic Record* 41 (March 1965): 54–57; *idem*, *Japan and a Pacific Free Trade Area* (Berkeley and Los Angeles: University of California Press, 1971), ch. 1; United Nations, *Economic Survey of Asia and the Far East*, 1957 (58. II. F. 1), (1958), p. 61, n. 1; Peter D. Drysdale, "Japanese Australian Trade: An Approach to the Study of Bilateral Trade Flows," (Ph. D. dissertation, Australian National University, 1967); *idem*, "Japan, Australia, New Zealand: The Prospect for Western Pacific Economic Integration," *Economic Record* 45 (September 1969): 321–342; Donald R. Sherk, "The Tendency toward Regionalism in the Pacific Trade Basin," *Developing Economics* 7, no. 3 (September 1969): 332–350; Yamazawa, "Intensity Analysis of World Trade Flow"; and Roemer, *U.S.-Japanese Competition in International Markets*, ch. 1.

[23] Hisao Kanamori, "Bōeki jiyūka to hikaku seisansei no kōzō (Trade liberalization and the structure of comparative productivity), *Keizai Hyōron* 9, no. 4 (March 1960): 14–24; *idem*, *Exports of Manufactures and Industrial Development of Japan*, United Nations Conference on Trade and Development (E / Conf. 46 / P / 12) (Geneva, 1964); and Japan, Economic Planning Agency, comp., *Economic Survey of Japan (1959–60)* (Tokyo: Japan Times), pp. 44–46 and pp. 256–269.

parative advantage.[24] Several remarks are in order on his analysis. First, his analysis was confined to seventy-four categories of manufactured goods, while our presentation above covers all the commodities internationally traded. Second, he calculated his index of the relative share for six countries: the United States, Canada, the United Kingdom, Sweden, Japan, and the European Common Market (EEC)——the six (original) EEC countries being regarded as a unit. Thirdly, the index he actually used is

$$Ii.k(c2')=\frac{Vi.k}{Vi..}\bigg/\frac{V_I.k}{V_I..}$$

where $Vi..=\sum_k Vi.k$, $V_I.k=\sum_i Vi.k$, and $V_I..=\sum_i\sum_k Vi.k$ and the summations of i and k are, respectively, extended over those countries above enumerated and over those manufactured goods he considered, while our index is

$$Ii.k(c2)=\frac{Vi.k}{Vi..}\bigg/\frac{V..k}{V...}$$

(For simplicity's sake, we shall assume here, and only here, that $Vi..$ denotes the value of the exports from country i of all the manufactured goods in world trade. The analysis here can be extended to a more general case without much difficulty.) Between the two indices, the following relation holds:

$$Ii.k(c2)=\frac{Vi.k}{Vi..}\bigg/\frac{V..k}{V...}$$
$$=\left(\frac{Vi.k}{Vi..}\bigg/\frac{V_I.k}{V_I..}\right)\left(\frac{V_I.k}{V_I..}\bigg/\frac{V..k}{V...}\right)$$

or

$$(3)\quad Ii.k(c2')=\frac{Ii.k(c2)}{I_I.k(c2'')}$$

where

$$I_I.k(c2'')=\frac{V_I.k}{V_I..}\bigg/\frac{V..k}{V...}=\frac{V_I.k}{\overline{V}_I.k(c2'')}$$

which, in turn, is

$$=\frac{V_I.k}{\sum_i \overline{V}i.k(c2)}=\frac{1}{\sum_i\left(\frac{Vi.k}{V_I.k}\right)\frac{1}{Ii.k(c2)}}$$

That is to say, $I_I.k(c2'')$ is a weighted harmonic average of $Ii.k(c2)$ with the weights $Vi.k/V_I.k$. (Note that $\sum_i Vi.k=V_I.k$.) Thus, if we use his phrasing, Balassa's index $Ii.k(c2')$ may be interpreted as revealing country i's comparative advantage (or disadvantage) in the export of commodity k as indicated by $Ii.k(c2)$ *in comparison with* that of the countries in group (or region) I taken together including country i $[I_I.k(c2'')]$.

Finally, he calculated the index $Ii.k(c2')$ for two three-year periods (1953-55 and 1960-62) and combined the results into one index. His index of relative export performance

[24] Bela Balassa, "Trade Liberalisation and 'Revealed' Comparative Advantage," *Manchester School of Economic and Social Studies* 33, no. 2 (May 1965) : 99–123. See also Balassa, *Trade Liberalization among Industrial Countries: Objects and Alternative* (New York: McGraw Hill Book Company, 1967), ch. 4 and appendix to ch. 4; *idem*, Introduction to *Studies in Trade Liberalization*: *Problems and Prospects for the Industrial Countries*, by Bale Balassa and associates (Baltimore: Johns Hopkins Press, 1967), pp. 3–14; and Thomas G. Parry "Trade and Non-Trade Performance of U.S. Manufacturing Industry: 'Revealed' Comparative Advantage," *Manchester School of Economic and Social Studies* 43, no. 2 (June 1975): 158–172.

$(EPi.k)$ is

$$EPi.k=\frac{1}{2}\Big[Ii.k(c2')^1+Ii.k(c2')^1\frac{Ii.k(c2')^1}{Ii.k(c2')^0}\Big]$$
$$=\frac{1}{2}[Ii.k(c2')^0+Ii.k(c2')^1]\frac{Ii.k(c2')^1}{Ii.k(c2')^0}$$

where superscripts 0 and 1, respectively, refer to the two periods he considered.

Our last example under the heading of *Hc*2 is from Kojima.[25] His index of country i's commodity export intensity in commodity k is

$$Ii.k(c2^*)=\frac{Vi.k}{Vi..}\Big/\frac{V..k-V.ik}{V...-V.i.}$$

By this index, he compares country i's commodity *export* structure $(Vi.k\,/\,Vi..)$ with the commodity *import* structure of the rest of the world $[(V..k-V.ik)\,/\,(V...-V.i.)]$.[26]

Note the difference between Kojima's index $Ii.k(c2^*)$ and Balassa's index $Ii.k(c2')$.[27] The latter's denominator $(V_I.k\,/\,V_I..)$ is the commodity *export* structure of region I to which country i belongs. In particular, if region I covers all the countries of the world (including country i), $V_I.k\,/\,V_I..=V..k\,/\,V...$, that is, $I_I.k(c2'')=1$, and hence $Ii.k(c2')=Ii.k(c2)$. Thus, in Balassa's interpretation which is also Kanamori's, $Ii.k(c2)$ is an indicator of country i's relative comparative advantage *vis-à-vis* the world average in the export of commodity k.[28]

It may be noted, however, that $V..k\,/\,V...$ $(k=1,\ldots,m)$ indicates not only the commodity export structure of the world but also its commodity import structure.[29] This leads to the second difference between Balassa's and Kanamori's analyses, on the one hand, and Kojima's analysis, on the other. While Balassa and Kanamori put more emphasis on the international comparison of the export behavior of various countries in the commodities traded internationally, in Kojima's analysis, emphasis was placed on the individual countries' export performance in various commodities.

*Hc*3: *M and C are independent in the margin*

The hypothesis *Hc*3 assumes that the geographical import structure of the world is independent of the commodity composition of world trade. In a formal sense, this case is symmetrical to the previous case. We are now interested in the import behavior of the countries in the world with regard to the commodities traded internationally, rather than

[25] Kojima, *Sekai keizai to Nihon bōeki* (World economy and Japan's foreign trade), ch. 7.

[26] The index $Ii.k(c2^*)$ was later used by Drysdale and Roemer. See Drysdale, "Japanese Australian Trade"; *idem*, "Japan, Australia, New Zealand"; and Roemer, *op. cit.*, ch. 1.

[27] To make the comparison possible, from now on we shall again assume that $k=1,\ldots,m$, or that all the commodities traded internationally are under considertaion.

[28] Kojima recognized the conceptual difference between his formulation in Kojima, *op. cit.*, and Kanamori's and Balassa's formulation. See, for example, Kojima, "Kōgyōhin bōeki no kata to ketteiin" (Patterns and determinants of industrial trade), *Keizai seichō to bōeki kōzō* (Economic growth and the structure of foreign trade), by Kiyoshi Kojima, Takuji Shimano, and Fukutarō Watanabe (Tokyo: Keisō Shobō, 1968), p. 22, n. 11. Some of his Kanamori-Balassa type analyses are Kojima, "Japan's Trade Policy"; *idem*, "Trends in Exports of Manufactured and Semi-Manufactured Goods from Developing Countries," *Hitotsubashi Journal of Economics* 8, no. 1 (June 1967): 1–26, which mainly uses the index $I_I.k\ (c2'')$; and *idem*, "Structure of Comparative Advantage in Industrial Countries: A Verification of the Factor-Proportions Theorem," *ibid.*, 11, no. 1 (June 1970): 1–29. These analyses are followed by Yamazawa, "Intensity Analysis of World Trade Flow"; and *idem*, "Structural Changes in World Trade Flows."

[29] World export of commodity k is world import of the commodity by definition.

their export behavior in the same regard. Thus, what was said above applies here as well when this symmetry is duly accounted for. For example, Kojima's modification of the intensity index $I.jk(c3)$ would be

$$I.jk(c3^{**})=\frac{V.jk}{V.j.}\bigg/\frac{V..k-Vjk.}{V...-Vj..}$$

viewed from the standpoint of country j as an importer.[30]

It may be noted that, in spite of this formal symmetry, Kanamori and Balassa did not propose the import specialization index and the index of the relative import share, respectively.[31] Their asymmetrical treatment of exports and imports poses an interesting empirical as well as theoretical question.[32]

*Hd*1: *X and M are independent for each C*

The hypothesis *Hd*1 assumes that in the trade of commodity k ($k=1, \ldots, m$), the geographical export structure of the world is independent of its geographical import structure (see $Pijk=Pi.kP.jk/P..k$ in line 8 of column A of the table). This hypothesis may be compared with the hypothesis *Hc*1 which assumes that the geographical export structure of world trade is independent of its geographical import structure (see $Pij.=Pi..P.j.$ in line 5 of column A). Any deviation from this hypothetical world of reference under the hypothesis *Hd*1 is indicated by the value of the index $Iijk(d1)$ greater or less than unity. In order to take into account the fact that a country does not export a commodity to itself internationally, the index necessitates modification. With regard to the export of country i, it will be

$$Iijk(d1^{*})=\frac{Vijk}{Vi.k}\bigg/\frac{V.jk}{V..k-V.ik}$$

and with regard to the import of country j, it will be

$$Iijk(d1^{**})=\frac{Vijk}{V.jk}\bigg/\frac{Vi.k}{V...-Vj.k}$$

The modified indices were utilized by Kojima and others.[33]

*Hd*2: *X and C are independent for each M*

The matrix with which we are concerned now is a two-dimensional matrix in which

[30] The index $I.jk(c3)$ was used by Yamazawa, "Intensity Analysis of World Trade Flow"; and *idem*, "Structural Changes in World Trade Flows." In addition, Drysdale defined the trade intensity index under the hypothesis *Hc*3 as

$$I.jk(c3^{*})=\frac{V.jk}{V.j.}\bigg/\frac{V..k-V.ik}{V...-V.i.}$$

from the standpoint of country i as an exporter. See Drysdale, "Japanese Australian Trade"; and *idem*, "Japan, Australia, New Zealand."

[31] In other words, they opted for the hypothesis *Hc*2, rather than the hypothesis *Hc*3. Balassa's reasoning is that "..., imports will be affected by intercountry differences in taste as well as by interindustry disparities in the degree of protection" (Balassa, "Trade Liberalisation and 'Revealed' Comparative Advantage," p. 103). "On the other hand, as long as all exporters are subject to the same tariff, data on relative export performance are not distorted by differences in the degree of tariff protection" (*ibid.*, p. 104).

[32] This same problem was investigated in a slightly different guise in Michael Michaely, "The Shares of Countries in World Trade," *Review of Economics and Statistics* 42, no. 3, pt. 1 (August 1960), pp. 307–317.

[33] Kojima, "Sekai keizai to Nihon bōeki" (World economy and Japan's foreign trade), ch. 7; Drysdale, "Japanese Australian Trade"; *idem*, "Japan, Australia, New Zealand"; and Roemer, *op. cit.* On the other hand, the unmodified index $Iijk(d1)$ was used by Yamazawa, "Intensity Analysis of World Trade Flow"; and *idem*, "Structural Changes in World Trade Flows."

a country's imports are classified according to their countries of orgin and the commodities imported. For country j's imports ($j=1,\ldots, n$), the trade intensity index in this case is $Iijk(d2)$ as defined in line 9 of column C of the table. It indicates the deviation from the hypothesis that country j's geographical import structure is independent of its commodity composition of imports. When viewed from the standpoint of country i as an exporter, it is one of five indices of export intensities used by Kojima.[34] And, elsewhere, he called it the index of the commodity intensity of bilateral trade, again seen from the point of view of country i as an exporter.[35]

*Hd*3: *M and C are independent for each X*

The matrix of our concern under the hypothesis *Hd*3 is a two-dimensional matrix in which a country's exports are classified according to their countries of destination and the commodities exported. Under the hypothesis *Hd*3, a country's geographical export structure is assumed to be independent of its commodity composition of exports. For the exports of country i ($i=1, \ldots, n$), the trade intensity index $Iijk\,(d3)$ is given in line 10 of column C of the table. This index was implied in Kojima's analysis and he later used it under the name of the index of the regional bias in country i's (export) trade.[36]

He: there is no three-factor interaction between X, M, and C

The three-factor interaction here refers to the interaction among the three kinds of classification of world trade (X, M, and C).[37] As far as the author is aware, the hypothesis *He* has never been explicitly applied to the analysis of international trade flows. We shall, therefore, discuss this hypothesis separately in the next section.

V. *An Expected Intensity of Trade*

Having examined various types of trade intensity indices, our next problem is which index to choose among these indices. If we follow the standard procedure of the contingency-table analysis, we should calculate the value of the chi-square statistic under each independence hypothesis to carry out a statistical test. In our analysis, however, this is not feasible since it is virtually impossible to ascertain the number of international transac-

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[34] Kojima, *op. cit.*

[35] Kojima, *Japan and a Pacific Free Trade Area*, p. 26. See also Kojima, "The Pattern of Triangular Trade among the U.S.A., Japan, and Southeast Asia," *Developing Economics*, preliminary issue no. 1 (March-August 1962): 48–74; *idem*, "Trade Arrangements among Industrial Countries: Effects on Japan," in *Studies in Trade Liberalization*, pp. 177–215; Japan, Ministry of International Trade and Industry, comp., *Foreign Trade of Japan 1961* (Tokyo: Japan External Trade Organization); Toshiaki Yoshihara, "Japan's Trade with Developing Countries——A Note Based on *Foreign Trade of Japan 1961*——," *Developing Economics*, preliminary issue no. 1 (March-August 1962), pp. 106–120; Roemer, *op. cit.*, ch. 1; and Parry, *op. cit.*, especially p. 166. Cf. Yamazawa, "Intensity Analysis of World Trade Flow," p. 70, n. 12, in particular Sij^h of formula (i).

[36] Kojima, "Trade Arrangements among Industrial Countries"; and *idem*, *Japan and a Pacific Free Trade Area*, ch. 1. See also Japan, Ministry of International Trade and Industry, comp., *op. cit.*; and Yoshihara, *op. cit.*

[37] It should be noticed that the definition of no three-factor interaction given in the table is different from the conventional definition in the three-dimensional contingency-table analysis. For the relation between the two definitions, see Kunimoto, *op. cit.*, ch. VI.

tions or their average size.[38] Thus, we cannot count on any help from statistics in this respect. Can we, then, expect a help from the side of economics? As we showed in the previous section, every trade intensity index—except *Iijk*(e)——has been utilized in one study or another. This would presumably mean that it is not possible *a priori* to say which index is best suited for the analysis of international trade flows. We may conclude, therefore, that the choice of index hinges on the nature of the problem to be tackled.

This should not be taken, however, to mean that the eleven indices given in the table are unrelated. On the contrary, there exist certain hierarchical and symmetrical relations among them.[39] To give an example, take the index *Iijk*(e) so far untouched. Between this index and the indices *Iij.*($c1$), *Ii.k*($c2$), and *Iijk* ($b3$), the following relation holds:

$$(4)\qquad Iijk(e)=\frac{Vijk}{V\ldots\dfrac{Vij.}{V..k}\,\dfrac{Vi.k}{V.j.}\,\dfrac{V.jk}{Vi..}}$$

$$(4\text{-a})\qquad =\frac{\dfrac{Vijk}{V.jk}\Big/\dfrac{Vi..}{V\ldots}}{\left(\dfrac{Vij.}{V.j.}\Big/\dfrac{Vi..}{V\ldots}\right)\left(\dfrac{Vi.k}{V..k}\Big/\dfrac{Vi..}{V\ldots}\right)}$$

$$(4\text{-b})\qquad =\frac{Iijk(b3)}{Iij.(c1)\,Ii.k(c2)}$$

With the aid of the relation derived above, we shall now explore the meaning of the trade intensity index *Iijk*(e).[40] For the convenience of exposition, we will consider it from the standpoint of country i as an exporter.

When (4-b) is compared with (4-a), we will see at once that all the three trade intensity indices on the right-hand side of (4-b) have the same denominator equal to $Vi..\,/\,V\ldots$. We may interpret this as indicating country i's average export capability or competitiveness in the world market, for country i establishes its share in world trade in competition with other countries of the world. On the other hand, the numerator of the index *Iij.*($c1$) ($Vij.\,/\,V.j.$) may be considered to indicate country i's export strength in the import market of country j since country i establishes its share in country j's import market through competition with other countries of the world. *Iij.*($c1$) then indicates country i's export competitiveness in country j's import market as compared with its average export strength. As we discussed in section II, if countries i and j are neighboring countries, geographic proximity will work in favor of country i's export to country j, which, in turn, will be shown by the value of *Iij.*($c1$) exceeding unity. To give another example, these two countries may belong to the same customs union. This will also lead to a high value for *Iij.*($c1$).

Similarly, the numerator of the index *Ii.k*($c2$) ($Vi.k\,/\,V..k$) may be thought of as

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[38] N being the number of international transactions of a given year and B their average size (in U.S. dollars), the chi-square statistic under the hypothesis $Hc1$, for instance, is

$$\sum_i\sum_j\frac{[Vij.-\bar{V}ij.(c1)]^2}{B\bar{V}ij.(c1)}$$
$$=N\sum_i\sum_j\left[\frac{\bar{V}ij.^2}{\bar{V}ij.(c1)}-1\right]$$

We cannot, therefore, calculate the value of the chi-square statistic without the knowledge of B or N. For more detail, see Kunimoto, *op. cit.*

[39] See Kunimoto, *op. cit.*, especially Table 7, p. 141.

[40] A more detailed discussion of the analysis to follow is given in Kunimoto, *op. cit.*, ch. VII.

indicating country i's export competitiveness in the trade of commodity k. If country i's factor endowment is especially suited to the production of commodity k, for example, country i's international competitiveness in this commodity $(Vi.k / V..k)$ will be stronger than its average export competitiveness $(Vi.. / V...)$ which will be "revealed" by the value of $Ii.k(c2)$ being greater than unity. Another example of a high value of $Ii.k(c2)$ would be the case in which a large neighboring country demands a huge amount of commodity k. Easy access to the large market for commodity k will place country i's export competitiveness in it above its average.

Thus, the denominator of $Iijk(e)$ as defined on the right-hand side of (4-b) may be interpreted as indicating an *expected* intensity of trade from country i to country j in commodity k as the product of country i's export competitiveness in the import market of country j and its export strength in commodity k both compared with its average export competitiveness.

On the other hand, the numerator of $Iijk(e)$ on the right-hand side of (4-b), i.e., $Iijk(b3)$, indicates country i's export competitiveness in commodity k in the import market of country j for this commodity $(Vijk / V.jk)$ as compared with country i's average export competitiveness $(Vi.. / V...)$.

In other words, any deviation of the index $Iijk(b3)$ from unity indicates the presence of the factors which promote (or hamper) the export from country i to country j in commodity k. It should be noted that among these factors are also factors which influence country i's export to country j generally, and factors which influence country i's export of commodity k at large. The trade intensity index $Iijk(e)$ as defined in (4-b) by normalizing $Iijk(b3)$ by $[Iij.(c1)Ii.k(c2)]$ indicates the presence of the factors which are genuinely specific in the export of commodity k from country i to country j. The effect of a uniform tariff reduction on the part of country j for the imports from country i by a formation of a customs union, say, will be seen by much the same increases of $Iijk(b3)$ and $Iij.(c1)$ with the value of $Iijk(e)$ virtually unchanged, while the reduction of import tariff of country j imposed against country i's export of commodity k will mainly show up in the increase of $Iijk(b3)$, and hence $Iijk(e)$. In the absence of the factors which are specific in the export trade of country i in commodity k directed to country j, the index $Iijk(e)$ will be equal to unity, and $Iijk(b3)$ to $[Iij.(c1)Ii.k(c2)]$, expected intensity of trade from country i to country j in commodity k.

VI. *Summary*

In this article, we examined various types of trade intensity indices from the framework of the three-dimensional contingency-table analysis in statistics. In section II, we offered an intuitive explanation to the index of geographic intensity of trade, this having been one of the most frequently used indices of the kind discussed here. In section III, we pointed out the two basic difficulties in applying the contingency-table analysis to a matrix of international trade flows and, then, presented eleven types of trade intensity indices which correspond to the standard independence hypotheses in the three-dimensional contingency-table analysis. In section IV, we examined each of these indices and gave references to some of the existing studies which have made use of them. In most of these studies, the index used was devised simply as a ratio of two ratios without recognizing that it was in fact based on the contingency-table analysis in statistics as we have demonstrated here clearly.

Finally, in section V, we pointed out that there exist certain relations among these trade intensity indices and explored the meaning of the index $Iijk(e)$ which has never been used before.

We could not fully discuss here the difficulties in applying the contingency-table analysis to an international trade matrix, the interrelations among the trade intensity indices, and economic implications of the individual indices. These problems will be discussed elsewhere.[41] Also there remains the task of actually analyzing the structure of international trade flows from the analytical framework we have proposed here.

[41] Many of these problems are dealt with in detail in Kunimoto, *op. cit.*