FACTOR MARKET DISTORTIONS AND FACTOR SUBSTITUTION IN TWO-SECTOR GROWTH MODELS

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I

Recent empirical studies on production functions have shown consistently that the value of the elasticities of substitution is less than unity in post-war US manufacturing[1] as well as the manufacturing industries in developing countries like Taiwan.[2] In spite of this, the modern theory of economic growth has used to stipulate that the elasticities of substitution being no less than unity as a sufficient condition for causality and stability in neoclassical growth models.[3] In a recent paper, I have relaxed this condition to conform with the recent empirical findings for the neoclassical models in which the possibility of factor market imperfection is not considered. The purpose of this paper is to extend the previous results on factor substitution to a more realistic situation where factor market imperfection is allowed.

The usual two-sector model with general saving function is specified briefly in Section II. In section III, the aggregate elasticity of factor substitution and the properties of its coefficients are derived and then causality theorems are established in accordance with these properties. Using the similar arguments as in Section III, the stability theorems are derived in Section IV. Section V shows some new causality and stability conditions which are derived by circumventing explicitly the capital intensity assumptions. In the last section, we compare our results with those in the previous paper, the model of which is a special case of the present one.

In general, it is shown that for the models considered in this paper, the aggregate elasticities of factor substitution can still be expressed as a weighted average of the sectorial elasticities, and that the system is causal and stable if any one of the sectorial elasticities is no less than certain fraction of unity. The previous results on factor market distortions and economic growth are also shown in more sharpened forms in this paper.

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[1] See Lovell [1973] and articles cited therein.

[2] See Lee and Huang [1975].

[3] The main results of neoclassical growth models are summarized in Stiglitz and Uzawa [1970].

II. *The Model*

The simple two-sector neoclassical models of economic growth are well-known (see, e.g., Burmeister and Dobell [1970]). The models with factor market distortions may be summarized as follows:[4]

Production functions:

$$y_j=f_j(k_j),\ j=1,\ 2, \tag{1}$$

$$f_j>0,\ f_j'>0,\ f_j''<0,\ j=1,\ 2 \tag{2}$$

$$f_j(0)=0,\ f_j'(0)=\infty,\ f_j'(\infty)=0$$

Full-employment conditions:

$$\ell_1 k_1+\ell_2 k_2=k \tag{3}$$

$$\ell_1+\ell_2=1$$

Non-distorted factor rewards in the first industry:

$$\omega+k_1=f_1(k_1)\,/f_1'(k_1) \tag{4}$$

Distorted factor rewards in the second industry:

$$\delta\omega+k_2=f_2(k_2)\,/f_1'(k_2) \tag{5}$$

Saving-Investment Equation:

$$y_1\ell_1=f_1'(k_1)[s_w\omega(\ell_1+a\ell_2)+s_r(k_1\ell_1+bk_2\ell_2)] \tag{6}$$

Growth Process:

$$\dot{k}/k=[(\ell_1 y_1)/k]-g \tag{7}$$

where $y_j\equiv Y_j/N_j=F^j(1,\ \mathrm{k}_j)$, the output labor ratio; $k_j\equiv K_j/N_j>0$, the capital-labor ratio; $\ell_j\equiv N_j/N$, the proportion of labor employed in the ith sector. $Y_1\equiv$investment goods; $Y_2\equiv$consumption goods; $\omega\equiv w/r$, the wage-rental ratio; $\delta\equiv a/b$, the distortion index, that is, the relative distortion (a) of the labor reward to that (b) of the capital reward; $\delta>0$. $k\equiv K/N$, the aggregate capital-labor ratio; $\dot{k}\equiv dk/dt$; $g\equiv\lambda+n>0$, the sum of the constant rate of depreciation (λ) and the constant growth rate of labor force ($n=\dot{N}/N$). s_i is the constant marginal propensity to save out of wage income ($i=w$), or of rental income ($i=r$), $0\leq s_w<1$, $0<s_r\leq 1$. Note that (6) follows from the definition of national income $Y=p_1Y_1+p_2Y_2=w(N_1+aN_2)+r(K_1+bK_2)$, and the saving-investment assumption that $p_1Y_1=s_w w(N_1+aN_2)+s_r r(K_1+bK_2)$, where p_i is the price of the ith commodity.

Let θ_{ij} be the ith factor share ($i=N$, labor, or $i=K$, capital) in the jth sector, $j=1, 2$. That is, $\theta_{N1}=\omega/(\omega+k_1)$, $\theta_{K1}=k_1/(\omega+k_1)$, $\theta_{N2}=a\omega/(a\omega+bk_2)$, and $\theta_{K2}=bk_2/(a\omega+bk_2)$. Then it can be shown by direct calculation that, $\theta_{Nj}+\theta_{Kj}=1$, and

$$\theta_{N1}-\theta_{N2}=\theta_{K2}-\theta_{K1}=\theta_{N1}\theta_{K2}-\theta_{N2}\theta_{K1}, \tag{8}$$

$$=\omega(bk_2-ak_1)/(\omega+k_1)(a\omega+bk_2), \tag{9}$$

Note that, from (4) and (5), k_1 is a function of ω while k_2 is a function of $\delta\omega$. Furthermore,

$$d\theta_{Kj}/d\omega=-d\theta_{Nj}/d\omega=\theta_{Nj}\theta_{Kj}(\sigma_j-1)/\omega \tag{10}$$

which is essentially the well-known Robinson-Amoroso relation extended to a model with factor market distortion, and $\sigma_j\equiv(\omega_j/k_j)(dk_j/d\omega_j)$, the elasticity of substitution in the jth sector, $\omega_1\equiv\omega$, $\omega_2\equiv\delta\omega$, and $\sigma_j>0$ by the neoclassical assumptions.

[4] See Johnson [1965], Batra and Casas [1971], Herberg and Kemp [1972].

For the sake of simplicity, we exclude the pathetic case in which any of the distributive shares of the factors of production vanishes.

III. *CAUSALITY CONDITIONS*

It is well known that the neoclassical system (1) to (6) for $a=b=1$ is causal if and only if ω can be solved uniquely in terms of k (see, e.g., Burmeister and Dobell [1970], pp. 115-120). This is also true for $a\neq 1$ or $b\neq 1$. Substituting (4) into (6), ℓ_1 may be expressed as

$$\ell_1=\frac{s_w a\omega+s_r bk_2}{(1-s_w)\omega+(1-s_r)k_1+(s_w a\omega+s_r bk_2)}, \tag{11}$$

which may be substituted into the solution of the simultaneous equations in (3) to have

$$k=\frac{k_1(s_w a\omega+s_r bk_2)+k_2[(1-s_w)\omega+(1-s_r)k_1]}{(1-s_w)\omega+(1-s_r)k_1+(s_w a\omega+s_r bk_2)}. \tag{12}$$

Dividing the numerator and the denominator by $(\omega+k_1)(a\omega+bk_2)$, (12) may be written as

$$\frac{k}{\omega}=\frac{a\theta_{K2}B+ab\theta_{K1}A}{b\theta_{N2}B+ab\theta_{N1}A}\equiv\frac{D}{C} \tag{13}$$

where $A\equiv s_w\theta_{N2}+s_r\theta_{K2}$ and $B\equiv(1-s_w)\theta_{N1}+(1-s_r)\theta_{K1}$, the weighted saving ratios and consumption ratios, respectively.

Let the aggregate elasticity of substitution be $\sigma\equiv\omega dk/(kd\omega)$. Then, differentiating $k=\omega D/C$ with respect to ω in (13), we have

$$\sigma=[CD+\omega(D^*C-C^*D)]/CD \tag{14}$$

where D^* (*or* C^*) denotes the derivative of D (or C) with respect to ω.
Since

$$\begin{aligned}
\omega D^*&=D_1(\sigma_1-1)+D_2(\sigma_2-1)\\
D_1&\equiv a[bA-(s_r-s_w)\theta_{K2}]\theta_{N1}\theta_{K1},\\
D_2&\equiv a[B+b(s_r-s_w)\theta_{K1}]\theta_{N2}\theta_{K2},\\
\omega C^*&=C_1(\sigma_1-1)+C_2(\sigma_2-1),\\
C_1&\equiv b[aA+(s_r-s_w)\theta_{N2}]\theta_{N1}\theta_{K1},\\
C_2&\equiv b[a(s_r-s_w)\theta_{N1}-B]\theta_{N2}\theta_{K2},
\end{aligned}$$

substituting into (14) and rearranging the terms, we have

$$\sigma=a_1\sigma_1+a_2\sigma_2+a_3 \tag{15}$$

where

$$\begin{aligned}
a_1&\equiv(CD_1-DC_1)/CD\\
&=abA\theta_{N1}\theta_{K1}[abA+a(1-s_r)\theta_{K2}+b(1-s_w)\theta_{N2}]/CD\\
a_2&\equiv(CD_2-DC_2)/CD\\
&=abB\theta_{N2}\theta_{K2}(B+as_w\theta_{N1}+bs_r\theta_{K1})/CD\\
a_3&\equiv[CD-(CD_1-DC_1)-(CD_2-DC_2)]/CD\\
&=ab(a\theta_{N1}\theta_{K2}-b\theta_{N2}\theta_{K1})[(1-s_w)s_r\theta_{N1}\theta_{K2}-s_w(1-s_r)\theta_{N2}\theta_{K1}]/CD,
\end{aligned}$$

and $a_1+a_2+a_3=1$, $a_1>0$, $a_2>0$. Moreover, it can be shown that

$$\begin{aligned}
a_1+a_3&=1-a_2\\
&=\{abB[as_r\theta_{N1}\theta_{K2}{}^2+bs_w\theta_{N2}{}^2\theta_{K1}]+(ab)^2\theta_{N1}\theta_{K1}A^2\}/CD>0,\\
a_2+a_3&=1-a_1\\
&=\{ab\theta_{N2}\theta_{K2}B^2+abA[a(1-s_w)\theta_{N1}{}^2\theta_{K2}+b(1-s_r)\theta_{N2}\theta_{K1}{}^2]\}/CD>0.
\end{aligned}$$

Thus, $0<a_1<1$, $0<a_2<1$ and $|a_3|<1$. Note that these results do not depend on the factor or value intensities.

Since the neoclassical system (1) to (6) is causal if and only if $\sigma>0$, if line L is defined as

$$L: \frac{\sigma_1}{b_1}+\frac{\sigma_2}{b_2}=1$$

where $b_i\equiv -a_3/a_i$, $b_i<1$, $i=1, 2$, and if the open half-space is defined as $L^+=\{(\sigma_1, \sigma_2)|(\sigma_1/b_1)+(\sigma_2/b_2)>1\}$, and the positive orthant of the (σ_1, σ_2) space as $\Omega^+\equiv\{(\sigma_1, \sigma_2)|(\sigma_1, \sigma_2)>0\}$, then we have

Causality Theorem I:

The necessary and sufficient condition that the neoclassical system be causal is that $(\sigma_1, \sigma_2)\in L^+\cap\Omega^+$.

It should be noted that if $a_3<0$, then $0<b_i<1$, and L is in the unit interval of the σ_1 and σ_2 axis; if $a_3\geq 0$, then $L^+\cap\Omega^+=\Omega^+$, and the system is causal for all $\sigma_1>0$ and $\sigma_2>0$. The following new sufficient conditions are immediate.

Causality Theorem II:

The neoclassical system (1) to (6) is causal if any one of the following conditions is satisfied:

(a) $\sigma_1\geq\max\{b_1, 0\}$,
(b) $\sigma_2\geq\max\{b_2, 0\}$.
(c) $\sigma_1+\sigma_2\geq\max\{b_1, b_2, 0\}$.

where for $b_i>0$, $1>b_i\equiv -a_3/a_i>0$.

Noting that $b_i\gtrless 0$ as $a_3\lessgtr 0$, the proof follows immediately from the properties of a_i's in (15). (c) is a generalized version of the Herberg-Kemp [1972] result that $\sigma_1+\sigma_2\geq 1$ is sufficient. Note that the causality region of (c) in the (σ_1, σ_2) space may overlap with the causality region of either (a) or (b), but not both, if $b_1\neq b_2$. It should be emphasized that these conditions are based on the parameters of the production functions and are independent of the factor or value reversals. If II(a), II(b) and II(c) are drawn in the (σ_1, σ_2) space along with Line L for $a_3<0$, we see immediately that these three conditions cover almost all of the causality region defined in Causality Theorem I for $a_3<0$.

The conditions for $a_3\geq 0$ yield the following slightly sharpened form of Herberg-Kemp [1972] results based on the behavioral conditons:

Causality Theorem III:

The neoclassical system (1) to (6) is causal if any one of the following conditions is satisfied:

(a) $k_2=k_1$;
(b) $s_r=s_w$, $bk_2=ak_1$;
(c) $s_r=s_w$, $(k_2-k_1)(bk_2-ak_1)>0$;
(d) $(s_r-s_w)(k_2-k_1)>0$ and $(k_2-k_1)(bk_2-ak_1)\geq 0$;
(e) $(k_2-k_1)[s_r(1-s_w)b-s_w(1-s_r)a]>0$;
(f) $(1-s_w)s_ra=s_w(1-s_r)b$.

The proof follows directly from (9) and the expression of a_3 in (15). Causality

Condition III(c) is also obtained in Batra and Casas ([1972], p. 531). Note that III(c) and III(d) require that if the consumption sector is relatively capital (or labor) intensive in the physical sense, then it should also be capital (or labor) intensive in the value sense.[5]

IV. *STABILITY CONDITIONS*

Similar to the case in which the factor market distortions are absent, the necessary and sufficient condition that the balance growth path of System (1) to (7) be unique and stable is that $d(\dot{k}/k)/dk=[d(\dot{k}/k)/d\omega][d\omega/dk]<0$ for all capital-labor ratio $k>0$. Since aggregate production has to be efficient,[6] that is, $\sigma=\omega dk/(kd\omega)>0$, the growth path under efficient production is unique and stable if $d(\dot{k}/k)/d\omega=d\phi/d\omega<0$, where $\phi\equiv\ell_1 y_1/k$. Dividing (11) by (12) and then dividing the resulting numerator and the denominator by $(\omega+k_1)(a\omega+bk_2)$, we have

$$\frac{\ell_1 y_1}{k}=\frac{bA\theta_{K1}}{\theta_{K2}B+b\theta_{K1}A}\frac{y_1}{\omega}$$
$$=abAf_1'(k_1)/D.$$

Differentiating both sides logarithmically with respect to ω and using the method similar to (14), we have

$$(\log\phi)^*=(-\theta_{N1}AD+\omega A^*D-\omega D^*A)/\omega AD,$$

where $\omega A^*D-\omega D^*A$ may be written in a form $-E_1(\sigma_1-1)-E_2(\sigma_2-1)$. Thus $(\log\phi)^*<0$ if and only if

$$e_1\sigma_1+e_2\sigma_2+e_3>0 \tag{16}$$

where

$$e_1\equiv E_1/\omega AD$$
$$=a[(b-1)A+s_w]A\theta_{N1}\theta_{K1}/\omega AD$$
$$=a\{bs_w\theta_{N2}+[bs_r-(s_r-s_w)]\theta_{K2}\}A\theta_{N1}\theta_{K1}/\omega AD$$
$$e_2\equiv E_2/\omega AD$$
$$=as_wB\theta_{N2}\theta_{K2}/\omega AD$$
$$e_3\equiv 1-e_1-e_3$$
$$=a[s_r(1-s_w)\theta_{N1}\theta_{K2}-s_w(1-s_r)\theta_{N2}\theta_{K1}]\theta_{K2}/\omega AD$$

and $e_2>0$. It may be calculated further that

$$e_1+e_3=1-e_2 \tag{17}$$
$$=[aB\theta_{K2}(s_r\theta_{N1}\theta_{K2}-s_w\theta_{N2}\theta_{K1})+bA\theta_{N1}\theta_{K1}]/\omega AD$$
$$=\{s_r\theta_{N1}\theta_{K2}D+as_w\theta_{N2}\theta_{K1}[s_w\theta_{N1}(b\theta_{N2}+\theta_{K2})+s_r\theta_{K2}(b\theta_{N1}+\theta_{K1})-\theta_{K2}\}/\omega AD$$

$$e_2+e_3=1-e_1$$
$$=a(1-s_w)A\theta_{N1}\theta_{K2}/\omega AD>0 \tag{18}$$

Thus $e_1<1$. In summary, the coefficients of (16) have the following properties: $e_1+e_2+e_3=1$, $e_2+e_3>0$, $e_1<1$, $e_2>0$. Note that e_1, e_2, e_1+e_3, and e_2+e_3 do not depend on the factor and value intensity differentials.

Let line M be defined as

[5] For these terminology, see, e.g., Jones [1971]. We need not assume here that the consumption sector be relatively more capital intensive in the physical sense than the investment sector.

[6] See Sato [1969].

$$M:\frac{\sigma_1}{c_1}+\frac{\sigma_2}{c_2}=1$$

where, $c_i\equiv -e_3/e_i$, $i=1, 2$, and for $c_i>0$, we have $1>c_i>b_i$. As in the previous section, if the open half-space is defined as $M^+\equiv\{(\sigma_1, \sigma_2)|(\sigma_1/c_1)+(\sigma_2/c_2)>1\}$, then we have

Stability Theorem I:

The necessary and sufficient condition that the balanced growth path in neoclassical system (1) to (7) be unique and stable is that $(\sigma_1, \sigma_2)\epsilon M^+\cap\Omega^+\cap L^+$.

In a particular case in which $bs_r\geq s_r-s_w$, that is, $b\geq 1-(s_w/s_r)$, we have $e_1>0$. Then the intercepts, c_i, of Line M are positive if $e_3<0$, and nonpositive if $e_3\geq 0$. The following new set of sufficient conditions may be proved.

Stability Theorem II:

If $bs_r\geq s_r-s_w$, then the growth path of system (1) to (7) is unique and stable if any one of the following conditions is satisfied:

(a) $\sigma_2\geq\max\{c_2, b_2, 0\}$,

(b) $\sigma_1\geq\max\{c_1, b_1, 0\}$, and $s_w\to 0$,

(c) $\sigma_1+\sigma_2\geq\max\{c_1, c_2, b_1, b_2, 0\}$, and $s_w\to 0$,

where, for $c_i>0$, $1>c_i\equiv -e_3/e_i>b_i$, $i=1, 2$, and for $b_i>0$, $1>b_i\equiv -a_3/a_i>0$ for $i=1, 2$.

(d) $\sigma_1+\sigma_2\geq 1+\alpha$, where $\alpha\equiv|e_2-e_1|/e_1$.

(e) $\sigma_2\geq\theta_{K1}$ and $s_r\geq s_w$,

(f) $\sigma_1+\sigma_2\geq 1+\alpha\theta_{K1}$, and $s_r\geq s_w$.

Proof: (a) is sufficient since, for $c_2>0$ (that is, $-e_3>0$), it can be shown easily that $c_2>b_2$. Hence, either $c_2>0>b_2$ or $c_2>b_2>0$, and for such c_2, (16) holds. If $c_2<0$ (that is, $-e_3<0$), then either $\sigma_2>b_2>0$ or $\sigma_2>0>b_2$ to satisfy the causality condition. (b) and (c) are sufficient since, as $s_w\to 0$, $e_1>-e_3$. In this case, for $c_1>0$, (that is, $-e_3>0$), it can be shown that $c_1>b_1$. Hence, either $c_1>0>b_1$ or $c_1>b_1>0$, and for such c_1, (16) holds. The rest of (b) and the proof of (c) may proceed as (a). (d) is sufficient since, for $\sigma_1\geq 1+\alpha-\sigma_2>0$ and $\sigma_2>0$, the LHS of (16) $\geq e_2+(e_2-e_1)\sigma_2+e_3>0$ for $e_2>e_1$ or the LHS of (16) $>e_2[1+((e_1-e_2)/e_1)-\sigma_2]+e_3=[e_2(e_1-e_2)+e_1(e_2+e_3)]/e_1>0$ for $e_1>e_2$. (e) is sufficient from (a) above: since, for $s_r\geq s_w$, we have $e_2+e_3-\theta_{N1}e_2=a\theta_{N1}\theta_{K2}[s_w(s_r-s_w)\theta_{N2}\theta_{K1}+s_r(1-s_w)\theta_{K2}]>0$, thus $\theta_{K1}>-e_3/e_2\equiv c_2$. (f) is sufficient by using a similar argument as (d). QED.

(a) is an extension of Herberg-Kemp [1972] conditon that $\sigma_2\geq 1$ and $s_r-s_w\leq s_rb$ are sufficient.[7] (e) is also an extension of the Batra-Casas [1971] condition that $\sigma_2\geq\theta_{K1}$, $\sigma_1\geq\theta_{K2}$, and $s_r=s_w$ are sufficient. (b), (c) and (e) are generalizations of the familiar conditions in the growth models without factor market distortions (see Drandakis [1963]). A similar formulation of (d) for usual two-secor models may be found in Sato [1969]. α in (d) and (f) is less than unity if $e_2<2e_1$: these conditions are not of much interest as α may exceed unity; nevertheless, they provide other examples of stability conditions which do not depend on the capital intensity assumptions.

Lastly the following conditions insure $e_1>0$ and $e_3\geq 0$:

(a) $s_r=s_w$, $bk_2\geq ak_1$,

(b) $bs_r\geq s_r-s_w>0$, $bk_2\geq ak_1$,

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[7] When $s_r=s_w$, $e_1>0$ for *any* b. Note that the expression of e_1 indicates asymmetry between a and b in the stability conditions.

(c) $s_r < s_w,\ s_r(1-s_w)b \geq s_w(1-s_r)a,\ k_2 \geq k_1$,

(d) $bs_r \geq s_r - s_w > 0,\ s_r(1-s_w)b \geq s_w(1-s_r)a,\ k_2 \geq k_1$.

These conditions in turn give rise to the following stability conditions which have been derived by Herberg and Kemp [1972]. Here as in Causality Theorem III, our interest lies mainly in our attempt in the next section to reformulate these results in terms of the "basic parameters of the production functions and savings functions".

Stability Theorem III:

The balanced growth path of System (1) to (7) is unique and stable if any one of the following conditions is satisfied:

(a) $s_r = s_w,\ bk_2 \geq ak_1,\ k_2 \geq k_1$,

(b) $bs_r \geq s_r - s_w > 0,\ bk_2 \geq ak_1,\ k_2 > k_1$,

(c) $s_r < s_w,\ s_r(1-s_w)b \geq s_w(1-s_r)a,\ k_2 > k_1$,

(d) $bs_r \geq s_r - s_w \geq 0,\ s_r(1-s_w)b \geq s_w(1-s_r)a,\ k_2 > k_1$.

Stability Condition III (a) is also obtained in Batra and Casas [1972]. Here Conditions III (a) to III (d) all require that consumption sector be relatively capital intensive in the physical sense ($k_2(\delta\omega) > k_1(\omega)$), and III (a) and III (b) in addition require that the consumption sector be relatively capital intensive in the value sense ($k_2(\delta\omega) > \delta k_1(\omega)$). If such assumptions on the factor intensities in the physical and value sense are imposed on the System (1) to (7), then b in condition $bs_r \geq s_r - s_w > 0$ may be interpreted as that the absolute distortion of the factor reward in the (distorted) industry which uses that factor more intensively can not be less than a positive fraction.

V. *SOME IMPLICATIONS OF DISTORTIONS*

We have seen that conditions in Causality Theorem II do not explicitly depend on the distribution indexes a and b, and that the conditions in Stability Theorem II require, except when the marginal propensity to save out of the profit income is no more than that out of the wage income ($s_r \leq s_w$), that, among other things, the absolute distortion of the captial reward in the consumption sector can not be less than a positive fraction ($b \geq (s_r - s_w)/s_r > 0$). This lower limit increases to 1 as s_w decreases to zero. The distortion index $\delta \equiv a/b$, however, still may take any positive number as there is no restriction on a. Thus all the sufficient conditions in Stability Theorem II also do not explicitly depend on distortion index δ, nor on the capital intensity assumptions.

This is not the case for Causality Theorem III and Stability Theorem III, where the conditions depend on the distortion index δ and capital intensity assumptions. As noted by Herberg and Kemp ([1972], p. 594) they appear to be of little value. However, as shown below, they are not as restrictive as they might be thought at the first sight.

Let $k_2(\omega) > k_1(\omega)$ for $\delta = 1$ and δ_1 be the value of the distortion index at which $k_2(\delta\omega) = k_1(\omega) = k$. Such δ_1 exists and is unique, and $1 > \delta_1 > 0$. Then obviously $\delta \gtreqless \delta_1$ as $k_2(\delta\omega) \gtreqless k_1(\omega)$. It is well-known that,[8] at the momentary equilibrium, reversals in the capital intensities in the physical and value sense ($k_2(\delta\omega) > k_1(\omega)$, $k_2(\delta\omega) > \delta k_1(\omega)$) do not

[8] See Mundlak [1970], Theorem 3. Some minor additional conditions such as $k_j > 0$ for $\omega > 0$, $k_j(0) = 0$, $k_j(\infty) = \infty$, are required.

exist if $\delta > \delta_1$ and $\sigma_j \geq 1$ for at least one j. Thus, we have

Causality Theorem IV:

The neoclassical system is causal if any one of the following conditions is satisfied:

(a) $(s_r - s_w)(\delta - \delta_1) \geq 0$ and $(\sigma_1 - 1)(\sigma_2 - 1) \leq 0$;

(b) $(s_r - s_w)(\delta - \delta_1) \geq 0$ and $(\sigma_j - 1)(\delta - \delta_1) \geq 0$, for $j=1$, or 2;

(c) $(\delta - \delta_1)\left(\dfrac{s_r(1-s_w)}{s_w(1-s_r)} - \delta\right) \geq 0$;

where $\delta \neq \delta_1$.

Causality Condition IV (a) or IV (b) is sufficient for III (c) and III (d); IV (c) is sufficient for Causality condition III (e) and III (f). Now IV (a) or IV (b) says that, in particular, if $s_r \geq s_w$, $\delta > \delta_1$ and one of the elasticities of factor substitution is no less than one and the other no more than one, or, if $s_r \geq s_w$, $\delta > \delta_1$, and at least one of the elasticities is no less than one, then the system is causal. Note that, our causality condition II (a), or II (b), is much simpler and more general than IV (a) and IV (b).

When the dynamic process is considered, δ_1 is a function of k, which changes over time. Since $k_2(\delta_1 \omega) = k$ and $k_1(\omega) = k$, we have

$$\frac{k}{\delta_1} \frac{\partial \delta_1}{\partial k} = (\sigma_1 - \sigma_2)/\sigma_1 \sigma_2.$$

where σ_j is evaluated at $k_1 = k_2 = k$. Thus δ_1 is monotonically increasing or decreasing function of k in accordance with $\sigma_1 > \sigma_2$ or $\sigma_1 < \sigma_2$ and converges to $\delta_1{}^o = \delta_1(k^o)$ as k converges to the balanced equilibrium capital-labor ratio k^o. Hence we have,

Stability Theorem IV:

The balanced growth path of System (1) to (7) is unique and stable if $bs_r \geq s_r - s_w \geq 0$, $\delta > \max\limits_{k \in R}\{\delta_1(k)\}$, $\sigma_j \geq 1$, for at least one j, $j=1, 2$, where $R = \{k | k_0 < k < k^o\}$, $k_0 \equiv k(t_0)$, the initial capital-labor ratio.

This condition is sufficient for Stability Conditions III (a) and III (b). Note that $\max\{\delta_1(k)\}$ exists if $\sigma_j \geq \varepsilon \geq 0$ for some ε, in particular, it is satisfied in the case for the family of CES production functions with variable factor substitution. Again, we note that Stability Theorem IV is already implied in our Stability Conditions II (a) and II (b).

VI. *AN IMPORTANT SPECIAL CASE*

In the absence of factor market distortions, $a=b=1$, the system (1) to (7) reduces to the conventional neoclassical two-sector growth model.

In this case, (13) becomes

$$\frac{k}{\omega} = \frac{(1-s_w)\theta_{K2} + s_w \theta_{K1}}{(1-s_r)\theta_{N2} + s_r \theta_{N1}} \equiv \frac{D}{C}$$

and a_i's in (15) reduces to $a_1 = A\theta_{N1}\theta_{K1}/CD$, $a_2 = B\theta_{N2}\theta_{K2}/CD$, $a_3 = (\theta_{N1} - \theta_{N2})Q/CD$, where $Q \equiv [(1-s_w)s_r \theta_{N1}\theta_{N2} - s_w(1-s_r)\theta_{N2}\theta_{K1}]/CD$. Causality Theorem I and II hold with these a_i's, and causality conditions III (b), III (c) and III (f) reduce to $s_r = s_w$ and III (d), III (e), to $(s_r - s_w)(k_2 - k_1) > 0$.

Similarly, e_i's in (16) can be written as $e_1 = a_1 s_w C/\omega A$, $e_2 = a_2 s_w C/\omega A$, $e_3 = Q\theta_{K2}/\omega AD$. After substitution, condition (16) reduces to

$$a_1\sigma_1+a_2\sigma_2+a_0>0$$

where $a_0\equiv Q\theta_{K2}/s_wCD=a_3-q$, and $q\equiv -Q/s_wC$. Thus conditions in Stability Theorem I and II hold without condition $bs_r\geq s_r-s_w$ and with e_i substituted by a_i, $i=1$, 2, and e_3 by a_0. Furthermore, Stability Conditions II (b) and II (c) also hold for the case $s_r\rightarrow 1$. Similarly stability conditions III (a), III (b), and III (d) reduce simply to the well-known Drandakis-Inada conditions: $s_r\geq s_w$ and $k_2\geq k_1$. A discussion of this special case has already been given elsewhere (see Hsiao [1975]).

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