FACTOR-PROPORTIONS THEORY RECONSIDERED*

By IPPEI YAMAZAWA**

I. Introduction

An increasing number of economists have recently seemed to consider the factor-proportions theory merely as a logical proposition which holds under a set of unrealistic assumptions and consequently have proposed new hypotheses for the determination of trade patterns. But none of them seem to have succeeded in providing a complete theoretical framework. The factor-proportions theory, well-established in the framework of the neo-classical theory of production and distribution, can explain the pattern of international trade and its change over time if it is adequately modified and extended to include various determinants of trade patterns which have been recently proposed. One of the aims of this paper is to review the recent literatures on trade patterns and to do justice to the factor-proportions theory in the light of recent development of the theory.

The other aim of this paper is to give attention to a relatively neglected area of the factor-proportions theory; namely the pattern of production specialization in trade-equilibria. In the factor-proportions theory the structure of comparative advantage between two countries before trade and the subsequent direction of trade once trade is opened are both determined by the differences in the factor endowment ratios; but, for trade equilibria, only one particular case has been analyzed by many writers. That is the case in which both countries continue to produce both commodities and factor prices are equalized between the two countries after trade is opened. There are, however, other patterns of trade-equilibria; they are, (1) either one of the two countries specializes completely in the production of one commodity while the other continues to produce both commodities, or (2) each country specializes completely in the production of that commodity in which it has a comparative advantage. In all these patterns of specialization factor prices are not equalized through trade.

The lack of attention paid to patterns involving complete specialization seems to be due to the observation that while complete specialization is common among less-developed countries, it is rare among advanced countries, i.e., most advanced countries produce a wide range of commodities from very labor-intensive to highly capital-intensive ones. But this is due to such factors as transportation costs, product differentiation, tariffs and other artificial impediments, which are not considered by the factor-proportions theory. But complete and incomplete specialization have different welfare implications from which different policies can be derived. In almost every country a preference for incomplete to complete specialization is observed.

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From the viewpoint of positive economics incomplete specialization is associated with a greater degree of flexibility in adjustment to a given exogenous change than complete specialization. Therefore, the concept of patterns of specialization is not merely a theoretical tool, but may also be usefully employed in analyzing various problems in the real world.

In the following section, a generalized approach to the factor-proportions theory which it is hoped will provide an adequate explanation for international trade in the real world will be discussed. The next three sections are devoted to my second aim. In Section III and IV the determination of patterns of trade and specialization is examined and the characteristics of their trade-equilibria are explored in a simple factor-proportions model. Patterns of specialization are extended to more general settings and their economic consequences mentioned above are explored in the last section.

II. Toward a Generalized Theory of Factor-Proportions

New hypotheses of trade patterns emphasize the strategic roles played by such factors as availability, representative demand, differences in the level of technology and factors associated with "Research and Development" in the determination of trade patterns. These new hypotheses are mostly in the framework of the traditional theory of specific factors with which they share a common shortcoming in their theoretical formulation; namely, they do not explain why a particular country is endowed with a particular specific factor. This shortcoming is not serious in the case of trade in primary commodities but is critical where trade in manufactures employing the intensively such artificial specific factors as labor skill and technology is considered.

The factor-proportions theory, on the other hand, depends on the difference in the relative abundance of such general factors as the capital stock and labor population among trading countries, reflecting generally the past accumulation of wealth or the stage of economic development of each country. The difficulty involved in the factor-proportions theory is its over-simplification. Although simplification is a necessary step of theorizing, theoretical economists have adhered to such convenient assumptions as constant returns to scale, common production techniques, homogeneous physical capital and labor and so on. The over-simplification limits the applicability of the theory to the complex realities of international trade. For example, the theory is incapable of providing an adequate explanation for the Leontief paradox and the observed fact that trade among advanced countries is increasing rapidly.

Four of the criticisms on the assumptions of the theory are to be considered.

(i) An invariable ranking of factor intensities of industries is required for the factor-proportions theory to hold as an explanation for the determination of trade patterns among countries with different factor endowment ratios; the possibility of a reversal of factor intensities invalidates the theory. This possibility was raised in connection with the Leontief paradox and Minhas claims to have proved it empirically. However, Minhas' methodology has been

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1 Linder [26], Posner [30], and Vernon [7], [39].
2 A specific factor is defined as a factor of production which is endowed in various but not all countries and used for the production of a particular commodity but not of others. On the other hand, general factors are endowed in every country and used for the production of all commodities.
3 Leontief [21], [22].
4 Minhas [25].
criticized by several writers, and the conflicting evidence has been presented by Ball, Lary, and others. In so far as commodities intensive in natural resources are excluded, it seems fair to say the early contributors to the theory were right in suggesting that the possibility is mainly of theoretical but not of much empirical importance.

(ii) The assumption of constant returns to scale has been used frequently due to its convenience, but it is well-established fact that some manufacturing industries are subject to increasing returns to scale so that the comparative advantage in such industries is not so much influenced by factor-proportions as by the size of the country. It seems to be more promising to explore the implications of increasing returns to scale in a more strategic model in which factor-proportions are neglected and the sizes of countries determine the patterns of trade and specialization.

(iii) Patterns of comparative advantage can be reversed if the difference of opposite direction in demand prevails. But the difference in demand patterns is not so much intrinsic to individual countries as reflecting their income per capita. This follows from the assumption of non-unitary income elasticities of demand so that some commodities are preferred to others at a higher income level. If the income elasticities of demand for capital-intensive commodities are found to be greater than unity and those for labor-intensive commodities less than unity, then capital-intensive commodities are demanded more in capital-rich countries with high income per capita whereas labor-intensive commodities are demanded more in labor-rich countries with low income per capita so that the difference in demand patterns tends to offset the difference in factor endowments. The significance of the demand patterns, however, does not consist in its effect of partially offsetting the effect of factor endowment differences on the determination of trade patterns but rather in its dynamic role whereby changes in the patterns of demand with the growth of per capita income lead changes in the industrial structure.

(iv) The most harmful criticism to the factor-proportions theory is its neglect of the difference in technology or labor skill between countries. Although these differences were recognized by Ohlin, it is only recently that theoretical economists have attempted to introduce them into the determination of trade patterns. Two approaches can be distinguished. One assumes that a country has created a new or superior technology, which provides the country with temporary oligopoly power in the product market until other countries acquire the technology by their own invention or through patent-purchase. The other approach, by way of comparison, assumes that each country has equal access to any technology but that it requires labor skill of different degrees instead of the usual homogeneous "labor". The difference in technology or labor skills causes the most important modification of the factor-proportions theory in the explanation of patterns of trade in manufactures.

Each approach emphasizes either differences in the level of technology or labor skills but the difference is only superficial since both superior technology and labor skills are the result of differences in research and development activities or education and training respectively.

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6 Leontief [23], and Chipman [6].
6 Ball [2], Lary [20], and Yahr [40].
7 Lerner [24] and Samuelson [33], [34].
8 Kojima's theory of agreed specialization [19].
9 Robinson [31].
10 Posner [30] and Vernon [7], [39].
11 Leontief [23], Keessing [14], [15], and Kenen [17], [18].
In other words both superior technology and labor skills are different forms of a more generally defined "capital" separate from physical capital stock such as machines and plants, since each of the three forms of capital offers flow of services for the production and provides its owner with an income stream in exchange. On the contrary, another primary factor of production, "labor" is defined as homogeneous unskilled labor. It is well observed that the less-developed countries are characterized by low endowment ratios of generalized capital stock to unskilled labor unit while the advanced countries by high ratios.

Patterns of trade and specialization between countries at different stages of wealth accumulation are strongly influenced by the proportion of unskilled labor and generalized capital. Therefore, less-developed countries will have a comparative advantage with commodities intensive in unskilled labor while the advanced countries will have a comparative advantage in the production of those commodities whose production is relatively intensive in either physical capital (plants or machinery) or skilled labor, or whose production requires a new or superior technology which has not been diffused to other countries.

The commodities of the former type include textiles, apparels, handicrafts, simple machineries and other miscellaneous products. The commodities of the latter type include not only chemicals and other heavy manufactures but also medicines, measuring and communication equipments, and electronic computers all of which are produced with the aid of skilled labor or new technology whose development requires many scientists and engineers, and have been considered as highly labor-intensive under our conventional simple capital-labor ratio. This generalization of the capital concept thus restores to the factor-proportions theory an ability to explain the trade patterns between the less developed and advanced countries and also provides us with a theoretical basis for rational trade and development policies for both countries.¹²

Division of the generalized capital into its three alternative forms is not uniform among countries, but seems to depend partly upon the stages of development or wealth accumulation (i.e., the generalized capital-labor ratio). Thus difference in the division of capital into its various form determines the patterns of trade among countries with similar and not very different generalized capital-labor ratios. One good example is the U.S.'s lead over the other advanced countries in R and D activities thus providing the U.S. with a comparative advantage in R & D-intensive commodities.¹³,¹⁴

The generalized capital approach to the theory has just begun and it is expected to provide a clue to a dynamic theory of comparative advantage. Nevertheless at this stage of development of the theory, it seems to be worthwhile to explore one of the static aspects of the theory and to examine its full implication, that is, patterns of production specialization under free trade.

In the theory of specific factor, complete specialization occurs when a country is not endowed with a specific factor (not available at an economic price) necessary in the production of its import-competing commodity. On the contrary in the factor-proportions theory the

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¹² Lary [20].

¹³ Johnson [13] has given a clear consideration on this subject.

¹⁴ If it is taken into consideration that physical capital is highly mobile internationally relative to unskilled and skilled labor, and that technology is diffused easily to the world except for R and D-intensive commodities, then the ratio of skilled to unskilled labor can be important determinant of the pattern of international trade. See Keesing [16] and Kenen [17] [18].
determination of specialization patterns is more complicated and results in more profound economic consequences.

III. A Simple Factor-Proportions Model

Assume two countries A and B, two commodities X and Y, and two homogeneous factors of production, capital K and labor L.

Ass. (i) Each country is endowed with a fixed amount of capital \((K_a, K_b)\) and labor \((L_a, L_b)\). Country B is relatively more abundant in capital than country A is.

\[ k_a < k_b, \quad k_i = K_i / L_i \quad (i = a, b) \]

Ass. (ii) Each commodity is produced with the aid of both capital \((K_x, K_y)\) and labor \((L_x, L_y)\). The production function in each industry is assumed to be linear and homogeneous so that average products of labor depend only on the factor-proportions \(k_j = K_j / L_j, j = x, y\). The output of each commodity may be written as

\[ X = F_x(K_x, L_x) = L_x f_x(k_x) \quad (1) \]

\[ Y = F_y(K_y, L_y) = L_y f_y(k_y) \quad (2) \]

Further it is assumed that the Y-industry is more capital-intensive than the X-industry for any level of factor price ratio.

\[ k_x < k_y \quad (3) \]

Ass. (iii) The marginal productivities of capital in terms of the product in each industry is

\[ \frac{dF_j}{dK_j} = \frac{df_j}{dk_j} = f_j', \quad j = x, y \quad (4) \]

and the corresponding marginal productivities of labor is

\[ \frac{dF_j}{dL_j} = f_j - k_j f_j', \quad j = x, y \quad (5) \]

All marginal productivities are assumed to be positive but diminishing, so that

\[ f_j' > 0, f_j'' < 0, \quad j = x, y \quad (6) \]

For any given factor price ratio (wage-rentals ratio) \(\omega\) the optimum factor-proportion in each industry is uniquely determined by

\[ \omega = f_x(k_x) / f_y(k_y) = f_x' / f_y' \quad (7) \]

Differentiating (7) with respect to \(\omega\), we get

\[ \frac{dk_j(\omega)}{d\omega} = -\left[ f_j'(k_j(\omega)) / f_j(k_j(\omega)) \cdot f_j'(k_j(\omega)) \right] \quad j = x, y \quad (8) \]

which, in view of (6), is always positive.

Ass. (iv) Under competitive conditions, the cost ratio \(c\) of commodity Y to commodity X is equal to the ratio of marginal productivities of capital in the X-industry to that in the Y-industry and is also a function of \(\omega\).

\[ c(\omega) = f_x'(k_x(\omega)) / f_y'(k_y(\omega)) \quad (9) \]
Logarithmically differentiating (9) and substituting (8), we obtain

$$\frac{1}{c(\omega)} \cdot \frac{d[c(\omega)]}{d\omega} = 1/[k_p(\omega) + \omega] - 1/[k_s(\omega) + \omega]$$

which, in view of the assumption (3), is always negative.

The relationship among the factor price ratio, the cost ratio and the factor-proportions in the two industries are illustrated by R-curve and $k_2$-and $k_s$-curves in the left-upper and lower quadrants of Fig. 1.

Ass. (v) Both capital and labor are fully employed

$$L_{x_i} + L_{y_i} = L_i, \quad i = a, b$$

$$K_{x_i} + K_{y_i} = K_i$$

which are combined into a single constraint

$$k_{x_i}(L_{x_i}/L_i) + k_{y_i}(L_{y_i}/L_i) = k_i, \quad i = a, b$$

The full-employment constraint (11) limits a range for the factor price ratio $\omega$ and also for the cost ratio $c$. To a given factor endowment ratio $k_i$, there corresponds the highest value of $\omega$, $\omega_{\text{max}}(k_i)$, and the lowest level of $c$, $C_{\text{min}}(k_i)$ which holds when all factors are employed in the $X$-industry and the lowest value of $\omega$, $\omega_{\text{min}}(k_i)$, and the highest level of $c$, $C_{\text{max}}(k_i)$, which holds when all factors are employed in the $Y$-industry, as illustrated in Fig. 1.\(^{15}\)

Only commodity $X$ is produced when the commodity price ratio $p$ is below the cost ratio $C_{\text{min}}(k_i)$ and the production of $X$ is decreased and that of $Y$ is increased when the commodity price ratio increases from $C_{\text{min}}(k_i)$ up to $C_{\text{max}}(k_i)$, beyond which only commodity $Y$ is produced. Thus the product ratio of two commodities supplied in each country is uniquely determined by the supply price ratio $p$ for its factor endowments ratio $k_i$,

$$Y_i/X_i = S_i(p, k_i), \quad i = a, b$$

and in view of the above relationships

if $p \geq C_{\text{max}}(k_i)$, $S_i(p, k_i) = \infty$

if $C_{\text{max}}(k_i) > p > C_{\text{min}}(k_i)$, $S_i(p, k_i) = c_i$ and

$$S_i(p, k_i) = [L_{y_i}f_y(k_{y_i})]/[L_{x_i}f_x(k_{x_i})], \quad dS_i/dp > 0$$

if $C_{\text{min}}(k_i) \leq p$, $S_i(p, k_i) = 0$

(13)

that is, it increases from zero to infinity over the range from $C_{\text{min}}(k_i)$ up to $C_{\text{max}}(k_i)$, as illustrated by $S_a$ and $S_b$ in the right-upper quadrant of Fig. 1. Since it is easily proved that

$$dC_{\text{max}}(k_i)/dk_i < 0, \quad \text{and} \quad dC_{\text{min}}(k_i)/dk_i < 0$$

and for a given price ratio $p$,

$$dS_i/dk_i > 0$$

(14)

the $S_i$-curve with a higher factor endowment ratio has lower values of the two critical cost ratios and is always located to the right of the $S_a$-curve.

\(^{15}\) This diagram was originally introduced by Samuelson [33] and has become familiar to every student of international trade. The two critical cost ratios are used most effectively by Johnson [10] and Onuki-Uzawa [38].
The product ratio supplied by the two countries combined is composed of the two supply functions $S_a$ and $S_b$, and is defined, in case the cost ranges of the two countries overlap with each other,

$$C_{\text{min}}(k_a) \leq C_{\text{max}}(k_b)$$

as

- if $p \geq C_{\text{max}}(k_a)$,
  $$S_{a+b}(p, k_a, k_b, L_a, L_b) = \infty$$

- if $C_{\text{max}}(k_a) > p \geq C_{\text{max}}(k_b)$,
  $$S_{a+b} = \frac{(Y_a + Y_b)}{X_a}, \quad dS_{a+b}/dp > 0$$

- if $C_{\text{max}}(k_b) > p > C_{\text{min}}(k_b)$,
  $$S_{a+b} = \frac{(Y_a + Y_b)}{(X_a + X_b)}, \quad dS_{a+b}/dp > 0$$

- if $C_{\text{min}}(k_b) \leq p < C_{\text{max}}(k_b)$,
  $$S_{a+b} = \frac{Y_b}{X_b}, \quad dS_{a+b}/dp > 0$$

(16)

It has a steadily increasing portion over the range between $C_{\text{max}}(k_a)$ and $C_{\text{min}}(k_b)$ and is always located between the two individual supply curves, $S_a$ and $S_b$; as illustrated in the right-upper quadrant of Fig. 1.

On the other hand, in the case where the cost ranges of the two countries does not overlap with each other,

$$C_{\text{min}}(k_a) > C_{\text{max}}(k_b)$$

the steadily increasing portion is interrupted by a vertical portion with constant product ratio over the range between $C_{\text{min}}(k_b)$ and $C_{\text{max}}(k_b)$, as

- if $C_{\text{max}}(k_a) > p > C_{\text{min}}(k_a)$,
  $$S_{a+b} = \frac{(Y_a + Y_b)}{X_a}, \quad dS_{a+b}/dp > 0$$

- if $C_{\text{min}}(k_a) \leq p \leq C_{\text{max}}(k_b)$,
  $$S_{a+b} = \frac{Y_b}{X_b}, \quad dS_{a+b}/dp = \text{constant}$$

- if $C_{\text{max}}(k_b) > p > C_{\text{min}}(k_b)$,
  $$S_{a+b} = \frac{Y_b}{(X_a + X_b)}, \quad dS_{a+b}/dp > 0$$

(17)

and is depicted as in Fig. 2.

So far we have discussed the supply side of the model. As to the demand side, the same pattern is assumed for both countries. This assumption, which is required for the derivation of the factor-proportions theory with the Leontief's definition of factor abundance, has
turned out to be a restrictive one.\textsuperscript{16}

Ass. (vi) The demand function, which is common to both countries, has unitary income elasticity so that the ratio of the two commodities demanded depends only upon the price ratio $p$ and is independent of the income per capita.

\[ \frac{Y}{X} = D(p), \quad D(0) = \infty, \quad D(\infty) = 0, \quad bD/dp < 0 \]  \hfill (18)

The negatively-sloped curve $D$ in the Fig. 1 and 2 depicts the demand function of each country $D_a$, $D_b$ and of two countries combined, $D_{a+b}$ at the same time.

The pre-trade equilibrium in each country is determined by

\[ S_i(p, k_i) = D(p) \]  \hfill (19)

which is illustrated in Fig. 1 and 2 by $Q_a$ and $Q_b$, the intersections of the $S_a$- and $S_b$-curves with the common $D$-curve respectively, and $R_a$ and $R_b$ along the $R$-curve corresponding to $Q_a$ and $Q_b$.\textsuperscript{17}

Thus it follows that before trade is opened labor is relatively cheap in labor-abundant country $A$, and country $A$ has a comparative advantage in the production of commodity $X$. On the other hand capital is relatively cheap in capital-abundant country $B$ and country $B$ has a comparative advantage in the production of commodity $Y$. The product ratio demanded and supplied ($Y/X$) is higher in country $B$ than in country $A$.

According to the traditional international trade model,

Ass. (vii) The transportation cost is negligible between the two countries and there is no artificial impediment to trade imposed by either of them so that the price level of each commodity is equal between the two countries after trade is opened.

Then the trade-equilibrium is determined by

\[ S_{a+b}(p; k_a, k_b, L_a, L_b) = D(p) \]  \hfill (20)

which is illustrated in Fig. 1 and 2 by $Q_*$, the intersection of the $S_{a+b}$-curve with the $D$-curve and $R_*$ along the $R$-curve corresponding to $Q_*$.\textsuperscript{16}

\textsuperscript{16} See Bhagwati [4] and Inada [9].

\textsuperscript{17} It is readily modified so as to introduce the difference in demand patterns between the two countries but the advantage of this diagram is the straightforward representation of the pre-trade and post-trade equilibria of both countries with the demand patterns common to the two countries.
Country A will export commodity X and import commodity Y, which is necessarily accompanied by the expansion of the X-industry and the contraction of the Y-industry. The opposite pattern of trade and specialization will result in country B. These shifts of production in response to the pre-trade comparative advantage will produce a tendency toward the equalization of the cost ratio and factor price ratio between the two countries.

IV. Patterns of Trade and Specialization

The pattern of specialization in trade-equilibrium is uniquely determined by the equilibrium price ratio $p^*$ with respect to two critical cost ratios, $C_{\text{min}}(k_a)$ and $C_{\text{max}}(k_b)$. If $C_{\text{max}}(k_b)$ exceeds $C_{\text{min}}(k_a)$ and the equilibrium price ratio $p^*$ falls in the overlapping portion of the two cost ranges as illustrated in Fig. 1 both countries will continue to produce both commodities and the cost ratio and factor price ratio and their absolute levels will be equalized completely between the two countries.

If, on the other hand, the equilibrium price ratio falls, in the range between $C_{\text{min}}(k_a)$ and $C_{\text{max}}(k_b)$ at the intersection $Q^{**}$ with the D-curve shifted leftward, as illustrated by $p^{**}$ in Fig. 1, country A will specialize completely in the production of its exportable commodity X while country B will continue to produce both commodities. The equalization of the cost ratios and factor price ratios will not be complete, since the equilibrium price ratio $p^{**}$ falls short of $C_{\text{min}}(k_a)$ and the factor price ratio of country B, $\omega^{**}$ exceeds that of country A, $\omega_{\text{max}}(k_a)$.

Finally, if the equilibrium price ratio falls in the non-overlapping range as illustrated by $p^{***}$ in Fig. 2, each country will give up the production of its importable commodity and specialize completely in the production of its exportable commodity. The equilibrium price ratio does not coincide with the cost ratio of either country and the factor price ratio in each country does not coincide with each other.

Instead of solving the trade equilibrium condition (20) straightforward, a complete set of solutions is obtained by examining the existence of excess demand or supply at the two critical cost ratios. Take the product ratios supplied and demanded at these critical cost ratios and denote them as $S^*(a), S^*(b)$ and $D^*(a), D^*(b)$ respectively as illustrated Fig. 1 and 2. If

\[ D^*(a) > S^*(a), \quad \text{and} \quad D^*(b) < S^*(b) \]

in the overlapping case as in Fig. 1, that is, there is an excess supply of commodity Y at the cost ratio $C_{\text{max}}(k_b)$ when country B specializes completely in commodity Y, while country A produces both commodities, and an excess demand for Y at the cost ratio $C_{\text{min}}(k_a)$, when country A specializes completely in commodity X while country B produces both commodities, the equilibrium price ratio will be between $C_{\text{max}}(k_b)$ and $C_{\text{min}}(k_a)$ and each country will produce both commodities in trade equilibrium. Table 1 shows all possible patterns of specialization of two countries according to

\[ D^*(a) \equiv S^*(a) \quad \text{(21)} \]
\[ D^*(b) \equiv S^*(b) \quad \text{(22)} \]

in addition to the condition of over-lapping and non-over-lapping cost ranges

\[ C_{\text{min}}(k_a) \equiv C_{\text{max}}(k_b). \quad \text{(15)} \]
A further consideration might be required concerning the characteristics of trade equilibrium, that is, how the absolute levels of the cost and factor prices are determined in the cases of complete specialization in at least one country. There will be a gap between the equilibrium price ratio and cost ratios in these cases, and this gap may seem conflict with the notion of the equalization of commodity prices under costless free trade. However, this conflict is easily resolved.

**Table 1**

<table>
<thead>
<tr>
<th>Overlapping case:</th>
<th>$D_e(a)&lt;S_e(a)$</th>
<th>$D_e(b)&gt;S_e(b)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\text{min}}(k_a)&lt;C_{\text{max}}(k_b)$</td>
<td>$A(X)$, $B(X, Y)$</td>
<td>$A(X)$, $B(Y)$</td>
</tr>
<tr>
<td>Non-overlapping case:</td>
<td>$D_e(a)&lt;S_e(a)$</td>
<td>$D_e(a)&gt;S_e(a)$</td>
</tr>
<tr>
<td>$C_{\text{min}}(k_a)&gt;C_{\text{max}}(k_b)$</td>
<td>$A(X)$, $B(X, Y)$</td>
<td>$A(X)$, $B(Y)$</td>
</tr>
</tbody>
</table>

Take for example a case illustrated by an equilibrium point $Q_{**}$ in Fig. 1, where the equilibrium price ratio $p_{**}$ coincides with the cost ratio of country $B$ but falls short of that of country $A$, $C_{\text{min}}(k_a)$. Since only commodity $X$ is produced commonly in both countries, its absolute cost is equalized between the two countries under costless free trade by means of the fluctuations of the exchange rate.\(^\text{18}\) The absolute cost of producing commodity $Y$ in country $A$ exceeds that in country $B$ so that commodity $Y$ is not produced in country $A$.

Although commodity $X$ is produced in both countries, the factor proportion is lower in country $A$ than in country $B$, and the marginal productivities of capital is higher in country $A$ while that of labor is higher in country $B$.

\[
(MPK)_a > (MPK)_b, \quad (MPL)_a < (MPL)_b
\]

Then the absolute level of the reward to capital, which is the product of the marginal productivities of capital with the absolute price of commodity $X$, is higher in country $A$ while that of labor is higher in country $B$.

Consider the case where each country specializes completely in the production of its exportable commodity, as shown by the equilibrium $Q_{***}$ illustrated in Fig. 2. The equilibrium price ratio $p_{***}$ is determined by the ratio of the cost of producing commodity $Y$ in country $B$ to the cost of producing commodity $X$ in country $A$ and there remains a gap in the cost ratios between two countries.

Since there is no commodity produced commonly in the two countries, there is no way of comparing marginal productivities of factors between them. Absolute levels of factor prices in each country depend uniquely on the absolute levels of prices of the commodities in which it completely specializes. If the demand situation in the world market is favorable for commodity $X$, then the absolute price level of commodity $X$ may be so high that both capital

\(^{18}\) Adjustments in the exchange rate is required for comparative advantage to be reflected correctly in the pattern of trade. This suggests in turn that a country can maintain a specialization pattern which does not reflect its comparative advantage by intervening in the exchange market. See also arguments in Section V.
and labor are paid higher rewards in country A than in country B in absolute terms (evaluated at the equilibrium rate of exchange).

We have seen so far that there are four patterns of specialization of two trading countries in trade equilibrium, (i) incomplete specialization in both countries, (ii) complete specialization in both countries, (iii) and (iv) complete specialization in one country but incomplete specialization in the other. Which pattern will result in under free trade depends upon various structural parameters of the two economies, such as the factor endowment ratios, the size of the countries, and the shape of the production and demand functions.  

Assume Cobb-Douglas production functions for each industry

\[ X = K^a L^{1-a}, \quad 1 > a > 0 \]  
\[ Y = K^\beta L^{1-\beta}, \quad 1 > \beta > 0 \]

the assumption of invariable ranking of factor intensities requires

\[ \alpha < \beta \]

The demand function is assumed to have unitary price elasticity in addition to unitary income elasticity

\[ Y/X = \mu \cdot p^{-1} \]

under which the average (and marginal) propensities to spend on commodity \( X \) and \( Y \) \( (m_x, m_y) \) are fixed regardless of the price ratio and \( \mu \) represents the degree to which commodity \( Y \) is preferred to commodity \( X \)

\[ \mu = m_y/m_x \]

Then the relationships (15), (21) and (22) can be specified more clearly with the aid of (23)-(25), i.e.

\[ k_a/k_b \geq \gamma, \quad \gamma = (1/\beta - 1)(1/\alpha - 1), \quad 1 > \gamma > 0 \]  
\[ k_a/k_b \geq \gamma [1 + (1-\delta)/\mu(1-\gamma)/(1+\delta/\mu)], \quad (28) \]  
\[ k_a/k_b \geq \gamma [1 + (1-\delta)/\mu(1-\gamma)/(1+\delta/\mu)], \quad (29) \]  

and the direction of the inequality in (28) is the same as that in (21) whereas the direction is reversed both between (15) and (27) and between (22) and (29).

Denote the equations in (27)-(29) as \( l_1, l_2 \) and \( l_3 \) respectively. Then we can represent the relationships of Table 1 by specialization regions in the \( (k_b, k_a) \)-plane in Fig. 3 according as to

\[ \gamma = (1-\alpha)/(1-\beta), \quad \lambda = L_b/L_a \]

19 Tinbergen [36], Meade [28] and Johnson [10] have stated in loose fashion that the pattern of incomplete specialization in both countries will result when factor endowments are not too divergent relative to the differences in factor intensities. In the Lerner-Pearce diagram with production isoquants in the two-factor quadrant, this can be shown by the factor endowments rays of two countries being inside the same diversification cone determined by the common tangent of a given factor price ratio. See Chipman [6] and also "equalization region" of Travis [37].

Oniki-Uzawa [38] and Bardhan [3] derive the conditions for specialization mathematically and illustrate them by 'specialization regions' in the quadrant of two countries' factor endowment ratios. However, it seems to be difficult to obtain more than the general characteristics of specialization regions under their general assumptions used in the previous section. It might, therefore, be worthwhile to specify production and demand functions in less general forms and derive conditions for specialization in a more specific way.
The position of $l_1$ is solely determined by the parameters of productions, whereas $l_2$ and $l_3$ diverge from $l_1$ to the extent that $(1-\alpha)m_xL_x$ is larger or smaller than $(1-\beta)m_yL_y$. When the structural parameters of the model satisfy the equality in (30), both $l_2$ and $l_3$ converge to $l_1$ as shown in Fig. 3c.

The economic meaning of the inequality (30) is easy to see. Under our assumption of the Cobb-Douglas production function and the demand function with unitary price- and income-elasticity, the allocation of labor between the two industries in a closed economy (i.e. in pre-trade-equilibrium) is represented by

\[
\frac{(1-\alpha)m_x}{(1-\beta)m_y} = 1
\]  

(30)

\[
\frac{\delta \lambda}{\mu} = \frac{(1-\alpha)m_xL_x}{(1-\beta)m_yL_y} \equiv 1
\]  

(31)
for any level of factor endowments ratio and for any factor price ratio and commodity price ratio. Thus the right-hand side of (31) represents the amount of labor employed in the \( X \)-industry of country \( B \) in comparison with the amount of labor employed in the \( Y \)-industry of country \( A \) before trade.

Now, a proposition is derived on the determination of the patterns of specialization under trade.

The parameters in production and demand functions and the size of a country determine the size of each industry before trade is opened.

In case (i) commodity \( Y \) is preferred to commodity \( X \), or country \( A \) is larger than country \( B \) just sufficient to offset the difference in labor’s share between the two industries, the size of the importable goods industries (i.e. the industries with which each country has a comparative disadvantage, the \( Y \)-industry in country \( A \) and the \( X \)-industry in country \( B \) will be equal before trade is opened between two countries.

Then if the factor endowment ratios differ between two countries by more than the difference in factor intensities between the two industries, then each country will give up the production of its importable commodity and specialize completely in the production of its exportable commodity, that is, country \( A \) will specialize in commodity \( X \) and country \( B \) in commodity \( Y \) after trade is opened.

If, on the other hand, the factor endowment ratios differ by less than the difference in factor intensities, then both countries will continue to produce both exportable and importable commodities.

In case (ii) commodity \( X \) is preferred to commodity \( Y \) or country \( B \) is larger than country \( A \), than the importable goods industry in country \( B \) will be larger than that in country \( A \) before trade, and there will be an intermediate range of the difference in factor endowment ratios within which country \( B \) will continue to produce both commodities while country \( A \) will specialize completely in commodity \( X \).

Finally in case (iii) commodity \( Y \) is preferred to commodity \( X \), or country \( A \) is larger than country \( B \), more than enough to offset the difference in labor’s shares between the two industries, then country \( A \) will have a larger importable goods industry \( \text{vis-a-vis} \) that of country \( B \) before trade, there will be another intermediate range of the factor endowments differences where the opposite pattern of specialization, incomplete specialization in country \( A \) and complete specialization in country \( B \), will result.\(^{20}\)

Although the convenient relationship (32) does not hold under more general production and demand functions, the above propositions suggest how the parameters on the demand and supply sides affect the patterns of specialization after trade.

\[ \text{V. Economic Consequences of Patterns of Specialization} \]

In the preceding two sections the determination of complete vs. incomplete specialization in a simple two-country, two-commodity, two-factor model was discussed. In this section it

\(^{20}\) Meade [28] ch. 23 presents the similar argument in a descriptive way and states that a country with smaller size, strong world demand for its export commodity and wide difference in factor endowment ratio with its trading partner would specialize completely in its export industry.
will be considered in a more general setting and the economic consequences of different patterns of specialization on international economic relations will also be discussed.

When a third commodity is introduced into the model, the difference in factor intensities which determines the patterns of specialization is that one in between the most capital-intensive commodity and the most labor-intensive one. Thus as we increase the number of commodities in our two-factor model, the likelihood of incomplete specialization will increase if the new commodity is either more capital-intensive or more labor-intensive than any one previously existing. Fig. 4 which reproduces the left-lower quadrant of Fig. 1 illustrates that country B can also be under incomplete specialization at a given world price ratio (and its corresponding factor price ratio \( w_* \)) if a third commodity Z is introduced.\(^{21}\)

![Fig. 4](image)

Complete specialization in the production of only one commodity be considered to be quite abstract from the real world of many commodities. In fact, however, a certain group of commodities have a narrow range of factor intensities and almost the same elasticities of substitution so that they may be treated as a single commodity with the same production function in the real world of where imperfections in markets exist. Complete specialization in such a group of commodities may be defined as “almost complete specialization.”\(^{22}\)

In a many-country world, factor price equalization holds among countries any pair of which produces any two common commodities at a given commodity and factor price ratio, and also all countries can produce commodities over the whole range.\(^{23}\) But it does not necessarily follow that complete specialization is unlikely in a many-country, many-commodity world. The likelihood depends upon the differences in factor endowment ratios among countries or the geographical distribution of the world’s total endowments of factors. For a given world commodity and factor price ratio the group of very capital-abundant countries might be located above and outside the bundle of the \( k_1 \)-curves in Fig. 4 while the other group of very labor-abundant countries might be located below and outside the bundle. The countries of the first group will have to specialize almost completely in highly capital-intensive commodities.

\(^{21}\) Travis [37] has developed the diagrammatical device of representing the likelihood of incomplete specialization in both countries in many-commodity, two-factor case. See his “equalization region” (pp. 1–38).

\(^{22}\) Hicks [8] introduced this concept when he applied the factor-proportions theory to the real world.

\(^{23}\) See Samuelson [35].
while the countries of the second group will have to specialize almost completely in highly labor-intensive commodities under free trade.\footnote{This is what Travis [37] pp. 39-67 showed by means of the estimation of his "empirical equalization region." He concludes that "under free trade many nations would specialize in relatively few commodities."} If, however, either group of countries is large enough to make the determination of world price ratio closer to its own cost range, then it will have a more diversified structure of production.

The existence of a minimum optimum scale of production, which is not necessarily the lowest point on the average cost curve but rather below the scale which production cannot take place economically, should be taken into consideration as a factor increasing the likelihood of complete specialization for countries of small and medium sizes even in the case where their capital-labor ratios are intermediate ones. On the other hand transportation cost, tariffs and other artificial impediments, and preference for domestically produced commodities in the case where product differentiation tends to increase the likelihood of incomplete specialization.

So far we have maintained the assumption of two homogeneous factors of production. Generally in case of $r$ factors, incomplete specialization requires $r$ commodities produced commonly between any pairs of countries.\footnote{See Samuelson [35].} However, it is harmful to the factor-proportions theory to increase the number of factors of production and to allow substitutability between factors in the production of each commodity, since in a general $r$-factor model the ranking of factor-intensities will not be a clear concept characterizing the productions of individual commodities. If more than two factors of production are distinguished and considered to be important, we have to emphasize two strategic factors and to follow the conventional two-factor approach, either by assuming complementarity among some of them or international mobility of the others, as was mentioned in Section II.

Complete and incomplete specialization in a country under free trade should be clearly distinguished because of the difference in welfare implication, and also because of the differences in the flexibility of adjustment of an economy to an exogeneous change.

It was suggested in the preceding section that under complete specialization in at least one trading country, there will be differences in factor prices and consequently the world economy will be short of a Paretian optimum. In this case free trade is only an incomplete substitute for factor movements to achieve the Paretian optimum for the world as a whole, and freer factor movement should be promoted to supplement free trade. Travis, as a result of empirical investigations he has undertaken, has shown that complete specialization would be most likely in many countries under free trade, in view of the production techniques and the disproportionate distribution of factors of production among countries, and recommends freer capital movement between countries.\footnote{See Travis [37]. Meade [28], ch. 27 also discussed the effect of factor movements in two countries under complete and incomplete specialization.}
view of their high complementarity with physical capital in major manufacturing industries, is a main reason for the alleged poor achievement of past economic aid to the less developed countries. It is strongly desired that the movement of these productive factors should be promoted so as to provide a balanced supply of physical and human capital. In the long run the transferred resources should be channelled to investment in education and training of the labor population of the less developed countries in order to achieve the same end.\(^2\)

Incomplete specialization observed in all trading countries does not necessarily imply a Pareto optimum, since it can be maintained by tariffs and artificial trade impediments. There is observed in almost every country a preference for incomplete specialization to complete one or a tendency to preserve import-competing industries, motivated by such considerations as self-sufficiency, infant-industry protection, or redistribution of incomes in favor of a particular group, rather than a desire for optimum resource allocation according to the law of comparative advantage. Under the present I.M.F. system with a pegged exchange rate, observed patterns of trade and specialization do not correctly reflect comparative advantages and they are strongly influenced by protective commercial policies of individual countries.\(^2\)

In positive economics, complete specialization is associated with inflexibility in adjustment to a given exogenous change, whereas more flexibility is provided under incomplete specialization.

As we have seen in Section IV, under complete specialization in both countries factor prices depend only upon the terms of trade. Thus the unfavorable change in world demand for a country’s export commodity will make its terms of trade deteriorate and then lower its factor prices. This seems to explain why primary-producing countries are now suffering from the slow rate of growth in their real income and also why there is a widening gap in the standards of living between primary-producing and industrial countries.\(^2\)

Similarly, under complete specialization in export production, any growth of productive factors or technical progress will have a pro-trade bias and an unfavorable pressure on its terms of trade.\(^2\) However, under incomplete specialization, these unfavorable tendencies will be mitigated by the reallocation of factors from the export industry to the import-competing industry.\(^2\)

Inflexibility in adjustment under complete specialization is well illustrated by Chenery’s

\(^2\) Another contrasting welfare difference between two specialization patterns is concerned with the welfare gains from forming a customs union. Typically there will be no trade creation gains but trade diversion loss in the case where member countries are subject to complete specialization before the formation of the customs union, whereas under incomplete specialization the trade diversion loss tends to be offset by trade creation gains. See Kojima [19].

\(^2\) See Travis [37] ch. 4 & 5. Arguments of effective rate of protection recently developed by, Johnson [12], Balassa [1] and others have cast light on the effect of tariff structure on the pattern of trade and specialization. As for an empirical study of Japan’s tariff structure along the same lines see Yamazawa [41].

\(^2\) This widening gap in real income between primary-producing countries and their industrialized trade partners has occasionally been cited as conflicting with the factor-proportions theory. See Myrdal [29]. But it is clear that the pattern of complete specialization but not of incomplete specialization should have been applied to this phenomenon.

\(^2\) Johnson [13].

\(^2\) This argument depends upon an implicit but not implausible assumption that an import-competing industry cannot be established without a considerable time lag to be competitive even where moderate tariff protection prevails.
"two-gaps-approach" in the estimation of aid requirement by which the investment-savings gap and the foreign exchange gap give different estimates for aid requirement for less-developed countries\textsuperscript{2} or Linder's "maximum-export hypothesis" for less developed countries.\textsuperscript{3} Both Chenery and Linder implicitly assume that the less-developed countries specialize completely in their primary export industries and the earnings from them are limited by inelastic world demand and unfavorable terms of trade so that there always exists a tendency for their export earnings to fall short of their import requirements and for additional savings squeezed by disabsorption policies to be left idle consequently fail to increase export earnings thereby failing to improve their trade balances.

If, on the other hand, those countries have import-competing manufacturing industries, they can utilize easily adaptable productive factors such as unskilled labor which were previously employed in unfavorable primary export earnings in more favorable directions. It may well be said that the flexility of adjustment provided by a more diversified production structure in advanced countries contributes to their stable balance of payments relative to those of primary-producing countries with highly-specialized production structure.

REFERENCES


\textsuperscript{22} Chenery [5].
\textsuperscript{23} Linder [27].


