THE PHASE OF UNLIMITED SUPPLIES OF LABOR

By KAZUSHI OHKAWA *and* RYOSHIN MINAMI*

Institute of Economic Research

In his path-breaking articles [5] [6] Arthur Lewis presented a modern version of the classical system, from Smith to Marx, in terms of his concept of "unlimited supplies of labor" at subsistence wages. His idea was to establish the kind of economics which would throw light upon the economic development problems of under-developed countries with surplus populations, because the neo-classical assumption of a limited supply of labor, in his understanding, would not be applicable to the problems of these countries. Since then, not a small number of authors have followed this line of thought in the field of economic development. Some of them succeeded in their efforts to develop and refine Lewis' original presentation. This paper attempts to make a further contribution along this line.

We shall be primarily concerned here with a generalized model of the phase of unlimited supplies of labor. This theoretical approach is taken due to our belief that the basic assumptions hitherto made in building two-sector models of the Lewis type seem to be not necessarily flexible and general enough to be consistent with historical realities. First, attempts will be made to examine the basic assumptions in general. In defining the characteristics of the subsistence sector in particular, special efforts will be made in order to specify the operational assumptions to be used for the discussion that follows. Secondly, we shall describe the main results of our investigation through the different characteristics of several cases, which are based on different assumptions respectively. In doing so, even cases of increasing wage rates will be discussed. Lastly, mathematical treatments of these models will be given to provide a more or less rigorous presentation. Because of technical limitations, our models will be built on rather simple assumptions, and yet we hope that these will contribute to clarifying the general nature of the phase of unlimited supplies of labor, and in particular, to illuminating the problem of possible solutions of various equilibrium growth paths.[1]

I. *The Basic Assumptions*

The economy is assumed to be of dual structure, being composed of two sectors:

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[1] For simplifying the discussion, the problem of output-equilibrium between the two sectors is not examined here. This does not imply that the possible changes in the terms of trade are unimportant. An illuminating analysis is given, among others, with respect to agriculture by Ranis and Fei [13]. One of the present authors discussed this problem elsewhere [11].

capitalists sector (sector I) and subsistence sector (sector II), following Lewis' original model and expressions. In examining the features of these sectors and the assumptions which are relevant to the discussion that follows, no particular difficulties are involved with regard to sector I.

The features of sector I can be assumed in a most simplified form. With a certain stock of technological knowledge available, the capitalists, (in whom are combined the functions of both enterprise and finance) carry out the production process by using available capital and labor. They follow the principle of profit-maximization and at the equilibrium state of perfect competition the wage rates equal the marginal product of capital, leaving no surplus for income distribution. The economy grows by their investments for production, being financed by their own savings out of profits, so long as the labor force is readily available. All the workers' earnings are assumed to be spent for their consumption. An equilibrium growth path can be discussed in a very simple form under these conditions, if we further assume a sustained equilibrium of savings=investments.

In the growth process of sector I described above, the rate of capital accumulation (ratio of investment to capital stock) will be determined by the two variables: profit rates and savings ratio. Given the savings ratio at a certain level and allowing no change, only the level of profit rates concerns us here. How can the profit rate be determined? It can only be endogenously determined within sector I, when either the amount of labor available or the wage rate to be paid is given from outside this sector. This answer constitutes the core of our problem.

Suppose a certain production function can represent the production process with a given technology. It would be easy to ascertain that our simple system is composed of four equations: production function, equality between wage rate and marginal product of labor, equality between profit rate and marginal product of capital and equilibrium of savings= investment. The number of variables contained in this system, however, are five: output, capital, labor, wage rate and profit rate; the output elasticities of capital and labor being both assumed given. Hence, one more variable is required from outside in order to complete the system. Which variable have we to select? The answer should depend upon the nature of the problem one may wish to analyze.

Lewis stressed the importance of giving the wage rate from outside, instead of the labor force, which is assumed as given in the neo-classical system. Following him, we define the basic feature of sector I to be its complete dependence on the wage rate which is determined outside the sector. To substantiate the given wage rate, Lewis introduced the classical notion of subsistence wages, which in our view seems to imply some vagueness, conceptually. This will be discussed in detail later, and it is sufficient at the moment to note that the unchanged wage rate based on the subsistence level is given central importance in his model.

Once sector I is given a certain level of the wage rate, the rate of profit and accordingly the rate of capital accumulation is determined in the system. At the same time the demand for labor at that wage rate will be determined. So long as the supply of labor to this sector exceeds the demand, we can say that an unlimited supply of labor exists at that wage rate. The sources of labor supply will be discussed in later pages. If all the required conditions mentioned above are adequately fulfilled in order to provide the capitalists with sufficient profit rates to initiate their enterprises, then the economy of sector I, it is argued, will expand in a sustained manner as capital accumulation continues until the sources of unlimited

supplies of labor are exausted. If we assume an economy which is completely free either from the diminishing returns due to natural resource limitations or from the effects which exogenous factors would bring into the economy, it is quite possible to identify a sustained equilibrium growth path with unchanged wage rates. Lewis rightly pointed out that throughout the entire path of such an expansion a capital-widening process takes place, the capital-labor ratio being held constant.

These are the main features of the phase of unlimited supplies of labor, centering on sector I. In our view, there is no conceptual difficulty with this approach.

Next, let us turn to a discussion of the features of sector II. Unlike the case of sector I, we have to face some difficulties in simplifying the appropriate assumptions which are required for characterizing this sector. This should be done in a way so as not to depart too far from historical realities. For one thing, the conventional use of the somewhat vague concept "subsistence level" is subject to different interpretations. For another, the historical and institutional circumstance, inherited from pre-modern stages, vary from one country to another. Nevertheless, with some hesitation, we have to make several bold assumptions for the sake of simplifying the basis for subsequent analysis.

It is most convenient, we believe, to begin with the problem Lewis himself left un-solved. After referring to the classical notion of subsistence level, he suggests a more objective index, namely, the average product of the peasant. But again he doubts the applicability of this objective standard by recognizing the case of tenant-peasants for, as he observed, the rent will probably be adjusted so as to leave them just enough for a conventional level of subsistence. May we cite his own conclusion. "It is not, however, of great importance to the argument whether earnings in the subsistence sector are determined objectively by the level of peasant productivity, or subjectively in terms of a conventional standard of living. Whatever the mechanism, the result is an unlimited supply of labor for which this is the minimum level of earnings [5, p. 409]."

It is our contention that the mechanism of the "objective standard" can imply different results from those of the mechanism of the "subjective standard." In this section, however, we take up only the subjective standard, leaving the discussion of the objective standard to the subsequent section. Here we would like to begin with the simple assumption that a certain level of minimum subsistence is inherited historically at the beginning of modernization from the pre-modern stage and that this level continues to prevail due to inertia during the phase under consideration.

First of all, sustained sources of unlimited supplies of labor must be provided from sector II. To meet this requirement throughout the entire phase under consideration, there must always exist a certain number in the labor force of this sector who are willing to be employed by the capitalists in sector I at unchanged subsistence wage rates. Two elements are contained in this situation: one is an ever-lasting source of the labor force and the other a maintenance of incentives for their outmigration. Here we are concerned with the latter, leaving the discussion of the former to later pages.

The incentive is assumed to be differentials between subsistence wages and earnings of labor employed in sector II. The differentials should be considered with appropriate allowances such as the extra living costs required for urban life and the psychological resistance or social inertia to out-migration, etc. These considerations, however, would not alter the nature of the problem nor the assumption that the differentials must be large enough to

sustain incentives for migration.[2] Implicit is a potential of earnings in sector II, which must be necessarily lower as compared with the subsistence wages of sector I.

Another feature remains to be discussed with respect to sector II: that is, the feature of maintaining such workers whose earnings are lower than the subsistence level. In order to explain these features of sector II, it is most convenient to assume that typically this sector is composed of traditional households or extended families,[3] that is, unseparated units of production and consumption. The heads of households are responsible for both carrying out production and maintaining all family members. Let us take a typical case of peasant farming. With given technological knowledge and a negligible amount of capital goods (which can be assumed, in theory, part of land because of their perfect complementarity with land), the household head carries out farming on his own land, by using his own family workers. Some sort of maximization principle (which must be different from the principle assumed for sector I) can be defined in one way or another, but we hesitate to do so here.[4] The minimum specifications required are that the household head maintain his dependent family workers whose production contributions are lower than the subsistence level and that he offer no resistance to their outmigration to sector I if they can find jobs there.

Figure 1 shows a schematic presentation of what has been said thus far. At point E on the horizontal axis the average product per worker (AP) equals the subsistence level (SL). In reality a certain margin ($AP>SL$) may be required at least to maintain the household at a traditional level of production and consumption corresponding to the subsistence level of individuals. For the sake of theoretical simplicity, however, the margin can be assumed as zero. Given a decreasing return of the production curve, it is assumed that the household can not maintain more dependent workers beyond this limiting point. At this point the

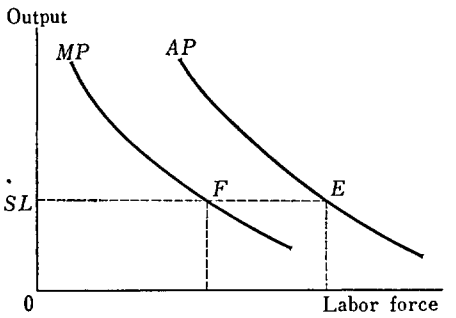
[2] In this respect, it is important to make a distinction between the head of households and dependent family workers. In the case of household head, the possibility of having job opportunities for his dependent family workers in sector I is an important additional factor to be considered in the discussion. Mazumdar presents an interesting argument in this respect in the light of Indian experience [7]. As will be stated later in the text, however, we maintain that during the phase under consideration the source of unlimited supplies of labor concerns solely the dependent family workers. If outmigration of household heads accompanying their family workers is required even in the initial phase of economic development for one reason or another, as Mazumdar argues, the situation of demand and supply of labor would present a completely different picture. In the light of Japanese experience throughout a century of economic development, the main body of outmigrated labor has consisted of dependent family workers [12].

[3] An interesting description of people's behavior is presented by Bauer and Yamey with respect to the extended family system. The main concern here, however, is not the extended size of the family but the paternal behavior of the household head. With respect to the latter feature, the prewar Japanese pattern was typical especially as it was characterized by the existence of the right of primogeniture.

[4] For the sake of simplicity we have assumed that no landlords exist in sector II. If they were to be incorporated in the discussion, various functions could be attributed to them in accordance with the institutional conditions of the economy under consideration. This would not, however, alter substantially the core of our statement that follows.

Some principle of maximizing total output can be assumed, for example. The equilibrium state in this case would be determined at the point of zero marginal productivity of labor. Suppose that such a principle of behavior had been dominant in pre-modern times and that it was inherited by the modern period due to historical inertia. This would be an attempt along the line of thought which is well-presented by Georgescu-Roegen [4]. This attempt seems very suggestive in specifying the assumptions hitherto made with respect to the behavior of traditional households. We did not follow this line of thought, however, because no agreement was obtained between us with regard to applying the assumption of zero marginal productivity of labor to the system under consideration.

FIG. 1



marginal product of labor (MP) is lower than SL. If this point is sustained continuously as capital accumulation goes on in sector I, we can define the most simple phase of unlimited supplies of labor. This is only possible by assuming that all the increase in the labor force due to population growth in sector II is absorbed by the expansion of sector I. Such a special case, however, cannot be assumed in general. Any points located to the left of point E can represent given initial conditions. Let us assume that the number in the labor force in sector II tends to decrease as capital accumulation goes on in sector I. These points move to the left and sooner or later they reach point F, where MP equals SL. Between the range demarcated by points E and F, the condition $SL>MP$ will be maintained, although the differential between the two tends to be narrowed. The crucial assumption here is that despite this change, the supply price of labor will be kept unchanged at the same subsistence level, because of the continued influence of historical inertia. If this is satisfied, the range above demarcated can define the boundary condition for the phase of unlimited supplies of labor.

The range was thus specified in terms of economic growth. It specifies at the same time the boundary condition for the existence of unlimited supplies of labor at every point of time during economic growth. In this case it implies a theoretically assumed schedule for the supply price of labor. We draw particular attention to the two-fold implication of this illustration for the sake of discussions that follow.

We want to make further comments on points E and F before concluding the discussion on basic assumptions. First, with regard to point E, the system assumes population growth as an exogenous variable. Unlike the Ricardian system, as we interpret it, the system makes no link between the concept of subsistence level and the movement of population, since population growth is not affected by changes in per-head income. Thus, given the rate of population growth and the demand for labor which is determined by the mechanism of sector I, the surplus labor, if any, is merely assumed to remain in sector II at the unchanged subsistence level, irrespective of the resultant changes in productivity. The subsistence sector is treated as if it were a sponge which can absorb the labor force unlimitedly. In order to eliminate this theoretical difficulty while retaining the basic feature of the system, we assumed previously a limiting point shown by E. Beyond this point to the right, one may assume a sort of decreasing supply curve expressed by AP. But we hesitate to do so, because SL is defined here as the minimum level of subsistence.

Secondly, F in Figure 1 marks the turning point towards the next phase. If capital

accumulation goes on beyond this point, the supply price of labor henceforth will be determined not by the subsistence level but by the marginal product of labor. It is argued by Lewis and some other authors that the mechanism of the economy is now characterized by neo-classical thought: the wage rates and the allocation of the labor force between the two sectors are determined in accordance with the principle that the marginal productivity of labor should be equal in both sectors. There is no difficulty here except one point which will be touched upon later.

II. *Alternative Approaches*

Throughout the previous discussion, it was assumed that a constancy of the subsistence level is possible despite changes in productivity. This assumption is given by dint of the historical inertia applied to the subjective standard. We believe that this is not the only possible assumption which can be made. Alternative approaches can be pursued by adopting different assumptions, and it is desirable to do so to arrive at a generalized model. Among these, the objective standard previously mentioned will be discussed first together with its relationship to the subjective standard. Secondly, and more importantly, a general case which includes increasing subsistence wages will be taken up.

To begin with the objective standard, the picture appears quite different from that described in the previous section. In order to meet the requirement of an unchanged level of subsistence wages, a constant average productivity of labor (and accordingly its marginal productivity) should be maintained in household production. This would be impossible, in general, if we assume the schedule illustrated by Figure 1 in the previous section. Even under a permissible assumption of an unchanged technology, labor productivity and accordingly the supply price of labor would increase as the number of family workers decreases following a greater demand for labor as a result of capital accumulation. However the objective standard can be applicable in the special case where productivity changes can practically be assumed nearly zero. In reality, in the initial phase of economic development, the demand for labor by sector I is relatively very small as compared with the total labor force employed in sector II. Therefore, labor supply from this sector is possible to a certain extent without causing any substantial effects to its productivity level. So long as this is practically possible within this narrow range, the supply price of labor can be assumed constant. And the notion of unlimited supply of labor can be applicable within the limited range thus practically defined.

The objective standard can also be understood in terms of per-head consumption. The consumption level in this respect is defined to be flexible in principle, instead of being rigid due to historical inertia, in relation to the current level of productivity. Enke suggests an interesting idea in this connection [3]. Subsistence wages, he argues, concern not the level of productivity of household production but the consumption level of the individuals. Because of the peculiar behavior of traditional households, the latter level, in practice, changes a little despite substantial changes in productivity. It seems doubtful that such a fact can be assumed as a general basis for establishing the notion of unlimited supplies of labor. It does suggest, however, a practical range within which a constant level of consumption can theoretically be assumed.

In this connection, the relationship between the objective standard and subjective standard is observed. Suppose that the initial condition for modern economic development is given at point E in Figure 1. At this point, according to our simplified definition, average productivity is equal to the subsistence level. Within a certain narrow range extended to the left, this equality can be assumed to be maintained more or less unchanged. To that extent as Lewis has stated, we can say that it is not of great importance to the argument whether earnings in the subsistence sector are determined objectively by the level of productivity, or subjectively in terms of the conventional standard of living. From the historical point of view, this seems to be a plausible argument. In our view, however, this is not generally defendable in theory. Any points within the range between E and F can be considered as the initial conditions of sector II and these points cannot necessarily satisfy the indifference condition of the two standards.

Once such a practical consideration is taken into our discussion, the previous assumption of an unchanged subsistence level due to historical inertia requires here further examination. We believe that this assumption is much more realistic than the objective standards. But this also cannot be entirely free from practical limitations. The labor supply schedule, illustrated by Figure 1, implies two dimensions: one is that potentially it is applicable at every point in time of the historical process of economic development and the other that historically it expresses a locus of successive equilibrium points under an unchanged situation during economic development. With respect to the former the assumption of an unchanged subsistence level can be valid. Concerning the latter, however, it can not necessarily be supported. During the long process of economic development, the various factors responsible for maintaining historical inertia may substantially change and the subsistence level may rise. But, if the disguised unemployed, whose marginal productivity is lower than the subsistence level, still exist, the mechanism of the economy may remain the same.

Now, these considerations lead us further to the next problem of a generalized model. In practice, the given variables in the system, such as population, technology as well as the subsistence wage are all subject to changes during economic growth. The theory of unlimited supply of labor, therefore, should be tested by taking these changes into consideration in order to identify the scope of its validity.

Let us begin with a problem: what about the effects of technological progress in general? In the previous section we assumed that the system is given a certain level of technological knowledge and that there is no technological progress in either sector. If technological progress takes place in sector I, what effect will this have on the system? If technological progress takes a simple type of neutral shift in the given production function,[5] it is obvious that all the benefits brought about will accrue to the capitalists. Given a constant savings ratio, technological progress of this type will simply accelerate the growth rate of such an economy. This has already been pointed out by Lewis on the assumption of no technological progress in sector II. The most simple and natural case would be one in which no technological progress is assumed in sector II because it is composed of subsistence households. In reality, however, traditional agriculture can make technological progress and this may contribute to

[5] An assumption of a neutral shift of the production function in sector I is too simple to represent historical reality. The problem of technological unemployment of the Marxian type may be raised here. If this is dominant, the sources of unlimited supplies of labor would be continuously reproduced to the extent that the economy could never arrive at the turning point. In our view, however, such an assumption does not fit well with historical reality.

a considerable extent to developing the economy as a whole.[6] Therefore, it is worthwhile examining the effects of technological progress in sector II.

Again, under the simple assumption of a neutral shift in the production function, it is easy to see the following effects: if the subsistence level is kept unchanged despite technological progress, the range of unlimited supplies of labor, defined in the previous section, will be made wider. This can be shown by a shift of point E to the right in Figure 1. In other words, the capacity of holding the labor force in sector II becomes greater to that extent. The potential source of unlimited supplies of labor would become richer, other things being equal. Such a situation can be assumed, if we can still apply the effects of historical inertia to the subsistence level. However, an assumption of a constant subsistence level despite technological progress can not be fully supported. If this level is raised as a result of technological progress, the picture would be quite different. It would appear as follows: the subsistence wage would be raised to that extent and, other things being equal, the profit rate would be lowered and the rate of capital accumulation decelerated in sector I. In general, in order to approach reality more closely, we have to consider the case where technological progress takes place in both sectors at unequal rates of shift. This will be examined in the next section.

The effect of an increase in the subsistence level deserves particular attention. We have observed that the subsistence level can be raised due to technological progress in sector II to the extent that the historical inertia cannot retain it. The trend of an increasing subsistence level, however, can occur independently, at least in theory, from technological progress. Again referring to Figure 1 in section I, suppose an upward shift of the subsistence level. Other things being equal, points E and F will move to the left, the new SL line being crossed by both the AP and MP curves at the higher levels. At this new level, however, the range of unlimited supplies of labor can be defined with respect to the new level of subsistence in a similar way as was done previously. If such a shift occurs continuously as a historical process, it will result in an increase in subsistence wages due to a shift of the subsistence level. It brings forth the process of capital deepening in sector I, more capital being used in relation to labor. The capital widening process thus disappears in the system. If we follow Lewis' original concept of unchanged subsistence wages, such a situation cannot be called the phase of unlimited supplies of labor. However, so long as it maintains the basic feature of the system, that is, the complete dependence on the wage rate which is determined outside sector I, this must be called, in our view, the phase of unlimited supplies of labor. With population growth and technological progress, how can the boundary condition be satisfied in the case of an increasing subsistence level? An attempt to arrive at the answer will be given in the next section.

Lastly, we want to say a few words on the turning point towards the next phase. The economy will arrive at this point in the long run, if economic development continues to raise the marginal productivity of labor in sector II to the extent that it equals the subsistence level, irrespective of whether the subsistence level remains constant or increases. In principle,

[6] In particular, in the light of Japanese experience with the initial phase of economic development, traditional agriculture based on household production grew at a considerable rate in terms of both output and productivity; technological progress had taken place and the level of living and wage rates increased to a certain extent. These responses occurred together with the increase in population [10]. In view of this, it seems that the features of models of the Lewis type are too rigorous to be applied to such historical realities.

the nature of the turning point in our generalized model remains the same as in Lewis' case. In our view, however, after the turning point, although the supply price of labor is now increasing, the economy will not necessarily eliminate the subsistence sector; there may still remain a considerable labor force to be absorbed by sector I. In this respect, such a historical period draws particular attention, being distinguished from a genuine phase of limited supplies of labor. However, this would have required an extensive discussion and deserves separate analysis.[7]

III. *Mathematical Presentation of a Generalized Model*

Under the condition of unlimited supplies of labor, the supply price of dependent laborers in sector II (w_2) is equal to the subsistence level (SL);

$$w_2=SL(t),$$

and at the same time the marginal productivity of labor is lower and the average productivity is higher than SL. If the production function in sector II is given as follows;

$$Y_2=A_2(t)F_2(N_2, R_2),$$

the following relation must be satisfied;

$$\delta\frac{Y_2}{N_2}<SL<\frac{Y_2}{N_2} \quad \text{or} \quad \delta\frac{Y_2}{SL}<N_2<\frac{Y_2}{SL},$$

where Y_2, X_2 and R_2 denote real output, employment and natural resources respectively. R_2 is assumed constant. δ and ε are output elasticities of labor and natural resources (assumed constant); i.e.,

$$\delta\equiv\frac{\partial Y_2}{\partial N_2}\Big/\frac{Y_2}{N_2}, \quad \varepsilon\equiv\frac{\partial Y_2}{\partial R_2}\Big/\frac{Y_2}{R_2},$$

where we assume the relation

$$\delta+\varepsilon=1$$

or linear-homogeneity of the production function, and

$$\delta, \varepsilon>0$$

or decreasing returns to each factor. $A_2(t)$ denotes the shifts in the production function. For simplicity's sake, we call it neutral "technological progress"; the rate of shift is assumed constant. It must be noted that SL is not necessarily constant, but may rise historically. For simplicity, however, we assume the rate of increase in SL as constant.

On the other hand the production function in sector I can be stated as follows:

$$Y_1=A_1(t)F_1(N_1, K, R_1),$$

where Y_1, N_1, K and R_1 denote respectively real output, employment, capital stock and natural resources in sector I. We assume linear-homogeneity and decreasing returns to each factor, or

$$\alpha+\beta+\gamma=1, \quad \alpha, \beta, \gamma>0$$

where α, β and γ are output elasticities of labor, capital and natural resources (assumed constant); i.e.,

$$\alpha\equiv\frac{\partial Y_1}{\partial N_1}\Big/\frac{Y_1}{N_1}, \quad \beta\equiv\frac{\partial Y_1}{\partial K}\Big/\frac{Y_1}{K}, \quad \gamma\equiv\frac{\partial Y_1}{\partial R_1}\Big/\frac{Y_1}{R_1}.$$

[7] In the light of the current situation of the Japanese economy, we are particularly interested in a phase of this type.

Elsewhere, Ohkawa has called it the phase of semi-limited supplies of labor. Enke presents a similar view and his analysis gives valuable suggestions [3]. Smithies' illuminating analysis of an economy with a flexible supply of labor also is suggestive in this respect [15].

Technological progress, expressed in $A_1(t)$, is assumed to go on at a constant rate. The equilibrium state of the economy is represented in the relations below;

$$w_1=\alpha\frac{Y_1}{N_1},\quad q=\beta\frac{Y_1}{K},$$

where w_1 and q denote the real wage rate and profit rate in this sector. In other words w_1N_1 is labor income, qK profit income, and the residue $Y_1-(w_1N_1+qK)$ is paid as rent to the owner of natural resources. Assuming saving and investment occurs only in profit income, we have

$$\dot{K}=sqK,$$

where s is average propensity to save out of profit.

The conditions of equilibrium between the two sectors are

$$w_1=w_2$$
$$L(t)=N_1+N_2,$$

where L is labor supply in the economy given from outside the system.

Hence we have a model of economc growth as the following:

(1) $Y_1=A_1(t)F_1(N_1, K, R_1)$

(2) $w_1=\alpha\frac{Y_1}{N_1}$

(3) $q=\beta\frac{Y_1}{K}$

(4) $\dot{K}=sqK$

(5) $Y_2=A_2(t)F_2(N_2, R_2)$

(6) $w_2=SL(t)$

(7) $w_1=w_2$

(8) $L(t)=N_1+N_2$.

The model has a unique solution, because the number of variables, Y_1, Y_2, K, N_1, N_2, w_1, w_2 and q are equal to the number of equations, if the initial values of N_1, N_2 and K are given.

From the equations (2) (6) and (7) the real wage rate in sector I is determined at the level $SL(t)$. Capital stock is given from (4). Capitalists employ a certain number of laborers to the extent that the profit rate reaches a maximum, under the conditions of given real wage rate, capital stock, natural resources and the production function (1). As the labor supply is given, employment in sector II is determined as a residue in equation (8), and the volume of output in this sector can be known from the production function (5).

Rewriting these equations in growth rate terms, we get

(1)′ $G(Y_1)=\bar{G}(A_1)+\alpha G(N_1)+\beta G(K)$

(2)′ $G(w_1)=G(Y_1)-G(N_1)$

(3)′ $G(q)=G(Y_1)-G(K)$

(4)′ $G(K)=sq$

(5)′ $G(Y_2)=\bar{G}(A_2)+\delta G(N_2)$

(6)′ $G(w_2)=\bar{G}(SL)$

(7)′ $G(w_1)=G(w_2)$

(8)′ $\bar{G}(L)=WG(N_1)+(1-W)G(N_2)$,

where W is defined as the ratio of employment in sector I to total employment; i.e.,

$$W \equiv \frac{N_1}{N_1+N_2},$$

From the equations (1)' (2)' (3)' (4)' (6)' and (7)', we get

(9) $$G(q)=\frac{s\gamma}{1-\alpha}\left\{\frac{1}{s\gamma}[\bar{G}(A_1)-\alpha\bar{G}(SL)]-q\right\}.$$

Defining the relation as follows

(10) $$\bar{q}=\frac{1}{s\gamma}[\bar{G}(A_1)-\alpha\bar{G}(SL)],$$

equation (9) becomes

$$G(q)=\frac{s\gamma}{1-\alpha}(\bar{q}-q).$$

The general solution of (10) is

$$q(t)=\frac{\bar{q}}{1-\frac{q(0)-\bar{q}}{q(0)}e^{-s\frac{\gamma}{1-\alpha}\bar{q}t}}.$$

Here

$$\lim_{t\to\infty} q(t)=\bar{q},$$

because $\frac{s\gamma}{1-\alpha}>0$. It means that the equilibrium $\bar{q}$ is stable. In this equilibrium state, the growth rates of output and input in sector I take the levels shown below;

$$\bar{G}(Y_1)=\bar{G}(K)=\frac{1}{\gamma}[\bar{G}(A_1)-\alpha\bar{G}(SL)]$$

$$\bar{G}(N_1)=\frac{1}{\gamma}[\bar{G}(A_1)-(1-\beta)\bar{G}(SL)].$$

It may be useful to define three cases as follows:

Case A; $\bar{G}(A_1)>\alpha\bar{G}(SL)$

Case B; $\bar{G}(A_1)=\alpha\bar{G}(SL)$

Case C; $\bar{G}(A_1)<\alpha\bar{G}(SL)$

Case A: The equilibrium levels in profit rate and the growth rates of output and capital are positive. The growth rate of employment in sector I is smaller than that of capital. In other words the economy attains steady growth with capital deepening.[8]

In the case assumed by one of the present writers elsewhere [8] [9], in which a constant real wage dominates, the states of equilibria are expressed as follows;

$$\bar{q}=\frac{1}{s\gamma}\bar{G}(A_1)$$

$$\bar{G}(Y_1)=\bar{G}(K)=\bar{G}(N_1)=\frac{1}{\gamma}\bar{G}(A_1),$$

or the economy develops along the course of capital widening.

Case B: In this case we have the relations,

$$\bar{q}=\bar{G}(Y_1)=\bar{G}(K)=0, \quad \bar{G}(N_1)<0.$$

This might be called a sort of 'quasi-stationary state' in the sense that profit rate, output and capital remain constant, but with capital deepening. Assuming no technological progress

[8] Employment in sector I increases, remains constant or decreases, if

$$\bar{G}(A_1)\gtreqless(1-\beta)\bar{G}(SL).$$

Hence, it decreases in Cases B and C; i. e.,

$$\bar{G}(A_1)\gtrless\alpha\bar{G}(SL).$$

and constant wages, however, we get

$$\bar{q}=\bar{G}(Y_1)=\bar{G}(K)=\bar{G}(N_1)=0,$$

or a situation in which output, capital and employment are all stagnant; in other words, the stationary state dominates.

Case C: In this case, as the equilibrium level in profit rate becomes negative, the model does not work.

We have assumed thus far that production in sector I depends on three factors; labor force, capital stock and natural resources subject to the following condition:

$$\alpha+\beta+\gamma=1.$$

It may be useful, however, to investigate the state of equilibria in the case where production depends only on labor and capital, not on natural resources. This implies an assumption of zero elasticity of output with respect of natural resources; i.e.,

$$\alpha+\beta=1.$$

Under these assumptions, equation (9) becomes

$$(9)' \quad G(q)=\frac{1}{\beta}[\bar{G}(A_1)-\alpha\bar{G}(SL)].$$

We have now the following three cases:

Case A: The growth rate of profit is positive, or the model has no unique solution. Assuming constant wages, (9)' becomes

$$G(q)=\frac{1}{\beta}G(A_1).$$

This may be similar to the case of given wages, as we interpret it, in the Fei-Ranis model [14].

Case B: The profit rate keeps its initial value $q(0)$, because its growth rate is zero. Hence the growth rates of other variables reach the following levels:

$$\bar{G}(K)=\bar{G}(Y_1)=\bar{G}(N_1)=sq(0).$$

This case, also assumed in K. Ara's model [1],[9] is consistent with steady growth accompanied by capital widening.

Case C: This model has no unique solution, the growth rate of profit being negative.

In summary, we can say the following:

1) The model has a stable solution under the conditions of neutral technological progress and increasing wage rates.

If the growth rate of technological progress is higher than the product of output elasticity of labor and the growth rate of real wages, the model has a solution characterized by steady growth and capital deepening. In the special case of technological progress with constant wages, it results in steady growth with capital widening.

If the growth rate of technological progress is equivalent to the product of output elasticity of labor and the growth rate of real wages, then the 'quasi-stationary' state with capital deepening manifests itself. Assuming constant wages in this case, the stationary state with capital widening appears.

If the growth rate of technological progress is lower than the above product, there is no economically meaningful solution.

2) The conclusions above may be changed, if we assume that output elasticity of natural

[9] The model depends on the more restrictive assumptions: no technological progress and constant wage rates.

resources is negligible.

In the case where the growth rate of technological progress is not equivalent to the product of the output elasticity of labor and the growth rate of real wages, there is no solution. If the equality is satisfied, the economy develops steadily along the course of capital widening.

We have investigated economic growth in the capitalists sector. Now we will be concerned with economic growth in the subsistence sector. From (8)′ we get a relation in which the growth rate of the labor force in sector II is determined, if the growth rate of labor supply is given outside the system and that of employment in sector I is determined in the following state of equilibrium;

$$(11)\quad \bar{G}(N_2)=\frac{1}{1-W}\bar{G}(L)-W\bar{G}(N_1).$$

Assuming the relation

$$\bar{G}(A_1)>\alpha\bar{G}(SL).$$

the equilibrium growth rate of employment in sector I becomes

$$(12)\quad \bar{G}(N_1)=\frac{1}{\gamma}[\bar{G}(A_1)-(1-\beta)\bar{G}(SL)].$$

Substituting this equation into (11), we get

$$(13)\quad \bar{G}(N_2)=\frac{1}{1-W}\bar{G}(L)-\frac{W}{\gamma}[\bar{G}(A_1)-(1-\beta)\bar{G}(SL)].$$

This means that the growth rate of employment in sector I is larger and that in sector II is smaller, if the growth rate of technological progress and output elasticity of capital are higher and that of the wage rate in sector II lower. The growth rate of employment in sector I is independent from that of labor supply, while the growth rate in sector I depends on it. Relations (5)′ and (13) determine the growth rate of output in sector II; i.e.,

$$(14)\quad \bar{G}(Y_2)=\bar{G}(A_2)+\frac{\delta}{1-W}\left\{\bar{G}(L)-\frac{W}{\gamma}[\bar{G}(A_1)-(1-\beta)\bar{G}(SL)]\right\}.$$

The model developed in this section describes the phase of economic growth in which the real wage rate is historically given. The boundary condition of this phase, as above mentioned, is

$$\delta\frac{Y_2}{SL}<N_2<\frac{Y_2}{SL}.$$

This may be always satisfied, once settled at the initial conditions, if the following relation exists;

$$G(N_2)=G(Y_2)-\bar{G}(SL)\quad\text{or}\quad G(N_2)=\frac{1}{\varepsilon}[\bar{G}(A_2)-\bar{G}(SL)].$$

Substituting (11) or (13) into this, we get

$$\frac{1}{1-W}[\bar{G}(L)-W\bar{G}(N_1)]=\frac{1}{\varepsilon}[\bar{G}(A_2)-\bar{G}(SL)]$$

or

$$\frac{1}{1-W}\left\{\bar{G}(L)-\frac{W}{\gamma}[\bar{G}(A_1)-(1-\beta)\bar{G}(SL)]\right\}=\frac{1}{\varepsilon}[\bar{G}(A_2)-\bar{G}(SL)].^{10}$$

[10] In the most simplified case, in which the levels of technology in sector II and subsistence level remain constant, it becomes

$$\bar{G}(L)=\frac{W}{\gamma}\bar{G}(A_1).$$

On the other hand, if the growth rate of the labor force in sector II is comparatively low (owing to moderate population increase of insufficient demand for labor in sector I), or technological progress is rapid while subsistence wages are stationary in sector II; i.e.,

$$\bar{G}(N_2)<\frac{1}{\varepsilon}[\bar{G}(A_2)-\bar{G}(SL)]$$

or

$$\frac{1}{1-W}[\bar{G}(L)-W\bar{G}(N_1)]<\frac{1}{\varepsilon}[\bar{G}(A_2)-\bar{G}(SL)]$$

or

$$\frac{1}{1-W}\left\{\bar{G}(L)-\frac{W}{\gamma}[\bar{G}(A_1)-(1-\beta)\bar{G}(SL)]\right\}<\frac{1}{\varepsilon}[\bar{G}(A_2)-\bar{G}(SL)]$$

then the economy will move from this phase into the 'neo-classical' one in which real wage rates and the allocation of the labor force between the two sectors are determined in accordance with the principle that marginal productivity of labor should be equal in both sectors.

In the case of a high growth rate of the labor force, moderate technological progress or a rapid increase in subsistence wages; i.e.,

$$\bar{G}(N_2)>\frac{1}{\varepsilon}[\bar{G}(A_2)-\bar{G}(SL)],$$

the phase will emerge in which real wage rates should be equal to average productivity in sector II.

These examinations suggest the conditions of transition from the phase of unlimited supplies of labor to the phase of neo-classical system: moderate increases in population and subsistence level, a rapid technological progress and negligible output elasticities with respect to natural resources in both sectors, and a large share of labor force in the capitalists sector.[11]

IV. *Concluding Remarks*

First, we support the thesis of unlimited supplies of labor in terms of both theory and history. The basic feature in theory is, we believe, that the modern sector depends on wage rates determined outside this sector, that is in the traditional sector. This feature fits well with historical realities in countries or phases with surplus populations.

Second, the key notion of subsistence level, however, contains some vagueness. Lewis' objective standard of productivity cannot generally be supported. The subjective standard is much more plausible. But the assumptions of an unchanged subsistence level and of no technological progress in the subsistence sector are too rigid to be applied to historical realities. Historical trends of increasing subsistence levels, together with technological progress, in our view, can be assumed consistently with the basic features of unlimited supplies of labor.

Third, such a generalized model, as examined under a simple assumption of a neutral technological shift in the production function of the Cobb-Douglas type, revealed in particular, among others, the following: the possibility of having stable solutions depends first upon the

[11] The transition betwen these possible phases of economic development was discussed by one of the present writers elsewhere [8] [9]. The mechanism of transition between phases in terms of the model developed in this paper would have required an extended elaboration. This deserves separate analysis.

output elasticity of natural resources, and secondly, upon the relation between the rate of technological shift and the product of output elasticity of labor and the rate of increase in wage rates.

Fourth, the most significant case among those having a stable solution, is that in which the output elasticity of natural resources is not negligible and the rate of technological shift is greater than the product of output elasticity of labor and the rate of increase in wage rates. In this case, the boundary condition for the phase of unlimited supplies of labor is definitely given in terms of respective rates of change in population, technology, subsistence level and of the sectoral shares of labor force employed.

Fifth, the Japanese historical experience, we believe, presents one of the best examples of long-term growth of an economy with a dual structure. An empirical examination of the above theoretical conclusions is highly desirable. This, however, would be the subject of a separate paper.

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