THE PERMANENT INCOME AND THE TRANSACTION DEMAND FOR MONEY BY HOUSEHOLDS*

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I. *Introduction*

Since Keynes analyzed the demand for money, the traditional tendency for investigating it, whether the investigation was theoretical or empirical, has been to concentrate attention explicitly or implicity on consumer's behavior, applying the analytical result to the demand for money in the economy as a whole. Before we decide finally what kind of aggregate demand function for money we choose, however, it seems necessary that we analyze at first the demand for money by households and by business firms, respectively, because in households' choise of various asset holdings key variables are the demand for money and the demand for liquid assets other than money, on the one hand, and business firms are mainly concerned with physical assets holdings and money holdings, on the other, and because it seems that both economic units have different processes from other on adjusting their demand for money to the supply of money to them, respectively.[1]

The purpose of this paper is to investigate the transaction demand for money in households by examining behavior of currency holdings observed in consumer's survey data in Japan, where consumers buy almost all consumption goods through the medium of currency, and seldom or never hold the checking account in the bank, so that behavior of balances of currency held by them reflects very well that of transaction demand for money.

Showing, at first, our hypothesis in regard to the expenditure elasticity of transaction demand for money, we shall investigate cross-section data in the consumer's survey, making them tell us various interesting facts regarding consumers' holdings of currency. Then we shall explain these facts simultaneously by the permanent income hypothesis. In the process, however, we shall introduce not only the permanent but also the transitory component of income into our demand function for money, to explain the long-run as well as the short-run behavior of transaction demand for money by households.

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* This paper was translated from Part One of author's mimeographed paper "The Demand for Money by Households" (in Japanese) Sept. 1961, which investigates not only households' demand for money for transaction motives but also that for asset motives. The author wishes to express his thanks for comments which were given by Professor Hiroshi Kawaguchi on the above mimeographed paper when it was reported at the 1961's meeting of the Japan Association of Theoretical Economics. The author studied also firms' demand for money in his article "The Demand for Money in Business Firms" *Economic Review*, Jan. 1962 (in Japanese).

[1] The author investigated these points in "Cyclical Growth and the Balance of Trade," *Economic Review*, April 1960, pp. 148–159 (in Japanese).

II. *Expenditure and the Transaction Demand for Money* ——*Hypothesis and Facts*——

Households get money into hand by receiving their income, releasing it daily for expenditure. Therefore their demand for money for transaction motives must be closely connected with consumption-expenditure. They are, in principle, able to choose freely a transaction period for income-receivings or expenditure-payments from possible periods for transactions, however, in practice, their choice of an employment opportunity means at the same time to pick up an income-receiving period, at least under the present economic system. Therefore the transaction period will be a given condition for the going household enjoying some job-opportunity, on the one hand, and there are only a limited number of payment periods, even if it could select a period among alternatives, on the other. Accordingly the usual hypothesis assuming a given transaction period is not always unrealistic. That is, we can assume without any fear of leaving from essential aspects of households' behavior that the key variable to analyze the transaction demand for money by households is consumption-expenditure.

There are three possibilities of the size of elasticity of demand for money in regard to consumption-expenditure, i. e., it is possible for it to be greater than unity, equal to unity, and less than unity. According to W. J. Baumol's and J. Tobin's analyses, the transaction demand for money changes in proportion to square root of turnover or expenditure, so that the ratio of cash balances to expenditure decreases with an increase in total expenditure; the expenditure elasticity of demand for money is 0.5.[2] This conclusion is derived from an application of an analysis of determination of optimal inventory[3] to that of optimal cash balances, presupposing that households and/or economic units transform soon income received into interest-bearing assets (for instance, interest-bearing deposits), and pay expenditure by selling (or withdrawing) them. It will be examined later.

An alternative hypothesis is that the expenditure elasticity of demand for money equals unity. Excepting the possibility of diseconomy of large-scale utilizing of money on *a priori* ground, there remain two possibilities; (1) households squeeze the economy of large-scale utilizing of money, and (2) they take advantage neither of the economy nor of the diseconomy of large-scale utilizing of money. The Baumol-Tobin model assumes the former type of their behavior, and the second hypothesis the latter, which we shall choose to analyze behavior of households' transaction demand for money in the following. Although it is possible that the ratio of currency balances held by households to their expenditure depends on other variables than expenditure and/or income, we assume that the key-variable is expenditure and/or income that influences the transaction demand for money essentially. Whether it is proportional to consumption-expenditure or not, to explain its behavior is closely related with how to illustrate behavior of consumption, so far as it is connected to the latter. In consequence it will keep relationship with income and other variables through the consumption

[2] W. J. Baumol, "The Transaction Demand for Cash: An Inventory Theoretic Approach," *Quarterly Journal of Economics*, Vol. LVI, Nov. 1952, pp. 545–556, "Marginalism and the Demand for Cash in Light of Operations Research Experience," *Review of Economics and Statistics*, Vol. XL, Aug. 1958, pp. 209–214. J. Tobin, "The Interest Elasticity of Transactions Demand for Cash," *Review of Economics and Statistics*, Vol. XXXVIII, Aug. 1956, pp. 241–247.

[3] See, for example, T. M. Whitin, *The Theory of Inventory Management*, 1957.

TABLE 1. THE CURRENCY-INCOME RATIO AND THE CURRENCY-CONSUMPTION RATIO BY INCOME GROUPS (Consumers in All Urban Areas, 1953)

%

Income Group (unit : yen)	The Currency-Income Ratio			The Currency-Consumption Ratio		
	$\frac{M_0}{Y}$	$\frac{M_0'}{Y}$	$\frac{\bar{M}_0}{Y}$	$\frac{M_0}{C}$	$\frac{M_0'}{C}$	$\frac{\bar{M}_0}{C}$
Average	37.28	38.56	37.92	38.91	40.25	39.58
0– 3,999	1,191.24	339.44	765.34	89.37	25.47	57.42
4,000– 7,999	89.75	52.64	71.20	52.18	30.61	41.40
8,000–11,999	62.29	48.62	55.46	50.00	39.03	44.52
12,000–15,999	49.63	46.87	48.25	46.40	43.82	45.11
16,000–19,999	41.46	40.90	41.18	41.31	40.75	41.03
20,000–23,999	37.56	38.97	38.26	37.99	40.14	39.07
24,000–27,999	35.55	38.51	37.03	38.06	41.23	39.65
28,000–31,999	34.75	38.10	36.42	37.31	40.91	39.11
32,000–35,999	31.55	35.63	33.59	35.04	39.57	37.31
36,000–39,999	29.64	34.67	32.16	33.55	39.24	36.40
40,000–43,999	28.65	34.00	31.32	32.93	39.08	36.01
44,000–47,999	27.44	32.59	30.02	32.10	38.13	36.12
48,000–51,999	23.24	32.09	27.67	28.13	38.85	33.49
52,000–55,999	23.73	31.37	27.55	28.79	38.07	33.43
56,000–59,999	22.05	32.38	27.22	29.10	42.73	35.92
60,000–	20.85	31.15	26.00	28.37	42.39	35.38

function.

Before proceeding on our way of analysis, let us show a few facts regarding households' transaction demand for money. In the Bureau of Statistics' *The Survey of Consumer's Income and Expenditure* we have by income groups such items as "balances of currency at the beginning of month" and "balances of currency at the end of month" in addition to other items as income, expenditures and so forth. By these materials we obtain Table 1, which shows the currency-income ratios as well as the currency-consumption ratios by income groups in 1953. 1953's figures are shown as the representative of those in other years, because the latter brings similar observations.[4]

Our observations obtained from Table 1 are as follows;

[Observation : 1] The ratio of balances of currency to disposable income declines as the income group moves from low to high income, whether we take balances of currency held at the beginning of month M_0 or those at the end of month M_0' to represent balances of currency held by consumer units.

[4] In post-war Japan consumers' income in urban areas in December was used to be several times as high as that in other months, because payments of bonus from firms to employees, including white-collar workers, became generalized among various parts of economy in December, which lost the original meaning of bonus payments and were usual wage payments rather than distribution of special profits earned to workers. Thus income ranges for grouping households were different in December from other months in some years, so that in the following we shall use averages of various data in January through November in each income group.

[Observation: 2] In addition we observe that the ratio of balances of currency at the beginning of month to disposable income (M_0/Y) is greater in low income groups, but is less than the ratio of balances of currency at the end of month to disposable income (M_0'/Y) in high income groups.
[Observation: 3] We find out in Table 1 a declining tendency of the ratio's of average of M_0 and M_0' to consumption ($\bar{M}_0/C$)'s from 57.4% in the lowest to 35.4% in the highest income group, though some irregular movements are found there. The ratio (M_0/C) shows also similar behavior over various income groups.
[Observation: 4] The ratio (M_0'/C), however, increases at first, then turns into a diminishing tendency, as the income group moves from low to high income.
[Observation: 5] And the ratio (M_0/C) is greater than the ratio (M_0'/C) in low income groups but the opposite is true in high income groups.

III. *Examination of Cross-Section Data (1)*

Are the above observed facts favorable to our hypothesis or to the Baumol-Tobin hypothesis?

Now, suppose the following relationship to explain the behavior of the transaction demand for money M_T;

(III. 1) $M_T/C=AC^r$.

And let us represent M_T by $\bar{M}_0$. According to the data shown in Table 1, the ratio ($\bar{M}_0/C$) is 57.42% in the lowest and 35.38% in the highest income group, and C's, which are not given in Table 1, are 13,382 yen and 43,785 yen per month in each of both groups, respectively. By inserting these magnitudes into (III. 1), we get

(III. 1)' $0.5742=A(13,382)^r$,

and

(III. 1)'' $0.3538=A(43,785)^r$.

From these two equations we can easily estimate the value of r, i.e.,

(III. 2) $r=-0.409$.

It is nearly equal to -0.5,[5] corresponding closely to the consequence obtained from the Baumol-Tobin hypothesis that the transaction holdings of money are propotional to square root of expenditure. Further examination of our observations in details attracts, however our attention to the fact that Observation 4 of behavior of the ratio (M_0'/C) is not yet explained by it. In addition, how does the Baumol-Tobin hypothesis illustrate Observation 2 and 5?

Before advancing our examination, let us make clear the characteristics of the Bureau of Statistics' Survey of Consumer's Income and Expenditure. The Survey asks consumer units picked up as samples to keep accounts during successive six months. A sample is, however, not always classified into the same income group in each of the six months. Namely samples are grouped in each month according to the size of their money income in the month, so that a consumer unit may move from an income group to another as its income changes. If a consumer unit remains in one income group so far as it continues to be a sample, the

[5] If we take in equation (III. 1) total expenditure including non-consumption-expenditure like tax payment instead of consumption-expenditure, we get -0.459 as the magnitude of r in the same year.

figure of balances of currency at the end of a month should be equal to that at the beginning of the next month, apart from the fact that samples are replaced half-yearly. And it seems that there is not any systematic difference between the yearly average of currency holdings at the beginning of month M_0 and that at the end of month M_0' in each income group under such classification of samples into income groups. Thus we can expect that there occurs a systematic difference between M_0 and M_0', because of moving of a consumer unit among various income groups. However, Observations 2 and 5 can not yet be explained by the characteristics of the method of Survey alone.

IV. *Examination of Cross-Section Data (2)*

Is there any single hypothesis by which all of the above Observations 1, 2, 3, 4 and 5 can be explained consistently?

First of all let us examine what relationship there is between M_0 or M_0' and disposable income by income groups in 1953 on Fig. 1-1. Here M_0 is greater than M_0' in low income groups, but M_0' becomes larger systematically than M_0 above a certain level of income and the difference between them is increasing as income grows large. We observe similar relationships between M_0 or M_0' and Y in other years, too, which we generalize to obtain Fig. 2, where line AA shows the relation between M_0 and Y, and line BB that between M_0' and Y. As mentioned above, a household does not necessarily remain at one income group, moving among various income groups from month to month. Namely it is ranked in a high income group, if it happens for it to earn relatively large income in a month, on the one hand, and it is classified into a low income group in another month, when its income happens to decrease, on the other. If, in this case, the variance of permanent income is less than that of actual income, there might, in a relatively high income group, exist relatively large number of consumer units getting positive transitory income accidentally besides the permanent income, compared with the number of consumer units obtaining less income than the permanent by chance.[6]

In Fig. 2 the transaction holdings of money at a level of permanent income Y_1, for instance, appear as currency holdings at the beginning of month under an actual level of income Y_2, where the actual amount of consumption indicated by line C_0-C_0 is less than the one which is spent when Y_2 is permanent. Line P_C indicates the permanent relationship between consumption and income. Because part of savings will probably be transformed into deposits, an addition to holdings of currency will be less than savings. At Y_2, for example, savings are expressed by the distance $a'b'$ or ab. But there the addition to holdings of currency will be relatively small, as Y_2 just represents the average actual income in a relatively high income group which brings less amount of consumption than Y_2 happens to be the permanent. Thus line BB lies below line P_M, which shows the relationship between the permanent income and the corresponding normal cash balances. As a result we can clearly explain why in Fig. 1-1 the straight line connecting the origin with the intersection of line (M_0-Y) with line $(M_0'-Y)$ lies above line $(M_0'-Y)$ in high income groups. It is in the income group where line (M_0-Y) intersects with line $(M_0'-Y)$ that the average actual income coincides with the average permanent income.

[6] M. Friedman: *A Theory of the Consumption Function*, 1957, pp. 34–35.

FIG. 1-1

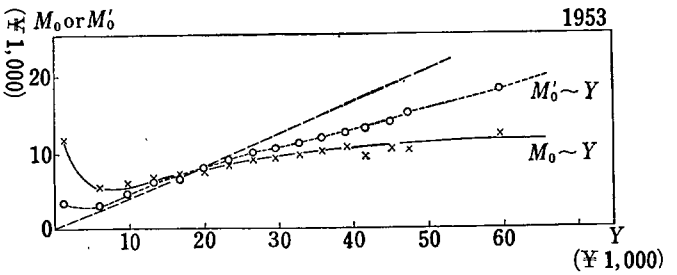


FIG. 1-2

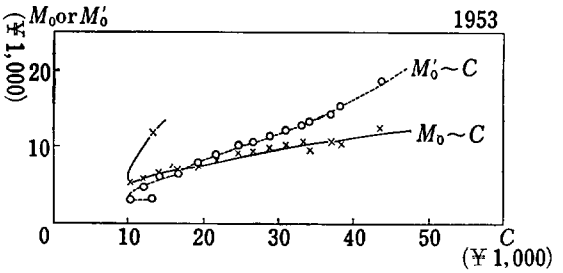
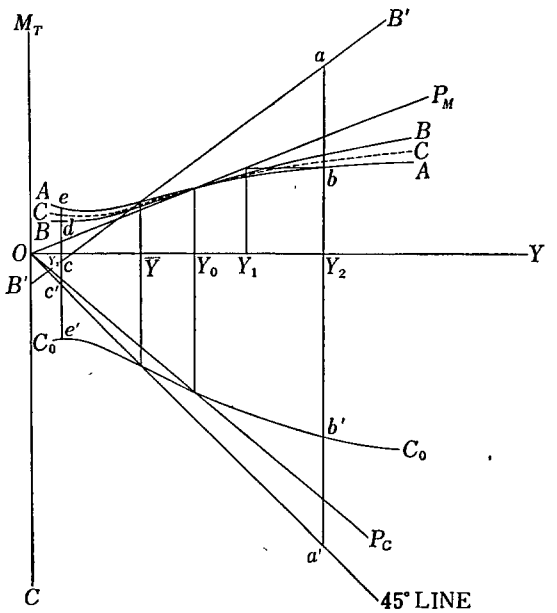


FIG. 2



On the other hand, the opposite phenomena occur in low income groups. That is, the lower the level of income, the more there exist consumer units whose income falls below the permanent accidentally. Their holdings of currency at the end of month will become less than those at the beginning of month, because part of balances of currency will be used to fill up part of the negative transitory income. They will, however, withdraw their deposits, sell their assets and/or borrow money, so that their holdings of currency at the end of month are greater than otherwise. In Fig. 2 deposits are drawn or assets are sold or money is borrowed by cd at Y_3, as the result that line BB lies above line $B'B'$, because the distance between line $B'B'$ and line AA means the amount of savings. Furthermore line BB is above line P_M, for actual income is lower than the permanent there, but the former lies below line AA, because consumer units use part of balances of currency for spending without refilling it by received income.[7]

Under the above conditions we could observe a relation between cash balances and income near to that embodied in line P_M, if we average holdings of currency indicated by line AA and magnitudes given by line $B'B'$ to get holdings of currency. But when we take average of holdings of currency obtained by line AA and those of line BB, the resultant line CC lies below line P_M in higher and above it in lower income than Y_0.

Regarding behavior of line AA or line BB, a further interesting phenomenon is observed. That is, line (M_0-Y) in Fig. 1-1 corresponding to line AA in Fig. 2 shows rather larger holdings of currency in the lowest income group than in other income groups.[8] This is because the lowest income group never include such consumer units that happen to be pushed

[7] In Fig. 2 we should note that there are positive savings at Y_0, where line AA intersects with line BB, so that Y_0 is greater than $\bar{Y}$, where there are no savings at all. Assume for simplicity that line C_0-C_0 is straight in neighborhood of both its intersections with 45° line and with line P_C. That is, assume the following observed consumption-income line for a moment;

* $C=a_0+a_1Y;$ $0<a_1<1.$

Furthermore we assume that the permanent component of consumption C_p is expressed by

** $C_p=\alpha Y_p;$ $0<\alpha<1,$

where Y_p is the permanent component of income and α is greater than a_1. 45° line on the consumption-income space is nothing but the following;

*** $C=Y.$

$\bar{Y}$, where there are no savings, is the solution of equations * and ***, so that

$$\bar{Y}=a_0/(1-a_1).$$

On the other hand Y_0, where actual consumption happens to coincide with the permanent component of consumption, so that line AA intersects with line BB, is obtainable to solve simultaneously both equations * and **, i. e.,

$$Y_0=a_0/(\alpha-a_1).$$

By our assumptions

$$1>\alpha>a_1>0,$$

so that

$$(1-a_1)>(\alpha-a_1)>0.$$

Thus

$$Y_0=a_0/(\alpha-a_1)>a_0/(1-a_1)=\bar{Y}.$$

Because we assume that α is less than unity, we get positive savings at Y_0. The author at first thought that savings are zero at Y_0. Mr. H. Kawaguchi suggested me that savings could be positive at Y_0, where line AA intersects with line BB. Owing to this suggestion I examined the matter again and led to the elaborated analysis of relationships among consumption, holdings of currecy and income shown in Fig. 2.

[8] In some years of 1951–1960 not only the lowest but also the next to the lowest income group shows similar tendency.

up from a low income group owing to positive transitory income, so that its holdings of currency become larger than those of upper but relatively low income groups, which include those consumer units whose holdings of currency are relatively low because of positive transitory income. Line $(M_0'-Y)$ in Fig. 1-1 or the corresponding line BB in Fig. 2 shows a similar tendency, although it is more moderate than that of line (M_0-Y) or line AA. This will be because the lowest income group will consist mainly of households fell from upper groups, so that there will occur withdrawing of much more deposits or selling much more assets or borrowing much more money in the average than in upper groups, in order to maintain their level of consumption and therefore balances of currency.

Next let us investigate connection between currency holdings and expenditure, shown in Fig. 1-2. We can say that it emerges from projecting the above relationship between currency holdings and income into the consumption function. Namely the span of observed consumption over the range between the lowest and the highest income group is less than that of observed income, and consnmption is greater than income in low income groups, as indicated by line (C_0-C_0) in Fig. 2. As a result co-ordinates of holdings of currency and consumption lies far from the origin, compared with those of holdings of currency and income. And there appear curves turning upwards and downwards with a shape of fish-hook in the lowest income group, because its amount of consumption is larger than the neighbors'. Observation 4 is brought about by the existence of the curve turning downwards in regard to M_0' and C. In fact it is obvious in Table 1 that the increasing tendency of (M_0'/C) occurs in income groups with greater consumption than disposable income.

Thus the various phenomena observed in cross-section deta based on the classification of income groups by the size of income received are explainable by the permanent income hypothesis. In addition we can say that the observed behavior of "currency holdings at the beginning of month" and "currency holding at the end of month" among income groups made the hypothesis tide over a new test and gave a support to it.

V. *Setting up Consumers' Transaction Demand Function for Money*

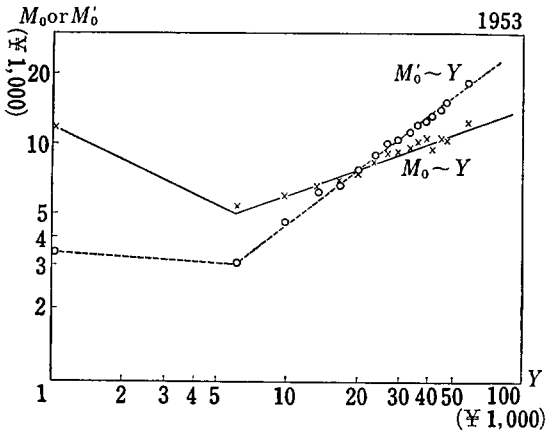
Since consumers' demand for currency, which seems to represent their demand for money for transaction motives, behaves as mentioned above, it will be a way to approach it that we follow M. Friedman's method used on investigating the consumption function. But a weak point existing in presuppositions of Friedman's permanent income hypothesis is to assume that the transitory components of income and consumption are uncorrelated with one another.[9] To adopt this assumption will be certainly a suitable way for finding out the most important aspect of consumers' behavior surrounded with complicated conditions of the actual world. In spite of that, it seems that there exists some part of consumers behavior in the short-run which will be interpreted by the transitory component of income. In addition, since non-human wealth is the accumulated sum of savings, which are largely influenced by the transitory component of income, it is apt to be affected by the transitory component of income in the short-run, although we can neglect its effect in the long-run owing to play of the law of averages. Specifically, we can say as follows regarding holdings of currency as a component of non-human wealth. Consumer's Survey data are classified into several groups on the basis of the amount of

[9] *Ibid.*, pp. 26–29.

income received, so that consumer units belonging to an income group in a month will hold balances of currency distributed at random at the beginning of the month. That is, we can expect that the transitory components of balances of currency are canceled each other within the group. Therefore, in each income group average balances of currency at the beginning of month will be nearly equal to the average permanent balances of the income group. On the other hand, currency holdings at the beginning of month are transformed into those at the end of month by being superimposed positive or negative influence upon it by the transitory component included in actual income in this month, so that on the average within an income group currency holdings at the end of month, we can say, consist of the permanent balances and the transitory balances, the latter of which depends not on all accumulated transitory components of income in the past but on the current transitory component of income. And it is observed in Fig. 1-1 that the transitory component of income gives positive effect upon that of balances of cash, because the slope of line (M_0-Y) is less than that of line $(M_0'-Y)$.

Now let us formalize the matter more exactly. At first Fig. 3 indicates us that relation-

FIG. 3.



ships between M_0 and Y and between M_0' and Y are log-linear except the lowest income group, which has different characteristics from others as mentioned above. Therefore the logarithmic values of actual balances of cash M_T and actual income Y are, we assume, composed of the permanent and the transitory component, respectively, as follows;

(IV. 1) $\log M_T \equiv m = m_p + m_t$,

and

(IV. 2) $\log Y \equiv y = y_p + y_t$,

where m_p or y_p represents the permanent and m_t or y_t the transitory component of the concerned variable. By our hypothesis that households take advantage neither of the economy nor of the diseconomy of large-scale utilizing of money we have

(IV. 3) $m_p = k_T + y_p$,

where k_T is constant. On the other hand we assume that m_t is an increasing function of y_t;

(IV. 4) $m_t = \gamma y_t$; $0<\gamma<1$,

where there is no constant term, because it seems that m_t vanishes, when y_t is zero. From

(IV. 4) we obtain

(IV. 4)' $m_t-y_t=(\gamma-1)y_t$.

And from (IV. 3) and (IV. 4)' we get the following;

(IV. 5) $m-y=k_T+(1-\gamma)(y_p-y)$,

or

(IV. 5)' $m-y=k_T-(1-\gamma)y_t$.

Secondly we can assume that the average balances of cash at the beginning of month in each income group coincide with the average permanent balances in that group, respectively, on the one hand, and that balances of cash per household at the end of month in each income group consist, on the average, of the permanent and the *current* transitory component, so that we infer from Fig. 3 the existence of the following equations;

(IV. 6.) $m_p=\alpha_0+\alpha_1(y_p+y_t)$,

and

(IV. 7.) $(m_p+m_t)=\beta_0+\beta_1(y_p+y_t)$.

If there are no correlations between m_p and m_t, between m_p and y_t, between y_p and y_t and between y_p and m_t, we can obtain

(IV. 8) $\alpha_1=(M_{m_py_p}/M_{yy})=(M_{y_py_p}/M_{yy})$

by regression of m_p on y on the one hand, and we can get

(IV. 9) $\beta_1=[M_{m_py_p}+M_{m_ty_t}]/M_{yy}=\alpha_1+(M_{m_ty_t}/M_{yy})$

by regression of m on y on the other hand, where M's with suffixes show covariances between the concerned variables indicated by the suffixes. From (IV. 8) and (IV. 9) we infer that if $M_{m_ty_t}$ is positive, then β_1 is greater than α_1, and contrarily if β_1 is greater than α_1, then $M_{m_ty_t}$ should be positive. We oberve in Fig. 3 that β_1 is greater than α_1, so that it is suggested that $M_{m_ty_t}$ is positive. This is favorable to our hypothesis that the transitory component of income has positive influence upon that of cash balances.[10]

Thirdly we get from equations (IV. 3) and (IV. 6)

(IV. 10) $y_p=(\alpha_0-k_T)+\alpha_1 y$.

And (IV. 10) gives us the following equation;

(IV. 10)' $y-y_p=(1-\alpha_1)y-(\alpha_0-k_T)$.

Taking the average of both sides of (IV. 10)' results in

(IV. 10)'' $\bar{y}-\bar{y}_p=(1-\alpha_1)\bar{y}-(\alpha_0-k_T)$,

where $\bar{y}$ or $\bar{y}_p$ expresses arithmetic mean of y or y_p, respectively. Now let us define τ as follows;

(IV. 11) $\bar{y}=\bar{y}_p+\tau$.

We substitute equation (IV. 11) into equation (IV. 10)'' to get

(IV. 12) $$\bar{y}=\frac{1}{1-\alpha_1}[(\alpha_0-k_T)+\tau].$$

Rewriting (IV. 12) gives us

(IV. 12)' $$y-\bar{y}=\frac{1}{1-\alpha_1}[(1-\alpha_1)y-(\alpha_0-k_T)-\tau].$$

Let us substitute (IV. 10)' into (IV. 12)' to obtain

[10] There could occur a question on identifying average cash balances at the beginning of month in each income group with the permanent component of cash balances, because the transitory components of income in the past are projected into the stock-variable, although the flow-variable has no projection of the past transitory components. We shall discuss this point later.

(IV. 13) $y_t=(1-\alpha_1)(y-\bar{y})+\tau.$

From (IV. 5)′ and (IV. 13) we get

(IV. 14) $m-y=k_T-(1-\gamma)\tau+b(\bar{y}-y),$

where b represents $(1-\gamma)(1-\alpha_1)$. If τ is equal to zero, i.e., if $\bar{y}$ coincides with $\bar{y}_p$, then (IV. 14) is transformed into the following;

(IV. 15) $m-y=k_T+b(\bar{y}-y).$

If this is the case, we can measure the demand function for money including not only the influence of the permanent component of income but also that of the transitory component of income upon the demand for money for transaction motives. In addition *we can fix the true, long-run relationship between the demand for cash balances and income, because we are able to estimate the size of* k_T *itself.*

VI. *A Measurement of Consumers' Transaction Demand Function for Money*

Before proceeding on measurement, we should give a solution to the following question referred already in footnote 10; is the transitory component of cash balances actually influenced not only by the current but also by the past transitory component of income, although we assumed in equation (IV. 4) that m_t depends only on the current transitory component of income y_t? It could be, we believe, affected by both. But the latter effect must be relatively minor, because the concerned transaction demand for money is deeply related with the amount of consumption, which is nothing but a flow-variable. Furthermore in cross-section data the magnitude of cash balances at the beginning of month will be nearly equal to that of permanent component of cash balances, as mentioned above. Therefore we proceed on measurement by assuming that in each income group arithmetic average of both cash balances at the beginning and at the end of month gives us the sum of permanent component and *current* transitory component of cash balances.

In order to measure the demand function for money, there is one more question to be solved. That is, what kind of figure should we use for representing $\bar{y}$ in equation (IV. 15)? First of all it is the simplest way to take average of disposable income given by Consumers' Survey, which is arithmetic mean of disposable income earned by consumers surveyed, and to convert it into logarithmic value. But the method is basically related not with geometric mean but with arithmetic mean, so that the estimate of $\bar{y}$ obtained by it will have some bias. In addition there could occur another kind of bias, because what we take is not population mean but sample mean. The first bias will be removed at least partly by such calculation that transforms sample arithmetic mean of disposable income in each income group into logarithmic value and then takes the average over all income groups. But it can not exclude the second shortcoming. The situation is not changed, even if we could compute geometric mean over all samples.

It is, however, possible to evade these defects and to estimate population geometric mean of disposable income. According to many empirical analyses of income distribution, we might be safely able to assume that the distribution of income among individuals is log-normal. Then we can estimate $\bar{y}$ from Consumers' Survey data. Denote by σ the standard deviation of y and by n the number of consumer units with y. We assume that disposable income obeys the log-normal distribution, so that

(V. 1) $$n=\frac{1}{\sqrt{2\pi}\,\sigma}e^{-\frac{1}{2\sigma^2}(y-\bar{y})^2}.$$

Let us define E as follows;

(V. 2) $$E=\int_{-\infty}^{\theta} n dy=\frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{\theta} e^{-\frac{1}{2\sigma^2}(y-\bar{y})^2}dy.$$

When we transform y into t by

(V. 3) $$t=\frac{y-\bar{y}}{\sigma}=\frac{1}{\sigma}y-\frac{\bar{y}}{\sigma},$$

we get

(V. 4) $$E=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\theta} e^{-\frac{t^2}{2}}dt.$$

Therefore we can estimate population parameters σ and $\bar{y}$ by getting the value of cumulative frequency distribution E in each income group from samples, converting it into the corresponding t and calculating regression between t and y. In each year of 1951–1960 data of consumers' disposable income classified by the size of income received show a good fit to the log-normal distribution except the first and the second group counted from the lowest income group. That is, we find out a fine linear line on the probability paper in each year. Estimates obtained are given in Table 2.[11] $\bar{R}$ is the correlation coefficient, and $\bar{S}$ is the standard error, each of them adjusted for the degree of freedom, respectively. It is interesting to observe in Table 2 that $\bar{Y}$, i.e., anti-logarithmic value of $\bar{y}$, takes near magnitude to disposable income in the income group where cash balances at the beginning of month M_0 become nearly equal to those at the end of month M_0'. The fact will be favorable to our explanation regarding the relationships between M_0 and Y and between M_0' and Y shown in Fig. 1-1 and Fig. 1-2.

Using $\bar{y}$'s thus obtained, we get the estimation of our demand function for cash shown in Table 3, where we do not include the lowest and the next income group to it to estimate the demand function, because they have the bias already noted. The figure put within

TABLE 2. ESTIMATION OF CONSUMERS' INCOME DISTRIBUTION

	$1/\sigma$	$\bar{y}/\sigma$	$\bar{R}^2$	$\bar{S}$	$\bar{y}$*	σ	$\bar{Y}$
1951	4.410	4.678	0.997	0.046	1.061	0.227	11,499 yen
1952	4.489	5.189	0.999	0.031	1.156	0.223	14,314
1953	4.657	5.797	0.998	0.045	1.245	0.215	17,564
1954	4.626	5.941	0.998	0.046	1.284	0.216	19,237
1955	4.499	5.816	0.998	0.044	1.293	0.222	19,624
1956	4.606	6.061	0.998	0.044	1.316	0.217	20,695
1957	4.288	5.754	0.998	0.047	1.342	0.233	21,972
1958	4.261	5.836	0.998	0.050	1.370	0.235	23,423
1959	4.333	6.038	0.998	0.049	1.394	0.231	24,746
1960	4.295	6.136	0.996	0.065	1.429	0.233	26,837

* Unit of $\bar{y}$ is the common logarithmic value of 10,000 yen.

[11] In the estimation we use common logarithm instead of natural logarithm. And the lowest and the next income group to it are excluded from the calculation to estimate parameters. We can estimate $(1/\sigma)$ and (y/σ) by regression of t on y or regression of y on t. Figures given in Table 2 are the average of the two estimates.

TABLE 3. ESTIMATION OF CONSUMERS' TRANSACTION DEMAND FUNCTION FOR MONEY

	k_T	K_T	b	$\bar{R}^2$	$\bar{S}$	M/Y*
1951	−0.302	0.4989	0.313 (0.024)	0.935	0.016	0.4761
1952	−0.269	0.5383	0.331 (0.027)	0.944	0.016	0.5059
1953	−0.377	0.4198	0.429 (0.015)	0.983	0.013	0.3792
1954	−0.405	0.3936	0.300 (0.023)	0.908	0.023	0.3694
1955	−0.404	0.3927	0.344 (0.027)	0.906	0.028	0.3622
1956	−0.410	0.3891	0.374 (0.026)	0.920	0.027	0.3560
1957	−0.425	0.3759	0.281 (0.013)	0.964	0.014	0.3513
1958	−0.429	0.3724	0.293 (0.014)	0.959	0.016	0.3456
1959	−0.404	0.3942	0.365 (0.018)	0.968	0.019	0.3573
1960	−0.398	0.3998	0.354 (0.028)	0.925	0.028	0.3629

* M/Y indicates the cash-income ratio calculated from sample means of cash balances and income.

parentheses under the estimate of b expresses its standard error. K_T represents the anti-logarithmic value of k_T. It remains stable around the level of 39% except in 1951 and 1952, where it is irregularily high. The high value of K_T in 1951 and 1952 will be investigated later. Let us assume that income is paid once a month and that consumer units spend a certain constant amount every day. Then the ratio of average cash balances to consumption should be 50%, and the ratio of the former to income is equal to 50% times the consumption-income ratio. If K_T is 40% under the assumed conditions, the consumption-income ratio should be 80%. But the normal consumption-income ratio of 80% seems to be low. Thus, when we assume that it is greater than 80%, the normal cash-income ratio of 40% will indicate either that consumers' income period is on the average less than a month or that consumers spend much more immediately after they received income than later or both. In Japan consumer units are usually paid their income once a month, so that the second reasoning will be, we infer, strongest to interpret the level of normal cash-income ratio.

Although K_T is sticky around 40% after 1953, there is a slightly declining tendency until 1958 and then it turns upward, if we investigate its behavior in detail. These tendencies seem to result from that $\bar{y}$ does not always coincide with $\bar{y}_p$ in every year. Let us define

(V. 5) $\quad k_T' \equiv k_T - (1-\gamma)\tau.$

Then it is obvious from (IV. 14) that if τ is not equal to zero, i.e., if $\bar{y}$ does not equal $\bar{y}_p$, the estimate of the constant term obtained from regression of $(m-y)$ on $(\bar{y}-y)$ is not that of k_T but that of k_T'. And if τ is positive, it will be less than k_T and *vice versa*. Therefore it seems that τ was increasing during the period of 1953 through 1958 and it became decreasing after 1958 and that such gaps between $\bar{y}$ and $\bar{y}_p$ will explain the tendencies observed in the estimates of K_T.

Now let us turn our attention to behavior of K_T before 1953. As shown in Table 3, the cash-income ratio was so high before 1953 that it seems to have behaved in different manner in this period compared with that after 1953. The shift was probably brought about by other factors than those considered so far. In Table 4 we can compare the ratio of currency holdings outside banks to national income (or consumption) in the economy as a whole (M_1/NI) (or M_1/CO) with the currency-disposable income (or consumption) ratio ($\bar{M}_0/Y$) (or $\bar{M}_0/C$) calculated from Consumer's Surveys.[12] Since the former ratio reflects behavior of currency holdings by consumer units as well as non-consumer units, and since there are great differences between the former and the latter, we can infer that the amount of currency held by business firms, to which we are not refer here, will play an important role on explaining behavior of (M_1/NI).[13] However the movement of ($\bar{M}_1/NI$) or ($\bar{M}_1/CO$) corresponds well to that of consumers' ratio ($\bar{M}_0/Y$) or ($\bar{M}_0/C$). Therefore we might assume that the

TABLE 4. THE ANNUAL RATIO OF CURRENCY HOLDINGS TO INCOME ETC. IN THE POST-WAR PERIOD

%

	$\frac{M_1^*}{NI}$	$\frac{M_1^*}{CO}$	$\frac{CO^*}{NI}$	$\frac{M_2^*}{NI}$	$\frac{\bar{M}_0}{Y}$	$\frac{\bar{M}_0}{C}$	$\frac{C}{Y}$
1946	18.62	20.17	92.30	6.12			
1947	16.98	17.97	94.54	9.97			
1948	13.38	15.07	88.76	13.48			
1949	10.63	12.87	82.60	15.19			
1950	9.80	13.82	70.89	15.64			
1951	8.96	13.43	66.70	15.60			
1952	8.84	12.22	72.35	18.42	3.91	4.09	95.63
1953	8.67	11.46	75.71	19.62	2.98	3.17	94.20
1954	8.30	10.55	78.71	19.77	2.94	3.18	92.56
1955	7.69	10.09	76.18	20.13	2.84	3.13	90.80
1956	7.60	10.53	72.13	21.68	2.77	3.13	88.23
1957	7.48	10.36	72.17	21.32	2.71	3.09	87.53
1958	7.65	10.36	73.88	23.29	2.67	3.06	87.35
1959	7.45	10.85	68.51	23.13	2.76	3.20	86.09
1960	7.39	11.44	64.64	23.38	2.76	3.25	85.11
1961	7.57	11.98	63.18	23.83			

Note: M_1: currency in circulation outside banks. M_2: balances of deposits excluding interbank deposits, where deposits cover current deposits, ordinary deposits, deposits at notice and special deposits. Source of data of both M_1 and M_2 for 1954–1961 is Bank of Japan: *Economic Statistics of Japan*, and data for 1946–1953 were estimated by the author.
NI: national income. *CO:* consumption. Source of data of *NI* and *CO* is Economic Planning Agency: *Report on National Income.*
* The figures are regarded with the fiscal year.

[12] Although figures to calculate the ratios ($\bar{M}_0/Y$) and ($\bar{M}_0/C$) are those in the calendar year, (M_1/NI) and (M_1/CO) are computed from yearly averages of currency holdings outside banks and national income in the fiscal year, which begins from April and ends in the next March.

[13] Regarding the demand for money by business firms, see S. Fujino "The Demand for Money in Business Firms," *op. cit.*

former represents the latter. Then how can we explain its declining tendency in the immediate post-war days? The weight of consumption in national income was very great in those days, as clearly known from Table 4. It will partly illustrate why the concerned ratio was high. But even the ratio of non-bank public holdings of currency to consumption was large and declining, so that we need other factors than it to interpret the concerned phenomenon. There could be several hypotheses for it.

[Hypothesis: 1] Generally speaking, inflation makes the total holdings of money, including not only currency holdings but also balances of deposits, decrease, other things being equal. (See (M_2/NI) in Table 4.) But it makes, on the other hand, the demand for physical goods increase, with the result that the propensity to hold those liquid assets, which have the highest convertability into them will be strengthened. Thus, even if the ratio of deposits to income decreases in the process of inflation, the currency-income ratio will increase.

[Hypothesis: 2] The abnormal conditions in the war as well as in the immediate post-war period decreased the demand for holdings of deposits and increased the demand for currency, because they intensified safety motives.

[Hypothesis: 3] This is a specific form of Hypothesis 2. The period when the currency-income ratio was high and declining coincides with that where necessaries were put on rations or where the control for rationing was gradually being abolished. Transactions through the black market throve in the economy under the rationing and the public were used to take advantage of chances to buy necessaries whenever they faced them. In such situation the demand for currency would become relatively large to consumption or income, because they attempted always to be ready to buy necessaries. As the economy went on way to recovery and the rationing was abolished, such demand for currency was decreased.

[Hypothesis: 4] All of the above three hypotheses implicitly assume that the demand for and supply of currency were in equilibrium in each year concerned. But we can suppose that there was the excess supply of currency until about 1952, which was absorbed into rising prices.

These four hypotheses will be combined to interpret the behavior of the cash-income ratio in the post-war period. When comparing the ratio (M_1/NI) or the ratio (M_1/CO) with the rate of change in prices, we find out positive correlation between them. The post-war inflation was related with the abnormal conditions of the economy in those days and obliged the rationing begun in the war-time to remain, so that we can say that Hypotheses 1, 2 and 3 are reflected in the positive correlation, and at the same time the phenomenon observed could be at least partly explained by Hypothesis 4.