LABOR, CAPITAL AND LAND IN ECONOMIC GROWTH

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I. *Introduction*

At the early stage of development in current theories of economic growth, the role of capital has received the greatest attention.[1] However, if we grant enough time for capitalists to rearrange their production methods, labor must also be taken into consideration, because labor can substitute for capital in producing a given level of output. Recently many objections have been raised against the postulate of fixed capital coefficient and the importance of variable factor ratio has grown.[2]

The purpose of this paper is to investigate the prosess of economic growth from the standpoint of marginal productivity principle. As productive factors, we shall take not only capital and labor, but also land or natural resources. By introducing the element of land into the system, we can apply our economic model effectively to the analysis of the Ricardian theory of economic growth.[3] In particular this should shed light on how types and properties of innovations are influenced also by the existence of land. Throughout this paper we shall always keep our attentions on the mechanism of income distribution, that is on the problem which was Ricardo's fundamental concern.

II. *Simplified Model*

In the following, we shall pose an ceonomy composed of three social classes, namely laborers, capitalists and landowners, and assume that capitalists are identified with entrepreneurs. We assume also that, neglecting the time span required for sale of output, factor costs to laborers and landowners are paid at the end of the production period. This latter assumption enables us to eliminate the cost of payments to laborers and land-owners from the amount of capital to be advanced. Further we shall leave aside the problem of relative prices, and assume that there exists a general price level for commodities and it

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The author is indebted to Professor W. Fellner, of the Yale University, for his helpful criticism and continuing encouragement. Any deficiencies remaining in this paper are, however, the sole responsibility of the author.

[1] R. F. Harrod, Towards a Dynamic Economics, 1948. E. D. Domar, Essays in the Theory of Economic Growth, 1957.

[2] For example, see W. Fellner, Trends and Cycles in Economic Activity, 1956, p. 143–145.

[3] This is perhaps one of the neglected problems in Harrod's growth economics. Harrod says, "I propose to discard the law of diminishing returns from the land as a primary determinant in a progressive economy" (R. F. Harrod, ibid., p. 20).

is always unitary. To make matters more managable, we have to add further that labor and land perfectly homogeneous, and that labor is measured in terms of the number of workers and land in terms of acre. Lastly it should be remembered that, unless a special reference is made, all the magnitudes used in what follows are measured jterms of an un production period.

Under these simplified assumptions, we define *labor productivity* as the net output per laborer produced per production period. Denoting the net output with Y and the number of the employed workers with N, labor productivity is shown simply with $\frac{Y}{N}$. Following the traditional mranliag productivity principle, let us put the following propositions:

(1) Other things being equal, labor productivity rises as the land per lavorer increases, but at a decreasing rate.

Let us call the land per laborer simply *land intensity* and denote it with $\frac{L}{N}$ where L is the land rented per period. Thus this proposition tells us that, if other things are equal, labor productivity is an increasing function of land intensity at a diminishing rate.

(2) Other things being equal, labor productivity rises as the capital to be advanced on the average per laborer increases, but at a decreasing rate.

Following the traditional definition, let us call the capital to be advanced on the average per laborer simply *capital intensity* and denote it with $\frac{K}{N}$, where K is the average amount of capital to be advanced in a production period. This is one of the fundamental proppositions in traditional capital theory,[4] and its validity can be testified only by the observation of reality, just as in the case of the former proposition. From this proposition, we may say again that labor productivity is *cet. parib.* an increasing function of capital intensity at a diminishing rate.

Owing to these two propositions, the following functional relationship can be presented between labor productivity, capital intensity and land intensity:

$$\frac{Y}{N}=F\left(\frac{K}{N}, \frac{L}{N}\right)$$

In the following, we call this F-function the labor productivity function or shortly the *productivity function.*

In order to simplify notations, it will be convenient to write as follows:

$$y=\frac{Y}{N}=\text{labor productivity}$$

$$k=\frac{K}{N}=\text{capital intensity}$$

$$l=\frac{L}{N}=\text{land intensity.}$$

Thus the labor productivity function can be rewritten as

$$y=F(k, l).$$

For the sake of simplicity, if we assume that this is infinitely continuous and differentiable,

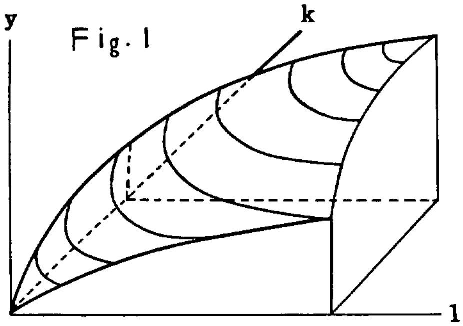
[4] Especially this proposition has been dwelt on in the exposition of Böhm-Bawerk in connection with the time-using character of highly mechanized production. Böhm-Bawerk, The Positive Theory of Capital (English trans., 1891).

then the following properties are obvious from our propositions:

$$\frac{\partial y}{\partial k}>0, \quad \frac{\partial^2 y}{\partial k^2}<0,$$

$$\frac{\partial y}{\partial l}>0. \quad \frac{\partial^2 y}{\partial l^2}<0,$$

With these properties in mind, we can immediately draw a diagram as in Fig.1. As is clear, the contours on the y-surface are shaped such that they are down-ward sloping and convex to the origin,. This is the principle of the "increasing marginal rate of substitution" between capital and labor.



Our first problem is to ask the economic implication of any given point on the y-surface, particularly its implication in the mechanism of income distribution. We shall next discuss in what direction such a given point on the y-surface moves in the course of economic growth and also its implication in the dynamic mechanism of income distribution. The analysis of technical progress, particularly its impact on income distribution and demand for factors will be made in the last chapter.

III. *Income Distribution under Constant Factor Ratios*

For the time being, we shall maintain the assumption of static condition in the sense that both capital and land intensities are kept constant. As will be seen later, this does not necessarily mean that labor, capital and land are all constant in their magnitudes. The only thing implied here is that the growth rates of labor, capital and land are the same.

Under free competition, capitalists are assumed to employ the most profitable method of production which realizes the maximum rate of profit. As the rate of profit is derived by substracting the (contract) payments to laborers and land-owners from the net output and diving it by the capital value to be advanced per period on the average, the rate of profit is given by

$$\frac{Y-wN-rL}{K}=\frac{Y-wN-rL}{N}\bigg/\frac{K}{N}=\frac{y-w-rl}{k}$$

where w is the wage rate per lavorer and r the rate of rent per acre of land.

Under the given productivity function, capitalists have to form two decisions as to the "scale of production". Firstly, they have to adopt that method of production which makes capital intensity realize a maximum rate of profit. This condition is obtained by a partial differentiation of the rate of profit with respect to capital intensity and making it zero, namely

$$\frac{\partial}{\partial k}\left(\frac{y-w-rl}{k}\right)=\frac{\frac{\partial y}{\partial k}k-(y-w-rl)}{k^2}=0$$

$$\therefore \quad y=\frac{\partial y}{\partial k}k+rl+w.$$

Secondly they must adopt that land intensity which realises the maximum rate of profit.[5] In the same way as capital intensity, this condition is given by the following equation,

$$\frac{\partial}{\partial l}\left(\frac{y-w-rl}{k}\right)=\frac{rk-\dfrac{\partial y}{\partial l}k}{k^2}=0$$

$$\therefore \quad r=\frac{\partial y}{\partial l}.^{6}$$

In the following, we assume that supplies of labor and land are infinitely inelastic with respect to the wage rate and rate of rent respectively. This means that laborers or land-owners are ready to accept any level of wage rates or rates of rent until full employment of labor or land is attained. We can of course pose the problem differently. For instance, we can assume, if we want, that a certain level of real wages is given or predetermined by a trade union. If this is the case, the problem is to ask the number of workers to be employed under the given wage rate. Which way the problem is to be posed is of course dependent on the question to be solved. It should be kept in mind, however, that as far as it is within a competitive capitalist economy, suppliers of productive factors cannot as a rule, claim to determine both the level of factor prices and the magnitudes to be employed simultaneously.

As N and L are given at factor markets on the one hand, and K are also given in the hands of capitalists on the other, both capital and land intensities to be chosen are also given from the begining, because we are assuming the full employment of these factors. Then our economic system under the maximum rate of profit becomes as follows:

$$(1.1) \qquad y=F(k,\ l)$$

$$(1.2) \qquad y=\frac{\partial y}{\partial k}k+rl+w$$

$$(1.3) \qquad r=\frac{\partial y}{\partial l}$$

$$(1.4) \qquad k=\bar{k}$$

$$(1.5) \qquad l=\bar{l},$$

where $\bar{k}$ and $\bar{l}$ denote respectively the capital intensity and land intensity under the full employment of labor, capital and land. Since there are five equations to determine five

[5] In the literature of traditional capital theory, this second decision has been seldom discussed. Wicksell, however, gave due consideration to this problem. K. Wicksell, Lectures on Political Economy, Vol. 1 (English Ed. 1934), p. 181.

[6] It should be kept in mind that the real value of capital intensity does not depend on the level of real wage rate (and also rate of rent), because K does not include the payments to factor costs by assumption. Therefore there does not arise the complicated problem with which Mrs. Robinson has met, i.e. shiftability of productivity function owing to changes in real wage rates (and real rates of rent). J. Robinson ,The Accumulation of Capital, 1956. The idetification of capitalists with entrepreneurs also excludes the dependence of capital intensity on the rates of interest.

variables y, k, l, w and r, our system is completely determined.[7]

The interpretation of this system is quite clear and easy. From (1.4) and (1.5), we can determine k and l. Substituting them into (1.1), y is also determined. Let us call it $\bar{y}$. As y, k and l are all known, we can also determine the value of derivatives in (1.2) and (1.3). Therefore, we can determine r from (1.3). Substituting it into (1.2), we can finally determine w. Thus we can determine all the variables uniquely.

What is then the economic meaning of derivatives $\frac{\partial y}{\partial k}$ and $\frac{\partial y}{\partial l}$? In order to answer this question, it is necessary to notice the following fact. From (1.1) we can derive

$$y \cdot N = \frac{Y}{N} \cdot N = Y = F(k, l) \cdot N = Y(K, L, N).^{8}$$

Let us multiply K, L and N respectively by the same positive number g. If Y is also increased by g times, then we say that the Y-function is homogeneous of the first degree. However, it is clear that k and l do not change by this multiplication, so that y also does not change. Thus, owing to this multiplication, Y is increased by g times, because F does not change and only N is increased by g times. Therefore, the Y-function is proved to be homogeneous of the first degree.

By the well known Euler theorem on homogeneous functions,[9] we can derive again

$$Y = \frac{\partial Y}{\partial K} K + \frac{\partial Y}{\partial L} L + \frac{\partial Y}{\partial N} N.$$

As long as the productivity function (1.1) is valid, this last equation is necessarily true under any circumstance. However, as will be clear immediately, each derivative on the right side of this equation is no more than the marginal productivity of capital, land and labor. This is shown as follows.

Under the condition of the maximum rate of profit, we have

$$\frac{\partial Y}{\partial K} = \frac{\partial}{\partial K}(y.\ N) = \frac{\partial y}{\partial k} \frac{\partial}{\partial K}\left(\frac{K}{N}\right) \cdot N = \frac{\partial y}{\partial k}$$

$$\frac{\partial Y}{\partial L} = \frac{\partial}{\partial L}(y.\ N) = \frac{\partial y}{\partial l} \frac{\partial}{\partial L}\left(\frac{L}{N}\right) \cdot N = \frac{\partial y}{\partial l}$$

$$\frac{\partial Y}{\partial N} = \frac{\partial}{\partial N}(y.\ N) = y + \frac{\partial F}{\partial N} N = y - \frac{\partial y}{\partial k} k - \frac{\partial y}{\partial l} l - w.$$

Thus, we can prove that the derivatives of productivity function with respect to capital and land intensities are respectively equal to the marginal productivity of capital and land and that the marginal productivity of labor is equal to the real wage rate. It has also been proved that there is no discrepancy in value between production and distribution in the sense that if each factor receives their remuneration according to their marginal

[7] As mentioned above, we can put the following equations instead of (1.4) and (1.5),

(1.4)* $w = \bar{w}$

(1.5)* $r = \bar{r}$.

Again, remember that there is no assurance of compatibility of such fixed $\bar{w}$ and $\bar{r}$ with the full emloyment of labor and land in the condition of the maximum rate of profit.

[8] $Y = Y$ (K, L, N) is sometimes called *production function* in modern economics.

[9] R. G. D. Allen, Mathematical Analysis for Economists, 1938, p. 317.

productivities, the whole net output is distributed to the production factors without residuals.[10]

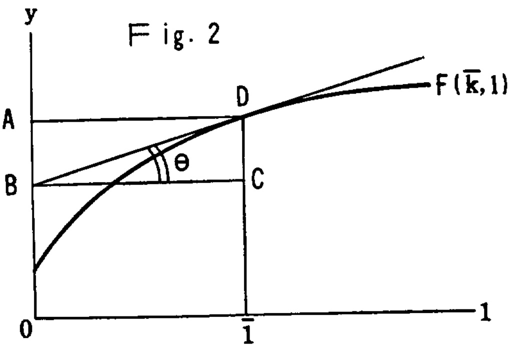


Fig.2 is intended to show the relationship between labor productivity and land intensity with capital intensity given at $\bar{k}$. If land intensity is also given at $\bar{l}$, then the tangent θ at this point on the curve F (k, l) shows the rate of rent to be paid to land-owners. As (1.3) says, this tangent is nothing but the marginal productivity of land. Since the rate of rent is given by this tangent, the relative share of land-owners among the net output per laborer is given by AB, because

$$\frac{\partial y}{\partial l}l=\frac{DC}{BC}BC=DC=AB.$$

Since OA is the labor productivity under the given $\bar{k}$ and $\bar{l}$, and since AB is the land-owner's relative share, $OA=AB=OB$ is the joint income of laborers and capitalists in the labor productivity. Therefore we have

$$\frac{\partial y}{\partial k}k+w=OB.$$

As is clear, the lower the land intensity, the higher the rate of rent and the smaller the joint income of laborers and capitalists.

In the same way, we can derive a diagram which shows the functional relationship between labor productivity and capital intensity with land intensity given. But this task will be left to the readers.

In order to see the mechanism of income distribution under the constant factor ratios more minutely, let us define the elasticities of productivity function with respect to capital and land intensities respectively as follows:

$$E_k=\frac{\partial y}{\partial k}\Big/\frac{y}{k}$$

$$E_l=\frac{\partial y}{\partial l}\Big/\frac{y}{l}.$$

From the equation (1.2), we can derive the equation

$$\frac{y-rl-w}{y}=\frac{\partial y}{\partial k}\frac{k}{y}=\frac{\partial y}{\partial k}\Big/\frac{y}{k}=E_k.$$

The left side of this equation is again rewritten as follows:

$$\frac{y-rl-w}{y}=\frac{Y-rL-wN}{Y}.$$

Since $(Y-rL-wN)$ is the total profits of capitalists, this last expression denotes the relative

[11] This is perhaps one of solutions to what we call "adding-up-problem" or imputation problem in distribution theory of income which has been so much discussed.

share of profit in the net output. We can say therefore that under the condition of maximum rate of profit, the relationship

$$\begin{array}{l}\text{relative share} \\ \text{of profit}\end{array} = \begin{array}{l}\text{elasticity of productivity function} \\ \text{with respect to capital intensity}\end{array}$$

must prevail.[11]

In the same way, we can establish the relationship

$$\begin{array}{l}\text{relative share} \\ \text{of rent}\end{array} = \begin{array}{l}\text{elasticity of productivity function} \\ \text{with respect to land intensity}\end{array}$$

under the condition of maximum rate of profit, but this derivation will be left again to the readers.

For obvious reasons, E_k and E_l are less than one. Let

$$E=E_k+E_l$$

and call E the total elasticity of productivity function If there should be a positive share of wages in the net output, E must be less than one. Indeed, from (1.2), we can have

$$y=\frac{\partial y}{\partial k}k+\frac{\partial y}{\partial l}l+w=E_k y+E_l y+w$$

$$\therefore \quad y=\frac{1}{1-E}w.$$

As both y and w are positive, it follows that E must necessarily be less than one. We shall use this last formula when we discuss the problem of stationary state in the Classical School.

IV. *Process of Economic Growth under Changing Factor Ratios*

So far we have been concerned only with the case of constant factor ratios. Under the condition of full employment of factors, this is feasible when labor, capital and land are all growing at the same rate.

Let us denote the capital and land intensities in the first period with k_1 and l_1 and those in the second period with k_2 and l_2. Further we denote the growth rates of these two ratios with G_k and G_l, namely

$$G_k=\frac{k_2-k_1}{k_1}\cdot \text{ and } G_1=\frac{l_2-l_1}{l_1}.$$

Using these notaions, the assumption in the previous chapter is simply $G_k=G_l=0$. If G_k is positive, this means that, under the full employment of factors, capital grows faster than labor, and if G_l is positive, it is that land increases faster than labor. If labor grows and land is held constant, G_l becomes negative, as in the case of Ricardo. Of course, we cannot say anything about the relative movements of these magnitudes *a priori*. In

[11] This relationship is equal to *Fellner's equation*

$$\frac{P}{O}=\frac{P}{V}\Big/\frac{O}{V}$$

where O is total output, P interst-plus-profit income and V total capital stock, because Ek is equal to the product of profit rate and the average productivity of capital .W. Fellner, ibid,. p. 122. Footnote 12.

particular it will be difficult to tell infallibly about the relative growth rates of labor and land. Admitting many possibilities of relative growth rates of factors, we try to formulate the Ricardian version of economic growth, and throw some light on the concept of the classical stationary state.

Before proceeding, let us make some assumption on the shape of productivity function. As was proved, the relative share of profit or rent in the net output is equal to the elasticities of productivity function with respect to capital intensity or land intensity. Then, how will these elasticities change as a consequence of changes in these intensities? They may sometimes increase and sometimes decline. For the sake of simplicity, however, and particularly for the sake of quantative qualification, we assume that, so long as the knowledge of alternative methods of production is constantly given, they are always constant. This amounts to saying that the relative shares in the net output remain unchanged under any factor ratio. Thus we may set up as

$$E_k=\text{constant and } E_l=\text{constant}.$$

It should be remembered that this assumption does not insist on the constant remuneration to each factors. What is meant here is simply that the marginal productivity of a factor changes proportionally to the changes in its average productivity, so that the ratio of marginal productivity of the factor to its average productivity does not chage under changing factor ratios.

From this assumption, it follows that the productivity function (1.1) is homogeneous of degree $E=E_k+E_l$.[12] For obvious reasons, total elasticity E is smaller than one. Some important economic consequences follow from this property immediately. They are that if capital and land internsities grow at the same rate, namely if $G_k=G_l>0$, (1) the ratio of profit rate to rate of rent does not change, but (2) the absolute level of both rates declines gradually.

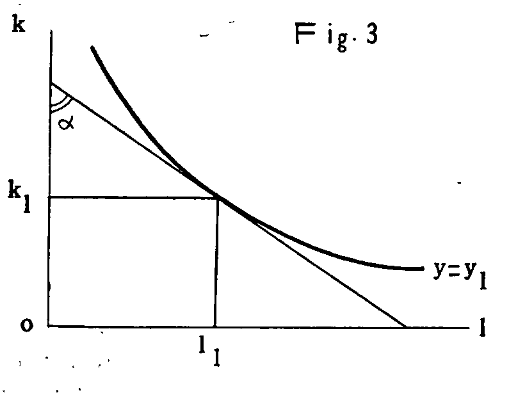


Fig. 3

In Fig.3, let us assume that capital intensity is given at k_1 and land intensity at l_1. Let us call the resulting labor productivity y_1, and denote a contour curve $y=y_1$ on this plane. Now, differentiating the equation (1.1) totally, we have

$$dy=\frac{\partial y}{\partial k}dk+\frac{\partial y}{\partial l}dl.$$

If this equation is applied to the contour curve $y=y_1$, it follows

$$dy_1=\frac{\partial y_1}{\partial k_1}dk_1+\frac{\partial y_1}{\partial l_1}dl_1=0.$$

[12] The proof os as follows. As E_k and E_l are constantly given, it follows

$$\frac{dy}{y}=E_k\frac{dk}{k}+E_l\frac{dl}{l}$$

Integrating this, we have again

$$\log y=\log A+E_k \log k+E_l \log l$$

$$\therefore \quad y=Ak^{E_k}l^{E_l},$$

where A is a parameter. Needless to say, this last equation is a homogeneous function of degree E_k+E_l.

From this we have again

$$\frac{\partial y_1}{\partial k_1}\Big/\frac{\partial y}{\partial l}=\frac{-dl_1}{dk_1}.$$

This last equaltion shows the marginal rate of substitution of capital for land. It follows therefore that the tangent α in this figure is equal to the ratio of profit rate to rate of rent, namely

$$\text{tangent } \alpha=\frac{\text{rate of profit}}{\text{rate of rent}}$$

Substituting E_k and E_l into the above equation, we have

$$\frac{dk_1}{k_1}\Big/\frac{dl_1}{l_1}=\frac{E_l}{E_k}=\sigma$$

As is well known, this last experssion is the elasticity of substitution of capital for land. Being E_k and E_l supposed to be constant by our assumption, this elasticity of substitution between capital and land is also constantly given.

In Fig.4, let us suppose that a pair of k_2 and l_2 is also located. The tangent α at the point S is equal to the ratio of profit rate to rate of rent. However, we can show again that, under the assumed conditions, the tangent β at the point T is equal to the tangent α.

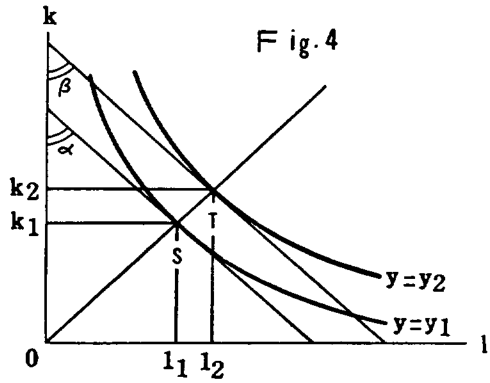


Fig.4

Since the elasticity of substitution of capital for land is constantly given, we have

$$\sigma=\frac{E_l}{E_k}=-\frac{dk_1}{k_1}\Big/\frac{dl_1}{l_1}=-\frac{dk_2}{k_2}\Big/\frac{dl_2}{l_2}$$

$$\therefore\quad \frac{dk_1}{dl_1}\Big/\frac{k_1}{l_1}=\frac{dk_2}{dl_2}\Big/\frac{k_2}{l_2}.$$

Since both pairs of k and l are on the same radius through the origin, it follows

$$\frac{k_1}{l_1}=\frac{k_2}{l_2}.$$

Thus we have finally

$$\frac{dk_1}{dl_1}=\frac{dk_2}{dl_2}=\text{tangent } \alpha=\text{tangent } \beta.^{13}$$

The second consequence that the absolute level itself declines is proved as follows. Let the point S in Fig.4 be moved to the point T. As far as the total elasticity E is less than one, the resulting labor productivity is less than capital intensity or land intensity in terms of growth rate. Since capital intensity or land intensity has increased more than labor productivity, the average productivity of capital or land must decline. So long as E_k and E_l are constantly given, the decline of the average productivity means the decline

[13] As is immediately clear, this conclusion is not confined to a special case, but any radius through the origin O cuts the curves in points where the tangents are parallel. Therefore, our conclusion is quite general.

of its marginal productivity, so that profit rate or rate of rent must decline absolutely.

It should be kept in mind, however, that this does not imply that profit rates or rates of rent must decline on evry point on a given contour curve $y=y_2$, as compared with a point S on $y=y_1$. Let us compare the point S, for example, with the point U in Fig. 5. The point U denotes a position such that a higher labor productivity is attained by increasing capital intensity with land intensity given. Clearly the average productivity of land is increased, so that the marginal prodictivity of land or rate of rent is also increased because E_l is constant. We know that rate of rent is decreased at the point T and increased at the point U, as compared with the point S. As far as the productivity function is continuous, there must exist a point V such that the rate of rent coincides with each other.

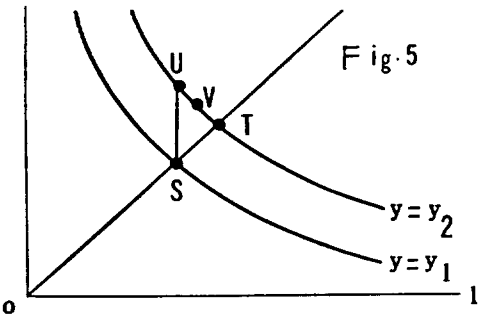


Fig. 5

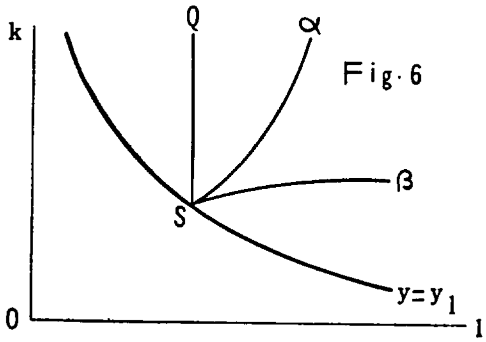


Fig. 6

If we repeat the same procedure on the higher productivity curves, we shall have an equi-rate-of-rent-curve α as denoted in Fig.6 which starts from some critical point S on productivity curve $y=y_1$. At the left side of this curve, it rises, and at the right side, declines. It should be remembered further that on the straight line Q, rate of rent rises at the same rate as labor productivity, because both the average and marginal productivity of land increases proportionally to labor productvity.

Mutatis mutandis,the same analysis may be applied also to capital intensity. A curve β is an equi-rate-of- profit-curve as compared with the point S.

Armed with these analyses, we can proceed to the Ricardian theory of economic growth. According to Mr. Edelberg, we may summarize the assumptions which underlie the Ricardian theory of economic growth as follows:[14]

(1) That the shape of productivity function is constantly given.
(2) That as the amount of land per laborer increases, the marginal productivity of land falls.
(3) That as the amount of capital per laborer increases, the marginal productivity of capital falls.
(4) That the growth of population obeys the subsistence law.

[14] V. Edelberg, "The Ricardian Theory of Profit, Economica," 1933, Feb., p. 51–74. To make matters more clear, I formulated his expressions in terms of our own terminologies, without alternation of the points.

(5) That the supply of land is constantly given.

(6) That the supply of saving is an increasing function of the rate of profit; but there is a sub-margin.

Let us assume, for a while, that the real wage rate is given at the subsistence level, and the rate of profit is so high that there is a strong motive for capitalists to accumulate their profits. Let us call the subsistence level of real wage rate $\bar{w}$. Since the total elasticity E of productivity function is constant by assumption, this means that there exists a subsistence level of labor productivity such as

$$\bar{y}=\frac{l-E}{l}\bar{w}.$$

In Fig.7, this is denoted by a stout curve $y=\bar{y}$. By assumption (6), there exists also a sub-margin of profit rate at which there is no motive for capitalists to accumulate their profits. This is denoted with an upward-rising dotted curve.[15] As far as labor productivity remains at $y=\bar{y}$, there is no growth of labor by assumption (4). By assumption (5), the supply of land is constant, so that the land intensity is also constant. At situation I, capital will be accumulated by assumption (6). So long as labor and land remain constant, situation I will move to II by capital accumulation. Only capital intensity has been increased, so that both real wage rate and rate of rent will be proportionally increased. If real wase rate is above the subsistence level $\bar{w}$ for a long time, population (therefore labor) must now increase in geometric ratio by assumption (4). Since land is constant, land intensity must decline accordingly. But for a while the rate of capital accumulation might be higher than that of labor, because there is a time-lag in the increase in laborers. If so, situation II will move to III. At situation III, the rate of profit becomes lower, and the growth rate of labor might be above the rate of capital accumulation. Both capital and land intensities may decline, and labor productivity also may decline absolutely until the subsistence level of real wage rate $\bar{w}=(l-E)\bar{y}$ is reached again. At situation IV, one round of the game between labor and capital has been finished.

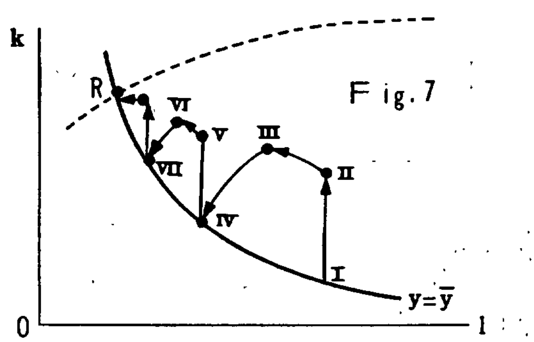


Fig.7

Since both situations I and IV are on the same contour curve $y=\bar{y}$, the real wage rate is the same as before, and since situation IV is on the left side of the same contour curve, rate of rent must increase and rate of profit must decrease. As far as there still exists a motivation for capitalists to accumulate capital at situation IV, the another new game will start again between labor and capital. The rule of the game is similar and another situation VII will result. In this way, the situation will move in the direction of the point

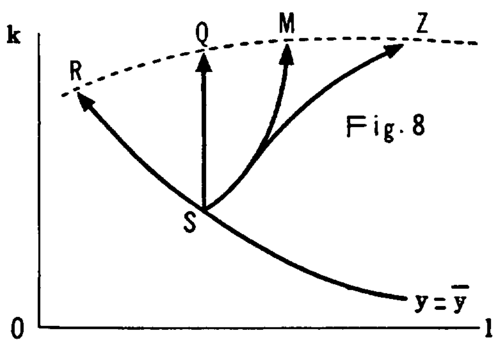
[15] "The motive will diminish with every diminution of profit; and will cease altogether when their profit are so low not afford them adequate compensation for the trouble, and the risk which they must necessarily encounter in employing their capital productively." D. Ricardo, Principles of Political Economy and Taxation, Everyman's Library, p. 72. The shape of this curve came from the same reasoning in the curve L in Fig.5.

R gradually. Let us call this the Ricardian point. At the Ricardian point, capitalist economy will be in a stationary state, because the situation has reached to the sub-margin of profit rate by assumption (6). The real wage rate is at the subsistence level, and rents continue at a higher and profit at a lower uniform level.

The conclusion that the labor class is neither winner nor loser of the game between labor and capital is substantially dependent upon the assumption (4) of the subsistence law of wages. The concept of subsistence, however, underwent a fundamental transformation between Smith and J.S. Mill.[16] If it is intrepreted as a "physiological" subsistence level, as shown by $\overline{w}$ in Fig.7, the stationary state must be terrible and dull, as in the case of Ricardo. It is the stationary state with a (relatively) large population and a low "natural" rate of wages. But if it is interpreted on "psychological" bases, it may be very pleasant and cultivated, because a (relatively) small population can enjoy a high "natural" level of wages.

The crucial point to notice in the present context is the relative growth rates of capital, labor and land in the course of economic growth. Indeed, even land or natural resources themselves cannot be deemed to be constant, because capitalists can cultivate new land or discover new resources by investing a part of their capital. The game is not only confined to labor and capital, but also land must come to be an active player.

A general score of the game in terms of remuneration to each factor can be shown in Fig.8. If there is a tendency toward the point Q, the benefits of capital accumulation will be favored between labor class and land-owners. This is simply the case of $G_k > G_l = 0$. If the situation is at M, only real wage rate has risen, and rate of rent has remained constant, as compared with the starting point S. The tendency towards the point Z is sometimes called a "state of bliss". Not only profit rate, but also rate of rent declines absolutely and the labor class will be in a state of bliss.



These are the pictures of the classical stationary state. It may be the Ricardian point or may be a state of bliss. But in the normal conditions of capitalist economy, it may be plausibly maintained that the growth rate of capital is generally larger than those of labor and land, so that the stationary state, whatever it may be, must come into existence sooner or later.[17]

[16] L. Robbins, "On a Certain Ambiguity in the Conception of Stationary Equilibrium," Economic Journal, 1930, June, p. 199–201.

[17] It should be remarked here that as far as E_k and E_l are constantly given, the relative share of each factor is constant even at a stationary state, so that the absolute share of capitalists must increase as the capital accumulation goes on. Ricardo, however, considers a possibility of the absolute decline in profit rates in the consequence of capital accumulation. D. Ricardo, ibid., p. 73.

V. *Innovation and Income Distribution*

So far our analysis has been carried on under the assumption of a given state of technology. A stationary state, however, can hardly be regarded as an approximation to an evolving world. As many histroians show, there have been ceaseless improvements and inventions, and they have served to "prevent the yield of capital from falling below some critical level".[18]

In this chapter, we are going to discuss the problems of innovations, in particular in its connection with the problems of income distribution. Let us now define an innovation in terms of an overall upward shift of y-surface in Fig.1, then an analytical expression of innovation is most clearly shown as follows. Let $\bar{k}$ and $\bar{l}$ be respectively the capital and land intensity over the relevant ranges of k and l. Further we difine F' and F'' as the productivity functions before and after an innovation. Thus by the definition, it follows

$$F''\ (\bar{k},\ \bar{l}) > F'\ (\bar{k},\ \bar{l}).$$

In general we may expect that innovations will help, in the long run, to raise the level of real wages and rents, because, other things remaining constant, the rate of profit on capital will be increased by introducing new methods of production, so that the rate of capital accumulation will be increased so much. But in order to discuss the effects of innovations quantatively, we shall maintain the assumption that the elasticities of productivity function are held constant again after an innovation. Needless to say, this does not imply that the elsticities do not undergo changes by innovations. On the contrary each elasticity will be changed by innovations, and to ask the rules of its change does constitute the main subject of the following analysis.

Let us begin with the definitions of types of innovations as follows:[19]

In terms of capital intensity:

(1) if E_k does not change, it is *capital-neutral,*

(2) if E_k becomes larger, it is *capital-using,*

(3) if E_k becomes smaller, it is *capital-saving.*

In terms of land intensity:

(1) if E_l does not change, it is *land-neutral,*

(2) if E_l becomes larger, it is *land-using,*

(3) if E_l becomes smaller, it is *land-saving.*

For the sake of convenience, further, we use the following notations:

	Before Innovation	After Innovation
E_k	E_k'	E_k''
E_l	E_l'	E_l''
E	$E'=E_k'+E_l'$	$E''=E_k''+E_l''$
y	y'	y''
w	w'	w''
r	r'	r''

[18] W. Fellner, op. cit., p. 138.

[19] The definition and classfication of innovation came from Mrs. Robinson. J. Robinson, ibid., partciularly Book VI.

Our first problem is to ask the relationship between labor productivity and real wage rate.

Let us define the growth rate of labor productivity as Gy and that of real wage rate as Gw, namely

$$Gy=\frac{y''-y}{y'} \text{ and } Gw=\frac{w''-w'}{w'}.$$

Since under the condition of the maximum rate of profit, we have

$$w=(1-E)y,$$

it follows that

$$Gy=Gw+\left(\frac{E''-E'}{l-E''}\right)\frac{w''}{w'}.^{20}$$

Thus the following results are obvious:

If $E''=E'$, then $Gy=Gw$.
If $E''>E'$, then $Gy>Gw$.
If $E''<E'$, then $Gy<Gw$.

Being $E=E_k+E_l$, we can summarize these conclusions in the following table:

	land-neutral	land-using	land saving
capital-neutral	$G_y=G_w$	$G_y>G_w$	$G_y<G_w$
capital-using	$G_y>G_w$	$G_y>G_w$	$G_y\gtreqless G_w$
capital-saving	$G_y<G_w$	$G_y\gtreqless G_w$	$G_y<G_w$

Remember that these conclusions are valid under any assigned k and l on the y''-surface.

As will easily be seen, if the case is the combination of capital-using and land-saving innovation or capital-saving and land-using innovation, any definite conclusion cannot be drawn. For instance let us take the former case. Capital-using innovation implies that the marginal productivity of capital becomes larger than its average prodictivity, so that if it is combined with land-neutral innovation, both real wage rate and rate of rent will increase at the smaller rate than rate of profit. But if it is combined with land-saving innovation, and if such land-saving bias is so strong that E'' becomes smaller thatn E', real wage rate will increase faster than labor productivity. *Mutatis mutandis,* it is also true of the case of the combination of capital-saving and land-using innovation.

Next problem is to ask the relationship between labor productivity and rate of rent. Let us denote Gr as the growth rate of rent, namely

$$Gr=\frac{r''-r'}{r'}.$$

We know already that under the condition of the maximum rate of profit, we have

$$r=\frac{\partial y}{\partial l}.$$

This condition must be true after as well as before innovation. Now to reach to quantatively definite conclusions here, we want to compare the situations which are characterized

[20] $Gy=\frac{y''}{y'}-1=\left(\frac{1-E'}{1-E''}\right)\frac{w''}{w'}-1=\left(\frac{1-E'}{1-E''}\right)\frac{w''}{w'}+Gw-\frac{w''}{w'}.$

From this last equation ,we obtain the above equation.

by the same land intensity. It does not matter whether such situations are realizable or not in reality. Thus it follows

$$Gy=Gr+\left(\frac{E_l''-E_l'}{E_l'}\right)\frac{r''}{r'}.$$ [21]

From this last equation, the following conclusions are clear:

If $E_l''=E_l'$, then $Gy=Gr$.
If $E_l''>E_l'$, then $Gy>Gr$.
If $E_l''<E_l'$, then $Gy<Gr$.

Under the condition of the same land intensity, we have again the following table:

	land-neutral	land-using	land-saving
capital-neutral	$G_y=G_w=G_r$	$G_r>G_y>G_w$	$G_w>G_y>G_r$
capital-using	$G_y=G_r>G_w$	$G_r>G_y>G_w$	$G_w\gtreqless G_y>G_r$
capital-saving	$G_w>G_y=G_r$	$G_r>G_y\gtreqless G_w$	$G_w>G_y>G_r$

What is then the effect of innovations on what we call capital-output ratio? This is the third question which we must discuss. Here again if we intend to compare two situations which are under the same profit rate, it is immediately clear that in case of capital-neutral innovations the capital-output ratio must be the same both bofore and after innovations, because the average productivity of capital must be equal in both situations. If they are capital-using, the capital-output ratio must increase and if they are capital-saving, it must decline.. Remember that these conclusions are quite independent of biases in land-innovations. Thus under the conditon of the same rate of profit, we have the following results:

Type of Innovation	Capital-Output Ratio
capital-neurtal	constant
capital-using	increasing
capital-saving	decreasing

Whether innovations raise capital-output ratio or not must be answewd only by observations of reality, but it should be careful that this answer has to be formed on the base of the same profit rate, because, other things being equal, the higher the rate of profit, the smaller the capital-output ratio, so that if we do not bear in mind this basic criterion, we might be led to misjudge, for example, capital-using innovation as lowering the capital-output ratio.

So far, we have been concerned with the discussion of properties of innovations, particularly in their connection with the problems of income distribution. As a last problem in this chapter, we want to shed some light on the problem of the rate of innovation itself.

As far as the elasticities of productivity function are held constant, it was proved that (see footnote 12)

[21] $Gy=\frac{y''}{y'}-1=\frac{E_l'}{E_l''}\frac{l''}{l'}\frac{r''}{r'}-1=Gr-\frac{r''}{r'}+\frac{E_l'}{E_l''}\frac{l''}{l'}\frac{r''}{r'}$,
where l' and l'' are the land intensity before and after innovation. By assumption l' is equal to l''. Thus we obtain the above equation in the text.

$$y=A^{E_k}l^{E_l},$$

where A is any structural parameter. Let us define the productivity functions before and after innovation as follows:

$$y'=A'k^{E'_k}l^{E'_l} \quad \text{.........before innovation}$$
$$y''=A''k^{E''_k}l^{E''_l} \quad \text{......after innovation.}$$

Since an innovation is defined in terms of an overall upward shift of productivity function, the progress rate in innovations can be most explicitely expressed in terms of upward shift in A, namely

$$Gi=\frac{A''-A'}{A'}.$$ [22]

It should be remembered that Gi has nothing to do at all with properties of innovations, but the movement in Gi is most crucial to the development of capitalist economy. If the retardation of progress rate in innovations appears, what bias they may have, a growing menace of the classical stationary state will assault the capitalist economy and make it fall into "secular stagnation". There might be perhaps an inherent rule in changes in the progress rate of innovations. But the detailed analysis of this problem is beyond the scope of this paper.

[22] Tinbergen presented the following economic model in his 1942's article:

$$u=e^t a^\lambda K^{1-\lambda}$$

where u is net output, a labor and K capital and land. e denotes, according Tinbergen, "element of technical development". J. Tinbergen, "Zur Theorie der langfristigen Wirtschaftsentwicklung"; Weltwirtschaftliches Archiv, Mai. 1942, s. 511-549.