MATHEMATICAL ANALYSIS OF THE METABOLISM AND ANALOGY OF ECONOMICS

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A thermodynamical analysis of the metabolism of matter and energy shows the resemblance of the mathematical form to that of economics. Therefore, the application of the mathematical method of economics to bio-physics is tried in the following. Further, there is the metabolic turnover of molecules and the turnover of cells which corresponds to the depreciation and repair of economics. Our body resembles a chemical plant or apparatus made of protein molecules and cells, which corresponds to the means of production. In this report the idea of metabolic turnover of the "negative entropy" is introduced.

Social economics might be seen as the highly extended form of the metabolism of free energy, because our labour is part of the output of the free energy and rewarded by the input of the means of consumption, which is necessary to maintain life. Therefore, the analogy seems to be a fundamental one. The idea of the "utility" of food is introduced too and, using the idea, the way of a quantitative analysis of the nutritive value is suggested.

I. *Introduction*

The analogy of economics and metabolism is rather old. Further, the progress of analysis in both fields revealed the close analogy, especially in the mathematical treatment. The writer of this paper has been trying a mathematical analysis of metabolism on the basis of thermodynamics [1][2] and found that the method of the mathematical analysis of economics can be applied to the study of metabolism.[3]

On the other hand, the study of mathematical biology should give us a model on which to extend the mathematical analysis of economics, although the social phenomena may be too complicated to be treated merely on such model. Nevertheless, we must rely upon some suitable model and the study of the living organism will supply useful suggestion in this respect.

Further, the analogy may not be only superficial but substantial and very important from the *methodological point of view*,[3] because labour means output of **F. E.** (free energy) of our body. The balance of payment and income in economic life may correspond to the balance of F. E., i. e. we obtain food at the cost of output of F. E. and supply F. E. by taking food (see Fig. **1.**). Therefore, *economic life may be considered a highly extended form of the metabolic turnover of F. E.*

In this paper the mathematical analysis of metabolism is described and the application of the mathematical method of economics to bio-physics suggested. Then the substantial relation between metabolism and economics will be discussed from the methodological point of view.

II. *The Phenomena of Life and its Analogy to Social Economy*

Let us first look at the resemblance of both phenomena.(3)

(i) Our body consumes various organic and inorganic substances, some of which are produced in our body, like hormone, enzymes, protein, nuclear acid, fats and others, and some of which are taken from the external world by the work of our muscles and digestive organs, like inorganic salt, vitamins, amino acids and others.

(ii) These substances are useful (4) to maintain life. The idea of nutritive value is well known but *quantitatively* the value of *caloric units* is mainly taken into account. The nutritive value of vitamins, iron and other inorganic substances and some amino acids is also taken into account but only *qualitatively*. There may exist the idea corresponding to *utility* or *welfare function* in economics which may be treated *analytically* and *quantitatively*.

(iii) There is consumption of F. E. to produce or absorb the necessary substances and consumption is *repaired* by production and intake. Even the absorption of glucose, which is the last stage of digested starch, is carried out by the "investment" of F. E. of ATP, an ester of phosphoric acid of high energy. Therefore, in the case of famine or when ill-fed, our organ loses the power to digest or absorb nutritious substances due to the lack of F. E., which corresponds to initial cost. On the contrary, the function of intestinal absorption will be dangerously damaged if over-fed.

The above is shown in Fig. **1.**, in which the energy is fed back to take the chemical energy from the external world.(4)(5) This "feeding back" is similar to business life, in which an enterprise is sometimes suppressed by the lack of the initial cost. Indeed our bodyc orresponds to a factory and ATP to capital.

(iv) There is the balance of the need and the supply. Superfluous protein, for instance, loses its amino-group and changes into carbon-hydrates correspond-

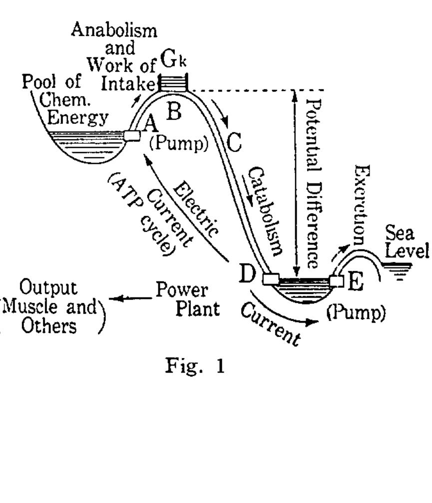


Fig. 1

ing to *consumer's goods*. On the other hand, the protein of our tissues, which corresponds to *producer's goods*, is destroyed by lack of protein, and the material is used to construct the other necessary part.

According to Professor Kida the relatively short legs of the Japanese are due to the lack of protein of high quality in food during growth. The body seems to lack protein to build legs, for we must use the material to construct the necessary part of our organs. Medical science may be considered good management in the balance of matter and F. E.

(v) Our body corresponds to our system of industry. Various substances are produced in every part of our body and supplied to other parts. On the other hand, the parts are also supplied from other parts. There is an *exchange* and *economy* of matter and energy. For instance, the production of protein corresponds to the *first department* of producer's goods. In this case as well ATP as protein is consumed. The consumption of the latter corresponds to the *depreciation* of producer's goods, in this case the chemical apparatus made of protein.

The ATP which is consumed, is *reproduced* again in our body and carbon-hydrates, protein and ATP are consumed for reproduction. Here, the carbon-hydrates correspond to consumer's goods and the reproduction of ATP to the *second department* of economics. Therefore, there is a close analogy between the two fields. For instance, labour is reproduced by the consumption of goods, just as carbon-hydrates in food.[2] This fact is important from the point of view of methodology (see Fig. **6**).

On the other hand the consumption of protein which is an example of "catabolism," corresponds to depreciation which is repaired by "anabolism."

(vi) *Depreciation* and the *repair* is the general aspect of life. For instance, reproduction is the *turnover* of the body itself, which depreciates during life, especially by reproduction itself.

If we take, however, the history of man-kind into account, depreciation in the individual body is repaired by other bodies. Therefore, those who enjoy youth enjoy the *turnover of the individual* body.

Therfore, one of the most prominent aspects of life is the turnover of molecules, of cells and of the individual body, so that the world of the living organism is repaired and *steadily maintained*. This is very important from the point of view of thermodynamics, for the F. E. on earth is constantly consumed by organisms.

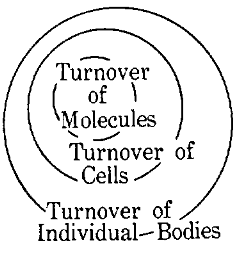
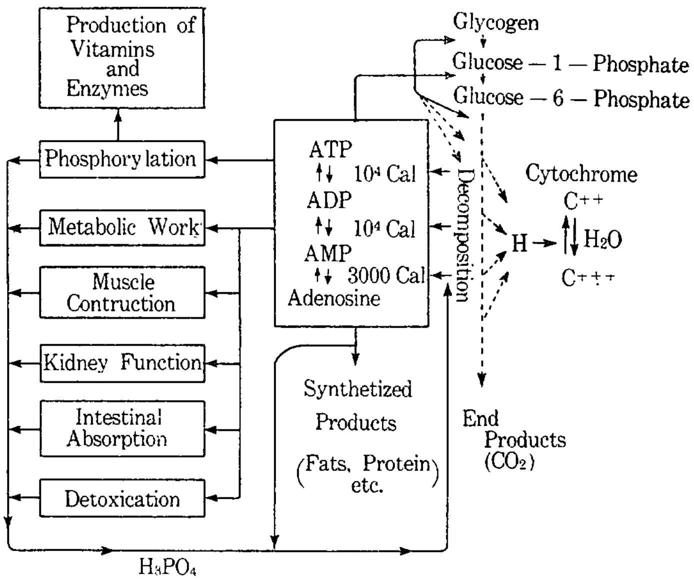


Fig. 2

The *steadiness* is similar to that of the river, which consumes the potential energy of water and also maintains steadiness on the "balance" of water.

In a similar manner the depreciation of the apparatus of a chemical plant, the value of **N. E.** (negative entropy) of our body is also depreciated (see **VI**). On the other hand this value of N. E. regulates the value of *F.E. of activation* of bio-chemical reactions. Therefore, the "catalytic action" of the organs, corresponding to the function of the chemical plant, is also depreciated and repaired. In this respect the writer has introduced the idea of the *metabolic turnover of N.E.*, corresponding to the depreciation and the repair of producer's goods in economics.(3)(5)(6)

(vii) Besides the "feed back" of F. E., there is the *circulation of matter* in our body, for instance the *chemical cycle* of ATP⇄ADP or the reduction and oxidation of enzymes. Fig. **3** shows the circulation of phosphates of adenosin in which ATP is included. The circulation is very



ATP : Adenosine triphosphate
ADP : Adenosine diphosphate
AMP : Adenosine monophosphate
According to V. R. Potter, Biological Energy Transformations and Cancer Problems (Advances in Enzymology 4, 201-256, 1944)

Fig. 3

complicated, in general, but is schematized in Fig. **4**. This is similar to the circulation of paper money in our society. In a similar manner, the matter of high energy is taken from the external world and excreted, so that our body corresponds to a pipe and is called an *open system*. But it is not an open system like the pipe through which the water of a tank flows. This is shown in Fig. **1**. The circulation of matter in our body is also similar to the "feed back" of energy, so that the system is half open and half closed.

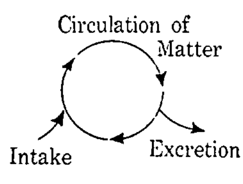


Fig. 4

(viii) The "feed back" of matter and energy is very similar to the management of our social life. Chemical processes in our body are combined like the system of gears (see **VI**), and, if we wish to promote a process, the effect is fed back and produces sometimes unexpected results. Here is the difference of the bio-chemical change from that in vitro. Therefore, if the knowledge of chemistry in vitro is applied mechanically the effect may be contrary to expectation, as in the controlled social economy.

There is *bad circulation* in our body. For instance, the appetite is diminished, if health is destroyed, and health is disturbed if the appetite is diminished. *Good management* by the physician will eliminate bad circulation.

(ix) There is the balance and stability of matter and F. E. in our metabolism. If the balance is disturbed, the function of our body is disturbed. We have seen that our body resembled a pipe, through which the matter of high chemical energy flows and the matter of low chemical energy is excreted. The balance seems to be favourable to the flow of matter (see Fig. **11**). In social life the balance of production and consumption is favourable to the movement of goods. There is the recovering action in our body as well as in our society. If the balance is disturbed, and cannot be recovered, a catastrophe occurs and finally death of our body.

Prof. Bertalanffy called the balance of our body *dynamic stability* or *equilibrium*. From the point of view of thermodynamics, this is not "thermal equilibrium." But stability can be seen in many transient phenomena of the *inanimate world*.(1)(2) I have studied such phenomena from the point of view of molecular statistics and noticed the stable equilibrium of the "second coordinate" which will be discussed later (see **VI**) in connection with the maximum principle.(7)

III. *Metabolism of Matter*(3)(4)

(i) There is the metabolic turnover of the chemical substances, as shown in fig. 4. Recently radioactive isotopes are used to study the turnover of matter in our body. For instance, organic compounds containing C^{14} or P^{32} are synthetized and introduced into a living body and the movement of these compounds is traced. The isotopes are synthetized to compounds of high energy by anabolism and then destroyed by catabolism.

Now let us take the behavior of a chemical element X, which is not necessarily a radioactive isotope, and let the notation i be a state of the element in our body, may be, for instance, a chemical combination like amino acid, protein as well as a biological one like cytoplasm or nucleoprotein. Let the quantity of the element X in the state i be n_i (X) and the rate of change of X from j to i be Q_{ji} (X), when the *chemical potential* of j, which will be defined afterwards, is *lower* than that of i, and is called the *rate of anabolism* from j to i. When the chemical potential of i is *higher* than that of k, the reaction from i to k is called the *catabolism* and the rate of this reaction is q_{ik} (X).

The phenomena of life may be represented by the net-work (see Fig. 5) of the chemical rates Q_{ji} (X) and q_{ik} (X), in which the diffusion of a component or absorption through the membrane is included. The reactions Q_{ji}(X) or q_{ik}(X), are not changeable independently but are combined like the gear of a machine, which is the characteristic feature of life and will be discussed later (see **VI**).

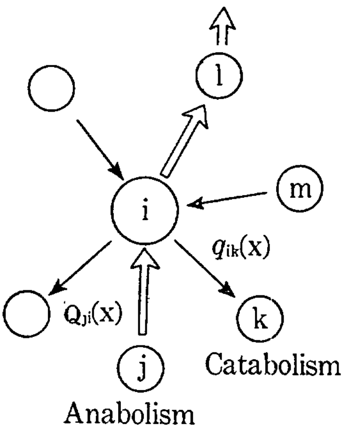


Fig. 5

Let $\dot{n}_i$ (X) be the timely rate of the change of n_i (X), then

$$\dot{n}_i(X)=\sum_j Q_{ji}(X)-\sum_l Q_{il}(X)-\sum_k q_{ik}(X)+\sum_m q_{mi}(X) \qquad (3.1)$$

If we sum up both sides of (3.1) with respect to i, then $\Sigma\dot{n}_i(X)$ is the rate of change of the total quantity of X in the living body. On the right side, the circulation of matter in the same body is cancelled and the rate of uptake and excretion will remain in the formula, because the living organism is an open system. If the rate of uptake of X is $Q_{ai}(X)$, where the notation a means the external world, and i means the absorbed state; and the rate of excretion of X be $q_{ak}(X)$, where k means the state of X before the excretion, then

$$\dot{M}(X)=\Sigma\dot{n}_i(X)=\Sigma Q_{ai}(X)-\Sigma q_{ka}(X) \qquad (3.2)$$

The first term of the right side, $Q_{ai}(X)$, corresponds to the import of the material X, for instance, and i specifies the state of the materials. It is analogous to the social economy, for instance, iron is imported in the state of an auto, a typewriter or other machine and instrument. The production of iron in the same country can also be included, for the underground material corresponds to the external world of the living organism. The second term of the right side of (3.2) corresponds to export and $\dot{M}$ (X) corresponds to the total increase of iron in the country.

If we take the summation of $\dot{M}(X)$ with respect to X, then the rate of increase of the total weight of the organism is

$$\dot{W}=\Sigma_X\dot{M}(X)=\sum_X\sum_{ai}Q_{ai}(X)-\sum_X\sum_{ka}q_{ka}(X). \qquad (3.3)$$

In many experiments of biology W is observed, as is well known.

(ii) Let us consider the experiment of the tracer with radioactive isotopes. $\alpha_i(X)$ is the percentage of the isotope in the state i. Then $n_i(X)\alpha_i(X)$ is the quantity of the isotope in the state i and $Q_{ji}(X)\alpha_j(X)$ is the rate of supply of isotope from j state. Then we obtain

$$\frac{d}{dt}\Big(n_i(X)\alpha_i(X)\Big)=\sum_j Q_{ji}(X)\alpha_j(X)-\sum_l Q_{il}(X)\alpha_i(X)-\Sigma q_{ik}(X)\alpha_i(X)+\Sigma q_{mi}(X)\alpha_m(X) \quad (3.4)$$

In the steady state $\dot{n}_i(X)=o$, so that

$$n_i(X)\dot{\alpha}_i(X)=\sum_j Q_{ji}(X)\alpha_j(X)-\sum_l Q_{il}(X)\alpha_i(X)-\sum_k q_{ik}(X)\alpha_i(X)+\sum_m q_{mi}(X)\alpha_m(X) \quad (3.5)$$

These equations can be solved in the form of

$$\alpha_i(X)=\Sigma A_{in}(X)e^{-\lambda_n(X)t} \quad (3.6)$$

and $\lambda_n(X)$ expressed by $Q_{ji}(X)$, $n_i(X)$ and $q_{ik}(X)$. Therefore, we obtain the detailed knowledge of $Q_{ji}(X)$, $n_{ik}(X)$, if we get the exact knowledge of $\lambda_n(X)$s.

This is only a preliminary report and a detailed analysis will be given in a following report.

(iii) The system of the net-work of Fig. **5** is too generalized and complicated and should be simplified. We first take a model of bio-chemical reactions which is most suitable. But this will be discussed in the following

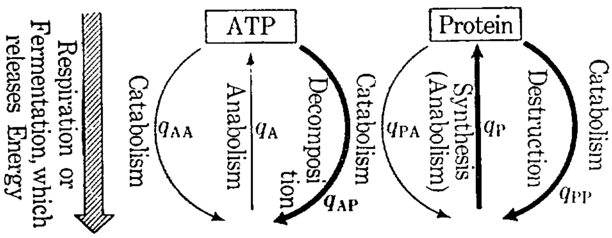


Fig. 6

report. Let us now take a simple model [2][3] into account, in which A_i of Fig. **7** corresponds to goods, and glucose to consumer's goods and protein, nuclear acid and other macro-molecules correspond to producer's goods. Fig. **6** shows the relation of the two departments. In department 1, glucose, etc. are produced from carbon-hydrates, and are decomposed by the action of ATP. In these reactions the protein and other macro-molecules are used as producer's goods and depreciated. The depreciation corresponds to the catabolism of those macro-molecules in our cells and tissues. The destruction by catabolism is repaired by the synthesis of anabolism which corresponds to department 1 in economics. In this reconstruction ATP is also required, which corresponds to *labour*. Therefore, the cycle of ATP$\rightleftarrows$ADP corresponds to the reproduction of labour and ADP to the state of labourer after work and ATP to the state of saturation after sleeping. Protein and other macro-molecules are also used as producer's goods in this reconstruction, thus the metabolic turnover may correspond to the depreciation and the repair of the productive goods.

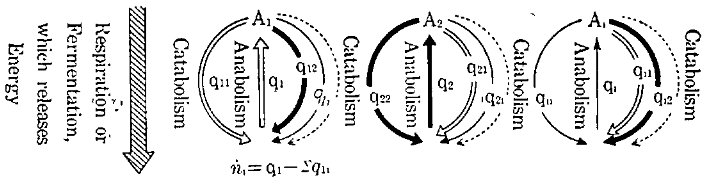
If q_{PP} and q_{PA} be the *mean rates* of catabolism of the macro-molecules during the reconstruction of the macro-molecules themselves and of ATP respectively and q_{AA} and q_{AP} the rate of change, ATP→ ADP, during the reproduction of ATP, which is combined with the decomposition of glucose, etc., and the synthesis of the macro-molecules respectively, then we have

$$\left.\begin{aligned}\dot{n}_A&=q_A-q_{AA}-q_{AP}\\ \dot{n}_P&=q_P-q_{PA}-q_{PP}.\end{aligned}\right\} \tag{3.7}$$

where n_A and n_P is the quantity of ATP and the macro-molecules and q_A and q_P is the rate of reproduction (see Fig. 6). So that the steady state, i. e. $\dot{n}_A=0$, $\dot{n}_P=0$, corresponds to the *simple reproduction*[2] of economics and

$$\dot{n}_A>0,\ \dot{n}_P>0$$

to the *expanding reproduction* (see **VIII**).



$\dot{n}_i=q_i-\Sigma q_{ii}$

$i=1, 2, 3 \cdots\cdots$

Fig. 7

We can generalize the model[5] to the coupling of many departments (see fig. 7).

(iv) *Chemical Cycle.* In the steady state

$$\dot{n}_i(X)=\sum_j Q_{ji}(X)-\sum_l Q_{il}(X)-\sum_k q_{ik}(X)+\sum_m q_{mi}(X)=0 \tag{3.8}$$

from (3.1). This equation means the *circulation* or the *cycle* of substances in the living organism. There are many types of such circulation, for instance, that of citric acid. Of course, matter is not conserved completely but is supplied and excreted (see Fig. 4). However, catabolism is repaired by anabolism and the *balance* of consumption and production is maintained. Therefore, circulation is kept in a *steady state*. Strictly speaking, the steady state in the living organism is in question, but the balance like the equation (3.8) is maintained in a short time in the living organism, although the organism may be fertile and decay in the long run.

When the organism is not in a steady state, the cycle changes and a *expanding* or *contracting* reproduction may be seen, according to external conditions. Therefore the living organism has self-movement and does not simply follow external action, because the chemical rates, $Q_{ji}(X)$ or $q_{ik}(X)$, are connecting and make a "feed back system." The living organism is called a *self-maintaining system* and this character may be due to the nature of the "chemical cycle."

A top has a self-movement too, and does not simply follow the external action. It is rotating on its axis and the rotation may correspond to the chemical cycle in the living organism.

There is the circulation of goods or commodities in our society, and this is also self-movement.

(v) What is the nature of the *coupling* of the chemical rates? These reactions are combined in *series* by the relation of supply and consumption.

(a) Therefore, the chemical rate is controlled by the law of supply, like the law of mass action, or by Fick's law of diffusion. In general, the rate of reaction and diffusion is determined by the *field of ch.p.* (chemical potential). introduced by the author(1)(2) in the living organism. The field is dependent on supply and consumption.

(b) On the other hand chemical reactions are combined in *parallel*, for instance, *endergonic* reaction, which absorbs energy, is combined with *exergonic* reaction, which releases energy, and then the former, which seems to be the *reversed course* from the thermodynamic point of view, is realized or maintained (see **VI**). The oxidation of fatty acids and the composition or decomposition of some amino acids is combined with the cycle of citric acid. The destruction of protein which corresponds to the depreciation of productive goods, is also combined with the reconstruction, for which the protein might be considered to play the role of the chemical apparatus.

These combined reactions may be due to the formation of the common *activated complex*, like

$$A+B+C+D+E+F+\ldots\ldots\rightarrow(A+B+C+\ldots\ldots)^{*}\rightarrow A'+B'+C'+\ldots\ldots$$

Therefore the reactions,

$$A+B\rightarrow A'+B',\quad C+D\rightarrow C'+D',\quad E+F\rightarrow E'+F',\ldots\ldots\ldots$$

are realized, some of which can not be realized independently.

(c) The activity of enzymes is controlled by the production of some substances, like the change of PH value or the stypic action of trombogen when the skin is wounded. The *fitness* of our body may be realized by such controlling action.(2)

IV. *Metabolism of F. E. (free energy)*

There is a *balance* of energy in our body, but it is more convenient to take F. E. into account in the case of the living organism, as there is a loss or dissipation of F. E. in the organism. The idea of F. E. contains the "utility" of energy (see the definition of (8.5)). There is no *loss of energy* itself. But the utility of the energy is depreciated. The depreciation is described by a change of F. E. of the living organism. The definition of F. E. will be given later. It may be mentioned here that the so-called *chemical energy* is not pure energy but F. E.

In **III** combined reaction is considered, in which F. E. released by exergonic reaction is transferred to endergonic reaction. In the transfer, the ATP cycle plays an important role. For instance, the released F. E. is transferred to reaction ADP→ATP, then the reaction ATP→ADP is coupled with the endergonic reaction (see Figs. **8** and **9**). In this case the action of ATP is similar to the electric power of Fig **1**, and endergonic reaction is promoted by pump action.

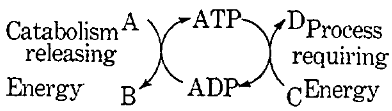


Fig. 8

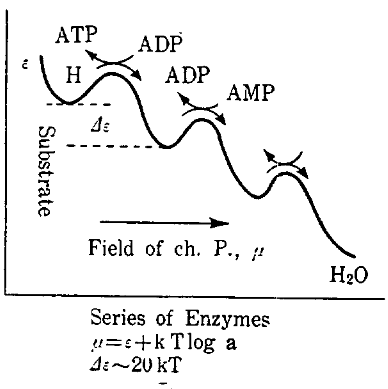


Fig. 9 Potential curve of the energy of the series of enzymes

Let us consider the balance of F. E. in our body.(4)(5) In Fig. **10**, Z_1 is the uptake of the F. E. and Z_2 is the one excreted. Then Z_1-Z_2 is utilized by the organism to *maintain life*. Let G_k be the F. E. of the organism, then

$$Z_1-Z_2=\dot{G}_k+D, \qquad (4.1)$$

where D is the F. E. consumed by the organism. We can spell (4.1) in the form

$$\dot{G}_k=Z_1-Z_2-D \qquad (4.2)$$

Z_1-Z_2 means F. E. of the external world which is consumed by the organism. If we consider the rate of decrease of F. E. of the external world, $-\dot{G}_a$, then

$$-\dot{G}_a=Z_1-Z_2. \qquad (4.3)$$

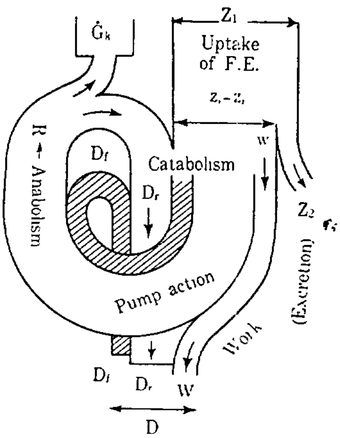


Fig. 10 Balance of F. E.

Therefore,

$$\dot{G}_a+\dot{G}_k=-D<0. \qquad (4.4)$$

That is, the *F. E. of the total system, i. e. the organism and the external world, decreases thermodynamically*. D may be a *positive definite* but verification might be a matter of future, by extending the H-theorem of Boltzmann (see **VI**).

(4.4) shows the decrease of F. E. of the total system but the increase of G_k, if the organism is in growth. *Growth may promote, however, the rate of decrease of* G_a+G_k *of the total system.* This is very interesting from the point of view of the maximum principle of **VI**. The fact, that $\dot{G}_k>0$ in growth, corresponds to the *accumulation* of capital in economics.(3)

Therefore, (4.4) shows that there is "no difficulty in applying thermodynamics" to the living organism.(2)

Let us now take the steady state into consideration. Then $\dot{G}_k=0$ and

$$-\dot{G}_a = Z_1 - Z_2 = D \tag{4.5}$$

form (4.1) and (4.3).

Let us consider the balance in detail. A part of F. E., $Z_1 - Z_2$, is used to do the work of muscles or of digestion, of absorption and of excretion. Let us denote this by W. The other part is used as pump action to promote synthetic reaction or anabolism. The F. E. of our body is reproduced by this reaction. Let us denote the rate of reproduction by R. There is a loss of F. E. during the work, W, and during the pump action, R, and this is spelled D_f. The relation of these quantities is expressed by Fig. **10**. Then

$$\left.\begin{aligned} Z_1 - Z_2 &= W + R + D_f, \\ \text{or} \quad R &= Z_1 - Z_2 - W - D_f, \end{aligned}\right\} \tag{4.6}$$

Then we obtain from (4.2) and (4.6)

$$\dot{G} = W + R + D_f - D.$$

Let us introduce the following relation

$$\left.\begin{aligned} D_r &= D - W - D_f \\ D &= W + D_f + D_r \end{aligned}\right\} \tag{4.7}$$

Then

$$\dot{G}_k = R - D_r \tag{4.8}$$

This is the fundamental equation of the thermodynamics of metabolism.(3)(5) (4.8) is expressed by Fig. **10** and D_r is the dissipation of F. E. due to catabolism. Our life is maintained by consuming the compounds of higher chemical energy, which is partly supplied from the external world, $Z_1 - Z_2$, and partly reconstructed by pump action, R. If part of the consumption, D_r, is compensated by reproduction, R, the organism will be in a *steady state*, that is

$$R = D_r \tag{4.5'}$$

and $\dot{G}_k = 0$ from (4.8). If $R > D_r$, the difference of F. E., $R - D_r$ is stored in the organism.

V. *Detailed Analysis*

The writer has introduced the following relation to F. E. of the organism,(1)(4)(5)

$$\dot{G}_k = \sum_{X}\sum_{i} v(X)\dot{n}_i(X) \tag{5.1}$$

from considerations of molecular statistics, where

$$v_i(X) = \frac{\partial G}{\partial n_i(X)} \tag{5.2}$$

is the ch. p. (*chemical potential*) of X in the state i (see note **1**). This definition is analogous to that of the *marginal utility* but the analogy is not so simple as shown later (see **VI**).

Note 1. For instance, the ch. p., v_i (C) of carbon of glucose, $C_6H_{12}O_6$, is different for every carbon atom. Therefore the total value of the ch. p. of glucose is

$$v=\sum_i v_i(C)+\sum_j v_j(O)+\sum_k v_k(H),$$

where i, j, k specifies the state of the chemical combination of each C, O, H respectively.

If we take (3.1) into account, then

$$\dot{G}_k=\sum_x\sum_i v_i(X)\{\sum_j Q_{ji}(X)-\sum_l Q_{il}(X)-\sum_k q_{ik}(X)+\sum_m q_{mi}(X)\}$$
$$=\sum_x\{\sum_{ij}(v_i(X)-v_j(X))Q_{ji}(X)-\sum_{ik}(v_i(X)-v_k(X))q_{ik}(X)\}+Z_1-Z_2, \quad (5.3)$$

where

$$\left.\begin{aligned} Z_1&=\sum_x\sum_{ai} v_i(X)\,Q_{ai}(X),\\ Z_2&=\sum_x\sum_{ka} v_k(X)q_{ka}(X). \end{aligned}\right\} \quad (5.4)$$

The meaning of $Q_{ai}(X)$ and $q_{ka}(X)$ is given by (3.2). Therefore, Z_1 is the uptake of F. E. from the external world, i. e. the F. E. of food and air, and Z_2 is the F. E. which is excreted. Let us write

$$D=-\sum_x\sum_{ij}(v_i(X)-v_j(X))Q_{ji}(X)+\sum_x\sum_{ik}(v_i(X)-v_k(X))q_{ik}(X). \quad (5.5)$$

Then we obtain the equation (4.2)

$$\dot{G}_k=Z_1-Z_2-D, \quad (4.2)$$

where D is the total consumption of the F. E. to maintain life. (4.2) is analogous to the equilibrium of economic life and Z_1-Z_2, D and $\dot{G}_k$ corresponds to *national income*, to *consumptions* and to *stocks* respectively.

The first term of (5.5) corresponds to R, so that

$$R=\sum_x\sum_{ij}(v_i(X)-v_j(X))Q_{ji}(X) \quad (5.6)$$

and the second term of D is

$$\sum_x\sum_{ik}(v_i(X)-v_k(X))q_{ik}(X)=D+R=W+D_r+D_f+R \quad (5.7)$$

from (5.6) and (4.7). Considering (4.7), let us define

$$\sum_x{}'\sum_{ik}(v_i(X)-v_k(X))q_{ik}(X)=$$
$$\sum_x\sum_{ik}(v_i(X)-v_k(X))q_{ik}(X)-(W+R+D_f)=D_r. \quad (5.8)$$

Then we obtain (4.8) again,

$$\dot{G}_k=R-D_r. \quad (4.8)$$

In the steady state, (4.5′) can be defined in the form

$$\sum_x\sum_{ij}(v_i(X)-v_j(X))Q_{ji}(X)=\sum_x{}'\sum_{ik}(v_i(X)-v_k(X))q_{ik}(X) \quad (5.9)$$

from (5.6) and (5.8). (5.9) shows the net balance of F. E. in our body.

Therefore $Q_{ji}(X)$ corresponds to the flow of goods during production and $q_{ik}(X)$ to that in consumption and they are similar to the quantities in the *activity analysis* of economics. $v_i(X)$ corresponds to the value (not the quantity of labour in this case, but the utility) of goods, and our body uses this value to maintain life. The substances which have lost the value or *utility* are excreted. But the definition of (5.2) is rather different from that of economics, because $n_i(X)$ is a thermodynamic quantity. It may, however, be possible to define $v_i(X)$ based on the quantities like $Q_{ji}(X)$ or $q_{ik}(X)$ which are not thermodynamic quantities and correspond to quantities in economics (see **VI**).

VI. *The Maximum Principle*[7]

In chemical kinetics the reaction rate usually depends on the difference of the chemical potential of this reaction (see note **2**). This is similar to the movement of goods depending on value.

Note 2. In the generalized chemical change $\Sigma A_i \rightarrow \Sigma B_k$, the ch. p. of the system of ΣA_i be v' and that of ΣB_k be v''. Then the rate of the reaction is proportional [2] to

$$e^{v'/kT} - e^{v''/kT}.$$

If the system is nearly in equilibrium, $v' \doteqdot v''$ and the rate of reaction is approximately proportional to $v' - v''$, i. e. the difference of the che. p..

In the living organism, however, the chemical reaction is not so simple, because the endergonic reaction is coupled with the exergonic one. Or the anabolism and catabolism is conjugated to each other and their rate of reactions is not independent (see **III** and **IV**). This corresponds to the relation of production and consumption of goods, because we consume goods, i. e. iron, oil and electricity, to produce goods.

Let $q_1, q_2 \ldots\ldots q_s$ be the parameters specifying the rate of such combined or coupled reactions of **III**. Then $Q_{ji}(X)$ and $q_{ik}(X)$ is represented in the following way,

$$\left.\begin{aligned} Q_{ji}(X) &= \Sigma_s A_{jis}(X) q_s \\ q_{ik}(X) &= \Sigma_s Q_{iks}(X) q_s \end{aligned}\right\} \tag{6.1}$$

Then, from (5.5)

$$D = \sum_s \sum_X \{ -\sum_{ij} (v_i(X) - v_j(X)) A_{jis}(X) + \sum_{ik} (v_i(X) - v_k(X)) a_{iks}(X) \} q_s = \sum_s \Delta v_s q_s, \tag{6.2}$$

where

$$\Delta v_s = \sum_X \{ -\sum_{ij} (v_i(X) - v_j(X)) A_{jis}(X) + \sum_{ik} (v_i(X) - v_k(X)) a_{iks}(X) \}. \tag{6.3}$$

Here, q_s may be dependent on Δv_s. But the relation is, in general, not so simple (see note **2**). There is, however, quasi-equilibrium, which is maintained in the living organism (see note **3**) and the linear relation between q_s and Δv_s may be assumed. Then D of (6.2) has the quadratic form with respect to q_ss. Therefore D may be a *positive definite* and, if so, the thermodynamic relation (4.4) can be verified.

Note 3. The energy change of each reaction, $i \rightarrow k$ of $j \rightarrow i$, may sometimes be very large but the energy is transferred to the combined reaction, so that the change of the ch. p. of the combined reaction is not large. Therefore, the relation between Δv_ss and q_ss will be linear and homogeneous. R and D_r is also the homogeneous quadratic function of q_ss.

Indeed many reactions in our body are nearly reversible and almost in equilibrium. There are some stages, which are irreversible. The loss of D_t in (4.6) may be due to such reactions.

Let us consider the steady state and transform R and D_r of (4.8) in the following form

$$\left.\begin{aligned} R&=\sum_s\sum_x\Sigma_{ij}A_{jis}(X)(v_i(X)-v_j(X))q_s=\sum_s\Delta v_s'q_s \\ D_r&=\sum_s\sum_x\Sigma_i'{}_k a_{iks}(X)(v_i(X)-v_k(X))q_s=\sum_s\Delta v_s''q_s \end{aligned}\right\}, \tag{6.4}$$

where

$$\left.\begin{aligned} \Delta v_s'&=\Sigma_x\Sigma_{ij}A_{jis}(X)(v_i(X)-v_j(X)) \\ \Delta v_s''&=\Sigma_x\Sigma_i'{}_k a_{iks}(X)(v_i(X)-v_k(X)). \end{aligned}\right\} \tag{6.5}$$

R and D_r are also assumed to have the quadratic form (see note **3**). Then the condition of the steady state, (4.5′), is defined in the form

$$\sum_s\Delta v_s'q_s=\sum_s\Delta v_s''q_s \tag{6.6}$$

which is equal to the relation (5.9).

Let a small variation be given to q_ss, maintaining the condition of steadiness, (6.6). Then, from the *physical consideration*, Z_1-Z_2 or D of (4.5) will show a maximum at a certain point in q_s-space. Mathematically speaking, D is quadratic and has a minimum, therefore, it is not possible to show that the section of two hyper-surfaces, (6.6) and (4.5), in q_s-space has a maximum. We assume, however, that the living organism consumes F. E. of the external world at a *maximum rate*. Therefore, in the steady state $\dot{G}_a$ of (4.3) is assumed to be a maximum from physical considerations.

Further, there are not only one maximum but many maximum points. Let us take, however, one maximum into consideration. Then the maximum condition is shown from (4.5)

$$\delta(Z_1-Z_2)=\delta D=0.$$

If we assume temporarily that W and D_f of (4.7) is constant, then

$$\delta D=\delta(W+D_r+D_f)=\delta D_r=0. \tag{6.7}$$

On the other hand from (4.5′) or (6.6)

$$\delta R=\delta D_r=0. \tag{6.8}$$

From (6.7) and (6.8) we obtain

$$\delta(R-\lambda D_r)=0, \tag{6.9}$$

where λ is Lagrange's coefficient.

R and D_r has the quadratic form, so that, from (6.4)

$$\left.\begin{aligned} \delta R&=2\Sigma_s\Delta v_s'\delta q_s \\ \delta D_r&=2\Sigma_s\Delta v_s''\delta q_s \end{aligned}\right\} \tag{6.10}$$

Therefore from (6.9) and (6.10)

$$\sum_s(\Delta v_s'+\lambda\Delta v_s'')\delta q_s=0 \qquad (6.9')$$

If we take (6.6) into consideration, $\lambda=-1$ and we get

$$\Delta v_s'=\Delta v_s''$$

or

$$\sum_x\sum_{ij}A_{jis}(X)(v_i(X)-v_j(X))=\sum_x\sum_{ik}{}'a_{iks}(X)(v_i(X)-v_k(X)) \qquad (6.11)$$

from (6.5). Or multiplying both sides of (6.11) with q_s we obtain

$$\sum_x\sum_{ij}A_{jis}(X)(v_i(X)-v_j(X))q_s=\sum_x\sum_{ik}{}'a_{iks}(X)(v_i(X)-v_k(X))q_s. \qquad (6.12)$$

This is the equation of the *detailed balance*, which is quite different from that of statistical mechanics, of the metabolic turnover of F. E. (see note **4**), and is similar to that of *simple reproduction* in economics. If it is possible to know the values of $A_{jis}(X)$, $a_{iks}(X)$ and q_s, then $v_i(X)$, or $v_k(X)$ can be known. These quantities refer to the *chemical potential of our body in the living state*, which might be obtained, if we were able to observe the rate of bio-chemical changes, $A_{jis}(X)q_s$, $a_{iks}q_s$ in detail. Here, $a_{iks}(X)q_s=q_{iks}(X)$ is the rate of consumption of the compounds containing X, which is used for the production, j→i, by the process s, and $A_{jis}(X)q_s=Q_{jis}(X)$ is the corresponding rate of production.

Note 4. Summing up both sides of (6.12) with respect to s, we obtain (6.6) or (5.9).

These values of $v_i(X)-v_j(X)$ and others are not those of the meat or the giblets on the market, as was described in the preceding.[2] They are quantities corresponding to the function of life. So that we can estimate the *function of life* itself in this manner.

Let us consider again the meaning of the *maximum principle*, which was discussed in the preceding report[2][5][7] (see Fig. **11**). There is no difference in the height of water flowing on both sides of the river, as water tends to equalize the surface. But there is a difference along the flow, for water must equalize the surface against the resistance of the stream. Let us call the difference or the rate of flow the *first coordinate*, and that, which can be equalized, the *second coordinate*.[4][5][7] In Fig. **11** (b), the second coordinate is not yet *equalized*. Then the surface must be equalized and the state (b) will return to (a). Then the rate of flow, which is the first coordinate, will be the *maximum*, and there is *equilibrium in the second coordinate* though the system is in transient state. This is a very important fact for the study of the transient state.

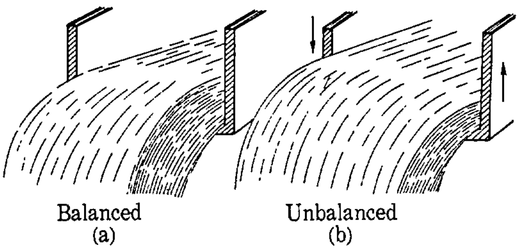


Fig. 11

A living organism is not in thermodynamic equilibrium but the matter is going through the body. Such flow corresponds to the first coordinate.

There is a *balance* or certain *equilibrium* which corresponds to the "second coordinates."

The economics of our social life is also phenomena of consumption and reproduction and a balance or equilibrium is seen which corresponds to the "equilibrium of the second coordinate." There is also a *net flow* of goods which corresponds to the first coordinate. Therefore, there is a fundamental difference between the equilibrium of economics and that of thermodynamics and the analogy of the ordinary theory of thermodynamics seems to be dangerous to the study of economics.

But quantities like $q_{ik}(X)$ or $Q_{ji}(X)$ and D, which are not of a thermodynamic nature (see note **5**), have a close correspondence with the quantities in economics. For instance, the definition of v like

$$\frac{\partial D}{\partial q_s}=2\Delta v_s \tag{6.13}$$

is analogous to the definition of the marginal utility (see **VII**). (6.13) is easily derived from (6.2), for D is the homogeneous quadratic function of q_s.

Note 5. Z_1-Z_2, R and D is not F. E. itself but its *rate of change*. In the same manner W is not the work generally but the work of a single day. Therefore, the *unit* is not that of energy but energy/time which has the dimension of power. From the point of view of thermodynamics they are "kinetic quantities" and not thermodynamic functions.

VII. *Utility in Bio-physics.*

It has been mentioned that there may be many maximums in a steady state. These maximums depend on the uptake of F. E. and the steadiness itself is the result of the balance of uptake and consumption, etc.

Let us use the analogy of economics in the analysis of metabolism.[3][4] In economic life, W can be defined by W_1+W_2, where W_1 is physiological, i. e. digestion, absorption and excretion, and W_2 is the "output of labour," which is *rewarded* by income (see Fig. **1.**). In order to obtain the *maximum utility* of nutritive substances, we consider at first "income and market price," and then digestion which depends on W_1. Thus, there is a close relation between the economics of social life and the physiological economics of matter and F. E.

Z_1-Z_2 has *utility* in maintaining life, by allowing the work of reproduction and repair of our body, R, as well as output, W, using the F. E. of Z_1-Z_2. Then

$$\frac{\partial(Z_1-Z_2)}{\partial q_s} \tag{7.1}$$

may correspond to *marginal utility*. It seems to me that the principal part of utility, U, or marginal utility of goods will be Z_1-Z_2 or the quantity of (7.1), because the principal utility of goods must be to "maintain life." U may thus be defined in the form

$$U=Z_1-Z_2+\sigma \tag{7.2}$$

where σ depends on personal or subjective matter. If we take the analogy with *welfare functions*, it corresponds also to Z_1-Z_2.

Now let us extract the "relation with society" and observe the organism purely biologically.

In (4.6) W corresponds to investment in F. E., Z_1-Z_2, and the latter corresponds to income. The relation between Z_1-Z_2 and W may be described by Fig. **12**. Therefore, if W is too large, $R+D_f=Z_1-Z_2-W$ becomes smaller and the depreciation of our body cannot be repaired. Then output W as well as intake Z_1-Z_2 becomes smaller. On the other hand, if W is too small, Z_1-Z_2 becomes small. Therefore, there may be an optimum partition between R and W, and then Z_1-Z_2 is the maximum. Then $R+W$ will be also a maximum and D_f a minimum. Therefore (6.12) may be obtained. Details will be described in the following report.

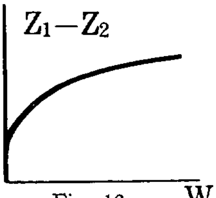


Fig. 12

In general, the living organism is not in a steady state, $\dot{G}_k>o$, when it is growing and recovering, and $\dot{G}_k<o$, when in senility and in ill health. There is, however, a steady state, if we choose adequately w_1, w_2, ..., the quality and the quantity of food, then a result similar to (6.12) may be obtained. But there may be many maximums according to the difference of the history of the organism in the experiment.

Details will be given in the following report.

The idea of the utility of nutritive substances is based on an experiment, in which food lacking tryptophane and tryptophane itself is given alternatively at an interval of 12 hours. Tryptophane is one of the essential amino acids. The total quantity of these amino acids is enough but the growth of rats used in the experiment was hindered. We can see from this that the utility of amino acids is diminished in such doses. The effect suggests also a new way of estimating the nutritive value of foods.(3)

VIII. *Depreciation and Repair of the Living Organism*

Let us take the turnover of F. E. into consideration based on the model of Fig. **6**. R of (4.5′) can be defined as R_1+R_2, where R_1 and R_2 corresponds to the reproduction of macro-molecules and of ATP respectively. In the same way D_r may be defined as $D_{r1}+D_{r2}$, where D_{r1} and D_{r2} corresponds to the destruction of macro-molecules and to the consumption of carbonhydrates, for instance, respectively. Then (4.5′) will be

$$D_{r1}+D_{r2}=R_1+R_2 \tag{8.1}$$

D_{r2} is transferred to ATP and, if we define it as V,

$$V=D_{r2}=V_1+V_2 \qquad (8.2)$$

where V_1 and V_2 is used to the reproduction of macro-molecules and of ATP respectively. On the other hand D_{r1} can be spelled

$$D_{r1}=C_1+C_2 \qquad (8.3)$$

where C_1 and C_2 is the depreciation of macro-molecules during the reproduction of the macro-molecules themselves and of ATP respectively. Then

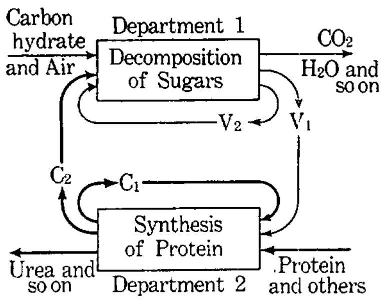


Fig. 13

$$\begin{aligned} C_1+V_1&=C_1+C_2=R_1 \\ C_2+V_2&=V_1+V_2=R_2 \end{aligned} \qquad (8.4)$$

This is analogous with simple reproduction in economics(2) (see Fig. **6** and the note **6**), and in adding both (8.4) and taking (8.2) and (8.3) into account, we again obtain (8.1).

Note 6. The quantity like the *surplus value* will not appear in the living body.

If we consider further the departments in detail, we have the relation of the balance of F. E. of (6.12).

Macro-molecules of the living organism are a chemical apparatus with excellent function due to their *orderliness*. This is specified by the value of F. E. from the view point of thermodynamics. Therefore the depreciation of F. E. of the macromolecules is due to that of the entropy part of F. E., for the F. E. (of Gibbs) is represented by

$$G=\chi-TS=\chi+T(-S) \qquad (8.5)$$

where χ is the heat function and $-S$ is N. E., and T the absolute temperature. The heat function or enthalpy is directly related to the *exchange of energy* itself. (8.5) shows that the *utility of the energy* is controlled by $T(-S)$. i. e. by N. E., $(-S)$.

The great value of N. E. in the living organism seems *queer* from the view point of the classical theory of thermodynamics. The value of the N. E. is, however, depreciated in *accordance* with the theory of thermodynamics or molecular statistics. On the other hand, it is reproduced as shown by (4.8). The mystery of life is thus clarified, if the idea of *metabolic turnover of N. E.* is introduced, corresponding to depreciation and repair in economics.

The idea of the *turnover of entropy* is first introduced by the author.(6) It is also important if we take the problem of fatigue into consideration. Ordinarily the consumption of glycogen, for instance, is taken into account. This, however, is not due to substances corresponding to consumer's goods only, but to the depreciation of N. E. in our body. In a state of fatigue we consume more energy than in a normal state. This is due to the depreciated function of our body, especially the function of *energy transfer*, as the loss in energy transfer, D_f of (4.6) is increased in such state. The depreciation of N. E. in our body, which corresponds to the chemical apparatus of macro-molecules, is not taken into consideration in the ordinary study of human labour.

IX. *Conclusion*

In human society, W, of output is rewarded in a complicated manner and there may be a difference in social science and biology at this point (see note **7**).

On the other hand, labour is stored in the form of instruments, machines and other means of production. The use of such storage requires technique and skill which is cultivated by training and study. Therefore, these are also a storage of human labour. Therefore we work with these stored instruments, technique and wisdom, and the products of labour are exchanged. Here is the difference between homo sapiens of today and other animals, and this is the reason of the complicated relation of output W_2, and the reward.

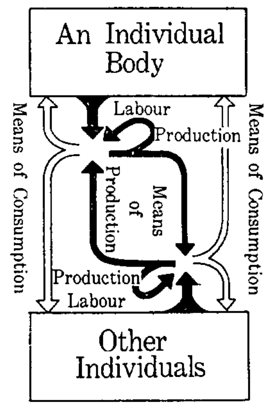


Fig. 14 Exchange of Output of Human F. E. from the Biophysical Point of View.

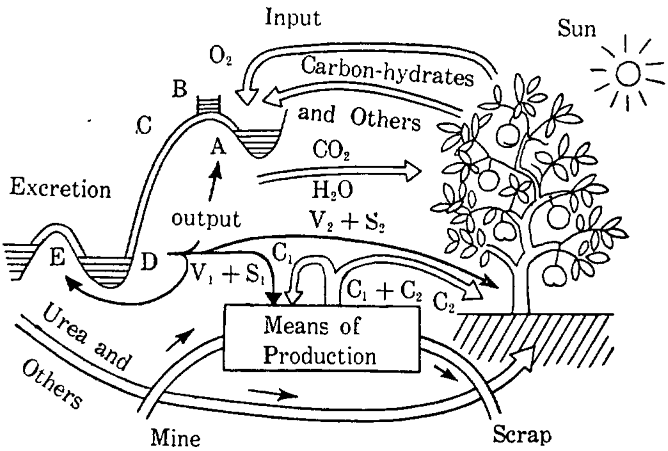


Fig. 15 Circulation of Matter and Free Energy in Nature

But the *fundamental* aspect of the balance of matter and F. E. to maintain life cannot be different in both phenomena. Therefore, the analogy is fundamental from the point of view of methodology.

Note 7. If $W=W_1$, and $W_2=o$, then $R+W_1+D=Z_1-Z_2$ of (4.6) corresponds to the basal metabolism, in which the quantity R is mainly due to the metabolism of entropy part.

The mathematical analysis given in this report is only a preliminary one, and will be extended in the near future.

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REFERENCE PUBLICATION

(Works of the Author)

(1) Thermodynamics of Metastability and the Phenomena of Life (in Japanese), *Kagaku*, **18**, No. 5 (1948).

Thermodynamics, Molecular Statistics and Biology (in Japanese), Nihon-Kagakusha, Kyoto (1949).

Thermodynamics of Transient Phenomena (in Japanese), Iwanami-Syoten, Tokyo (1950).

Attempts at Analysis of Metabolism (in Japanese), *Busseiron-Kenkyu* No. 30 (1950), No. 60 and 62 (1953).

(2) Thermodynamic Method in Biology, *Annals of the Hitotsubashi Academy, Supplement* **1**, p. 9 (1951).

Fitness and Natural Phenomena (in Japanese), *Hitotsubashi Review* (Ikkyo-Ronso) **22**, No. 3 (1949).

(3) Mathematical Analysis of Metabolism and the Analogy of Economics (in Japanese), *Hitotsubashi Review* (Ikkyo Ronso) **29**, No. 6 (1953).

The description is revised in this report.

(4) Metabolic Turnover of Entropy and Energy and its Mathematical Analysis I. (in Japanese), *Bull. Kobayashi Inst.* **III**, No. 2 (1953).

(5) Thermodynamic Analysis of Life I, II and III. *Journal of the Phys. Soc. of Japan*. **8**, No. 6, P. 697 (1953).

(6) The Idea of Negative Entropy (in Japanese), *Bull. Kobayashi Inst.* **I**, No. 4 (1951); *Kagaku*, **22**, No. 2 (1952).

Metabolic Turnover of Negative Entropy and the Living Organism (in Japanese), *Kagaku*, **23**, No. 3 (1953).

(7) The Maximum Principle of Transient Phenomena and its Application to Biology (in Japanese), *Bull. Kobayashi Inst.* **I**, No. 2, (1951).

Boltzmann's H-Therem and Dissipation Function, *Bull. Kobayashi Inst.* **II**, No. 3–4 (1952).