Discussion Paper Series

No.194

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December 2006

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A 21st-Century COE Program

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The Role of "Leads" in the Dynamic OLS Estimation of Cointegrating Regression Models

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December, 2006

Abstract

In this paper, we consider the role of "leads" of the first difference of integrated variables in the dynamic OLS estimation of cointegrating regression models. We demonstrate that the role of leads is related to the concept of Granger causality and that in some cases leads are unnecessary in the dynamic OLS estimation of cointegrating regression models. Based on a Monte Carlo simulation, we find that the dynamic OLS estimator without leads substantially outperforms that with leads and lags; we therefore recommend testing for Granger non-causality before estimating models.

JEL classification: C13; C22

Key Words: Cointegration; dynamic ordinary least squares estimator; Granger causality

*Hayakawa is a JSPS research fellow and acknowledges its financial support.

†Kurozumi's research was partially supported by the Ministry of Education, Culture, Sports, Science and Technology under Grants-in-Aid No. 17203016 and 18730142.

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1 Introduction

Since the seminal work of Engle and Granger (1987), cointegrating regressions have become one of the standard tools in analyzing integrated (I(1)) variables. Although the ordinary least squares (OLS) estimator is consistent in the presence of a serial correlation in the error term and/or a correlation between the regressors and cointegration errors, it is well known that the OLS estimator contains the so-called second-order bias. In the literature, there are three typical estimators that deal with this problem: the fully modified OLS estimator proposed by Phillips and Hansen (1990), Park's (1992) canonical cointegrating regression estimator, and the dynamic OLS (DOLS) estimator of Phillips and Loretan (1991), Saikkonen (1991), and Stock and Watson (1993). These three estimators are known to be asymptotically equivalent and efficient. In this paper, we focus on the DOLS estimator among the three estimators and consider the role of "leads" of the first difference of the integrated variables in DOLS. We investigate the case where leads are unnecessary for the DOLS method, and by using the Monte Carlo simulation, we demonstrate that in such a case, we can expect the improvement of the DOLS estimator in terms of the mean squared error (MSE) by excluding leads from the regressors.

2 Relation between Leads and Granger Causality

We consider a typical cointegrating regression model as follows:

$$\begin{aligned} y_t &= \alpha + \beta' x_t + u_{1t} = \theta' z_t + u_{1t} \qquad (1) \\ \Delta x_t &= u_{2t} \end{aligned}$$

where $\theta = [\alpha, \beta']'$, $z_t = [1, x_t']'$, x_t is an n-dimensional I(1) vector, and $u_t = [u_{1t},\, u_{2t}']'$ is a stationary process that satisfies the condition of the multivariate invariance principle. Under the regularity condition given in Saikkonen (1991), u_{1t} is expressed as

$$u_{1t} = \sum_{j=-\infty}^{\infty} \Pi_j' u_{2t-j} + v_t \qquad (2)$$

where $\sum_{j=-\infty}^{\infty} \|\Pi_j\| < \infty$ and v_t is a stationary process such that $E(u_{2s} v_t) = 0$ for all s and t. See also Brillinger (1981). By inserting (2) into (1), the model can be expressed as

$$y_t = \alpha + \beta' x_t + \sum_{j=-K}^{K} \Pi_j' \Delta x_{t-j} + \dot{v}_t \qquad (3)$$

where $\dot{v}_t = v_t + \sum_{|j|>K} \Pi_j' \; u_{2t-j}$ and K is known as the lead-lag truncation parameter. Saikkonen (1991) showed that the OLS estimator of β based on (3) does not suffer from the second-order bias and is efficient in a certain class of distributions.

Let us consider the case where

$$\Pi_j = 0 \quad \text{for} \quad \forall j < 0. \tag{4}$$

In this case, the model becomes

$$y_t = \alpha + \beta' x_t + \sum_{j=0}^{K} \Pi_j' \Delta x_{t-j} + \dot{v}_t \tag{5}$$

and then we do not have to include the leads of Δx_t as regressors. We therefore expect an improvement of the finite sample efficiency by estimating (5) because we do not have to include extra regressors. In this case, we note that condition (4) is related to the concept of Granger causality. According to Sims (1972) and Proposition 11.3 in Hamilton (1994), condition (4) holds if and only if u_{1t} does not Granger-cause u_{2t}. In other words, it is possible to efficiently estimate the cointegrating regression model without any leads of the first difference of integrated variables if the past values of u_{1t} do not help to predict u_{2t}. Therefore, we recommend that the null of Granger non-causality be tested before estimating the cointegrating regression model.

Tests for Granger non-causality can be conducted by approximating the process of u_t by a finite-order vector autoregressive model: $u_t = \Psi_1 u_{t-1} + \Psi_2 u_{t-2} + \cdots + \Psi_p u_{t-p} + e_t$. Let $\hat{u}_t = [\hat{u}_{1t}, u_{2t}']'$, where $\hat{u}_{1t} = u_{1t} - (\hat{\theta} - \theta)' z_t$ is the regression residual from (1) with $\hat{\theta}$ as the OLS estimator of θ. We then estimate

$$\hat{u}_t = \Psi_1 \hat{u}_{t-1} + \Psi_2 \hat{u}_{t-2} + \cdots + \Psi_p \hat{u}_{t-p} + \hat{e}_t \tag{6}$$

and test the hypothesis that $\Psi_{1,21} = \Psi_{2,21} = \cdots = \Psi_{p,21} = 0$ where $\Psi_{j,21}$ is the $(2,1)$ block of Ψ_j and $\hat{e}_t = e_t - (I_{n+1} - \Psi_1 L - \cdots - \Psi_p L^p)[z_t'(\hat{\theta} - \theta), 0]'$ and L being the lag operator. Although $\hat{u}_{1t}$ includes an estimation error, its effect is asymptotically negligible. In fact, we can show that for $j = -p, \cdots, p$,

$$\frac{1}{T} \sum_{1 \le t, t-j \le T} \hat{u}_{1t} \hat{u}_{1t-j} = \frac{1}{T} \sum_{1 \le t, t-j \le T} u_{1t} u_{1t-j} + O_p\left(\frac{1}{T}\right),$$

$$\frac{1}{T} \sum_{1 \le t, t-j \le T} \hat{u}_{1t} u_{2t-j}' = \frac{1}{T} \sum_{1 \le t, t-j \le T} u_{1t} u_{2t-j}' + O_p\left(\frac{1}{T}\right)$$

while for $j \ge 0$,

$$\frac{1}{\sqrt{T}} \sum_{1 \le t, t-j \le T} \hat{u}_{t-j} \hat{e}_t' = \frac{1}{\sqrt{T}} \sum_{1 \le t, t-j \le T} u_{t-j} e_t' + O_p\left(\frac{1}{\sqrt{T}}\right).$$

by the asymptotic technique explained in, for example, Chapters 17–19 in Hamilton (1994). If the evidence of Granger non-causality is observed by tests based on (6), we can expect the finite sample efficiency gain by excluding the leads of Δx_t from (3) and estimating (5).

We may also consider verifying condition (4) by investigating whether or not the regression error from (5) is serially uncorrelated. For this purpose, the portmanteau tests are available as explained in Lütkepohl (1993).

To demonstrate the case where the model can be expressed as (5), we consider the following case:

$$u_{1t} = \sum_{j=0}^{\infty} \phi_j \varepsilon_{1t-j} \quad \text{and} \quad u_{2t} = \varepsilon_{2t} \quad \text{where} \quad \sum_{j=0}^{\infty} j|\phi_j| < \infty$$

$$\text{and} \quad \varepsilon_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \sim iid \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \Sigma_{22} \end{pmatrix} \right]. \tag{7}$$

We then decompose ε_{1t} as

$$\varepsilon_{1t} = \varepsilon_{1\cdot 2t} + \tilde{\varepsilon}_{2t} \tag{8}$$

where $\varepsilon_{1\cdot 2t} = \varepsilon_{1t} - \sigma_{12}\Sigma_{22}^{-1}\varepsilon_{2t}$ and $\tilde{\varepsilon}_{2t} = \sigma_{12}\Sigma_{22}^{-1}\varepsilon_{2t}$. Note that $\varepsilon_{1\cdot 2t}$ is uncorrelated with all the leads and lags of ε_{2t} and $\tilde{\varepsilon}_{2t}$. Using this decomposition, u_{1t} can be expressed as

$$\begin{aligned} u_{1t} &= \sum_{j=0}^{\infty} \phi_j \varepsilon_{1\cdot 2t-j} + \sum_{j=0}^{K} \phi_j \tilde{\varepsilon}_{2t-j} + \sum_{j=K+1}^{\infty} \phi_j \tilde{\varepsilon}_{2t-j} \\ &= v_t + \sum_{j=0}^{K} \Pi_j' \Delta x_{t-j} + \sum_{j=K+1}^{\infty} \phi_j \tilde{\varepsilon}_{2t-j} \end{aligned} \tag{9}$$

where $\Pi_j' = \phi_j \sigma_{12}\Sigma_{22}^{-1}$ and $v_t = \sum_{j=0}^{\infty} \phi_j \varepsilon_{1\cdot 2t-j}$. Since $E(\varepsilon_{2s}\varepsilon_{1\cdot 2t}) = 0$ for all s and t, it is evident that v_t is uncorrelated with Δx_{t-j} for all j. The regression form in (5) is obtained by inserting (9) into (1).

To investigate the finite sample performance of the dynamic OLS estimator without leads, we conduct a Monte Carlo experiment. We consider the case of $n = 1$ and assume that u_{1t} follows a first-order autoregressive model with the AR coefficient ρ, while u_{2t} is an *iid* sequence. We set $T = 100$, $\sigma_{11} = \Sigma_{22} = 1$, $\sigma_{12} = \sigma_{21} = 0.4, 0.8$, and $\rho = 0.1, 0.5, 0.9$. The computation was conducted by using the GAUSS matrix language, and the number of replications is 10,000 for all the cases. The simulation results are summarized in Table 1 (further simulation results are available from the author upon request).

For the choice of K, we use the general-to-specific method by Ng and Perron (1995) with 1% and 5% significant levels and information criteria, i.e., the Akaike information criterion (AIC) and the Bayesian information criterion (BIC).

From Table 1, we observe that the dynamic OLS estimator without leads substantially outperforms that with leads and lags in all the cases. In particular, in terms of the MSE, the MSE of the DOLS estimator without leads are approximately half of that with leads and lags in many cases.

3 Conclusion

In this paper, we considered the role of leads of the first difference of the I(1) regressors in the dynamic OLS estimation. We demonstrated that leads are not necessary in cointegrating regression models when the cointegrating regression error does not Granger-cause the first difference of the I(1) regressors. Based on a Monte Carlo simulation, we found that the dynamic OLS estimator without leads substantially outperforms that with leads and lags when leads are, in fact, unnecessary.

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Table 1: Simulation Results

$T = 100,\ \sigma_{12} = 0.4$		GS001		GS005		AIC		BIC	
ρ		L&L	Lags	L&L	Lags	L&L	Lags	L&L	Lags
	BIAS	0.00231	0.00191	0.00138	0.00076	0.00156	0.00062	0.00229	0.00174
0.1	Std. Dev.	0.05592	0.03787	0.07115	0.04301	0.04820	0.03975	0.03448	0.03486
	MSE	0.00313	0.00144	0.00506	0.00185	0.00233	0.00158	0.00119	0.00122
	BIAS	0.01305	0.01239	0.00525	0.00482	0.00678	0.00378	0.01690	0.01178
0.5	Std. Dev.	0.09897	0.06823	0.12299	0.07647	0.10103	0.07482	0.06346	0.06415
	MSE	0.00997	0.00481	0.01515	0.00587	0.01025	0.00561	0.00431	0.00425
	BIAS	0.09278	0.09473	0.07786	0.07110	0.08037	0.06462	0.11620	0.09531
0.9	Std. Dev.	0.39449	0.27095	0.46348	0.29522	0.46084	0.30779	0.29436	0.27233
	MSE	0.16423	0.08239	0.22087	0.09221	0.21883	0.09891	0.10015	0.08324

$T = 100,\ \sigma_{12} = 0.8$		GS001		GS005		AIC		BIC	
ρ		L&L	Lags	L&L	Lags	L&L	Lags	L&L	Lags
	BIAS	0.00244	0.00268	0.00043	0.00083	0.00158	0.00068	0.00385	0.00269
0.1	Std. Dev.	0.03526	0.02498	0.04597	0.02889	0.03121	0.02612	0.02297	0.02324
	MSE	0.00125	0.00063	0.00211	0.00084	0.00098	0.00068	0.00054	0.00055
	BIAS	0.00913	0.00916	0.00312	0.00352	0.00460	0.00215	0.01402	0.00865
0.5	Std. Dev.	0.06438	0.04777	0.07958	0.05145	0.06938	0.05101	0.04811	0.04557
	MSE	0.00423	0.00237	0.00634	0.00266	0.00483	0.00261	0.00251	0.00215
	BIAS	0.14712	0.14187	0.12359	0.11137	0.12629	0.10198	0.18267	0.13415
0.9	Std. Dev.	0.30634	0.23202	0.32937	0.22818	0.33710	0.22879	0.27612	0.23326
	MSE	0.11549	0.07396	0.12376	0.06447	0.12959	0.06275	0.10961	0.07240

Note: "GS001", "GS005", "AIC", "BIC" denote the dynamic OLS estimator with K chosen by the general to specific approach with 1% and 5% significant levels, AIC, and BIC, respectively. "L&L" denotes the dynamic OLS estimator with leads and lags, and "Lags" denotes the dynamic OLS estimator without leads.