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Abstract

Carlsson and van Damme (1991, 93) presented a notion of a *global game*, which is an incomplete information game where the actual payoff structure is affected by a realization of a common shock and where each player gets noisy private information of the shock. For n -person symmetric games with two possible actions characterized by strategic complementarity, they showed that equilibrium play in a global game with vanishing noise is uniquely determined. The concept of global games is important not only as a theory of the most refined notion of equilibrium but also as a theory of coordination failures under private information. From this viewpoint, this paper makes the theory of global games more general and more applicable to such problems. The implications of the theory of global games are investigated in two specific models: a speculative attack model and a network externality model. It is shown that both the monetary authority in the speculative attack model and the central planner in the network externality model will prefer the equilibrium in a global game with small noise to the worst equilibrium in the corresponding complete information game. Therefore, they will welcome the existence of small noise, if they apply mini-max principle to multiple equilibrium problems.

Key words: global game; coordination failure; speculative attack; network externality

JEL classification: C73, D82, D84, F32

1. Introduction

In many coordination failure problems, it seems more plausible to assume that players do not exactly know other players' types. Investors in a foreign exchange market may have some private information on balance-of-payments or on monetary authorities' intention to defend a fixed exchange rate (Krugman 1979). Potential users of the new technology characterized by network externalities may face some uncertainty in other users' preference (Farrell and Saloner 1985) or in productivity of the new technology.

In their seminal theoretical paper Carlsson and van Damme (1991, 93) presented a notion of a *global game*, which is an incomplete information game where the actual payoff structure is affected by a realization of a common shock and where each player gets noisy private information of the shock. For *n*-person symmetric games with two possible actions characterized by strategic complementarity, they showed that equilibrium play in a global game with vanishing noise is uniquely determined.

The concept of global games is important not only as a theory of the most refined notion of equilibrium but also as a theory of coordination failures under private information. From this viewpoint, in Section 2 of this paper I make the theory of global games more general and more applicable to such problems. First, Carlsson and van Damme (1991) assumed that the prior on the common shock is uniform and the payoff function is linear about the shock.[1] I relax these assumptions. Secondly, in order to get a unique equilibrium we do not need to assume that the radius of the support of the noise converges to zero. Convergence of the variance of the noise to zero is sufficient. Thirdly, the global game theory singles out an equilibrium not only when players observe shocks with infinitesimal noise but also when they observe the sum of a common shock and an infinitesimal idiosyncratic shock and each player's payoff depends on this sum.

In the subsequent two sections, I apply the theory of global games to coordination failure problems. In Section 3, I study a speculative attack model under incomplete information. It is

[1]Carlsson and van Damme (1990) study a more general case for two player game.

shown that a monetary authority, who applies mini-max principle to multiple equilibrium problems, will prefer to keep investors' information on monetary authorities' intention to defend a fixed exchange rate incomplete. In Section 4, by using a network externality model, I study welfare implications of small noise. In the network externality model, I assume that the payoff function is linear about both the realization of a common shock and the number of other players who choose a new technology. I also assume that the common shock and noises are independent normal random variables. Under these assumptions, we can derive stronger results than in preceding sections: the Bayesian Nash equilibrium is uniquely determined for a certain set of pairs of standard deviation of common shock and that of noises.

2. *n*-person Symmetric Global Games

Suppose there are n players, $i = 1,..., n$. The set of all the players is denoted by N. Players' payoffs are affected by a common shock s. Before the players simultaneously decide their actions, each player i observes the shock with noise:

(1) $$\theta_i = s + \varepsilon_i.$$

The shock s is drawn from a distribution on $[\underline{s}, \bar{s}]$ with a strictly positive and continuously differentiable density h. The noises $\varepsilon_1, \varepsilon_2,..., \varepsilon_n$ are independent of s and have a joint distribution Φ with a continuous and bounded density φ. φ is symmetric about any ε_i and ε_j : $\varphi(\varepsilon_1,.., \varepsilon_i,.., \varepsilon_j,.., \varepsilon_n) = \varphi(\varepsilon_1,.., \varepsilon_j,.., \varepsilon_i,.., \varepsilon_n)$ for any $i, j \in N$ and any $\{\varepsilon_1, \varepsilon_2,..., \varepsilon_n\}$. The support of Φ is contained in a ball around zero with radius γ. The support of each player's private information is denoted by $[\underline{\theta}, \bar{\theta}]$.

Each player i has two possible actions, α and β. Every player's payoff has an identical functional form and is affected by other player's action in the same way. Each player's marginal gain by choosing α instead of β is expressed as

$$v\left(\frac{m}{n-1}, s\right)$$

where m denotes a number of the other players who choose α. s is a realization of the common shock. $v\,(m\,/\,(n-1), s\,)$ is differentiable and strictly increasing in s. The game is characterized by strategic complementarity: $v\,(m\,/\,(n-1), s)$ is strictly increasing in $m\,/\,(n-1)$. We also assume that $v\,(m\,/\,(n-1), s\,)$ satisfies $v\,(0, \underline{s}\,) > -\infty$, $v\,(1, \bar{s}\,) < +\infty$,

(2) $\quad v\,(0, \bar{s}\,) > 0,$

and

(3) $\quad v\,(1, \underline{s}\,) < 0.$

Inequalities (2) and (3) imply that for high enough shocks, action β is strictly dominated by α, and for low enough shocks, action α is strictly dominated by β.

We first study Nash Equilibria of the complete information games that correspond to our incomplete information games. Suppose there is no noise in observation of the shock s and s is common knowledge. Let $\underline{s}_{\alpha\beta}$ denote the unique solution of

(4) $\quad v\,(1, \underline{s}_{\alpha\beta}) = 0,$

and let $\bar{s}_{\alpha\beta}$ denote the unique solution of

(5) $\quad v\,(0, \bar{s}_{\alpha\beta}) = 0.$

Then we have inequalities: $\underline{s} < \underline{s}_{\alpha\beta} < \bar{s}_{\alpha\beta} < \bar{s}$. If $\underline{s} \le s < \underline{s}_{\alpha\beta}$, there is a unique Nash equilibrium $B = (\beta_1,..., \beta_n)$, in which all the players choose β. If $\bar{s}_{\alpha\beta} < s \le \bar{s}$, then $A = (\alpha_1,..., \alpha_n)$, in which all the players choose α, is the unique Nash equilibrium. If $\underline{s}_{\alpha\beta} < s < \bar{s}_{\alpha\beta}$, there are two Nash equilibria in pure strategy, A and B, and one Nash equilibrium in mixed strategy, in which each player assigns identical probability q to action α. q is determined by

(6) $$\sum_{m=0}^{n-1} v\left(\frac{m}{n-1}, s\right)\binom{n-1}{m} q^{m}\,(1-q\,)^{n-1-m} = 0.$$

Figure 1 summarizes the Nash equilibrium correspondence of the complete information games.

When $\underline{s}_{\alpha\beta} < s < \bar{s}_{\alpha\beta}$, both A and B are strict equilibria, i.e. equilibria in pure strategies in which each player actually looses if he deviates unilaterally. Therefore both equilibria satisfy the conditions imposed by almost all the refined equilibrium notion such as perfectness (Selten 1975) and strategic stability (Kohlberg and Mertens 1986).[2]

Next we study Bayesian Nash equilibria of the incomplete information games. Let $q_i(\theta_i)$ denote player i's probability of taking action α when he observes θ_i. Player i's behavioral strategy is a function $q_i(\cdot)$ from $[\underline{\theta}, \bar{\theta}]$ to [0, 1]. When all the players except i follow strategies $\{q_1(\cdot),..., q_{i-1}(\cdot), q_{i+1}(\cdot),..., q_n(\cdot)\}$, player i with private information θ_i expects the following marginal gain by choosing α instead of β:

$$(7) \quad V_i(\theta_i) \equiv$$

$$\sum_{m=0}^{n-1} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} v\left(\frac{m}{n-1}, s\right) \left[\sum_{\lambda_{-i}^m \in \Lambda_{-i}^m} \left[\prod_{j \in \lambda_{-i}^m} q_j(s+\varepsilon_j) \prod_{k \notin \lambda_{-i}^m \text{ and } k \in N_{-i}} \{1 - q_k(s+\varepsilon_k)\} \right] \right]$$

$$\frac{h(s)\,\varphi(\varepsilon_1, \ldots, \varepsilon_{i-1}, \theta_i - s, \varepsilon_{i+1}, \cdots, \varepsilon_n)}{\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} h(s)\,\varphi(\varepsilon_1, \ldots, \varepsilon_{i-1}, \theta_i - s, \varepsilon_{i+1}, \cdots, \varepsilon_n)\, d\varepsilon_1 \cdots d\varepsilon_{i-1} d\varepsilon_{i+1} \cdots d\varepsilon_n ds}$$

$$d\varepsilon_1 \cdots d\varepsilon_{i-1} d\varepsilon_{i+1} \cdots d\varepsilon_n ds$$

where N_{-i} is the set of all the players except i, $\{1, 2,..., i-1, i+1,..., n\}$. λ^m_{-i} denotes a selection of m players from N_{-i} and Λ^m_{-i}, the set of all such selections. Since $\theta_j = s + \varepsilon_j$, the fraction in equation (7) is the conditional joint density of the shock s and the types of player i' s opponents, $\theta_1,..., \theta_{i-1}, \theta_{i+1},..., \theta_n$, given player i' s type θ_i. A Bayesian equilibrium is a set of strategies $\{q_1(\cdot),..., q_n(\cdot)\}$ such that, for each player i and every possible value of θ_i, behavioral strategy $q_i(\theta_i)$ maximizes player i' s expected payoff, that is, $q_i(\theta_i) = 1$ if $V_i(\theta_i) > 0$, $q_i(\theta_i) = 0$

[2]The strict equilibrium is the most refined equilibrium notion that is discussed in van Damme (1991a). van Damme (1991b) surveys non-cooperative game theories that select a unique equilibrium from two strict equilibria in 2 $\times$ 2 games.

if $V_i(\theta_i) < 0$, and $q_i(\theta_i) \in [0, 1]$ if $V_i(\theta_i) = 0$. On the existence of a Bayesian Nash equilibrium in behavioral strategies, we claim the following:

Proposition 1. *In our incomplete information games, there exists a Bayesian Nash equilibrium point in behavioral strategies.*

Proof. Theorem 1 in Milgrom and Weber (1985) shows that if a game satisfies the two regularity conditions, Equicontinuous payoffs (R1) and Absolutely continuous information (R2), then there exists a Bayesian Nash equilibrium point. Finiteness of the number of actions for each player is sufficient to imply R1. Under Assumption 1, the joint distribution of $(s, \theta^1, \theta^2,...,\theta^n)$ as well as the marginal distribution of each θ^i and s has a density. This implies R2. ∎

Our main results concerning the characteristics of the Bayesian Nash equilibria are summarized as follows:

Theorem 1. *Let s^G denote the unique solution of the equation:*

$$(8) \quad G(s^G) \equiv \sum_{m=0}^{n-1} \frac{1}{n} v\left(\frac{m}{n-1}, s^G\right) = 0.$$

s^G satisfies $\underline{s}_{\alpha\beta} < s^G < \bar{s}_{\alpha\beta}$. For every $\delta \in (0, \min[s^G - \underline{s}_{\alpha\beta}, \bar{s}_{\alpha\beta} - s^G])$ there exists positive γ, such that whenever the support of Φ is contained in a ball around zero with radius γ, the equilibrium behavioral strategy $q_i(\theta_i)$ for every $i \in N$ satisfies $q_i(\theta_i) = 0$ for all $\theta_i \in [\underline{\theta}, s^G - \delta)$, and $q_i(\theta_i) = 1$ for all $\theta_i \in (s^G + \delta, \bar{\theta}]$.

For the proof we will use the following lemmas.

Lemma 1. *Under Assumption 1, the unconditional probability of the event "player i's type θ_i is lower than other m players' types and higher than $n-1-m$ players' types" is equal to $1/n$ for every $m \in \{0, 1, .., n-1\}$:*

$$(9)\quad \binom{n-1}{m}\underbrace{\int_{-\infty}^{+\infty}\int_{\varepsilon_i}^{+\infty}\cdots\int_{\varepsilon_i}^{+\infty}}_{m\text{ integrals}}\underbrace{\int_{-\infty}^{\varepsilon_i}\cdots\int_{-\infty}^{\varepsilon_i}}_{n-1-m\text{ integrals}}\varphi(\varepsilon_1,\ldots,\varepsilon_n)\,d\varepsilon_1\cdots d\varepsilon_{i-1}d\varepsilon_{i+1}\cdots d\varepsilon_n d\varepsilon_i=\frac{1}{n}.$$

Proof. Let π be a permutation of $\{1, 2,..., n\}$. Π denotes the set of all the n! permutations. Let Z_π be the event that the permutation of the players sorted in increasing order of their noise ε_j coincides with π. For example, if $\pi = \{n, n-1,...,1\}$, then $Z_\pi = \{\varepsilon_n < \varepsilon_{n-1} <\cdots< \varepsilon_1\}$. Since Z_π and $Z_{\pi'}$ are mutually exclusive whenever $\pi \neq \pi'$, and the density function $\varphi(\varepsilon_1, \varepsilon_2,..., \varepsilon_n)$ is bounded and symmetrical about any ε_i and ε_j, the probability of Z_π is equal to $1/n!$ for all $\pi \in \Pi$. For every $m \in \{0, 1,.., n-1\}$, there are $(n-1)!$ permutations in which the $n-m$ th number is equal to i. Therefore the probability of the event that ε_i is $n-m$ th smaller in $\{\varepsilon_1, \varepsilon_2,..., \varepsilon_n\}$ is equal to $(n-1)!/n! = 1/n$. The left-hand side of equation (9) denotes the probability of this event. ∎

Lemma 2. *Let $h_\gamma^+(s)$ and $h_\gamma^-(s)$ denote the maximum and minimum value of $h(\cdot)$ on $[s-\gamma, s+\gamma]$. There exists a constant k such that for any $\gamma \in (0, (\bar{s} - \underline{s})/2]$ and $s \in [\underline{s}+\gamma, \bar{s}-\gamma]$,*

$$(10)\quad \frac{h_\gamma^+(s)}{h_\gamma^-(s)} \le 1 + k\gamma,$$

and

$$(10')\quad \frac{h_\gamma^-(s)}{h_\gamma^+(s)} \ge 1 - k\gamma.$$

Proof. Since $h(s)$ is strictly positive and continuously differentiable, there exist $\max_{s\in[\underline{s},\bar{s}]}|h'(s)|$ and positive $\min_{s\in[\underline{s},\bar{s}]}|h(s)|$. Let $k = (2\max_{s\in[\underline{s},\bar{s}]}|h'(s)|)/(\min_{s\in[\underline{s},\bar{s}]}|h(s)|)$, and we get the two inequalities. ∎

We turn to the proof of Theorem 1.

Proof of Theorem 1. Our assumptions on the marginal payoff function $v(m/(n-1), s)$ imply that equation (8) always has a unique solution which satisfies $\underline{s}_{\alpha\beta}<s^G<\bar{s}_{\alpha\beta}$.

If we choose γ as $\gamma < (\underline{s}_{\alpha\beta} - \underline{s})/2$, then type $\theta_i \leq (\underline{s}_{\alpha\beta} + \underline{s})/2$ is sure that s is smaller than $\underline{s}_{\alpha\beta}$ and prefers β to α. It implies that there is no equilibrium with $q_i(\theta_i) > 0$ for any $i \in N$ and any $\theta_i \in [\underline{\theta}, (\underline{s}_{\alpha\beta} + \underline{s})/2]$. By a similar proof, we can show that if $\gamma < (\bar{s} - \bar{s}_{\alpha\beta})/2$, there is no equilibrium with $q_i(\theta_i) < 1$ for any $i \in N$, and any $\theta_i \in [(\bar{s}_{\alpha\beta} + \bar{s})/2, \bar{\theta}]$.

Now we study the equilibrium behavior for $\theta_i \in ((\underline{s}_{\alpha\beta} + \underline{s})/2, s^G - \delta)$ and $\theta_i \in (s^G + \delta, (\bar{s}_{\alpha\beta} + \bar{s})/2)$. First we look for simple equilibria of the form

$$(11) \qquad q_i(\theta_i) = \begin{cases} 0 & \text{if } \square_i < x \\ 1 & \text{if } \square_i > x. \end{cases}$$

In such equilibria, type θ_i expects the following marginal gain by choosing α_i instead of β_i.

$$(12) \quad F_i(\theta_i, x) \equiv$$

$$\sum_{m=0}^{n-1} \binom{n-1}{m} \underbrace{\int_{-\infty}^{+\infty}\int_{x-s}^{+\infty}\cdots\int_{x-s}^{+\infty}}_{m \text{ integrals}} \underbrace{\int_{-\infty}^{x-s}\cdots\int_{-\infty}^{x-s}}_{n-1-m \text{ integrals}} v\left(\frac{m}{n-1}, s\right)$$

$$\frac{h(s)\,\varphi(\varepsilon_1,\ldots,\varepsilon_{i-1},\theta_i - s,\varepsilon_{i+1},\cdots,\varepsilon_n)}{\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} h(s)\,\varphi(\varepsilon_1,\ldots,\varepsilon_{i-1},\theta_i - s,\varepsilon_{i+1},\cdots,\varepsilon_n)\, d\varepsilon_1\cdots d\varepsilon_{i-1} d\varepsilon_{i+1}\cdots d\varepsilon_n ds}$$

$$d\varepsilon_1\cdots d\varepsilon_{i-1} d\varepsilon_{i+1}\cdots d\varepsilon_n ds\,.$$

By Lebesgue's Theorem of Bounded Convergence (see Weir 1973), the continuity of $\varphi(\cdot)$ implies that $F_i(\theta_i, x)$ is continuous in θ_i. Therefore, the equilibrium x must satisfy $F_i(x, x) = 0$. By a simple translation, we get

$$(13) \quad F_i(x, x) \leq \sum_{m=0}^{n-1} \binom{n-1}{m} \max\left[\frac{h_\gamma^+(x)}{h_\gamma^-(x)} v\left(\frac{m}{n-1}, x+\gamma\right), \frac{h_\gamma^-(x)}{h_\gamma^+(x)} v\left(\frac{m}{n-1}, x+\gamma\right)\right]$$

$$\underbrace{\int_{-\infty}^{+\infty}\int_{\varepsilon_i}^{+\infty}\cdots\int_{\varepsilon_i}^{+\infty}}_{m \text{ integrals}} \underbrace{\int_{-\infty}^{\varepsilon_i}\cdots\int_{-\infty}^{\varepsilon_i}}_{n-1-m \text{ integrals}} \varphi(\varepsilon_1,\ldots,\varepsilon_n)\, d\varepsilon_1\cdots d\varepsilon_{i-1} d\varepsilon_{i+1}\cdots d\varepsilon_n d\varepsilon_i\,.$$

Lemma 1, 2 and the fact that v $(\cdot, s)$ is strictly increasing in s, imply that for small enough γ, the right-hand side of the equation is smaller than G $(s^G) = 0$ on $x \in ((\underline{s}_{\alpha\beta} + \underline{s})/2, s^G - \delta)$. Therefore this class of equilibria with $x \in ((\underline{s}_{\alpha\beta} + \underline{s})/2, s^G - \delta)$ does not exist. A similar proof applies for $x \in (s^G + \delta, (\bar{s}_{\alpha\beta} + \bar{s})/2)$.

Next we study more general equilibria. Suppose that for any $\gamma > 0$ there exists a pair of a distribution Φ and a Bayesian Nash equilibrium profile $\{q_1(\cdot),..., q_n(\cdot)\}$; the support of Φ is contained in a ball around zero with radius γ and the equilibrium profile satisfies $q_i(\theta_i) > 0$ for some $i \in N$ and some $\theta_i \in ((\underline{s}_{\alpha\beta} + \underline{s})/2, s^G - \delta)$. In the equilibrium, $V_i(\theta_i)$, type θ_i's expected marginal gain by choosing α instead of β, is expressed by equation (7). Let $\Theta_i{}^*$ be the set $\{\theta_i | V_i(\theta_i) \geq 0$ and $(\underline{s}_{\alpha\beta} + \underline{s})/2 < \theta_i < s^G - \delta\}$ and let $\Theta^* = \cup_{i \in N} \Theta_i{}^*$. Since $q_i(\theta_i)$ is an optimal strategy, $V_i(\theta_i) \geq 0$ if $q_i(\theta_i) > 0$. Therefore if $q_i(\theta_i) > 0$ for some $i \in N$ and some $\theta_i \in ((\underline{s}_{\alpha\beta} + \underline{s})/2, s^G - \delta)$, then Θ^* is non-empty. For $\gamma < (\underline{s}_{\alpha\beta} - \underline{s})/2$, Θ^* have a lower bound that is greater than $(\underline{s}_{\alpha\beta} + \underline{s})/2$. Lebesgue's Theorem of Bounded Convergence implies that $V_i(\theta_i)$ is continuous in θ_i. These two facts imply that $\inf(\Theta^*) \in \Theta^*$ for small γ. Without loss of generality, suppose $V_i(\inf(\Theta^*)) = 0$. Since all the players whose type θ_j is smaller than $\inf(\Theta^*)$ will play β, $V_i(\inf(\Theta^*))$ is equal to or smaller than $F_i(\inf(\Theta^*), \inf(\Theta^*))$, which is smaller than zero for small enough γ. A contradiction. A similar proof with a contradiction applies for $x \in (s^G + \delta, (\bar{s}_{\alpha\beta} + \bar{s})/2)$. ▮

The intuition for the nonexistence of the simple equilibrium that is defined by equation (11) with $x \in [\underline{\theta}, s^G - \delta)$ or $x \in (s^G + \delta, \bar{\theta}]$ is very simple. Suppose there exists such an equilibrium with $x \in [\underline{s}_{\alpha\beta}, s^G - \delta)$, and player i gets private information θ_i that is infinitesimally higher than x. Since player i' s private information is not so favorable for action α, he would choose α only when he expects many other players choose α. Infinitesimal noises make such coordination impossible. If the support of the noises is small compared with the support of the common shock and the density of the common shock is smooth, the private information θ_i will convey almost no information on ε_i. Player i' s posterior joint distribution of his noise and the other players' noises is very close to the prior joint distribution. Therefore, he expects that one half of the other players observe θ_j to be lower than x and he can not rely on other players' coordination in choosing α. The critical point s^G depends on each player's expectation of the relative position of his noise with the other players' noises. Since the unconditional probability of the event "player i's type θ_i is lower than other m players' types and higher than $n-1-m$ players' types" is equal to $1/n$ for every $m \in \{0, 1, .., n-1\}$, s^G is determined by equation (8).[3]

Next we generalize our results. First we show

Corollary. *Assume that the support of* Φ *is contained in a ball around zero with radius* γ, *which is smaller than* $\min\{ (\underline{s}_{\alpha\beta} - \underline{s})/2, (\overline{s} - \overline{s}_{\alpha\beta})/2 \}$. *Then for every* $\delta \in (0, \min [s^G - \underline{s}_{\alpha\beta}, \overline{s}_{\alpha\beta} - s^G])$ *there exists positive* $\overline{\sigma}_\varepsilon^2$ *such that whenever* σ_ε^2 (*the unconditional variance of* ε_i) *is positive and smaller than* $\overline{\sigma}_\varepsilon^2$, *each players' behavioral strategy* $q_i(\theta_i)$ *satisfies* $q_i(\theta_i) = 0$ *for any* $\theta_i \in [\underline{\theta}, s^G - \delta)$, *and* $q_i(\theta_i) = 1$ *for any* $\theta_i \in (s^G + \delta, \overline{\theta}]$.

Proof. For $\theta_i \in [\underline{\theta}, (\underline{s}_{\alpha\beta} + \underline{s})/2]$ and $\theta_i \in [(\overline{s}_{\alpha\beta} + \overline{s})/2, \overline{\theta}]$, the same proof as in Theorem 1 applies. For $\theta_i \in ((\underline{s}_{\alpha\beta} + \underline{s})/2, s^G - \delta)$ and $\theta_i \in (s^G + \delta, (\overline{s}_{\alpha\beta} + \overline{s})/2)$, we first look for simple equilibria of the form

$$q_i(\theta_i) = 0 \text{ if } \theta_i < x \text{ and } q_i(\theta_i) = 1 \text{ if } \theta_i > x.$$

Chebyshev's inequality implies $\text{Prob}[|\varepsilon_i| \geq \sigma_\varepsilon^{0.5}] \leq \sigma_\varepsilon$. By this inequality, we get

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[3]If the number of players is two or the function $v(m/(n\text{-}1), s)$ can be expressed as the sum of a linear function of $m/(n\text{-}1)$ and a function of s, the unique equilibrium behavior in a global game with vanishing noise will be identical with the unique equilibrium behavior that is selected out of multiple equilibria of the corresponding complete information game by risk-dominance criterion of Harsanyi and Selten (1988). But, in general, the two equilibria differ. (See Carlsson and van Damme 1991.)

(14) $F_i(x, x) \leq$

$$\sum_{m=0}^{n-1}\binom{n-1}{m}\Bigg\{\max\Bigg[\frac{h^{+}_{\sigma_\varepsilon^{0.5}}(x)}{(1-\sigma_\varepsilon)\,h^{-}_{\sigma_\varepsilon^{0.5}}(x)+\sigma_\varepsilon h^{-}_{\gamma}(x)}v\left(\frac{m}{n-1}, x+\sigma_\varepsilon^{0.5}\right),$$

$$\frac{h^{-}_{\sigma_\varepsilon^{0.5}}(x)}{(1-\sigma_\varepsilon)\,h^{+}_{\sigma_\varepsilon^{0.5}}(x)+\sigma_\varepsilon h^{+}_{\gamma}(x)}v\left(\frac{m}{n-1}, x+\sigma_\varepsilon^{0.5}\right)\Bigg]$$

$$\int_{|\varepsilon_i|<\sigma_\varepsilon^{0.5}}\underbrace{\int_{\varepsilon_i}^{+\infty}\cdots\int_{\varepsilon_i}^{+\infty}}_{m\text{ integrals}}\underbrace{\int_{-\infty}^{\varepsilon_i}\cdots\int_{-\infty}^{\varepsilon_i}}_{n-1-m\text{ integrals}}\varphi(\varepsilon_1,\ldots,\varepsilon_n)\,d\varepsilon_1\cdots d\varepsilon_{i-1}d\varepsilon_{i+1}\cdots d\varepsilon_n d\varepsilon_i$$

$$+\max\Bigg[\frac{h^{+}_{\gamma}(x)}{(1-\sigma_\varepsilon)\,h^{-}_{\sigma_\varepsilon^{0.5}}(x)+\sigma_\varepsilon h^{-}_{\gamma}(x)}v\left(\frac{m}{n-1}, x+\gamma\right),$$

$$\frac{h^{-}_{\gamma}(x)}{(1-\sigma_\varepsilon)\,h^{+}_{\sigma_\varepsilon^{0.5}}(x)+\sigma_\varepsilon h^{+}_{\gamma}(x)}v\left(\frac{m}{n-1}, x+\gamma\right)\Bigg]$$

$$\int_{|\varepsilon_i|\geq\sigma_\varepsilon^{0.5}}\int_{\varepsilon_i}^{+\infty}\cdots\int_{\varepsilon_i}^{+\infty}\int_{-\infty}^{\varepsilon_i}\cdots\int_{-\infty}^{\varepsilon_i}\varphi(\varepsilon_1,\ldots,\varepsilon_n)\,d\varepsilon_1\cdots d\varepsilon_{i-1}d\varepsilon_{i+1}\cdots d\varepsilon_n d\varepsilon_i\Bigg\}$$

Lemma 1 and 2 imply that for small enough $\sigma_\varepsilon{}^2$, the right-hand side of the equation is smaller than $G(s^G) = 0$ on $x \in ((\underline{s}_{\alpha\beta} + \underline{s})/2,\ s^G - \delta)$. Therefore this class of equilibria with $x \in ((\underline{s}_{\alpha\beta} + \underline{s})/2,\ s^G - \delta)$ does not exist. A similar proof applies for $x \in (s^G + \delta,\ (\bar{s}_{\alpha\beta} + \bar{s})/2)$. For more general class of equilibria, the same proof as in Theorem 1 applies. ∎

Now, we study the games in which each player's payoff is affected by the sum of a common shock and an infinitesimal idiosyncratic shock. Suppose ε_i is not a noise but an idiosyncratic shock and player i' s payoff is affected by $\theta_i = s + \varepsilon_i$ instead of s.

$$v\left(\frac{m}{n-1}, \theta_i\right).$$

We keep all the other assumptions from our original model. In this new model, equations, (7), (12), and inequality (13) will hold if we replace both the term $v\,(m/\,(n-1), s)$ in equations (7) and (12) and the term $v\,(m/\,(n-1), s+\gamma\,)$ in inequality (13), with $v\,(m/\,(n-1), \theta_i)$. This fact implies that Proposition 1 and Theorem 1 are still true.

3. A Speculative Attack Model

In this section we apply the theory of global games to a speculative attack model in which investors' information on monetary authorities' intention to defend a fixed exchange rate is incomplete. Consider a foreign exchange market of a small open country with a fixed exchange rate system. The market is opened two times, at the first and the second period. There are three types of participants, investors, the domestic monetary authority, and pure traders. There are two types of assets, domestic and foreign currency. Both currencies bear zero nominal interest. Neither foreign residents nor foreign monetary authorities do not hold domestic currency. All the investors are domestic residents.

There are n risk neutral investors. At the beginning of the first period, each investor owns A/n unit of domestic currency. Investors consume all their assets in the second period. To buy or sell one unit of foreign currency, investors pay $c/\,2$ unit of foreign currency as transaction cost. In order to buy consumption goods, domestic currency is required. Therefore, if an investor purchases one unit of foreign currency in the first period, he will resell it in the second period and the total transaction cost will be c.[4] In the first period, each investor choose one of two actions, converting all the domestic currency into the foreign currency in the first period and repurchase the domestic currency in the second period, or holding domestic currency until the second period. We call the first action speculation and the second non-speculation. Each speculating investor will incur transaction cost cA/n. Let k denote the number of investors who speculate in the first period.

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[4]If we assume that the investors hold assets with interest, then c will consist of the transaction cost and interest rate differential between the two countries.

Then the total demand for foreign currency by the investors amounts to kA/n.

Net purchase of foreign currency by pure traders is equal to the current account deficit of the country. Let D denote the current account deficit in the first period. D is a random variable with cumulative density function $H(\cdot)$. The investors do not observe a realization of D when they make decisions on speculation in the first period.

We model the monetary authority's behavior as follows. Let R denote the potential reserves the monetary authority is willing to use to defend the initial fixed rate. If the demand for foreign currency is greater than R in the first period,

$$R < \frac{k}{n} A + D,$$

then the domestic currency will be devaluated $100e$ percent in the second period. Otherwise the fixed rate will be kept constant in the second period. R is a random variable with a support $[\underline{R}, \overline{R}]$. Each investor gets private information on R:

$$\theta_i = R + \varepsilon_i$$

where ε_i denotes noise.

The random variables D, R, and ε_i satisfy the following conditions.

A1) $H(\cdot)$, the cumulative density function of D is continuously differentiable and satisfies

$$H'(x) > 0 \qquad \text{for all } x \in [\underline{R} - A, \overline{R}],$$

$$H(\underline{R} - A) > 0,$$

$$H(\overline{R}) < 1,$$

$$-c + e\{1 - H(\underline{R})\} > 0,$$

and

$$-c + e\{1 - H(\overline{R} - A)\} < 0.$$

A2) R is drawn from a distribution on $[\underline{R}, \overline{R}]$ with a strictly positive and continuously differentiable density.

A3) The noises $\varepsilon_1, \varepsilon_2, ..., \varepsilon_n$ are independent of s and have a joint distribution Φ with a continuous and bounded density φ. φ is symmetric about any ε_i and ε_j : $\varphi(\varepsilon_1, .., \varepsilon_i, .., \varepsilon_j, .., \varepsilon_n) = \varphi(\varepsilon_1, .., \varepsilon_j, .., \varepsilon_i, .., \varepsilon_n)$ for any $i, j \in N$ and any $\{\varepsilon_1, \varepsilon_2, ..., \varepsilon_n\}$. The support of Φ is contained in a ball around zero with radius γ.

To simplify the model, we assume that in the first period the authority keeps the initial fixed rate and is willing to supply the amount of foreign currency that the market wants to buy, $kA/n + D$. If we assume that the authority stops selling foreign currency as soon as it uses up R, we would have to model the dynamic process in the first period market.

We assume that

$0 < c < e < 1$.

Therefore, if an investor expects devaluation, he will have an incentive to convert his domestic currency into foreign currency in the first period and repurchase the domestic currency in the second period. The price of the consumption good is set in world markets and the foreign price level is constant and equal to one. The initial fixed exchange rate is normalized to be one. Under these assumptions investors' payoff is summarized by Table 1.

The sequence of events in our model is summarized as follows.

1) Nature chooses the potential amount of intervention, R and the current account deficit of the first period, D.

2) Each investor gets private information on R . He observes neither D nor R.

3) The foreign exchange market of the first period is opened. The investors simultaneously decide whether speculate or not. The monetary authority keeps the initial fixed rate and supplies the amount of foreign currency that the market wants to buy, $kA/n + D$, where k denotes the number of investors who speculate.

4) If monetary authority's foreign currency supply in the first period is greater than R, the authority will devaluate the domestic currency $100e$ percent in the second period. Otherwise the authority

will keep the initial fixed rate. In both cases, the authority sustains the fixed rate within the second period and is willing to supply the amount of foreign currency that the market wants to buy. The investors consume all their wealth.

We study investor *i*'s marginal gain by choosing speculation instead of non-speculation. Let m denote a number of the other investors who speculate. If investor i does not speculate, the probability of devaluation will be equal to $1-F\,(R - m\,A\,/\,n)$. Therefore, investor *i*'s marginal gain by choosing speculation instead of non-speculation is expressed as

$$v\left(\frac{m}{n-1}, -R\right) = -c + e\left\{1 - H\left(R - \frac{n-1}{n}\frac{m}{n-1}A\right)\right\}.$$

We first consider the complete information case in which the investors observe R before participating the first period market. Let $\underline{R}_{\alpha\beta}$ and $\overline{R}_{\alpha\beta}$ be defined by

$$(16) \qquad -c + e\,\{1 - H\,(\underline{R}_{\alpha\beta})\} = 0,$$

and

$$(17) \qquad -c + e\,\{1 - H\,(\overline{R}_{\alpha\beta}) - A\} = 0.$$

If $\underline{R}_{\alpha\beta} < R < \overline{R}_{\alpha\beta}$, there will be two Nash equilibria in pure strategy. The one in which all the investors speculate and the probability of the devaluation is $1-F\,(R - A)$ and the other in which no investor speculates and the probability of the devaluation is $1-F\,(R)$.

Next we study incomplete information case. If we replace $-R$ with s, the marginal gain function $v\,(m\,/(n-1), -R\,)$ will satisfy all the assumptions in Section 2. Therefore we can apply *Theorem* 1 to our speculative attack model. The critical value of the potential intervention R^G is determined by

$$(18) \qquad \sum_{m=0}^{n-1} \frac{1}{n} H\left(R^G - \frac{m}{n}A\right) = 1 - \frac{c}{e}.$$

R^G satisfies $\underline{R}_{\alpha\beta} < R^G < \overline{R}_{\alpha\beta}$. Assume that noise is very small. Then *Theorem* 1 implies that if an investor gets private information that is lower than R^G, he will speculate. And if an investor gets private information that is higher than R^G, he will not speculate.

If actual R is smaller than R^G, the probability of the devaluation will be $1-F(R-A)$. And if actual R is greater than R^G, the probability of the devaluation will be $1-F(R)$. So we must be interested in the factors determining R^G. Equation (18) implies that as transaction cost c decreases or as the expected width of devaluation e increases, R^G will become higher.

In order to explicitly solve equation (16), (17), and (18), let us specify the probability distribution of the current account deficit. Suppose that the distribution is highly concentrated around $D = D^*$.[5] Then we can get approximate solutions of equation (16), (17), and (18):

$$\underline{R}_{\alpha\beta} \approx D^*,$$

$$\overline{R}_{\alpha\beta} \approx D^* + A,$$

$$R^G \approx D^* + \left(1 - \frac{c}{e}\right)A .$$

In this example, the relative sizes of the transaction cost and the possible devaluation rate determine the critical amount of foreign reserves. If the transaction cost is close to the size of the possible devaluation, the monetary authority will only need to prepare reserves as much as the expected current account deficit in order to defend the fixed exchange rate.

Assume that the monetary authority can choose one of two situations, the complete information case and the incomplete information case with small noise. The authority has to make a decision before the authority knows realization of R, which is exogenously determined. Also assume that the authority applies mini-max principle to multiple equilibrium problems. That is, the authority holds the worst case in whole account. Then the monetary authority will prefer the incomplete information situation with small noises over the complete information situation.[6]

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[5]We assume that $H(D^*-\varepsilon)$ is close to 0 and $H(D^*+\varepsilon)$ is close to 1 for a small value $\varepsilon > 0$.

[6]Bhattacharya and Weller (1992) constructed a model in which the central bank, when it intervenes in the foreign exchange market, chooses not to reveal precisely what their targets are. In their model, investors are assumed to be risk averse. Therefore, the impreciseness of investors'

4. The Network-Externality Model with Normal Random Shocks

Since there is no rigorous microeconomic foundation in our speculative attack model, it is difficult to evaluate national welfare in each equilibrium. In this section, by using a network externality model, we analyze welfare implication of small noise.

Consider a technology characterized by network externalities. There are n identical agents. The agents simultaneously decide whether to adopt the technology (action α) or not to adopt (action β). The more the agents coordinate to adopt the technology, the higher their utility. The payoff of the agents who do not adopt the technology is normalized to be zero. If agent i chooses α, his payoff is

$$v_i\left(\frac{m}{n-1}, s\right) = -c + r\,\frac{m}{n-1} + s \tag{19}$$

where m denotes the number of the other agents who choose α and s denotes a common shock that affects the productivity of the technology. r is positive. We assume the following information structure. Each agent i observes θ_i that consists of a common shock s and a noise ε_i:

$$\theta_i = s + \varepsilon_i.$$

s and ε_i are independent normal random variables with means zero and standard deviations $\sigma_s > 0$, $\sigma_\varepsilon > 0$ respectively. The noises, $\varepsilon_1, \ldots, \varepsilon_n$, are independent of each other. Each agent i knows neither other agents' private information nor the composition of θ_i.

Since the support of each noise is not bounded, we can not directly apply Theorem 1 to this model. But as Carlsson and van Damme (1993) show for two player game, under the normal distribution assumption we can derive stronger results than in Theorem 1: the Bayesian Nash equilibrium is uniquely determined for a certain set of $(\sigma_\varepsilon, \sigma_s)$.

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information increases central bank's ability to manipulate the exchange rate. For general discussion on central bank secrecy, see Cukierman (1992).

To begin with, we consider the complete information games that correspond to our incomplete information game. Suppose there is no noise and s is directly observable. If $s < c - r$, there is a unique Nash equilibrium B in which all users choose β. If $s > c$, there is a unique Nash equilibrium A in which all users choose α. If $c - r < s < c$, there are two pure-strategy Nash equilibria and one mixed-strategy Nash equilibrium.

Now we study our incomplete information game. For low enough θ_i, $E[v_i(1, s) | \theta_i] < 0$ and action α (to adopt the technology) is strictly dominated. Conversely, for high enough θ_i, $E[v_i(0, s) | \theta_i] > 0$ and action β (not to adopt the technology) is strictly dominated.

First we look for simple equilibria defined by equation (11). That is, all the agents set a common critical value x. In such equilibria, agent i who observes θ_i expects the following gain by choosing α instead of β.[7]

$$(20) \qquad F_i(\theta_i, x) = -c + \frac{\sigma_s^2}{\sigma_s^2 + \sigma_\varepsilon^2}\theta_i + \frac{r}{n-1}\sum_{j \in N_{-i}} \mathrm{Prob}\left[\theta_j \geq x \mid \theta_i\right]$$

$$= -c + \frac{\sigma_s^2}{\sigma_s^2 + \sigma_\varepsilon^2}\theta_i + \frac{r}{n-1}\sum_{j \in N_{-i}} \int_{\frac{1}{\sigma_z}\left(x - \frac{\sigma_s^2}{\sigma_s^2 + \sigma_\varepsilon^2}\theta_i\right)}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{u^2}{2}\right] du$$

where σ_z^2 denotes $\mathrm{Var}[\theta_j | \theta_i] = (2\sigma_s^2\sigma_\varepsilon^2 + \sigma_\varepsilon^4)/(\sigma_s^2 + \sigma_\varepsilon^2)$. Since function F_i is continuous and $\partial F_i / \partial\theta_i > 0$ for all θ_i and all x, F_i implicitly defines agent i' s reaction function: when all of agent i' s opponents take a common switching value x, then the optimal switching value for player i, θ_i, is determined by $F_i(\theta_i, x) = 0$. The necessary and sufficient condition of an equilibrium switching value x is

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[7]Note that

$$E[s \mid \theta_i] = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_\varepsilon^2}\theta_i, \qquad E[\theta_j \mid \theta_i] = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_\varepsilon^2}\theta_i,$$

and $\mathrm{Var}[\theta_j \mid \theta_i] = \dfrac{2\sigma_s^2\sigma_\varepsilon^2 + \sigma_\varepsilon^4}{\sigma_s^2 + \sigma_\varepsilon^2}\theta_i$.

(See Hoel 1962, p. 200.)

$F_i(x, x) = 0.$

$F_i(x, x)$ is strictly concave up at $x > 0$ and concave down at $x < 0$. Therefore, we get a sufficient condition for a unique equilibrium switching value x:[8]

$$(21)\quad \frac{\mathrm{d}F_i\,(x,\,x\,)}{\mathrm{d}x}\ \Big|_{x\,=\,0} = \frac{\sigma_s^{\,2}}{\sigma_s^{\,2}+\sigma_\varepsilon^{\,2}} - \frac{r}{2\sqrt{2\pi}\,(n-1)}\ \frac{\sigma_\varepsilon^{\,2}}{\{\,2\,\sigma_s^{\,2}\,\sigma_\varepsilon^{\,2}+\sigma_\varepsilon^{\,4}\,\}^{\frac{1}{2}}\ \{\,\sigma_s^{\,2}+\sigma_\varepsilon^{\,2}\,\}^{\frac{1}{2}}} \geq 0.$$

In the same way as in Theorem 1, we can show that under this condition there is no other Bayesian Nash equilibrium of a more complicated form. The shaded region of Figure 2 represents the set of $(\sigma_\varepsilon, \sigma_s)$ that satisfies the sufficient condition (21). If condition (21) is not satisfied and $F_i\,(0, 0) = -\,c + r\,/\,2$ is close to zero, then we will have three simple form equilibria.

To carry out welfare analysis, consider agent i's expected payoff evaluated before the revelation of θ_i. Suppose all the agents, including agent i, take a simple behavioral strategy defined by equation (11) with a switching value x. Then agent i expects the following payoff:

$$(22)\qquad W_i\,(x\,) = \int_x^{+\infty} F_i\,(\theta_i\,,\,x\,)\ \psi(\theta_i\,)\ \mathrm{d}\theta_i$$

where $\psi(\theta_i\,)$ denotes the density function of θ_i. By a differentiation, we get

$$(23)\qquad \frac{\mathrm{d}W_i\,(x\,)}{\mathrm{d}\,x} = -\,F_i\,(x\,,\,x\,)\ \psi(x\,) - \frac{r}{n\,-1}\sum_{j\,\in\,N_{-i}}\int_x^{+\infty} g\,(x\,\mid\theta_i\,)\ \psi(\theta_i\,)\ \mathrm{d}\theta_i$$

where function $g\,(\theta_j \mid \theta_i\,)$ is the conditional density of θ_j given θ_i. The second term in the right hand side of equation (23) denotes a *spillover effect*: a decrease in the switching value of all but one agent bestows an external benefit upon the remaining agent.[9] This fact implies that the unique Bayesian Nash equilibrium is not Pareto optimal and all the agents will gain by coordinating to decrease their common switching value.

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[8]This condition is almost identical with Carlsson and van Damme's (1990).

[9]For a more detailed discussion on spillover effects, see Cooper and John (1988).

Next we compare our Bayesian Nash equilibrium with Nash equilibria of the corresponding complete information games. Suppose there is no noise and s is directly observable. Then the economy can get stuck at a set of inefficient equilibria in which no agent adopts the technology at all $s \in (-\infty, c\,]$. In such a worst case scenario the expected welfare of a representative agent evaluated before the revelation of s is

$$\int_{c}^{+\infty} (-c + r + s)\, h(s)\, \mathrm{d}s$$

where $h(s)$ is the density function of s. On the other hand, in the incomplete information game with infinitesimal noises there is a unique Bayesian Nash equilibrium. Equation (20) implies that the switching value is close to $c - r / 2$. This value is identical with the solution of equation (8). Equation (20) and (22) imply that the expected welfare of a representative agent in this equilibrium is

$$\int_{c-\frac{r}{2}}^{+\infty} (-c + r + s)\, h(s)\, \mathrm{d}s + O(\sigma_\varepsilon)$$

which is greater than the expected welfare in the worst case of the complete information games for small enough σ_ε. Therefore a central planner, who applies mini-max principle to multiple equilibrium problems will prefer the incomplete information situation with small noises over the complete information situation.

5. Concluding Remarks

As more economists interested in coordination failure problems, the question how one of multiple equilibria is selected becomes more important. By generalizing Carlsson and van Damme's theory of global games, this paper studied a selection mechanism under incomplete information. The implications of the theory of global games are investigated in two specific models: a speculative attack model and a network externality model.

Even when the monetary authority possesses relatively large foreign exchange reserves, the speculative attack can be a self-fulfilling equilibrium under the complete information situation. Each investor finds speculation profitable, when he believes that many other investors will speculate. Suppose that there is a common shock and each investor gets noisy private information. The noise is small. Then each investor expects that one half of other investors get worse news for speculation than his. This expectation makes coordinative speculative attack impossible. It is shown that the monetary authority in the speculative attack model and the central planner in the network externality model will prefer the equilibrium in a global game with small noise to the worst equilibrium in the corresponding complete information game. Therefore, they will welcome the existence of small noise, if they apply mini-max principle to multiple equilibrium problems.

References

Bhattacharya, Utpal, and Paul Weller, "The Advantage to Hiding One's Hand: Speculation and Central Bank Intervention in the Foreign Exchange Market." *CEPR Discussion Paper #737*, Center for Economic Policy Research, London, 1992.

Carlsson, H., and E. van Damme, "Global Games and Equilibrium Selection," *Econometrica* 61 (1993), 989-1018.

Carlsson, H., and E. van Damme, "Equiibrium Selection in Stag Hunt Games," CentER for Economic Research, Tilburg University, the Netherlands, mimeo, 1991.

Cukierman, Alex, *Central Bank Strategy, Credibility, and Interdependence: Theory and Evidence.* (Cambridge: MIT Press, 1992)

Cooper, R., and A. John, "Coordinating Coordination Failures in Keynesian Models," *Quarterly Journal of Economics* 103 (1988), 441-63.

Farrell, J., and G. Saloner, "Standardization, Compatibility, and Innovation," *Rand Journal of Economics* 16 (1985), 70-83.

Harsanyi, J.C., and R. Selten, *A General Theory of Equilibrium Selection in Games* (Cambridge: MIT Press, 1988).

Hoel, P.G., *Introduction to Mathematical Statistics.* (New York, 1962).

Kohlberg, E., and J.F. Mertens, "On the Strategic Stability of Equilibria," *Econometrica* 54 (1986): 1003-38.

Krugman, P., "A Model of Balance-of-Payments Crises," *Journal of Money, Credit, and Banking* 16 (1979), 311-25.

Milgrom, P.R., and R.J. Weber, "Distributional Strategies for Games with Incomplete Information," *Mathematics of Operations Research* 10 (1985), 619-32.

Selten, R., Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games," *International Journal of Game Theory* 4 (1975), 25-55.

van Damme, E., *Stability and Perfection of Nash Equilibria*, 2d ed., rev. and enl. (Berlin: Springer-

Verlag, 1991a).

van Damme, E., "Equilibrium Selection in 2 X 2 Games," Center for Economic Research Discussion Paper No.9108, Tilburg University, the Netherlands, 1991b.

Weir, A.J., *Lebesgue Integration and Measure* (Cambridge, Great Britain: Cambridge University Press, 1973).

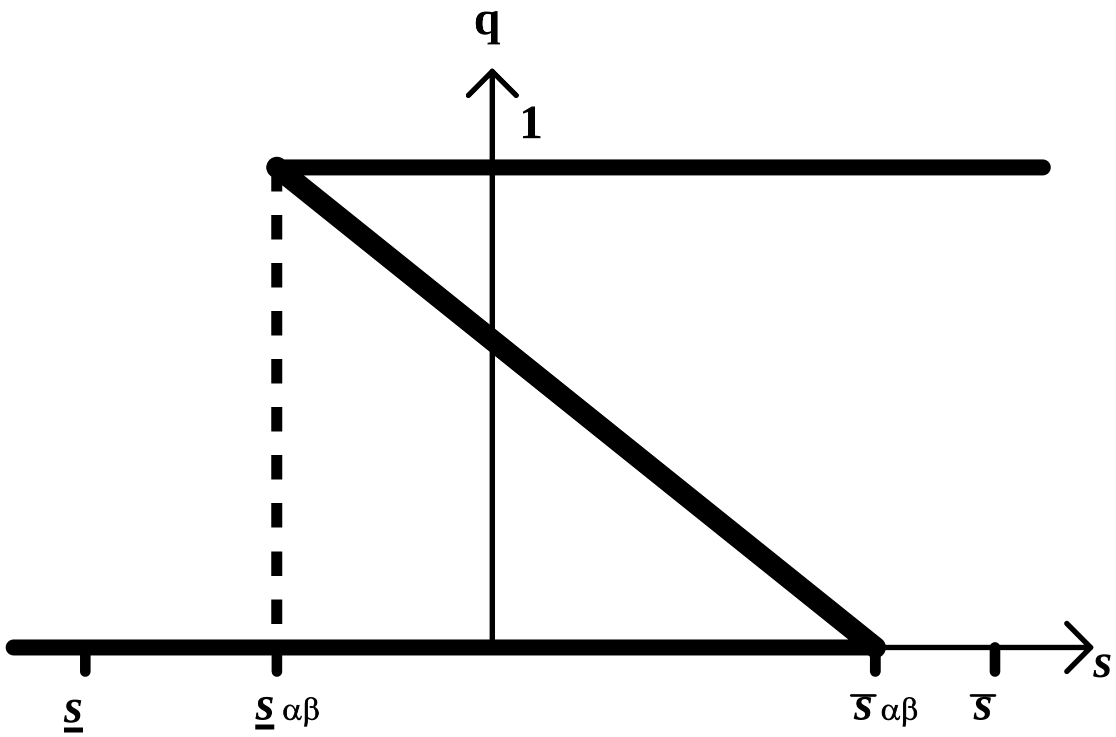

Figure 1. The Nash-equilibrium correspondence of the complete information games.

	under devaluation	under no devaluation
speculate	$(1-c)A/n$	$(1-c)A/n$
not specuate	$(1-e)A/n$	A/n

Table 1. Investors' payoff in the speculative attack model

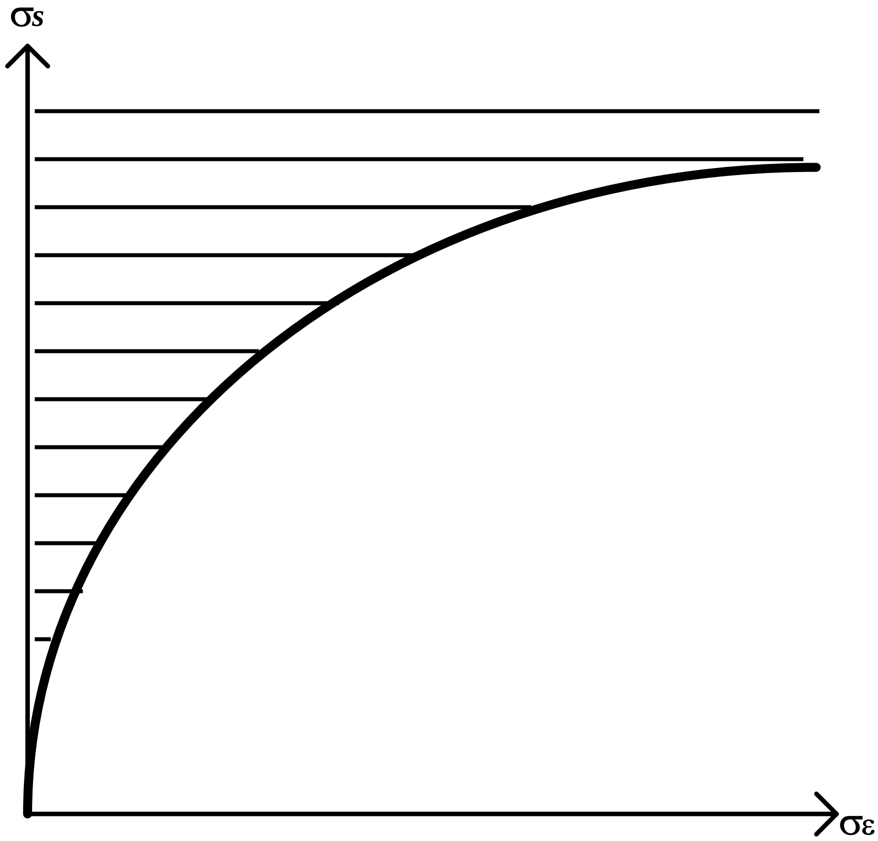

Figure 2. The set of (σ_ε, σ_S) that satisfies the sufficient condition (21) for a unique equilibrium.