The Speed of Convergence in a Two-Sector Growth Model with Health Capital*

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Abstract

In this paper we will show that for empirically plausible parameter values, a two-sector growth model contained health capital can yield a slow speed adjustment process. Calibrating the model, we demonstrate that in the case of a capital deepening externality in the health sector has relatively weak impact on additional health capital production and income tax rates which finance public health expenditure are at realistically reasonable levels, a slower speed of convergence occurs. Such slower adjustment process is consistent with the standard empirics on convergence. Consequently, we stress the good harmonization between a calibration-based theoretical prediction and the corresponding evidence.

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1 Introduction

In this paper we will show that for empirically plausible parameter values, the present macroeconomy contained health capital accumulation can yield a slow speed adjustment process. As a main contribution of this paper for the research fields of health and economic growth, we give weight to the good harmonization between a calibration-based theoretical prediction on a convergence phenomenon and the corresponding empirical results.

Numerous attempts have been made by researchers to investigate evidence theoretically or empirically on the phenomenon of convergence. In particular, modern literature on growth and convergence emphasizes the importance of the results from empirical studies and tries to replicate their evidence. In contrast to such movements, the early literature (largely in 1960s' works) on convergence has focused mainly on theoretical properties of the neoclassical growth models (see e.g. Sato, 1963).

The seminal empirical works by Barro and Sala-i-Martin (1992), Mankiw *et al.* (1992) and others obtain estimates of the speed of convergence of about 2-3%. However, as pointed out by Islam (1995), Caselli *et al.* (1996), Evans (1997) and others, thorny econometric problems are contained in the early contributions including Barro and Sala-i-Martin (1992). Following Eicher and Turnovsky (1999) and Turnovsky (2002), we can summarize such problems as follows.

- Omitted variables (country-specific effects);
- The endogeneity of the dependent variables;
- Measurement errors.

By these econometric problems, estimates for the convergence coefficient are downwardly biased. When properly addressed the problems, it is quite likely that the resultant convergence coefficient increases to 2% and over (see e.g. Islam, 1995; Caselli *et al.*, 1996; Evans, 1997). In view of the recent findings reported by Bond *et al.* (2001), however, it seems reasonable to suppose that benchmark range of the speed of convergence is in around 2-3%.[1]

[1]Islam (1995) employs a panel data method with fixed effects and obtains estimates of about 5% for non-oil and of about 10% for OECD countries. Caselli *et al.* (1996) find a rate of convergence of about 10% by introducing the GMM estimation technique. In addition, Evans (1997) yields estimates of about 6% for a whole sample of 48 countries and of about 16% for U.S. states. In more recent work, Bond *et al.* (2001) discuss the efficiency of the standard GMM estimator and present a more plausible way. Namely, they follow Arellano and Bover (1995) and Blundell and Bond (1998) and argue that a system GMM estimator is more efficient than the standard panel GMM one. This is because that the standard first-differenced GMM estimator is poorly behaved when time series are persistent and the number of time series observations is small. By employing the system GMM estimator, they reacquire estimates of the convergence speed of roughly 2-3% which accord with the original estimates including Barro and Sala-i-Martin (1992), Mankiw *et al.* (1992) and others.

On the other hand, in recent years, several theoretical articles have been also devoted to the study of the speed of convergence (see e.g. Albelo, 1999; Barro and Sala-i-Martin, 1997; Eicher and Turnovsky, 1999; Gokan, 2003; Ortigueira and Santos, 1997; Russo, 2002; and Turnovsky, 2002).[2] The importance of theoretical concern for convergence is reported by Ortigueira and Santos (1997, p. 383):

The speed of convergence provides important information in testing a model on the relative emphasis that should be placed on the steady-state behavior and transitional dynamics. If the speed of convergence to a steady state or balanced growth path is high, then the long-run behavior of the model should be determined by its predictions at the steady state. However, if this convergence rate is low, then transitional dynamics may play a relevant role in ascertaining the predictive power of a model even if long-run considerations are called into the analysis.[3]

It is well known that the one-sector neoclassical growth model ("Ramsey model") exhibits the rate of convergence varies between 7-15% depending on variations of the inverse of the intertemporal elasticity of substitution and the two-sector endogenous growth model with human capital ("Lucas model") generates the rate is above 15%.[4] Accordingly, we find that these calibrated convergence speeds are considerably high in comparison with the typical estimates of empirical literature noted above. That is to say, the adjustment time of the standard endogenous growth models with transitional dynamics seems like fairly short. In addition to the technical difficulties for complex dynamical system, Mino (2000) indicates that one of the reasons why a large number of endogenous growth models increase interest to the steady-state equilibrium is in such conventional facts obtained in the calibration analyses.

In this paper, we propose a simple model of two-sector endogenous growth with health capital, in which the accumulation of health capital is completely

[2]As an influential theoretical literature, we should mention the following two papers. (1) Ortigueira and Santos (1997) are most influential paper in the field of growth and convergence. They show that the introduction of adjustment costs may reduce dramatically the speed of convergence (i.e. about 2% levels) and assert the empirical relevancy of the model. In addition to such an important finding, their paper also provides a comprehensive and a practical survey for the field of growth and convergence. On the other hand, (2) Turnovsky (2002) investigates convergence properties in a one-sector growth model under more general conditions impose on both production and utility functions. He introduces the three distinct elasticities and analyzes the response of the rate of convergence to variations of the elasticities. The three elasticities are (i) the intratemporal elasticity of substitution in goods production, (ii) the intertemporal elasticity of substitution in consumption and (iii) the intratemporal elasticity of substitution in consumption. Accordingly, it is proved that the convergence coefficient is highly sensitive to the elasticities of (i) and (ii), but less so to that of (iii).

[3]Turnovsky (2002, p. 1766) makes similar assignment for the relation between convergence process and long-run steady-state equilibrium.

[4]For both results, see e.g. Ortigueira and Santos (1997).

performed by two external factors, and examine its convergence properties and empirical relevancy. Applying the calibration technique, we will show that the model has two important features: (i) under well known parameters are fixed and unknown ones are free, the model exhibits a higher speed of convergence to the steady-state equilibrium; (ii) if we respect the data facts on income tax rate across countries and suppose a situation of small external effect in health capital production, the model generates in turn a slower adjustment process. The former is consistent with the various calibration results for the standard endogenous growth models. The latter feature is more important for our concern. Namely, although the present model bears the earmarks of the endogenous growth models, it also fits for empirical verification.

To get empirically coherent rates of convergence, in general, various endogenous growth models require the introduction of something specific factor including *adjustment costs* into the specifications. In contrast, our model has no use for introducing the specific factor to maintain empirical relevancy of the convergence coefficient. As a matter of fact, even when the external effect from capital deepening has a significant impact for the production of health capital, we can show that the speed of convergence remains a low value. In particular, for the extended model employing an isoelastic preference, such slow adjustment process becomes more clear. Specifically, a larger value of the inverse of the intertemporal elasticity of substitution generates a slower speed of convergence.

The rest of the paper is organized as follows. Section 2 explains briefly a simple baseline model presented by Hosoya (2003). In section 3, we first investigate theoretically the speed of convergence for the corresponding model. And then we provide that the results of calibration and discuss empirical relevancy of the model. Section 4, following Hosoya (2004), extends the baseline model and re-examine convergence properties. Section 5 concludes.

2 Structure of the baseline model

We briefly review in this section a two-sector endogenous growth model with health capital presented by Hosoya (2003). An interesting feature of his model is that the evolution of health capital is completely determined by external factors for individuals; i.e. one is *government health expenditure* and the other is the *capital deepening externality.* Formally, the representative agent maximizes Eq. (1) under constraints of Eqs. (2)-(4):

$$\max_{C(t)} \int_0^{+\infty} \ln C(t) e^{-\rho t} dt, \quad \rho > 0, \tag{1}$$

subject to[5]

$$\dot{K} = Y - C - G, \tag{2}$$

$$Y = K^{\alpha}(HL)^{1-\alpha}, \quad \alpha \in (0,1), \tag{3}$$

$$G = \tau Y, \quad \tau \in (0,1), \tag{4}$$

where ρ is the subjective discount rate, K, Y, C, H and L represent physical capital, output, consumption, health capital and labor, respectively. The labor supply is assumed to be constant and we normalize $L = 1$, hence all variables give per-capita quantities. Parameters α and τ denote respectively the share of physical capital in goods production and the proportional income tax rate. Therefore, Eq. (4) implies that government public health expenditure (G) is financed by income tax (τY) imposed on private agent. The government balances its budget at each point in time. As noted before, the level of H as given, the agent's dynamic optimization yields the growth rate of per-capita consumption:

$$g_C \equiv \frac{\dot{C}}{C} = \alpha(1-\tau)\left(\frac{K}{H}\right)^{\alpha-1} - \rho, \tag{5}$$

where g_x denotes the equilibrium growth rate of placeholder x.

Next, we need to explain the evolution of health capital. Assuming that individuals' health levels are enhanced by both government health expenditure and the capital deepening externality. That is

$$\dot{H} = AG\left(\frac{\bar{K}}{\bar{H}L}\right)^{\epsilon}, \quad \epsilon \in (0,1), \tag{6}$$

where $A > 0$ is a constant parameter related to the efficiency of health capital production. The RHS of Eq. (6) consists of two input factors; i.e. G and $\bar{K}/(\bar{H}L)$.[6] The latter represents the social average level of the physical capital/effective labor ratio which brings about the external effects of capital deepening for health capital accumulation. Such the effects represent a social benefit derived from an improvement in living standards.[7] Since $L = 1$, substituting the relation of $G = \tau Y = \tau K^{\alpha} H^{1-\alpha}$ into Eq. (6) leads to the following dynamical process of health capital accumulation:

$$\dot{H} = A\tau K^{\alpha} H^{1-\alpha}\left(\frac{\bar{K}}{\bar{H}}\right)^{\epsilon}. \tag{7}$$

At the equilibrium, $\bar{K}$, $\bar{H}$ must be set equal to K, H, respectively. Therefore, Eq. (7) is rewritten as

$$g_H \equiv \frac{\dot{H}}{H} = A\tau\left(\frac{K}{H}\right)^{\alpha+\epsilon}. \tag{8}$$

[5] In the following we omit the time argument t.

[6] $\bar{K}$ and $\bar{H}$ are the social average level of physical and health capital.

[7] More detailed discussion for this point, see Hosoya (2003).

On the balanced growth path (BGP), g_Y, g_C, g_K and g_H are all equal to g; i.e. $g \equiv g_Y = g_C = g_K = g_H$. Applying $g = g_H$ to Eq. (8), we can derive $K/H = (g/A\tau)^{1/(\alpha+\epsilon)}$. Putting this relation into Eq. (5), we obtain the equilibrium growth rate along the BGP. That is,

$$g = \alpha(1-\tau)\left(\frac{g}{A\tau}\right)^{\frac{\alpha-1}{\alpha+\epsilon}} - \rho. \tag{9}$$

From Eq. (9), we find that the equilibrium growth rate at the BGP depends on the parameters (α, τ, A, ϵ, ρ). In addition, the following holds.

Proposition 1 (Existence, Uniqueness and Stability) *There exists in this baseline model a unique equilibrium with a positive solution of g, and the corresponding dynamical system is locally saddle-path stable.* $\square$

Proof : See Hosoya (2003, Proposition 2 and Proposition 3). $\blacksquare$

3 Speed of convergence

3.1 Theoretical investigation

To investigate the convergence speed, we introduce new stationary variables; $X \equiv C/K$, $Z \equiv K/H$. Differentiating these variables with respect to time we obtain $\dot{X}/X = \dot{C}/C - \dot{K}/K$ and $\dot{Z}/Z = \dot{K}/K - \dot{H}/H$. Using Eqs. (2)-(5) and Eq. (8), two differential equations on X and Z are transformed by

$$\frac{\dot{X}}{X} = X + (\alpha-1)(1-\tau)Z^{\alpha-1} - \rho, \tag{10}$$

$$\frac{\dot{Z}}{Z} = -X + (1-\tau)Z^{\alpha-1} - A\tau Z^{\alpha+\epsilon}. \tag{11}$$

We define here the Jacobian related to the dynamical system of Eqs. (10) and (11) as a 2×2 matrix:

$$J \equiv \begin{bmatrix} \frac{\partial \dot{X}}{\partial X} & \frac{\partial \dot{X}}{\partial Z} \\ \frac{\partial \dot{Z}}{\partial X} & \frac{\partial \dot{Z}}{\partial Z} \end{bmatrix}.$$

Therefore, the characteristic polynomial for this two dimensional system is

$$\text{Det } (J^* - \lambda I) = \text{Det} \begin{bmatrix} X^* - \lambda & (\alpha-1)^2(1-\tau)X^*(Z^*)^{\alpha-2} \\ -Z^* & [(\alpha-1)(1-\tau)(Z^*)^{\alpha-1} - A\tau(\alpha+\epsilon)(Z^*)^{\alpha+\epsilon}] - \lambda \end{bmatrix} = 0,$$

where the asterisk ($*$) denotes the steady-state value and λ is an eigenvalue of J^*.

Of course, "I" represents the two dimensional identity matrix.[8] Consequently we have

$$\lambda^2 + (V_1 + V_2 - X^*)\lambda - (\alpha V_1 + V_2)X^* = 0, \tag{12}$$

where $V_1 \equiv (1-\alpha)(1-\tau)(Z^*)^{\alpha-1}$ and $V_2 \equiv A\tau(\alpha+\epsilon)(Z^*)^{\alpha+\epsilon}$. Eq. (12) has two roots which are represented by λ_1 (negative sign) and λ_2 (positive sign), respectively. A negative root λ_1 is given by the following expression:

$$\lambda_1 = \frac{(X^* - V_1 - V_2) - [(V_1 + V_2 - X^*)^2 + 4(\alpha V_1 + V_2)X^*]^{1/2}}{2}. \tag{13}$$

Defining $\tilde{\lambda}$ as $\tilde{\lambda} = -\lambda_1$. As a result, the speed of convergence (i.e. convergence coefficient) for the present model is given by $\tilde{\lambda}$.

3.2 Numerical experiments of transitional paths

We can analyze the speed of convergence quantitatively if we specify each parameter value containing in Eq. (13). Our benchmark parameter values are (ρ, α, τ, A, ϵ)=(0.02, 0.3, 0.1, 0.1, 0.2). These values imply a rate of convergence ($\tilde{\lambda}$) from Eq. (13) of 0.085 per year (8.5%). Then, to pass a 90% of adjustment process, it takes approximately in 27 years.[9] This result represents that in the benchmark case the convergence speed $\tilde{\lambda}$ is relatively high compared with the standard empirical findings (see Barro and Sala-i-Martin, 1992; Mankiw *et al.*, 1992).[10]

$* * * * *$ **[Table 1 here]** $* * * * *$

[8] The steady-state values of X and Z (i.e. X^* and Z^*) are calculated as follows. Using the computation software (we now use GAUSS 5.0 throughout this paper), we can solve the following nonlinear simultaneous equations on (X^*, Z^*) under the given parameter values:

$$X^* = \Psi^*(\rho, \alpha, \tau, Z^*),$$
$$Z^* = \Omega^*(\rho, \alpha, \tau, A, \epsilon).$$

In the baseline model, the system of (X^*, Z^*) is specified as (from Eqs. (10) and (11))

$$X^* - (1-\alpha)(1-\tau)(Z^*)^{\alpha-1} - \rho = 0,$$
$$\alpha(1-\tau)(Z^*)^{\alpha-1} - A\tau(Z^*)^{\alpha+\epsilon} - \rho = 0.$$

In Appendix, we will characterize the existence of equilibrium value for Z^*.

[9] The $100 \times \beta\%$ of adjustment time from initial value $t = 0$ to the steady-state is calculated by $\Delta\ (\beta) \equiv \ln(1-\beta)/\lambda_1$.

[10] The influential empirical literature of Barro and Sala-i-Martin (1992), Mankiw *et al.* (1992) and so on reports the convergence speed of around 2-3% in the various sets of cross-country estimations. de la Fuente (1997) provides a selective survey of the empirical literature on growth and convergence.

Following Barro *et al.* (1995), let us summarize the results of sensitivity analysis for the baseline model. Typical cases are reported in Table 1. The convergence coefficient $\tilde{\lambda}$ is not so sensitive to variations in ρ and α. For example, if we set the values of the other parameters at their benchmark values, then $\tilde{\lambda}$ rises from 8.5% to 9.0% if ρ increases to 0.03, and it rises to 9.1% if α increases to 0.35.

On the other hand, the speed of convergence is more sensitive to variations in τ, A and ϵ. For example, $\tilde{\lambda}$ rises from 8.5% to 11.7% if τ increases to 0.2, and it rises to 12.2% if A increases to 0.2. Moreover, $\tilde{\lambda}$ rises to 10.0% if the degree of capital deepening externality (ϵ) increases to 0.3.

Regarding the model specification, we should mention the following two relations: (i) the relation between tax rate and the convergence coefficient; (ii) the relation between the degree of externality and that coefficient. Figure 1 shows the relation (i).[11] As well known, Barro's (1990) model of public spending and endogenous growth represents that the relation between growth and government size (i.e. tax rate) is a non-monotonic. In other words, the non-monotonicity implies a hump-shaped relation. In our model, such a result is also observed in the relation on tax rate and the rate of convergence (see Figure 1). Under the relevant parameter set, we can confirm that the *peak* of the speed of convergence is achieved in the range of tax rate of 0.65-0.7. However, in an empirically valid tax range (at least not exceeding 0.5), the convergence coefficient is a monotonic increasing function of tax rate (i.e. $f' > 0$, $f'' < 0$ as denoted $\tilde{\lambda} \equiv f(\tau)$), and its level is much lower than in around the *peak* point. Anyway, we may regard the present result (Figure 1) as a "convergence version" of Barro's (1990) result, because there is a positive correlation between the rates of economic growth and convergence.

$*****$ **[Figure 1 and Figure 2 here]** $*****$

Next, Figure 2 shows the relation (ii). [12] We can observe that the capital deepening externality affects monotonically the speed of convergence. This reflects a natural result that a positive external effect in health capital production raises the rate of economic growth. As pointed out earlier, however, the size of the convergence coefficient obtained above is significantly larger than conventional empirics of the fields. In the following, we will try to fit our calibrating results with the standard empirical findings without introducing the specific factors including adjustment costs, capital mobility and uncertainty.[13]

[11] Except for A and ϵ, we follow the benchmark values. As for A, based on Lucas's calibration of his two-sector model with human capital, we set A=0.05 (see Lucas, 1988, p. 26). In addition, we assume that the impact of the capital deepening externality (ϵ) on the accumulation of health capital will become a quite small and adopt a lower value ϵ=0.05 compared with the benchmark case.

[12] Following the case of Figure 1, we set A=0.05. The values of the other parameters are fixed at their benchmark values except for ϵ.

[13] See for example Ortigueira and Santos (1997), Barro *et al.* (1995), den Haan (1995) and Mino (2000). Among them, Mino (2000) provides a prominent survey and classifies various

3.3 Matching up theory with empirical facts

Recent influential papers on empirics of growth and convergence have reported evidence that individual/regional economies converge at a rate of about 2-3% per year (see e.g. Barro and Sala-i-Martin, 1992, 2003; Mankiw *et al.*, 1992). Taking a number of econometric issues into account, as explained in section 1, such empirical estimates are problematic. More recent papers including Islam (1995), Caselli *et al.* (1996) and Evans (1997) have discussed the accuracy of the benchmark estimates of about 2-3%. In consequence, they make a point that true estimates increase to at least 2% and over. According to the recent contribution of Bond *et al.* (2001), however, they assert that true estimates are shifted back to 2-3% levels under employing the system GMM estimation technique.

Even if any question remains about the accuracy of the estimates, it is clear that the speed of convergence is much slower than what obtained in the calibration studies for the standard endogenous growth models (see e.g. Ortigueira and Santos, 1997). Our principal analytical object in this section is to investigate that whether the present model of two-sector endogenous growth satisfies the qualification requirements from various empirical results, or not (i.e. the feasibility of slower adjustment process).

$*****$ **[Table 2 here]** $*****$

In order to examine the convergence properties carefully, we now suppose that the capital deepening externality in the health capital sector has not so a large impact for additional health production. Moreover, we assume that tax rate, which finances government spending on public health, is relatively lower than previously adopted levels. This assumption will be supported by the following important data facts. Table 2 shows that averaged public health expenditure (% of GDP) in several income groups in 1990-2000. We obtained the data from *World Development Indicators* 2003 *CD-ROM*. Table 2 indicates that the share of public health expenditure (% of GDP) is not so high in several income groups, in particular, non-OECD groups' average is about 3%. The share corresponds exactly to the tax rate of the present model (i.e. $\tau \times 100$ (%)). By these actual data, we can confirm a reasonable range of tax rate for our calibration is in about below 5%.

Given the relevant parameter set of $(\rho, \alpha, A) = (0.02, 0.3, 0.05)$, we can compute the speed of convergence under the circumstances with "low tax" and "small externality". Typical computation results are shown in Table 3-1. For example, if we specify $\tau = 0.03$ and $\epsilon = 0.10$, then the convergence coefficient is 3.3%. Such a coefficient value is acceptable in terms of estimates of the convergence coefficient from the large number of empirical works mentioned before. Then, if we assume a relatively higher degree of external effect of $\epsilon = 0.20$, a slower speed of adjustment process of $\tilde{\lambda} = 2.8\%$ is realized in the case at

theoretical models on growth and convergence.

$\tau = 0.01$. Furthermore, even in a 5% income tax case (which is close to OECD groups' average), there are some possibilities of realizing 3% levels of the convergence coefficient. In fact, for the extended Solow model with OECD sample, Murthy and Chien (1997) report estimate of the annual rate of convergence ($\tilde{\lambda}$) is 3.8%.[14] This estimation supports our computation, and in particular recommends us the relevancy of the 5% income tax case.

$* * * * *$ **[Table 3-1 and Table 3-2 here]** $* * * * *$

As an additional experiment, we provide the results of sensitivity analysis which are based on the previous calibration presented by Table 3-1. The sensitivity results are summarized in Table 3-2. In this experiment, we change the preceding values of α and A only, and update them to $\alpha = 0.35$ and $A = 0.07$. Other parameters are unchanged. These update values also agree with those typically employed in the macroeconomic literature. Firstly, increases in physical capital share (α) and the efficiency of health production (A) appear in relatively faster adjustment processes. From the results of Table 3-2, it should be found that empirically supported slower convergence processes are feasible in this alternative parameter setting. As a concrete example, if we specify $\tau = 0.03$ and $\epsilon < 0.10$, the speed of convergence remains at the levels of 3%.

From what has been discussed above, we can conclude that a slower speed of convergence is feasible in our model when the degree of capital deepening externality stays within the confines of low values. The robustness of the results was confirmed through a brief sensitivity analysis. Since we assumed conventional parameter set, especially for tax rate (τ) supported by the data, not only the results obtained here match well the standard estimates of $\tilde{\lambda}$ in the empirical literature on growth and convergence but its numerical backgrounds are also desirable. The empirical validity of the calibrated convergence coefficient is favorable to the present model of two-sector endogenous growth including health capital. To harmonize further the results of model calibration with standard empirical evidence, we will slightly extend the baseline model in the next section.

4 Introducing a CIES preference: the role of θ

In this section, we modify the agent's utility function and extend the analysis of the baseline model. The analysis follows Hosoya (2004). The instantaneous utility $U(C)$ is assumed to be a familiar form of constant intertemporal elasticity of substitution (CIES) function; $U(C) = (C^{1-\theta} - 1)/(1-\theta)$. Here, θ is the inverse of the intertemporal elasticity of substitution.

[14] Their extended Solow model contains physical capital, human capital and technological know-how as productive inputs for production function.

Then, the intertemporal utility function of Eq. (1) is altered by

$$W = \int_0^{+\infty} \frac{C^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt, \quad \theta > 0, \ \rho > 0,$$

where W denotes a lifetime utility which is represented by the integral sum of the instantaneous utility with discount factor. By applying standard dynamic optimization, we can find that in the transformed dynamical system Eq. (10) is only changed:

$$\frac{\dot{X}}{X} = X + \frac{(\alpha - \theta)}{\theta}(1-\tau)Z^{\alpha-1} - \frac{\rho}{\theta}. \tag{14}$$

Accordingly, the revised system for the extended model is represented by Eqs. (11) and (14).[15] As well as the baseline model, we can obtain the following characteristic polynomial which corresponds to the revised system:

$$\text{Det } (J^* - \lambda I) = \text{Det} \begin{bmatrix} X^* - \lambda & \frac{(\alpha-\theta)}{\theta}(\alpha-1)(1-\tau)X^*(Z^*)^{\alpha-2} \\ -Z^* & [(\alpha-1)(1-\tau)(Z^*)^{\alpha-1} - A\tau(\alpha+\epsilon)(Z^*)^{\alpha+\epsilon}] - \lambda \end{bmatrix} = 0.$$

Then we have

$$\lambda^2 + (V_1 + V_2 - X^*)\lambda - \left(\frac{\alpha}{\theta}V_1 + V_2\right) X^* = 0, \tag{15}$$

where $V_1 \equiv (1-\alpha)(1-\tau)(Z^*)^{\alpha-1}$ and $V_2 \equiv A\tau(\alpha+\epsilon)(Z^*)^{\alpha+\epsilon}$. Solving Eq. (15) with respect to λ, we can also obtain a negative root λ_1. That is,

$$\lambda_1 = \frac{(X^* - V_1 - V_2) - [(V_1 + V_2 - X^*)^2 + 4(\frac{\alpha}{\theta}V_1 + V_2)X^*]^{1/2}}{2}. \tag{16}$$

After all, from Eq. (16), the convergence coefficient is also given by the original expression of $\tilde{\lambda} = -\lambda_1$.

Based on the previous analysis in section 3, let us examine convergence properties of the extended model. In this analysis, the magnitude of θ has a very significant role for the determination of the speed of convergence. Except for τ, ϵ and θ, we set the other parameters to (ρ, α, A)=(0.02, 0.3, 0.05). For income tax rates and the degree of external effects, we refer to the previous values and vary in a relevant range; 0.03-0.05 for τ and 0.05-0.20 for ϵ.

The measurement for the magnitude of the intertemporal elasticity of substitution has extensively discussed in the empirical literature on consumption theory. However, there is no consensus value for the empirical estimate of θ. As a provisional but a comparatively reliable parameter range, a large number of the empirical literature obtains $\theta > 1$.[16] Taking account of the evidence, we

[15] The extended model with a CIES utility function has also a unique equilibrium, and the corresponding two dimensional dynamical system exhibits local saddle-path stability.

[16] See for example Lucas (1987), Epstein and Zin (1991), Patterson and Pesaran (1992), Cooley (1995) and Ogaki and Reinhart (1998).

shall pick up the following four values: θ=1.5, 2, 5 and 10, for calibration.[17] Under the situation of low tax (supported by the data) and small externality as in subsection 3.3, these cases also replicate empirically reasonable slower speeds of convergence. Typical results are shown in Table 4.

$* * * * *$ **[Table 4 and Figure 3 here]** $* * * * *$

According to a great deal of macroeconomic literature including typical real business cycle (RBC) models, the standard size of θ varies in a range 1.5-2. Then the rates of convergence in the extended model exhibit 2.5-3.3% per year. Remembering the evidence of the rates of convergence, such figures are very favorable in terms of empirical relevancies. Figure 3 displays a brief relation, and compares $\tilde{\lambda}$ among typical cases. We can see that the size difference of θ has a relatively large impact on the convergence coefficient. In this respect, along with income tax rate (public health expenditure) and the external effects, the size of θ has very important role for determining the rate of convergence. Ortigueira and Santos (1997, p. 390) obtained a similar result and stated that the speed of convergence $\tilde{\lambda}$ increases substantially with decrements in θ.

5 Concluding remarks

This paper has discussed a macroeconomic convergence problem based on a two-sector endogenous growth model with health capital accumulation. By calibrating for the analytical solutions of the model, we obtained the following two vectorial results: (i) relatively higher speed of convergence process arises from standard parameter constellations as well as a large number of the endogenous growth models; (ii) in contrast to (i), when incorporating actual data on public health expenditure into parameter candidates for the model calibration and assuming relatively small degree of external effects in the health sector, we can confirm the emergence of slower speed of convergence process.

In terms of the empirical evidence, we emphasized the importance of the latter result. Especially we found that the introduction of a CIES utility function deteriorates the rate of convergence under the circumstances with empirically valid low tax and small externality. At any hand, one significant contribution of this paper was to show the emergence of slower adjustment process even when employing an endogenous growth framework.

[17] As might be expected, the logarithmic utility employed in the baseline model corresponds to the case of $\theta = 1$.

Appendix

To prove the existence of Z^*, let us rewrite the nonlinear equation on Z^*. That is,

$$\alpha(1-\tau)(Z^*)^{\alpha-1} = A\tau(Z^*)^{\alpha+\epsilon} + \rho. \tag{A1}$$

We denote here the LHS and the RHS of Eq. $(A1)$ by $\Upsilon_1(Z^*)$ and $\Upsilon_2(Z^*)$, respectively:

$$\Upsilon_1(Z^*) = \alpha(1-\tau)(Z^*)^{\alpha-1}, \tag{A2}$$

$$\Upsilon_2(Z^*) = A\tau(Z^*)^{\alpha+\epsilon} + \rho. \tag{A3}$$

From Eq. $(A2)$, the functional properties of $\Upsilon_1(Z^*)$ are

$$\Upsilon_1'(Z^*) = \alpha(\alpha-1)(1-\tau)(Z^*)^{\alpha-2} < 0, \tag{A4}$$

$$\Upsilon_1''(Z^*) = \alpha(\alpha-1)(\alpha-2)(1-\tau)(Z^*)^{\alpha-3} > 0. \tag{A5}$$

Due to Eqs. $(A4)$ and $(A5)$, $\Upsilon_1(Z^*)$ is a concave up decreasing function of Z^*. While from Eq. $(A3)$ the functional properties of $\Upsilon_2(Z^*)$ are

$$\Upsilon_2'(Z^*) = A\tau(\alpha+\epsilon)(Z^*)^{\alpha+\epsilon-1} > 0, \tag{A6}$$

$$\Upsilon_2''(Z^*) = A\tau(\alpha+\epsilon)(\alpha+\epsilon-1)(Z^*)^{\alpha+\epsilon-2} \gtrless 0, \quad if \quad \alpha+\epsilon \gtrless 1. \tag{A7}$$

Eq. $(A6)$ shows that $\Upsilon_2(Z^*)$ is an increasing function, but the second derivative (Eq. $(A7)$) delivers us an additional information on the functional properties of $\Upsilon_2(Z^*)$. That is to say, the functional form is a concave up or a concave down whether $\alpha+\epsilon$ is larger than, or smaller than unity. In each case, however, we can confirm that two functions $\Upsilon_1(Z^*)$ and $\Upsilon_2(Z^*)$ inevitably intersect in the relevant plane only once. Therefore, the existence and the uniqueness of a positive Z^* are proved.

References

[1] Albelo, C. D. A. (1999) "Complementarity between physical and human capital, and speed of convergence," *Economics Letters*, 64, 357-361.

[2] Arellano, M. and Bover, O. (1995) "Another look at the instrumental variable estimation of error-components models," *Journal of Econometrics*, 68, 29-52.

[3] Barro, R. J. (1990) "Government spending in a simple model of endogenous growth," *Journal of Political Economy*, 98, $S103$-$S125$.

[4] ———. and Sala-i-Martin, X. (1992) "Convergence," *Journal of Political Economy*, 100, 223-251.

[5] ———. and ———. (1997) "Technological diffusion, convergence, and growth," *Journal of Economic Growth*, 2, 1-27.

[6] ———. and ———. (2003) *Economic Growth*, 2nd ed., Cambridge, MA: MIT Press.

[7] ———., Mankiw, N. G. and Sala-i-Martin, X. (1995) "Capital mobility in neoclassical models of growth," *American Economic Review*, 85, 103-115.

[8] Blundell, R. and Bond, S. (1998) "Initial conditions and moment restrictions in dynamic panel data models," *Journal of Econometrics*, 87, 115-143.

[9] Bond, S., Hoeffler, A. and Temple, J. (2001) "GMM estimation of empirical growth models," CEPR Discussion Paper No.3048.

[10] Caselli, F., Esquivel, G. and Lefort, F. (1996) "Reopening convergence debate: a new look at cross-country growth empirics," *Journal of Economic Growth*, 1, 363-390.

[11] Cooley, T. F. (ed.) (1995) *Frontiers of Business Cycle Research*, Princeton, NJ: Princeton University Press.

[12] de la Fuente, A. (1997) "The empirics of growth and convergence: a selective review," *Journal of Economic Dynamics and Control*, 21, 23-73.

[13] den Haan, W. J. (1995) "Convergence in stochastic growth models: the importance of understanding why income levels differ," *Journal of Monetary Economics*, 35, 65-82.

[14] Eicher, T. S. and Turnovsky, S. J. (1999) "Convergence in a two-sector nonscale growth model," *Journal of Economic Growth*, 4, 413-428.

[15] Epstein, L. G. and Zin, S. E. (1991) "Substitution, risk aversion, and the temporal behavior of consumption and asset returns: an empirical analysis," *Journal of Political Economy*, 99, 263-286.

[16] Evans, P. (1997) "How fast do economies converge?," *Review of Economics and Statistics*, 79, 219-225.

[17] Gokan, Y. (2003) "The speed of convergence and alternative government financing," *Journal of Economic Dynamics and Control*, 27, 1517-1531.

[18] Hosoya, K. (2003) "Tax financed government health expenditure and growth with capital deepening externality," *Economics Bulletin*, 5, 1-10.

[19] ———. (2004) "Health capital accumulation and macroeconomic convergence," *Iryo To Shakai (Journal of Health Care and Society)*, 14, 139-146, (in Japanese).

[20] Islam, N. (1995) "Growth empirics: a panel data approach," *Quarterly Journal of Economics*, 110, 1127-1170.

[21] Lucas, R. E. (1987) *Models of business cycles*, Oxford: Basil Blackwell.

[22] ———. (1988) "On the mechanics of economic development," *Journal of Monetary Economics*, 22, 3-42.

[23] Mankiw, N. G., Romer, D. and Weil, D. N. (1992) "A contribution to empirics of economic growth," *Quarterly Journal of Economics*, 107, 407-437.

[24] Mino, K. (2000) "Speed of convergence in growth models: an overview," *Kokumin-Keizai Zasshi (Journal of Economics and Business Administration)*, 181, 15-30, (in Japanese).

[25] Murthy, N. R. V. and Chien, I. S. (1997) "The empirics of economic growth for OECD countries: some new findings," *Economics Letters*, 55, 425-429.

[26] Ogaki, M. and Reinhart, C. M. (1998) "Measuring intertemporal substitution: the role of durable goods," *Journal of Political Economy*, 106, 1078-1098.

[27] Ortigueira, S. and Santos, M. S. (1997) "On the speed of convergence in endogenous growth models," *American Economic Review*, 87, 383-399.

[28] Patterson, K. D. and Pesaran, B. (1992) "The intertemporal elasticity of substitution in consumption in the United States and the United Kingdom," *Review of Economics and Statistics*, 74, 573-584.

[29] Russo, B. (2002) "Taxes, the speed of convergence, and implications for welfare effects of fiscal policy," *Southern Economic Journal*, 69, 444-456.

[30] Sato, R. (1963) “Fiscal policy in a neo-classical growth model: an analysis of time required for equilibrating adjustment,” *Review of Economic Studies*, 30, 16-23.

[31] Turnovsky, S. J. (2002) “Intertemporal and intratemporal substitution, and the speed of convergence in the neoclassical growth model,” *Journal of Economic Dynamics and Control*, 26, 1765-1785.

Table 1: Sensitivity under the benchmark case

Changing parameter	Benchmark	Sensitivity	Convergence coefficient ($\tilde{\lambda}$)
ρ	0.02	0.03	9.0%
α	0.3	0.35	9.1%
τ	0.1	0.2	11.7%
A	0.1	0.2	12.2%
ϵ	0.2	0.3	10.0%

Table 2: Public health expenditure 1990-2000 (% of GDP)

Income group	Sample	$(G/Y)\times 100$ (%)
Low	43	2.2 (1.25)
Lower middle	34	3.2 (1.54)
Upper middle	24	3.6 (1.21)
OECD	24	5.9 (1.13)
Average		3.7

Note: Standard deviations are shown in parentheses.
Source: World Development Indicators 2003 *CD-ROM.*

Table 3-1: Tax rate, externality and convergence coefficient

Tax rate (%)	Externality (ϵ)	Convergence coefficient ($\tilde{\lambda}$)
1	0.05	2.5%
1	0.10	2.6%
1	0.15	2.7%
1	0.20	2.8%
1	0.30	3.1%
1	0.40	3.5%
3	0.05	3.1%
3	0.10	3.3%
3	0.15	3.5%
3	0.20	3.8%
5	0.05	3.6%
5	0.10	3.9%

Note: Fixed parameters are $\rho = 0.02$, $\alpha = 0.3$ and $A = 0.05$.

Table 3-2: Tax rate, externality and convergence coefficient (Sensitivity)

Tax rate (%)	Externality (ϵ)	Convergence coefficient ($\tilde{\lambda}$)
1	0.10	2.6%
1	0.20	3.0%
1	0.30	3.5%
3	0.05	3.4%
3	0.10	3.7%

Note: Fixed parameters are $\rho = 0.02$, $\alpha = 0.35$ and $A = 0.07$.

Table 4: Tax rate, externality and convergence coefficient in the extended model

	Tax rate (%)	Externality (ϵ)	Convergence coefficient ($\tilde{\lambda}$)
$\theta = 1.5$	3	0.10	2.7%
	3	0.20	3.0%
	5	0.05	3.2%
	5	0.10	3.3%
$\theta = 2$	3	0.10	2.5%
	3	0.20	2.7%
	5	0.05	2.9%
	5	0.10	3.0%
$\theta = 5$	3	0.10	1.8%
	3	0.20	2.1%
	5	0.05	2.3%
	5	0.10	2.4%
$\theta = 10$	3	0.10	1.5%
	3	0.20	1.7%
	5	0.05	1.9%
	5	0.10	2.0%

Note: Fixed parameters are $\rho = 0.02$, $\alpha = 0.3$ and $A = 0.05$.

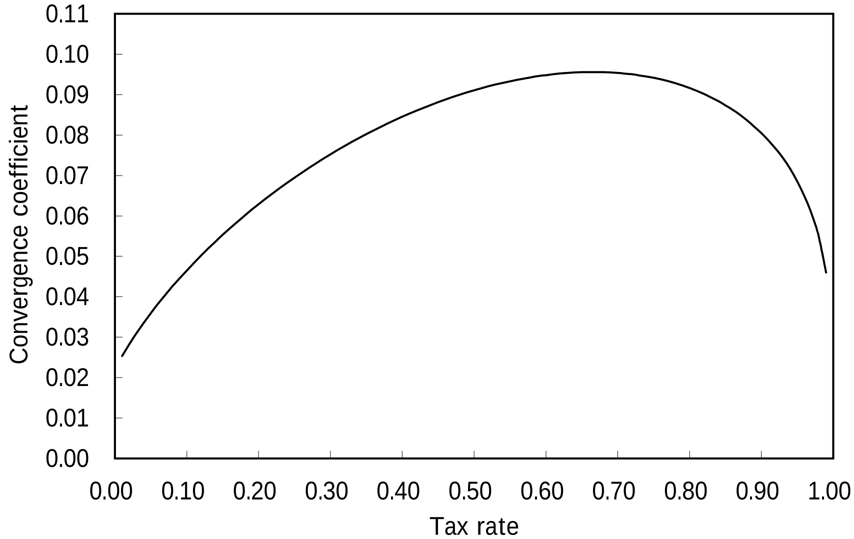

Figure 1: Tax rate and convergence coefficient

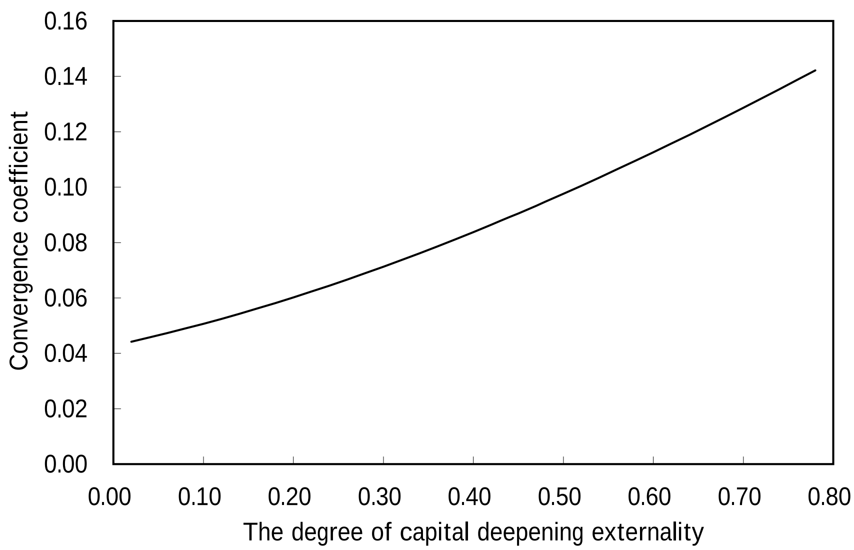

Figure 2: Externality and convergence coefficient

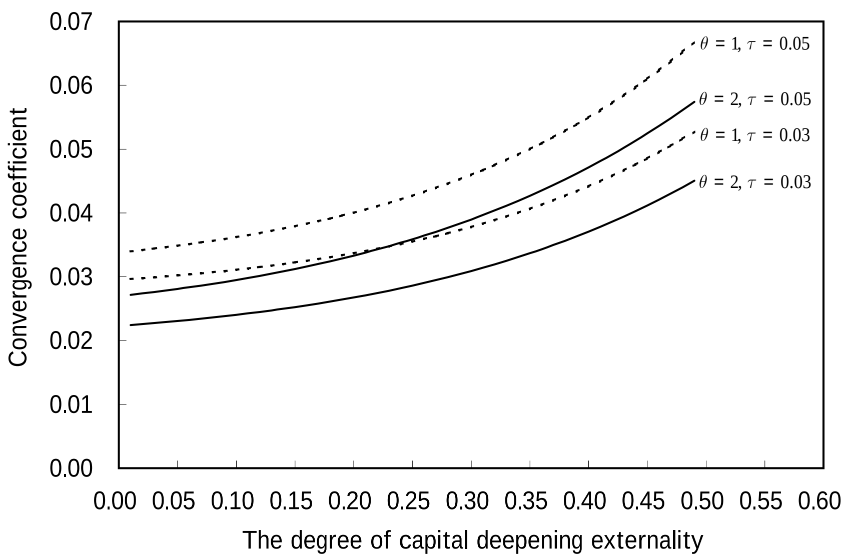

Figure 3: Intertemporal elasticity and convergence coefficient