# The Effects of Emission Permits on Growth and the Environment*

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This version: August 2001

**Abstract**

We develop an overlapping generations model of growth and the environment in which industrial firms produce environmentally harmful emissions. A government controls the emissions by assigning emission quotas to firms, and permits could be issued and freely traded as financial instruments across firms on the basis of the quotas. We show that an environmental policy that decreases an aggregate number of emission quotas could degenerate economic growth and lower environmental quality in the long run. We also show the implications of this result for environmental policy.

*Key words:* Emission permit, Economic growth, Environmental quality, Overlapping generations.

*JEL Classification*: D91, Q28, O11.

*This paper is forthcoming in *Environmental and Resource Economics*. I thank two anonymous referees for insightful comment, which greatly improved the paper. Any remaining errors are my own.. This research was financially supported by Asahi Glass Foundation and Grants-in-Aid for Scientific Research from the Ministry of Education, Culture, Sports, Science and Technology..

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# 1 Introduction

An emission permits system accompanied by the assignment of quotas on emission levels is one of the most effective instruments to control environmentally harmful emissions. A government assigns quotas on emissions to polluters (for example, industrial firms), and polluters are issued emissions permits on the basis of quotas and freely trade them in a market to in order to satisfy their need to produce emissions. If a government assigns permits of lower limits to polluting industries, the emission levels decrease but the business activity of polluters may be hampered. Thus, analyzing the effects of emission permits is important in view of sustainable development, which is a central economic issue in many countries.

A large number of studies have been conducted on environmental policy and economic growth. However, most of them are related to environmental tax policy;[1] few consider emissions permits and economic growth despite the increasing concern on tradable emission permits for protection of the environment. Exceptions are Stokey (1998) and Grimaud (1999), who introduce emissions permits as an instrument of achieving optimal capital allocation and environmental protection. However, they do not analyze how a decrease in the aggregate number of permits affects growth and the environment, which has been one of the most significant issues in environmental policy since the Kyoto Congress in 1997. In this paper, we intend to consider this unresolved issue.

To execute our aim, in Section 2 we develop an overlapping generations model of growth and the environment, which is based on studies by John and Pecchenino (1994) and John et al. (1995). In particular, we assume that industrial firms produce environmentally harmful emissions, and

[1] Examples are as follows: Lighanrt and van der Ploeg (1994), van der Ploeg and Withagen (1991), and Mohtadi (1996) focus on optimal emission charge in the context of a dynamic pollution problem embedded in the Ramsey model, while John et al. (1995), Marini and Scaramozzino (1995), Fisher and van Marrewijk (1998), and Jouvet, Michel, and Vidal (2000) examine optimal tax schemes to decentralize an intergenerationally efficient allocation in an overlapping generations model of growth and the environment. Bovenberg and Smulders (1995, 1996), Bovenberg and de Mooij (1997), and Bovenberg and Heijdra (1998) examine the effects of environmental tax policy and/or reform on growth and welfare.

that these emissions are regulated by assignment of quotas on emissions. In Section 3, we show that, for some number of quotas, there exist two nontrivial steady state equilibria: the one is unstable equilibrium with low capital and low environmental quality, and the other is stable equilibrium with high capital and high environmental quality. An equilibrium path would initially display environmental deterioration and capital accumulation and later exhibit environmental improvement and capital accumulation; it finally converges to a stable steady state.

In Section 4, we focus on stable steady state equilibrium and then show the main result: a decrease in the number of quotas on emissions may be harmful to both growth and the environment. A decrease in the number of quotas on emissions has two effects: positive and negative income effects. The former is an increase in environmental assets bequeathed by the previous generations. This positive effect leads to an increase in savings and investment in the environment, which enhances capital accumulation and environmental improvement. The latter is a decrease in private assets. This negative effect leads to a reduction in savings and investment for environmental maintenance, which in turn lowers capital and environmental quality. For some cases, the negative effect overcomes the positive one; a decrease in the quotas that aim to reduce the flow of emissions results in capital dissipation and environmental deterioration in the long run. We also show the implication of this result for environmental policy. In Section 5, we provide the concluding remarks.

# 2 The Model

Consider an infinite-horizon economy composed of perfectly competitive firms and finitely-lived agents. A new generation (called generation $t$) is born in each period $t = 1, 2, ...$, and lives for two periods, youth and old age. We assume no population growth and normalize the size of each generation as unity. Agents are identical in each generation.

There is a continuum of identical firms. They are perfectly competitive profit maximizers that produce final good $Y_t$ using the production function

$$Y_t = \tilde{A}(K_t)^{\alpha}(L_t)^{1-\alpha}z_t, \tag{1}$$

where $\tilde{A}$ is a productivity scalar, $K_t$ is the total quantity of capital in period $t$, $L_t$ is the total employment in period $t$, $z_t$ is the intensity of pollution, and $\alpha \in (0,1)$ is a constant parameter. We do not make an index for each firm except for the case in which we need indexation to explain the behavior of each firm clearly. Capital depreciates in the process of production at the rate $\delta \in [0,1]$.

The activity of production leads to a flow of environmentally harmful emissions

$$P_t = Y_t(z_t)^{\theta}, \tag{2}$$

where $\theta > 0$ is constant. A larger $\theta$ implies more emissions given the final output. Elimination of $z_t$ between the production function (1) and the emission function (2) leads to

$$\begin{aligned} Y_t &= (\tilde{A})^{\frac{\theta}{1+\theta}}(K_t)^{\frac{\alpha\theta}{1+\theta}}(L_t)^{\frac{\theta(1-\alpha)}{1+\theta}}(P_t)^{\frac{1}{1+\theta}} \\ &= A(K_t)^{\alpha_K}(L_t)^{\alpha_L}(P_t)^{\alpha_P} \end{aligned} \tag{3}$$

where $A \equiv (\tilde{A})^{\frac{\theta}{1+\theta}}$, $\alpha_K \equiv \alpha\theta/(1+\theta)$, $\alpha_L \equiv \theta(1-\alpha)/(1+\theta)$, and $\alpha_P \equiv 1/(1+\theta)$. This production function has constant returns to scale since $\alpha_K + \alpha_L + \alpha_P = 1$.

The long-lived government assigns in each period quotas on emissions to firms in order to control their emissions. Let $S > 0$ be the aggregate number of quotas assigned to firms in each period and let $S^i > 0$ be the number of quotas assigned to firm $i$ where $\int S^i di = S$. We could interpret that the participants in the world congress (for example, the Kyoto Congress in 1997) decide the total number of quotas, $S$, assigned to each state, and that each state distributes the quotas to domestic firms in order to execute the agreement.[2] Emissions permits could be issued

[2] There is no international trade of emissions permits in our model.

and freely traded as financial instruments between firms on the basis of the quotas. There is a competitive market for these permits, where the unit price is $q_t$. Each firm is a price taker in the market. If firm $i$ emits $P_t^i < S^i (> S^i)$ units, then it can sell $S^i - P_t^i$ units (buy $P_t^i - S^i$ units) of permits in the market at the price $q_t$.

In each period, firms choose $K_t$, $L_t$, and $P_t$ to maximize the profit $\pi_t$ :

$$\pi_t = A(K_t)^{\alpha_K}(L_t)^{\alpha_L}(P_t)^{\alpha_P} - \rho_t K_t - w_t L_t + q_t(S - P_t)$$

where $\rho_t$ is the rental price of capital and $P_t - S$ is the net demand of permits. Let denote $k_t \equiv K_t/L_t$ and $p_t \equiv P_t/L_t$. Then, the first order conditions of profit maximization are

$$\rho_t = \alpha_K A(k_t)^{\alpha_K - 1}(p_t)^{\alpha_P}, \tag{4}$$

$$w_t = (1 - \alpha_K - \alpha_P)A(k_t)^{\alpha_K}(p_t)^{\alpha_P}, \tag{5}$$

$$q_t = \alpha_P A(k_t)^{\alpha_K}(p_t)^{\alpha_P - 1}, \tag{6}$$

where (4) - (6) state that firms hire capital ($K_t$) and labor ($L_t$) and emits pollution ($P_t$) until the marginal products equal the factor prices. Constant returns to scale and perfect competition taking together mean that payments to factors of production will exhaust every profit-maximizing producer's revenue, leaving the endowment of permits, $q_t S$, for profits. We assume that these profits are distributed to young agents by the long-lived government.

The budget equation of the long-lived government in period $t$ is $\tau_t^l = q_t S$, where $\tau_t^l$ is the aggregate amount of transfers to households. The long-lived government cannot control both the price $q_t$ and the quotas $S$ since the price $q_t$ is determined in the market and the quotas $S$ is decided by the world congress. Thus, it cannot affect the amount of lump-sum transfer. The task of the long-lived government is to transfer the revenue of emission trading from firms to households in each period.

Agents born in period $t$ have preferences over consumption in old age, $c_{t+1}$, and an index of the quality of the environment when they consume, $E_{t+1}$. These preferences are represented by the utility function $\ln c_{t+1} + \mu \ln E_{t+1}$, where $\mu > 0$ is a parameter of environmental concern. The larger $\mu$ implies more concern for the environment.

Young agents are each endowed with one unit of labor which they supply to firms inelastically. They divide their wage, $w_t$, and the lump-sum transfer from the long-lived government, $\tau_t^l$, between savings for consumption in old age, $s_t$, and investment in the environment, $m_t$. In old age, agents supply their savings to firms and earn the gross return $(1 + r_{t+1})$.

Environmental quality is an intergenerational public good that is reduced by emissions caused by firms, $P_t$, but that can be improved by maintenance investment, $m_t$. This mechanism is expressed by

$$E_{t+1} = (1 - b)E_t - \beta P_t + \gamma m_t,$$

where $b \in (0, 1)$ measures the speed of the autonomous change in environmental quality, $\beta > 0$ is a parameter which evaluates the effect of emissions on environmental quality, and $\gamma > 0$ is a parameter that represents the efficiency of environmental maintenance. The second term in the right-hand side, $\beta P_t$, is an externality caused by firms.

Our formula of environmental quality is based on the work of John and Pecchenino (1994) and John et al. (1995) but differs from theirs in the assumption of environmental externality. They assume that the source of environmentally harmful externality is consumption by past generations: $E_{t+1} = (1 - b)E_t - \beta c_t + \gamma m_t$. They adopt this formula to focus on consumption externality across generations. On the other hand, we assume that firms cause the flow of emissions during the process of production, and that the harmful effects of emissions on the environment accumulate toward the future. We adopt this formula to examine the regulation of emissions by firms by introducing tradeable emission permits.

Following John and Pecchenino (1994) and John et al. (1995), we assume that a short-lived government representing the young chooses the maintenance investment $m_t$ and savings $s_t$ to maximize the utility of generation $t$ on the condition that the old are not made worse off by this decision.[3] Given the wage, $w_t$, the return on savings, $r_{t+1}$, environmental quality at the beginning of period $t$, $E_t$, the quantity of emissions by firms, $P_t$, and the lump-sum transfer $\tau_t^l$, the lifetime choice problem of a representative agent in generation $t$ is:

$$\max_{\{s_t,m_t\}} \ln c_{t+1} + \mu \ln E_{t+1}$$

subject to

$$s_t + m_t = w_t + \tau_t^l, \tag{7}$$

$$c_{t+1} = (1 + r_{t+1})s_t, \tag{8}$$

$$E_{t+1} = (1 - b)E_t - \beta P_t + \gamma m_t, \tag{9}$$

$$s_t, m_t \geq 0,$$

where (7) and (8) are budget constraints and (9) is an environmental equation. The first order conditions of this problem are (7), (8), (9), and

$$\gamma\mu c_{t+1} \leq (1 + r_{t+1})E_{t+1};\ \text{an equality holds if } m_t > 0. \tag{10}$$

(10) implies that the marginal rate of substitution between consumption and environmental quality, $E_{t+1}/\mu c_{t+1}$, is equal to or greater than the marginal rate of transformation, $\gamma/(1+r_{t+1})$.

# 3 Equilibrium

This section investigates existence and stability of the equilibrium.

There are two input markets: one for capital and one for emission permits. A capital market clearing condition is $s_t L_t = K_{t+1}$, which says that total savings by young agents in generation $t$,

[3] We should notice that there are two types of government in our model: long-lived and short-lived governments.

$s_t L_t$, must equal their purchase of used capital stock from old agents in generation $t-1$, $(1-\delta)K_t$, plus their own addition to the future stock, $K_{t+1} - (1-\delta)K_t$. A market clearing condition of emissions permits is $P_t = S$, which says that the total amount of emissions, $P_t$, must equal the total number of tradable emissions permits, $S$, which is based on quotas on emissions. Since $L_t = 1$ for all $t$,[4] these two conditions are rewritten as

$$s_t = k_{t+1}, \tag{11}$$

$$p_t = S. \tag{12}$$

The markets for renting and purchasing physical capital are competitive; the opportunity cost of owning equipment for one period should equal the relevant rental rate. Then, an arbitrage condition of the form $r_{t+1} + \delta = \rho_{t+1}$ or

$$1 + r_{t+1} = 1 - \delta + \rho_{t+1} \tag{13}$$

holds in equilibrium.

**Definition:** An *equilibrium* is a sequence $\{c_t, E_t, s_t, m_t, p_t, \rho_t, w_t, q_t, k_t, r_t\}_{t=1}^{\infty}$ such that, in each period, (i) agents maximize utility subject to the constraints, (ii) firms maximize profits, and (iii) markets clear, given the initial condition $\{k_1, E_1\}$.

An equilibrium sequence $\{c_t, E_t, s_t, m_t, p_t, \rho_t, w_t, q_t, k_t, r_t\}_{t=1}^{\infty}$with the initial condition $\{k_1, E_1\}$ is characterized by the first order conditions of profit maximization, (4) - (6), the first order conditions of utility maximization, (7) - (10), two input markets clearing conditions, (11) and (12), and the arbitrage condition, (13).

[4] In the labor market, the supply is one unit and the demand is $L_t$ units. Thus, $L_t = 1$ is the labor market clearing condition.

By summarizing (4)-(13), the equilibrium path with the initial condition $\{k_1, E_1\}$ is characterized by the sequence $\{k_t, m_t, E_t\}_{t=1}^{\infty}$ which satisfies

$$\gamma\mu k_{t+1} \leq E_{t+1}, \text{equality holds if } m_t > 0 \tag{14}$$

$$m_t + k_{t+1} = (1-\alpha_K)A(S)^{\alpha_P}(k_t)^{\alpha_K}, \tag{15}$$

$$E_{t+1} = (1-b)E_t - \beta S + \gamma m_t. \tag{16}$$

(14) corresponds to the first-order condition of the utility maximization, (10).[5] (15) is a rewrite of the budget equation in youth, and (16) is a rewrite of an environmental equation. The *steady state equilibrium* is an allocation such that $\{k, m, E\}$ are stationary along the equilibrium path. In the steady state, it must hold that $m > 0$; if $m = 0$, then (16) is reduced to $E = -\beta S/b < 0$ which contradicts the inequality constraint $E > 0$.[6]

In what follows, we first consider the cases of $m_t = 0$ and $m_t > 0$, respectively. After that, we show that there exists a nontrivial stable steady state equilibrium with $m > 0$ under a certain condition.

**Zero Maintenance Case**

When $m_t = 0$, (14) - (16) are rewritten as

$$\gamma\mu(1-\alpha_K)A(S)^{\alpha_P}(k_t)^{\alpha_K} \leq (1-b)E_t - \beta S, \tag{17}$$

$$k_{t+1} = (1-\alpha_K)A(S)^{\alpha_P}(k_t)^{\alpha_K}, \tag{18}$$

$$E_{t+1} = (1-b)E_t - \beta S. \tag{19}$$

The equilibrium path of capital and environmental quality under zero maintenance is charac-

[5] When a generation chooses to invest in the environment, (14) holds with equality; there is no trade-off between capital (growth) and the environment. This positive relation holds since a generation chooses consumption and environmental quality to equate the marginal rate of substitution between consumption and environmental quality to the marginal rate of transformation in view of its utility maximization.

[6] A log-linear utility function requires $E > 0$.

terized by (17) - (19). The inequality (17) is a zero maintenance condition,[7] which means that generation $t$ chooses to invest nothing in the environment if a pair of $k_t$ and $E_t$, which is given for generation $t$, satisfies (17). Figure 1 depicts a zero maintenance curve. In the region above this curve, environmental quality is sufficiently high and/or a level of capital stock is sufficiently low so that a generation chooses not to engage in environmental maintenance. We call such region a zero maintenance area.

In the zero maintenance area, (18) yields the locus of capital stock characterized by

$$k_{t+1} \geq k_t \Leftrightarrow k_t \leq (1 - \alpha_K) A(S)^{\alpha_P} (k_t)^{\alpha_K}.$$

As depicted in Figure 1, in the zero maintenance area capital stock continues to increase (decrease) on the left (right) side of the line characterizing the steady state level of capital stock, $k = \{(1 - \alpha_K) A(S)^{\alpha_P}\}^{1/(1-\alpha_K)}$.

(19) yields the locus of environmental quality characterized by

$$E_{t+1} \leq E_t \Leftrightarrow E_t \geq -\beta S / b.$$

Environmental quality continues to decrease over time because of lack of maintenance investment and the externality of emissions. Thus, the equilibrium path with zero maintenance will break through the zero maintenance curve; that is, there is no steady state equilibrium with $m = 0$. For some period $t$, a new generation born in that period will find it worthwhile to invest in the environment. We therefore next consider the equilibrium path with positive maintenance.

**Positive Maintenance Case**

When $m_t > 0$, (15) - (16) are reduced to

$$E_{t+1} = (1 - b) E_t - \beta S + \gamma \{(1 - \alpha_K) A(S)^{\alpha_P} (k_t)^{\alpha_K} - k_{t+1}\}.$$

[7] We obtain this condition by replacing $k_{t+1}$ and $E_{t+1}$ in (14) with $k_t$ and $E_t$.

With (14), the above equation is rewritten as

$$k_{t+1} = G(k_t) \equiv \frac{(1-b)\mu}{1+\mu} k_t - \frac{\beta S}{\gamma(1+\mu)} + \frac{(1-\alpha_K)A(S)^{\alpha_P}}{1+\mu}(k_t)^{\alpha_K}. \tag{20}$$

The positive maintenance condition is $m_t = (1-\alpha_K)A(S)^{\alpha_P}(k_t)^{\alpha_K} - k_{t+1} > 0$, or

$$k_{t+1} < \tilde{G}(k_t) \equiv (1-\alpha_K)A(S)^{\alpha_P}(k_t)^{\alpha_K}. \tag{21}$$

Therefore, the equilibrium path of capital under positive maintenance is characterized by (20) and (21).

Figure 2 depicts the functions $G(k_t)$ and $\tilde{G}(k_t)$ in the $k_t - k_{t+1}$ space. Below $\tilde{G}(k_t)$, the equilibrium path displays positive maintenance. The function $G(k_t)$ may cut the $45^0$ line twice, which means that there may exist two nontrivial steady state equilibria. In what follows, we derive the range of quotas $S$ that ensures the existence of multiple steady state equilibria with positive maintenance.

**Proposition 1:** *There exist two nontrivial steady state equilibria if and only if $S \in (0, \bar{S})$ where*

$$\bar{S} \equiv \left\{ \frac{(1-\alpha_K)A\alpha_K}{1+b\mu} \right\}^{\frac{1}{1-\alpha_K-\alpha_P}} \left\{ \frac{(1-\alpha_K)(1+b\mu)\gamma}{\alpha_K \beta} \right\}^{\frac{1-\alpha_K}{1-\alpha_K-\alpha_P}}.$$

**Proof:** Let $\tilde{k}$ denote $k$ which satisfies $\tilde{G}(k) = k$ (see Fig. 2). Direct calculation leads to:

$$\tilde{k} = \{(1-\alpha_K)A(S)^{\alpha_P}\}^{\frac{1}{1-\alpha_K}}. \tag{22}$$

We first show that $\tilde{G}(k) > G(k)$ for any $k \in (0, \tilde{k})$. We then show that $G(k) = k$ has two solutions $k^H$ and $k^L$ where $0 < k^L < k^H < \tilde{k}$ if and only if $S \in (0, \bar{S})$.

The inequality $\tilde{G}(k) > G(k)$ is equivalent to

$$\frac{\mu}{1+\mu}(1-\alpha_K)A(S)^{\alpha_P}(k)^{\alpha_K} - \frac{(1-b)\mu}{1+\mu}k + \frac{\beta S}{\gamma(1+\mu)} > 0.$$

Since the third term on the left-hand side is positive, the above inequality holds if

$$\frac{\mu}{1+\mu}(1-\alpha_K)A(S)^{\alpha_P}(k)^{\alpha_K} \geq \frac{(1-b)\mu}{1+\mu}k,$$

which is rewritten as $(1-\alpha_K)A(S)^{\alpha_P} \geq (1-b)(k)^{1-\alpha_K}$. The left-hand side of this inequality is constant while the right-hand side increases in $k$. Thus, we have $\tilde{G}(k) > G(k)$ for any $k \in (0, \tilde{k})$ if $(1-\alpha_K)A(S)^{\alpha_P} \geq (1-b)(\tilde{k})^{1-\alpha_K}$, that is,

$$(1-\alpha_K)A(S)^{\alpha_P} \geq (1-b)(1-\alpha_K)A(S)^{\alpha_P}.$$

Since the above inequality always holds, we find that the inequality $\tilde{G}(k) > G(k)$ always holds for $k \in (0, \tilde{k})$.

Let $\hat{k}$ denote $k$ which satisfies $G'(k) = 1$. For the purpose of ensuring the existence of two solutions for $k \in (0, \tilde{k})$, it is necessary and sufficient to show the range of $S$ which satisfies $G(\hat{k}) > \hat{k}$ (see Figure 2); i.e.,

$$\frac{(1-b)\mu}{1+\mu}\hat{k} - \frac{\beta S}{\gamma(1+\mu)} + \frac{(1-\alpha_K)A(S)^{\alpha_P}}{1+\mu}(\hat{k})^{\alpha_K} > \hat{k}. \tag{23}$$

Since $\hat{k}$ satisfies $G'(\hat{k}) = 1$, this condition leads to

$$\hat{k} = \left\{\frac{(1-\alpha_K)A(S)^{\alpha_P}\alpha_K}{1+b\mu}\right\}^{\frac{1}{(1-\alpha_K)}}. \tag{24}$$

Substituting (24) into (23) and rearranging, we obtain

$$\bar{S} \equiv \left\{\frac{(1-\alpha_K)\hat{A}\alpha_K}{1+b\mu}\right\}^{\frac{1}{1-\alpha_K-\alpha_P}} \left\{\frac{(1-\alpha_K)(1+b\mu)\gamma}{\alpha_K\beta}\right\}^{\frac{1-\alpha_K}{1-\alpha_K-\alpha_P}} > S.$$

Thus, there exist two solutions of $G(k) = k$ in the range of $k \in (0, \tilde{k})$ if and only if $S \in (0, \bar{S})$.

**Q.E.D.**

The source of multiple equilibria is environmental externality of emissions, $-\beta P$. If there is no external effect, $\beta = 0$, then the scalar system characterizing the equilibrium path under positive maintenance (20) has a unique nontrivial steady state equilibrium. Externality of emissions is a key factor that generates multiple equilibria.

Let $e^H(e^L)$ denote the equilibrium with high (low) capital, $k^H(k^L)$. At $e^H(e^L)$ equilibrium, the slope of $G(\cdot)$ function is less (greater) than one (see Fig. 2); that is, the $e^H(e^L)$ equilibrium is stable (unstable). Suppose that the equilibrium path enters the positive maintenance area in period $T$. If $k_T < k^L$, the equilibrium path converges to the trivial steady state equilibrium with $k = 0$. If $k_T = k^L$, the equilibrium path continues to stay at $e^L$ equilibrium. If $k_T \in (k^L, \tilde{k})$, the equilibrium path monotonically converges to $e^H$ equilibrium.

A possible equilibrium path is as follows: Given sufficiently low $k_1$ and sufficiently high $E_1$, the equilibrium path initially displays environmental deterioration and capital accumulation in the zero maintenance area and later exhibits environmental improvement and capital accumulation in the positive maintenance area. The equilibrium path finally converges to the stable steady state equilibrium, $e^H$. This possible path displays a U-shaped relationship between growth and the environment, which is identical to the path shown in John and Pecchenino (1994).

# 4 The Effects of Emission Permits on Growth and the Environment

In this section, we consider a situation in which a world congress decided to decrease the quotas on emissions in each country. We investigate how the change in the number of quotas on emissions affects capital accumulation and environmental quality in the long run. In particular, we focus on the nontrivial stable steady state equilibrium $e^H$.

**Proposition 2:** *$\partial k/\partial S > 0$ and $\partial E/\partial S > 0$ hold at $e^H$ equilibrium if $S \in (0, \hat{S})$ where*

$$\hat{S} \equiv \left\{ \frac{(1-\alpha_K)A\alpha_K}{1+b\mu} \right\}^{\frac{1}{1-\alpha_K-\alpha_P}} \left\{ \frac{\alpha_P(1+b\mu)\gamma}{\alpha_K\beta} \right\}^{\frac{1-\alpha_K}{1-\alpha_K-\alpha_P}} < \bar{S}.$$

**Proof:** Differentiating (20) with respect to $k$ and $S$ and evaluating it at the steady state with $k = k^H$, we obtain

$$\{1 - G'(k^H)\}dk = \frac{1}{1+\mu}\left\{-\frac{\beta}{\gamma} + (1-\alpha_K)A\alpha_P(S)^{\alpha_P - 1}(k^H)^{\alpha_K}\right\}dS.$$

The sign of the differential coefficient of $dk$, $1 - G'(k^H)$, is positive since $G'(k^H) < 1$.

We next examine the sign of the differential coefficient of $dS$. Since $\hat{k} < k^H$ holds, we have

$$\begin{aligned}
-\frac{\beta}{\gamma} + (1-\alpha_K)A\alpha_P(S)^{\alpha_P-1}(k^H)^{\alpha_K} &> -\frac{\beta}{\gamma} + (1-\alpha_K)A\alpha_P(S)^{\alpha_P-1}(\hat{k})^{\alpha_K} \\
&= -\frac{\beta}{\gamma} + (1-\alpha_K)A\alpha_P(S)^{\alpha_P-1}\left\{\frac{(1-\alpha_K)A(S)^{\alpha_P}\alpha_K}{1+b\mu}\right\}^{\frac{\alpha_K}{(1-\alpha_K)}}.
\end{aligned}$$

The last equality holds from (24). Thus, the differential coefficient of $dS$ is positive if the last equation is positive, that is,

$$\left\{\frac{(1-\alpha_K)A\alpha_K}{1+b\mu}\right\}^{\frac{1}{1-\alpha_K-\alpha_P}}\left\{\frac{\alpha_P(1+b\mu)\gamma}{\alpha_K\beta}\right\}^{\frac{1-\alpha_K}{1-\alpha_K-\alpha_P}} > S.$$

Let $\hat{S}$ denote the left-hand side of the above inequality. We can immediately find $\hat{S} < \bar{S}$. Then, we have $dk/dS > 0$ if $S \in (0, \hat{S})$. Since $E = \gamma\mu k$ holds, we have $dE/dS > 0$ if $S \in (0, \hat{S})$.

**Q.E.D.**

This proposition implies that, although a decrease in the number of quotas on emissions reduces the flow of environmentally harmful emissions, $p$, it could eventually result in capital dissipation and environmental deterioration in the long run. To understand the result, consider the constraints of generation $t$, (7), (8), and (9), which are reduced to

$$\frac{c_{t+1}}{1+r_{t+1}} + \frac{E_{t+1}}{\gamma} = w_t + \tau_t^l + \frac{1}{\gamma}\{(1-b)E_t - \beta S\}.$$

The right hand side is the total income of generation $t$. The first term, $w_t$, is wage income, and the second term, $\tau_t^l$, is the lump-sum transfer from the long-lived government. These two are private goods assets exogenously given for generation $t$. The third term, $\{(1-b)E_t - \beta S\}/\gamma$, is an environmental asset bequeathed from generation $t-1$ to generation $t$. In equilibrium, a decrease in $S$ has two negative income effects: a reduction in wage income, $w_t$, and the transfer, $\tau_t^l = q_t S$, since $w_t$ and $q_t S$ are increasing in $S$ (see (5) and (6)). These negative income effects lead to a reduction in investment for environmental maintenance, which in turn lowers future environmental quality. On the other hand, a decrease in $S$ has a positive income effect in equilibrium: an increase in environmental assets, $\{(1-b)E_t - \beta S\}/\gamma$, which improves future environmental quality. At $e^H$ equilibrium, the negative effect overcomes the positive one if $S \in (0, \hat{S})$.

Proposition 2 has the following implication for environmental policy. It is often argued that a country should be assigned a smaller number of quotas on emissions to control environmentally harmful emissions. In our model, such an argument is not necessary true. If the initial assignment of quotas satisfies $S \in (0, \hat{S})$, then a decrease in $S$ results in environmental deterioration. Therefore, an environmental policy that aims to decrease a flow of emissions is not necessarily beneficial to environmental preservation in the long run.

# 5 Conclusion

In this paper, we develop an overlapping generations model of growth and the environment base on the work of John and Pecchenino (1994) and John et al. (1995). They focus on consumption externality across generations and who a tax-transfer scheme in order to achieve an intergenerationally efficient allocation. In contrast to them, this paper focuses on environmental deterioration caused by emissions of industrial firms and examines how a decrease in quotas on emissions affects growth and the environment. We find that, although an environmental policy that decreases the number of quotas reduces the flow of environmentally harmful emissions, it could result in environmental deterioration in the long run. The result implies that we must take account of the long-run consequence of an emissions permits system when we introduce it as an instrument of environmental preservation.

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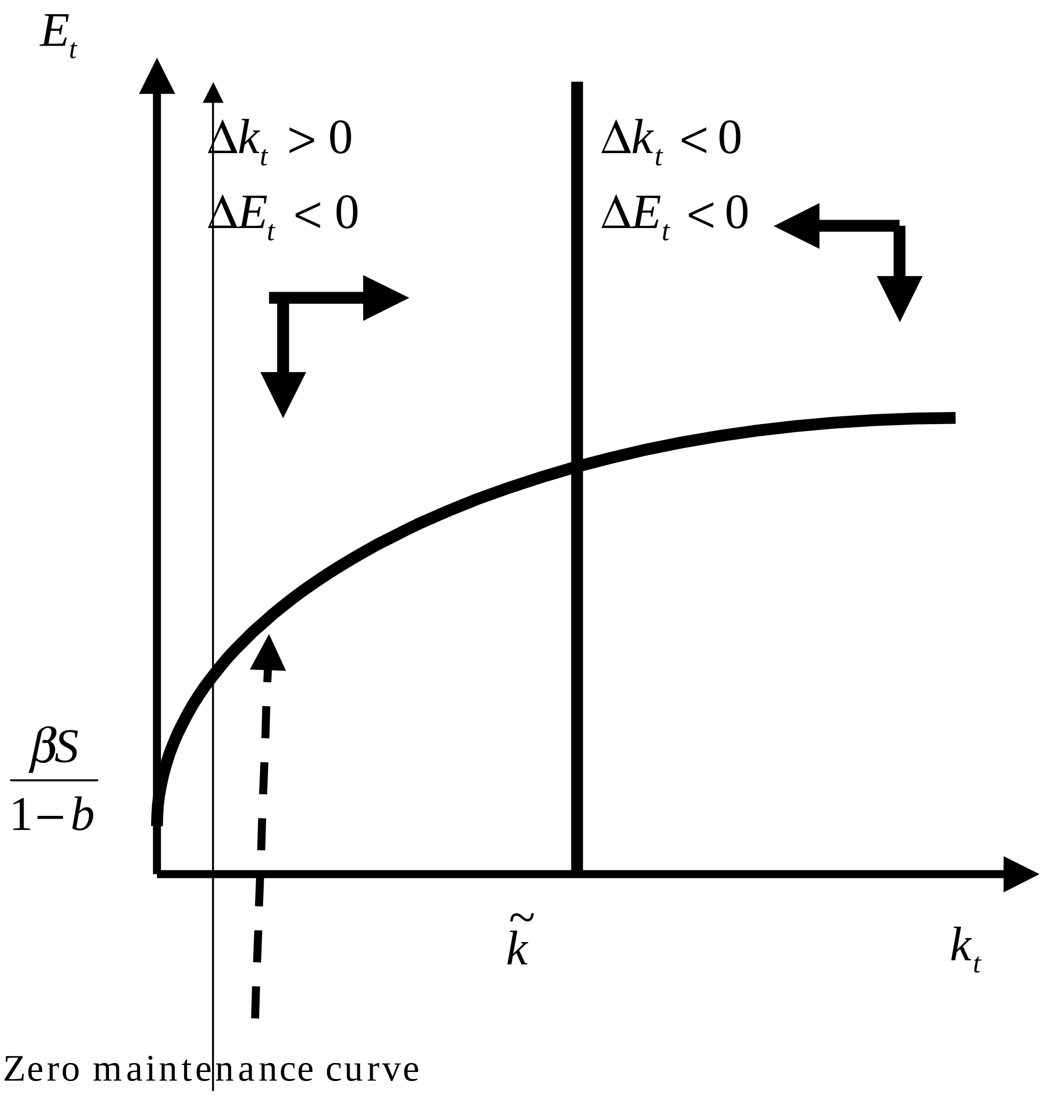
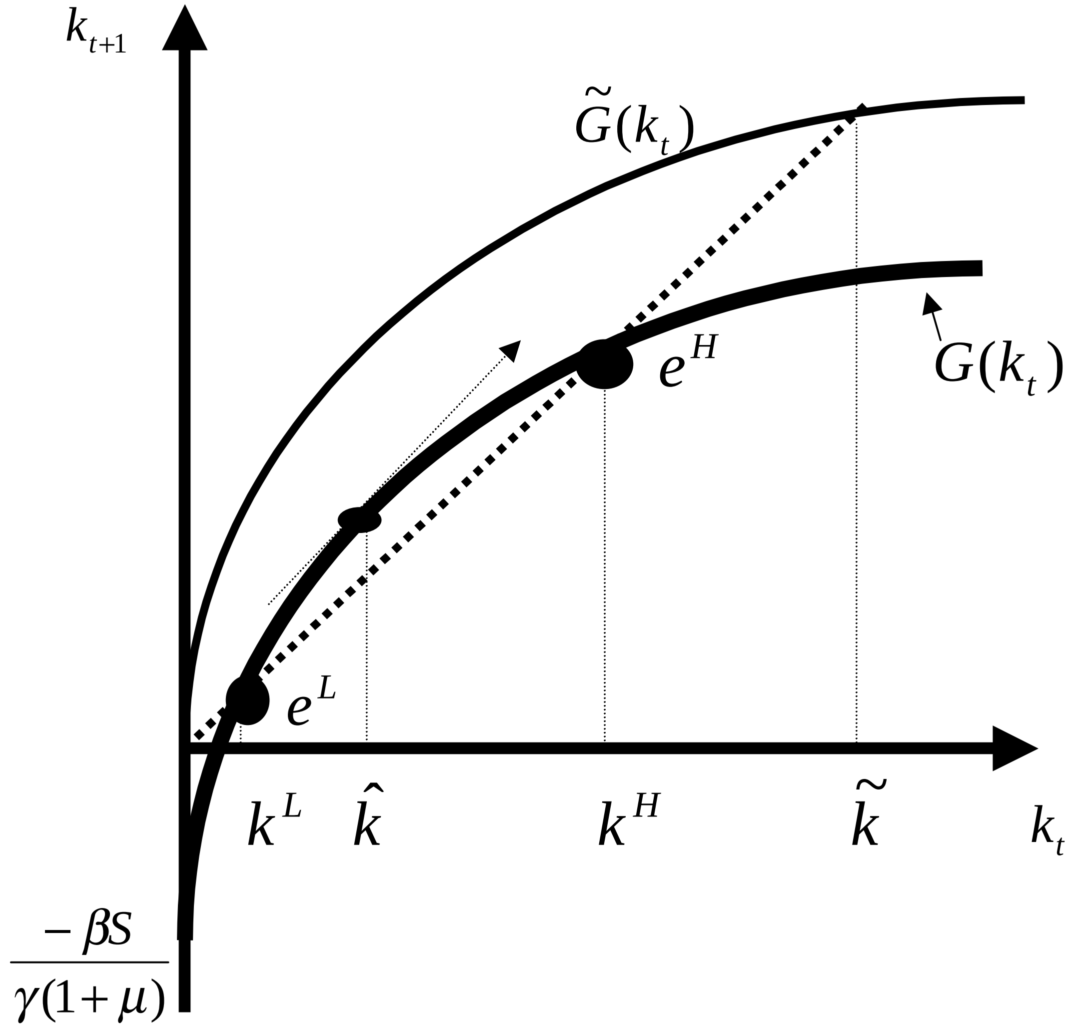
Figure 1



$E_t$
$\Delta k_t > 0$
$\Delta E_t < 0$
$\Delta k_t < 0$
$\Delta E_t < 0$
$\frac{\beta S}{1-b}$
$\tilde{k}$
$k_t$
Zero maintenance curve

Figure 2



$k_{t+1}$
$\tilde{G}(k_t)$
$e^H$
$G(k_t)$
$e^L$
$k^L$
$\hat{k}$
$k^H$
$\tilde{k}$
$k_t$
$\frac{-\beta S}{\gamma(1+\mu)}$