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Random Walk or A Run
—Market Microstructure Analysis of the Foreign Exchange Rate Movements based on Conditional Probability—*

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Abstract:

Using tick-by-tick data of the dollar-yen and euro-dollar exchange rates recorded in the actual transaction platform, a "run"—continuous increases or decreases in deal prices for the past several ticks—does have some predictable information on the direction of the next price movement. Deal price movements, that are consistent with order flows, tend to continue a run once it started i.e., conditional probability of deal prices tend to move in the same direction as the last several times in a row is higher than 0.5. However, quote prices do not show such tendency of a run. Hence, a random walk hypothesis is refuted in a simple test of a run using the tick by tick data. In addition, a longer continuous increase of the price tends to be followed by larger reversal. The findings suggest that those market participants who have access to real-time, tick-by-tick transaction data may have an advantage in predicting the exchange rate movement. Findings here also lend support to the momentum trading strategy.

JEL: F31, F33, G15

Key words: Foreign exchange rate, electronic broking system, random walk, market microstructure.

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1. Introduction

The foreign exchange market remains sleepless around the clock. Someone is trading somewhere all the time—24 hours a day, 7 days a week, 365 days a year. Analyzing the behavior of the exchange rate has become a popular sport of international finance researchers, while global financial institutions are spending millions of dollars to build real-time computer trading systems (program trading). High-frequency, reliable data are the key in finding robust results for good research for academics or profitable schemes for businesses.

Program trading typically uses some algorithm to find a "trend" that is to increase or decrease the price (exchange rate) for several minutes to several hours. The computer program produces buy or sell orders depending on a detected trend. Finding a trend is usually based on a continuous increase or decrease (a run) of the price in the past several minutes to several hours. This type of trading is known as a momentum trading strategy (as opposed to a contrarian strategy). What is attempted in this paper is to construct a very naïve strategy, try to detect a trend by a run, and make a bet on the next move. A program trading strategy is much more sophisticated than the naïve strategy of just detecting a run and making a bet. So, even if the conditional probability is estimated to be different from 0.5, then it is just an example of such a trading, and any profit opportunity detected there would be underestimate. On the other hand, all exercises here would not be proof for profitability, because exercises are conducted in the in-sample manner. However, the point is to examine whether the conditional probability of increase (or decrease) could be different from 0.5—the prediction of the random walk hypothesis.

In order to mimic a program trading strategy, a data set has to be exactly the same as the dealers'. Recently, the actual transactions data have become available. The EBS data set is a historical record of firm quotes and transactions prices recorded at one-second slice. The information is almost the same as what dealers' had in the information. One of the advantages of this paper is the use of tick by tick (a one-second slice, to be precise) data of the EBS, which covers a large share of spot exchange rate transaction in the world. The data from the transaction platform is much more reliable and desirable, as shown in Ito and Hashimoto (2006).

The most popular data set among researchers through various sources, FXFX screen of Reuters indicative quotes, is not appropriate for this purpose because quotes are input by dealers for delivering the market condition. However, they are information only, without any commitment for trade. Reliability of indicative quotes to capture the whole picture of a market reality is much less than firm quotes. Firm quotes (ready to trade) and transaction price data are simply not available in the FXFX screen.[7]

Many academic researchers and officials believe that the exchange rate changes follow a random walk. The literature is full of papers failing to refute a random walk hypothesis, in that the best predictor of the price in the future is the current price. Most papers are written using daily, hourly, or minute-by-minute frequency, with data using FXFX screen of Reuters. However, the literature does not prove non-existence of any profit opportunity in real world, since the FXFX screen does not represent actual trading possibilities and tick-by-tick, conditional trading strategy is not examined properly in the existing literature.

A particular test conducted in this paper is a test using a concept of a "run"—a continuous increase or decrease in deal or quote price (exchange rate) changes. The probability of the next exchange rate change being positive has to be one-half, if the increase or decrease is a random event, irrespective of a history of exchange rate changes. However, momentum traders tend to perceive that the probability of next change being positive is higher than a half. Therefore, the test of random walk is the conditional probability of next change after a run of the same signs in the exchange rate changes.[8] Second, the size of the next change is examined. In particular, it is examined whether the step size of the next change may become different in case of a run.

There are three kinds of innovations in this paper. First, our data set is much better than any other data set used in the literature, because it is a record of actual trading platform. Second, the frequency is one-second slice—almost tick-by-tick. Third, a very simple test is devised so that a test is not subject to model uncertainty or a structural break. It is possible to test whether expected percentage change (direction times step size) can be estimated in a regression model, using both price and transaction volume information, to test a "no profit condition," as Ito and Hashimoto (2007). However, the regression model relies on an assumption of stable structural parameters.

Major findings with respect to price behavior, for both the dollar-yen and euro-dollar, are as follows. First, quote prices are less likely to continue rising or falling even if there exists a run. For quote prices, a conditional probability of a run

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7 See Goodhart and O'Hara (1997: p.78) for detailed arguments on the advantage of using the actual trading data, and Goodhart, Ito, and Payne (1996) and Goodhart and Payne (1996) for an early attempts to make use of transactions data.

8 Bouchaud, Gefen, Potters and Wyart (2004) and Lillo and Farmer (2004) examined price movements in stock market and found that the conditional probability of the sign of the next order is higher than 0.5.

(movement in the same direction) is lower than 0.5 for most cases. Second, deal prices are more likely to continue falling or rising. For deal prices done on the bid (ask) side, conditional probability that the price continues falling (rising, resp.) is higher than 0.5. Third, the step size of price change in a run is constant in general. The size of price fall is larger than that of price increase in the case of bid quote prices. On the contrary, the size of price increase is larger than that of price fall in the case of ask quote prices. Fourth, in the case of deal prices, the size of price increase is larger than that of price fall for the bid-side deal prices, (and *vise versa* for the ask-side deal prices). Finally, the size of correction after a run is found to depend on the number of continuous price increase (decrease) and on the cumulative price change in a run.

The sample period covers from January 1998 to October 2003. Saturdays, Sundays, Mondays and days when interventions were conducted by the monetary authorities (Bank of Japan, Federal Reserve Bank, and ECB) were dropped from the data.

The rest of this paper is organized as follows: Section 2 describes the data. Section 3 shows patterns of transactions (quotes) in foreign exchange market based on conditional probability. In section 4, the size of price change at each transaction (quote revision) is examined conditional on a run of price increase/decrease. In section 5, probability of price increase/decrease is calculated conditional on bid-ask spread. Section 6 concludes the paper.

2. The data

Recently, almost all spot foreign exchange transactions of major currencies are now carried out through electronic broking systems. The EBS, whose data we use in the analysis below, has a strong market share (in absolute terms and in comparison to other electronic broking systems such as Reuter D-3000) in the yen/dollar rate and the Euro/dollar rate.[9] The EBS is a provider of trading technology, and the quotes and transactions are shown continuously, 24 hours a day. The EBS screen shows the "firm bid" and "firm ask", the bid and offer that are committed to trade if someone on the other side is willing to trade at that price.[10]

A "firm ask" ("firm bid") means that the institution that posts the quote is ready to sell (buy, resp.) the shown currency (e.g., the dollar in exchange for the quoted yen).

[9] Reuters have significant market shares in exchanged related to sterling, Canadian dollar, and Australian dollars.
[10] For the general reference on the microstructure of the foreign exchange market, see Goodhart and O'Hara (1997), Lyons (1995) and Lyons (2001).

The ask quote is almost always higher than the bid quote.[11] When the deal is done at the ask side, it means that the firm ask (ready to sell) quote is "hit" by a buyer. When the deal is done at the bid side, the firm bid quote is "hit" by a seller. Therefore the ask deal is a buyer-initiated deal, and the bid deal is a seller-initiated deal, according to the description in Berger, et al (2005).[12] If ask (bid) deals are continuously hit, then the ask (bid) deal prices tend to move up (down), because of the buy (sell, resp.) pressure.

The data used in the analysis includes information on price of the USD-JPY and EUR-USD currency pairs from January 1998 to October 2003. It contains information of best bid, best ask, bid-side deal prices, and ask-side deal prices. Every data point is on the one-second time slice. Bid and ask quote prices are recorded at the end of the time slice. Any bid/ask quote price movements within a second are not recorded. When there are multiple trades within one second, "lowest given price" and "highest paid price" will be shown. Highest paid is the deal price done on the ask side and the lowest given is the price done on the bid side within the one second.

As part of facilitating an orderly market, EBS requires any newly linked institution to secure a sufficient number of other banks that are willing to open credit lines with the new comer. A smaller or regional bank may have fewer trading relationships, thus not as many credit relationships. Then the best bid and ask for that institution may be different from the best bid and ask of the market. A smaller or regional bank may post more aggressive prices (higher bids or lower asks) because they will have relatively fewer credit relationships, implying that they will see fewer dealable prices generally.

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[11] However, in the EBS data set, the reversal (bid higher than ask) can happen, when the two parties do not have credit lines each other, and there is no third party that has credit lines to the two quote posting parties. The EBS system facilitates, as part of the dealing rules, each institution to control bilateral credit lines. Namely, each EBS-linked institution sets credit lines (including zero) against all other potential counter-parties. Therefore, an institution faces a restriction of bid, offer, or deal from other institutions. When bid and offer rates are posted for the system, they are not necessarily available to all participants of the EBS system. The EBS-registered trader's screen shows the best bid and best offer of the market and best bid and best offer for that particular institution. In normal times, the best bid of the market is lower than the best offer of the market. Otherwise, some institution that has positive credit lines with both institutions on the bid and ask sides will be able to make profits by arbitrage.

[12] The buyer-initiated trades (the seller-initiated trades) used in Berger et al. (2005) corresponds to the number of deals on ask side (the number of deals on bid side) in our paper, respectively. The order flow, the net excess of buyer-initiated trades in Berger et al. corresponds to the *netdeal* in our paper. Berger et al. had access to the data of actual transaction volumes---proprietary data of EBS---while we use the number of seconds in which at least one deals was done. The number of deals, rather than the signed (actual) volume, is good enough proxy for the volume of transaction. In fact, the actual transaction volume is not revealed to participants other than parties involved, so that they would not be able to be used in prediction of price movement in real time.

This trend means that "hot potatoes" (Lyons (1997)) are less important now, and a cool supercomputer is increasingly important. In other words, dealers' tactics to transform order flows from the corporate sector into the interbank market may be less influential than before, and the dealers' behavior in posting firm bids and asks through the electronic broking system is more influential then before.[13]

3. Patterns of Quote and Deal prices

3.1. Run

In this section, the directional patterns of quote and deal price changes will be examined, calculating conditional probabilities of sign of changes. In the foreign exchange market, quote prices fluctuate as limit orders are newly entered, hit or withdrawn, and deal prices fluctuate as deals are done at different prices. By taking a first difference of prices, p_t-p_{t-1}, a series of binary choices consists of sings of price changes is generated. For example, … - + + + - - + + - + + + + - - - + - + + - - …. is a new series.[14]

Many theoretical works derive or assume that the exchange rate follows a random walk process. The random walk hypothesis implies that probability of price increases, irrespective of history, in the next change should be 0.5 (given the size of increase/decrease in prices is symmetric). Thus, probability that the price is revised higher for *n* successive trade (that is, the sign of price change is positive for *n* successive quotes) equals 0.5^n. Similarly, probability that price is revised lower for n successive trade is 0.5^n.

The probability, calculated from the new data series, that the next price revision is positive (negative) conditional on the *n* successive positive (negative) price revisions is as follows:

$$P_n(+\,|\,++,,,+(n\ pluses)) = \frac{N(+\,|\,++,,,+,-)}{N(+\,|\,++,,,+,-) + N(-\,|\,++,,,+,-)}, \qquad (1)$$

where $P_n(\ |\ )$ is the calculated conditional probability of continuing a run, given the N successive price change in the same direction; N(|) is the number of successive price

13 Our interviews (in November 2003) with banks with substantial foreign exchange trading in London revealed that they had reduced the degree of discretion of dealers and shifted proprietary trading to the specialized section. Computer models have replaced dealers' instincts.

14 Methodology employed in this paper follows the up-down analysis of Figure 6 in Mizuno, Kurihara, Takayasu and Takayasu (2003), where they used CQG quote data for a test of trend existence.

changes; “+” means positive price revisions (price going up), and “–“ means negative price revisions (price going down). The “run”—the number of successive changes in the same direction—is defined as those after the change in the direction N+1 times earlier.[15]

If $P_n(+|++,,,+,-)$ is higher than 0.5 and statistically different from a null of $(0.5)^n$, then the process is said to have “a momentum.” Given a history of a run—positive (negative) changes n times in a row—it is more likely that the price will move up (down, resp.) again in the next change. If it is less than 0.5, then the process is said to have a nature of “mean reversion,” that is a sign reversal in the next change is more likely, given a history of a run.

In the data, the conditional probability of a run may be different from theoretical probability with an assumption of random walk. Table 1 shows the number of samples of a run. For example, N=2 (+) shows the number of runs that have two consecutive increases in the prices. Five kinds of prices are considered in this sample: (1) bid quote; (2) ask quote; (3) mid-price quote; (4) bid-side quote; and (5) ask-side quote.

<u>Table 1</u>

In the following, we calculate the conditional probability of equation (1) for five different kinds of prices: bid quote, ask quote, mid-price of bid and ask quotes, bid-side deal price, and ask-side deal price. The probability is calculated up to 13 successive (either positive or negative) changes. In calculating probability, Saturdays, Sundays and Mondays are excluded from the data set.[16]

3.2. Results

Figures 1-1 through 1-5 show the conditional probability of price changes in USD/JPY, Figures 2-1 through 2-5 show the conditional probability of EUR/USD. In each figure, the horizontal axis shows numbers of successive positive/negative price revisions. The probability at $n=0$ can provide an unconditional test of the random walk hypothesis. The conditional probability at $n=j$, for example, indicates the probability of price increase/decrease in the next price change after j successive price hike (down).

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15 Therefore there is no “double counting” of a run. That is, a run of four successive changes (++++) in the same direction contain two successive changes and three successive changes as a subset of the four successive change, they are not counted in the N=2, or N=3 definitions.

16 Mondays are excluded because many national holidays fall into Mondays. The patterns of price changes on Monday are slightly different from other business days.

3.3. Conditional probability, USD/JPY

Figures 1-1 and 1-2 show the calculated conditional probability of bid and ask price for currency pair of USD/JPY, respectively. As is clear from these figures, similar patterns of price movement are seen in both bid and ask prices. At *n=0*, the probability of positive revision (white square) is slightly above 0.5 and the probability of negative revision (black circle) is slightly below 0.5 for bid price. This means that the bid price is more likely to be revised higher. The result is opposite for the ask price: At *n=0,* the probability of negative revision (black circle) is slightly above 0.5 and the probability of positive revision (white square) is slightly below 0.5 for ask price. The ask price is more likely to be revised lower. Putting two results together, it implies that at *n*=0, the bid-ask spread tends to narrow, but the difference from neutral (P=0.5) is very slight.

At n between 1 and 9, the conditional probability of a run, either pluses or minuses, is substantially below 0.5 for both bid and ask prices. This implies that there is a mean-reversion tendency both for bid and ask prices. Probabilities of successive positive price changes (white square) is higher than that of negative price changes (black circle) for bid price (and *vice versa* for ask price), while they are far below 0.5. As n becomes larger, the probability tends to converge toward 0.5.

The conditional probability of mid-price for USD/JPY is shown in figure 1-3. The pattern of the mid-price probability is an average of bid and ask price patterns. No significance is detected between the probabilities of positive price revisions and negative revisions. For *n=0,* the conditional probability is almost exactly 0.5, i.e., consistent with a random walk hypothesis. However, for n between 1 and 7, the conditional probability is significantly below 0.5, indicating mean reversion, but as n becomes larger, price revisions become closer to unpredictable (P=0.5). Conditional probability becomes significantly larger than 0.5 for a run of more than 10 successive positive price revision (white square). Negative quote changes are mean reversion in a small-number (n<6) run, but become a momentum after a large-enough-number run.

Figures 1-4 and 1-5 show the conditional probability of bid-side deal and ask-side deal prices of USD/JPY, respectively. It is clear from these figures that patterns of the conditional probability for deal prices are quite different from those of quote prices in some important ways.

Figures 1-1—1-5

The probability of having a run of ask-side deal price increases (white square in

Figure 1-5) is higher than 0.5, and so is a run of bid-side deal price decreases (black circle in Figure 1-4). Once the price starts to rise on the ask side—that is, the buyer initiated deals—then the price increases tend to continue (P>0.5). Similarly, once the price starts to decline on the bid side—that is, the seller-initiated deals—then the price decreases tend to continue (P>0.5). These results are quite in contrast to results implied by quote price movements, and quite consistent with a real-world conventional view that there are some moments in time when a momentum is formed. When the buyers are eager to hit the ask quotes, and starts to drive up the prices, then this creates a momentum to push prices up further. Having deals is important in this process. Similarly, when sellers hit bid-prices, driving down the prices, with deals, this creates downward momentum. These movements support a view that momentum strategy is a winning strategy while the run continues. The run tends to continue for two to nine ticks.

The positive price change for bid-side deal prices (white square in Figure 1-4) does not have a tendency of a run, but a mean reversion, and so does the negative price change for ask-side deal prices (black circle in Figure 1-5).

As for deal bid prices, for $0 \leq n \leq 4$, conditional probability of negative price revision is around 0.55-0.58. Then it gradually declines to 0.53 at $n=8$. The probability of longer than 9-successive negative price revisions is not significantly different from 0.5. In contrast, the probability of positive price revision is 0.42-0.43 for $n=1$ and 2, and it remains stable around $P=0.45$ up to $n=9$. The probability of longer than 10-successive positive price revision is not significantly different from 0.5. Once the bid-side deal prices (seller-initiated deals) starts to decline, then a run of the same sign tends to continue.

Conditional probability of ask deal price has the exactly opposite pattern to that of bid deal price. The conditional probability of positive price revision is around 0.56-0.58 and higher than that for negative price changes. One of the contrasts to deal bid price is that the longer than 7-successive positive price revisions disappears.

3.4. Conditional probability, EUR/USD

Figures 2-1 and 2-2 show the conditional probability of bid and ask prices, respectively, for currency pair of EUR/USD. The patterns are quite comparable to those of the USD/JPY. In both ask and bid price revisions, the conditional probability at $n=0$ equals almost 0.5, indicating that the quote price moves more or less randomly. The conditional probability is lower than 0.5 at n>0, indicating that it is not likely that the quote prices tend to show mean reversion—the change following positive change tends to be negative, and vice versa. However, the conditional probability converges to 0.5 as n becomes higher, and power of mean reversion wanes.

Figure 2-3 shows the conditional probability of mid-price quote for EUR/USD. The pattern of the probability is similar to that of the USD/JPY. First, the conditional probability pattern shows that the quote price change shows no unconditional expected change (P_0=0.5), while it shows mean reversion at 1<n<6. Second, conditional probability becomes neutral (P=0.5) after n>7. Third, conditional probability of 10 or longer successive positive price revisions (white square) becomes significantly larger than 0.5. The last feature is in contrast to USD/JPY where *negative* price revisions become a momentum after n > 10.

Figures 2-4 and 2-5 show the conditional probability of bid-side and ask-side *deal* prices, respectively. Similar to USD/JPY, a run tends to continue (P>0.5) for a negative run of bid-side deal price, and for a positive run of ask-side deal prices. Again, buying pressure (buyer-initiated deals) generates a positive run, and selling pressure (seller-initiated deals) generates a negative run. As n becomes larger, the conditional probability converges toward 0.5.

The positive price change for bid-side deal prices does not have a tendency of a run, but rather a mean reversion, and so does the negative price change for ask-side deal prices.

Similar analysis for the EUR/JPY has been conducted, but the patterns are the same, and results are less robust (wider standard error) so that it is not presented here.

<u>Figures 2-1—2-5</u>

In summary, several salient features are detected regarding the conditional probability of a run. First, the USD/JPY and EUR/USD show similar patters with regard to patterns of conditional probability for a run for five different kinds of prices. Second, patterns for quote price changes are quite different from patterns for deal price changes. The quote prices tend to show mean reversion (i.e., conditional probability of a run is less than 0.5), regardless of a positive run or a negative run, either for bid or ask quotes. For deal prices, buyer-initiated deal prices (ask-side deal prices) tend to have a continuous positive run (i.e., conditional probability of a run of positive changes to continue exceeds 0.5), and seller-initiated deal prices (bid-side deal prices) tend to have a tendency of a continuous negative run. The contrast is striking, and this shows that any study based on quote prices will be misleading in describing how transactions are proceeding in the market. Once a “momentum” measured by deal prices fueled by buying pressure or selling pressure is formed, the momentum tends to continue.

4. Size of price change in the continuous quotes/deals

4.1. Acceleration or Deceleration?

In this section, the size of price change is examined. When a run is detected (i.e., ask-side positive run and bid-side negative run), is it more likely that the step of increase or decrease becomes larger (acceleration in momentum)? This question is interesting for the following reasons.

As the momentum trading works for several ticks (ask-side deals and bid-side deals), that is, the conditional probability is above 0.5, those who detect this momentum may want to join the bandwagon. It may be conjectured that those one-side movement may gather force until the momentum stops and possibly followed by a sharp reversal. The process of momentum and an eventual stop may be a reflection of a (rational) "bubble" phenomenon, if a stop means a correction of a significant degree, following acceleration of momentum. A rational bubble requires a process that a higher degree of correction has to be compensated by a larger step toward the end of the bubble process. This kind of a pattern is known as a stochastic bubble.

In contrast, a run may stop without having a reversal in the price. The run results in finding a new equilibrium level. Pattern 1 is a typical bubble process if the increase is followed by a significant amount of drop. Pattern 2 indicates a process that is a convergence to a new equilibrium after digesting some fundamental news. In order to distinguish the process, we first examine whether the step of increase or decrease would become larger or smaller as the run continues.

Patterns 1, 2

In the following, we analyze the size of price change in successive quotes/deals.

$$\textit{Size of price change} = \textit{price}(t+n) - \textit{price}(t) \qquad (2)$$

We calculate the size of price change for the phase of price increase only (or price decrease only) for *n* successive quotes/deals. Four types of prices are used for calculation: bid, ask, midprice, deals done on the bid side and deals done on the ask side.

Results are shown in figures 3-1 through 3-5 for USDJPY; and figures 4-1 through 4-5 for EURUSD. In each figure, the vertical axis shows the cumulative price increment (or decrease) and the horizontal axis shows the successive price increase (or decrease). The white circles show the price path of continuous price increase and black circles show the price path of continuous price decline. The symmetric dashed straight lines indicate the smallest cumulative change of n-successive price increase (decrease). The minimum price increase/decrease is called "pip" and its size is 0.01 for USD/JPY; and 0.0001 for EUR/USD.

Figure 3-1 shows a run of rise or fall of bid quotes for USD/JPY. As seen in the figure, the price paths of both the continuous rise and fall are linear up to n=10, indicating that the size of incremental increase or decrease is almost constant at each quote change. The size is slightly larger than the minimum increment ("pip") when bid prices are falling, while it is almost equal to the minimum increment, when bid prices are rising. The decreasing step for the bid quote becomes much larger when $n>10$.

Figure 3-2 shows the price rise or fall of ask quotes for USD/JPY. Again, the change in the price is linear up to n=10. The size of ask quote change is larger when the price is going up than when the price is going down. They are almost linear up to n=10. In contrast to the case with bid quote, the size of price increases is larger than the size of price decreases.

Figure 3-3 shows the Midprice of bid and ask quotes for USDJPY. Again, the price paths of price increase/fall are linear up to n=12, and they are almost symmetric. The decrease of the mid-price quotes after n=12 accelerates, while no such acceleration is detected in the rise of mid-price quote.

Figures 3-4 and 3-5 show the rise and fall of deal price run. The pattern shows the contrast to the run of bid/ask quotes. The path of price increase (white circle) for deal bid is well above the straight line, indicating that the size of continuous price increase is larger than the continuous price fall in the run of bid-side deals, and while the sizes of continuous price fall is larger than sizes of price increase in the run of ask-side deals. In figure 3-5, at n=12 or larger, it is found that the ask-side deal price fall accelerates: when transaction price continues to fall for more than 12 times, the selling price starts falling faster.

As argued above, the price increase with buying pressure tends to exhibit a positive run of the ask-side deal price and the price decrease with selling pressure tends to exhibit a negative run of the bid-side deal price. For these price changes no acceleration is detected.

<u>Figures 3-1—3-5</u>

Figures 4-1 through 4-5 show the price paths for EURUSD. Figure 4-1 shows the price increment/fall of bid quotes. Similar to the case for bid quotes of USDJPY, the price paths of continuous rise/fall are linear up to *n*=12, and the sizes of price fall (black circle) is a bit wider than that of price increase.

Figure 4-2 shows the price paths of ask quotes. In contrast to the bid quotes, the size of price increase is larger than the size of price fall. The paths of price rise/fall are linear up to *n*=8. The pace of price increase accelerates after *n*=10, indicating that the ask quote price increases once the quote price start increasing. Figure 4-3 shows midprice of bid and ask quotes of EURUSD. Again, the price paths of price increase/fall are linear and almost symmetric.

<u>Figures 4-1—4-5</u>

Figures 4-4 and 4-5 show the price path of a run of deal price changes. The size of price increase is slightly wider than the size of price fall for deals on bid side, and vise versa for deal prices on ask side. In both cases, the price paths are almost linear and there is no evidence that the price soars once deal price starts increasing.

In summary, there is no difference in patterns of price paths between the two currency pairs. It is found that the price increases (decreases) tend to accelerate as price continues to rise (fall). The size at each price change is mostly constant regardless of its price—bid, ask, midprice, deals done on the bid side, or deals done on the ask side.

4.2. Size of Correction after a run

In this section, we test whether the size of an opposite movement at the termination of a run becomes larger as the number of a run so far is larger and whether the cumulative increases (decreases) during the run becomes disproportionately larger. The following specifications are adopted for examinations:

$$|\Delta p_{n+1}| = \alpha + \beta n + \varepsilon_{n+1} \qquad (3)$$

After the run with the length of *n*, the direction of the price movement reverses, shown in Δp_{n+1}. If $\beta=0$, then there is no relation between the size of a correction and the length of a run. The run is more likely associated with a transition from an old equilibrium to a new equilibrium. On the other hand, if $\beta>0$, then the longer the run, the larger the correction. This will be a suggestive evidence of a speculative bubble. This would suggest a scenario that the price deviates from a fundamental value and then crashes back to a fundamental value. Recall panels 4-1 and 4-2 for the two cases, respectively.

The second specification of the same test is to use the cumulative price change rather than the just a number of the successive changes.

$$|\Delta p_{n+1}| = \alpha + \beta \sum_{j=1}^{n} \Delta p_j + \varepsilon_{n+1} \qquad (4)$$

Where $\sum_{j=1}^{n} \Delta p_j$ denotes the cumulative price change of the run in the length of n. The interpretation of β is the same as equation (3). In addition, if β =1, then all the cumulative change is wiped out in one change after the run.

Table 2-1 and Table 2-2 summaries the regression results of equation (3) and (4), respectively. In each regression, β is estimated for either a run with positive price changes only (increase), or a run with negative price changes (decrease). “All” shows the estimated coefficient of β using all of the run.

<u>Tables 2-1, 2-2</u>

As shown in Table 2-1, β is estimated significantly positive in all cases regardless of the currency pairs. This implies that the longer the run, the larger the correction. We also find that the size of a correction is larger after the run with price increase than the run of price fall for ask and/or deal price on the ask side. On the other hand, the size of a correction is larger after the price decrease run than the price increase run for bid and/or deal price on the bid side. Note that price increases here means the yen depreciation vis-à-vis the US dollar, and the dollar depreciates vis-à-vis the euro.

The estimation results of equation (4) is shown in Table 2-2. Again, β is estimated significantly positive in all cases regardless of the currency pairs. That is, the larger the cumulative price increases (decreases), the size of price revision after the run will be larger. We also find the similar patterns in the coefficient—the size of correction is larger after the run of price increase than the run of price fall for ask and/or deal price on the ask side, and vice versa for bid and/or deal price on the bid side.

5. Concluding remarks

In this paper, a random walk hypothesis is refuted in a simple test of a run—continuous increases or decreases in quote prices and deal prices—using tick-by-tick exchange rate data. Several questions exploiting the tick-by-tick data have been examined in this paper. Will the changes in ask (bid) deal prices, as well as ask (bid) quote prices, be influenced by the previous transactions? If a deal price goes up from its previous deal, then will the next deal price be more likely to go up or not? How long do prices continue to increase (decrease)? How about the size of price changes? We have examined the patterns of quote activities and transaction activities in foreign exchange market using a very rich data set.

The main findings are as follows. First, conditional probability that the price is to be revised in the same direction is lower than 0.5 for both bid and ask quotes. Therefore, the quote price moves more like in a mean-reverting manner. On the contrary, deal prices are more likely to continue rising or falling. For deal prices done on the bid side, conditional probability that the price is revised lower exceeds 0.5. Similarly, for deal prices done on the ask side, conditional probability that the price is revised higher exceeds 0.5. Second, we do not see the size of price increment/fall at each transaction (quote) is widening or shrinking as a run continues. Again, regardless of currency pairs, the size of price fall is larger than that of price increase in bid-quote prices, and *vice versa* in ask-quote prices. In contrast, size of price increase is larger than that of price fall for deals done on the bid side, and *vise versa* for deals done on the ask side. Third, we estimate the size of price correction after the continuous price increase (decrease). The absolute size of price correction is larger when a run becomes longer. It is also found that the absolute size of price correction is larger when the cumulative change in price in a run is larger.

What is found at a very high-frequency level in exchange rate markets is that the exchange rate mostly follows random walk, but there appears a so-called mini bubble once exchange rate starts rising (falling). This was evident in the conditional probability of a deal price run, and the size of correction after a run.

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Table 1 Summary of “Run” January 1998 to October 2003

Number of samples of a run with + or - USD/JPY													
		N=1 (all)	N=2	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=11	N=12
Quote bid	+	4820958	1648403	569926	211046	82612	33827	14411	6486	3070	1469	712	367
	-	4442196	1269642	382374	126624	44893	16890	6677	2775	1166	516	237	105
Quote ask	+	4563975	1275940	364736	113306	37615	13315	4991	1953	793	326	143	50
	-	4957516	1669481	569214	209820	82457	34124	14782	6658	3152	1559	802	166
Quote mid	+	8015204	3182504	1234233	508938	218742	97330	44664	21086	10224	5086	2595	1339
	-	8006548	3173848	1235730	515430	226005	103228	48520	23515	11717	6034	3187	1688
Ask-side deal	+	1694588	942219	527967	293037	159842	85665	45239	23521	12075	6149	3137	1582
	-	1298931	546562	249941	115173	52812	24132	11018	5027	2278	1037	464	209
Bid-side deal	+	1242564	519298	234787	106710	48202	21554	9637	4295	1903	842	391	195
	-	1654012	930746	528965	297994	165346	90096	48344	25515	13357	6926	3585	1873

Number of samples of a run with + or - EUR/USD													
		N=1 (all)	N=2	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10	N=11	N=12
Quote bid	+	4016461	1243324	412360	151179	59367	24715	10729	4818	2269	1116	559	290
	-	3874695	1101558	351711	124955	47409	18987	7965	3431	1491	680	320	152
Quote ask	+	3960877	1107694	340999	116400	42782	16736	6925	2976	1324	599	281	149
	-	4108672	1255490	411685	150236	58990	24535	10788	4863	2261	1059	509	257
Quote mid	+	6455384	2542775	1006191	434786	195691	92044	44395	21993	11248	5913	3201	1788
	-	6427969	2515360	995392	433340	197693	93880	45516	22567	11404	5848	3024	1546
Ask-side deal	+	1616044	845883	449193	235488	121269	61310	30574	15071	7376	3633	1773	846
	-	1376109	605948	288292	137957	65417	30597	14202	6544	3017	1381	649	313
Bid-side deal	+	1350884	593361	280617	132924	62322	28902	13136	5920	2675	1184	536	250
	-	1599561	842038	450945	240016	125793	64663	32762	16432	8097	3962	1962	976

Table 2-1

	Bid			Ask			Deal bid			Deal ask		
	increase	decrease	all	increase	decrease	all	increase	decrease	all	increase	decrease	all
USDJPY												
β	6.93E-04 ***	8.39E-04 ***	7.57E-04 ***	1.08E-03 ***	5.90E-04 ***	7.95E-04 ***	2.22E-04 ***	1.44E-04 ***	3.64E-04 ***	1.85E-04 ***	2.06E-04 ***	3.60E-04 ***
(s.e.)	6.44E-06	7.89E-06	5.02E-06	8.37E-06	7.02E-06	5.39E-06	1.01E-05	1.17E-05	7.83E-06	1.01E-05	1.16E-05	7.46E-06
nob	3108100	3108100	6216201	3219839	3219839	6439678	687279	687279	1374558	712094	712095	1424189
EURUSD												
β	4.76E-06 ***	3.23E-06 ***	4.04E-06 ***	4.53E-06 ***	4.13E-06 ***	4.31E-06 ***	9.15E-07 ***	1.31E-06 ***	1.86E-06 ***	1.83E-06 ***	8.37E-07 ***	2.07E-06 ***
(s.e.)	4.38E-08	4.70E-08	3.21E-08	5.26E-08	4.70E-08	3.51E-08	7.38E-08	8.34E-08	5.62E-08	8.00E-08	6.57E-08	5.25E-08
nob	3313249	3313250	6626499	3406970	3406970	6813940	915094	915095	1830189	927990	927990	1855980

Note: *** indicates the significance level at 1%.

Table 2-2

	Bid			Ask			Deal bid			Deal ask		
	increase	decrease	all	increase	decrease	all	increase	decrease	all	increase	decrease	all
USDJPY												
β	1.17E-01 ***	2.24E-01 ***	1.78E-01 ***	2.67E-01 ***	1.12E-01 ***	2.02E-01 ***	3.27E-02 ***	4.55E-02 ***	4.36E-02 ***	4.95E-02 ***	3.55E-02 ***	4.60E-02 ***
(s.e.)	4.45E-04	4.02E-04	2.98E-04	4.04E-04	4.89E-04	3.14E-04	4.98E-04	7.96E-04	4.74E-04	6.91E-04	5.73E-04	4.50E-04
nob	3108100	3108100	6216201	3219839	3219839	6439678	687279	687279	1374558	712094	712095	1424189
EURUSD												
β	6.69E-02 ***	1.16E-01 ***	9.27E-02 ***	1.69E-01 ***	6.59E-02 ***	1.23E-01 ***	2.22E-02 ***	3.25E-02 ***	2.97E-02 ***	4.18E-02 ***	2.13E-02 ***	3.43E-02 ***
(s.e.)	3.43E-04	3.31E-04	2.38E-04	3.36E-04	3.73E-04	2.50E-04	4.91E-04	6.63E-04	4.15E-04	6.00E-04	4.46E-04	3.79E-04
nob	3313249	3313250	6626499	3406970	3406970	6813940	915094	915095	1830189	927990	927990	1855980

Note: *** indicates the significance level at 1%.

Figure 1-1: Conditional probability of bid price, USDJPY

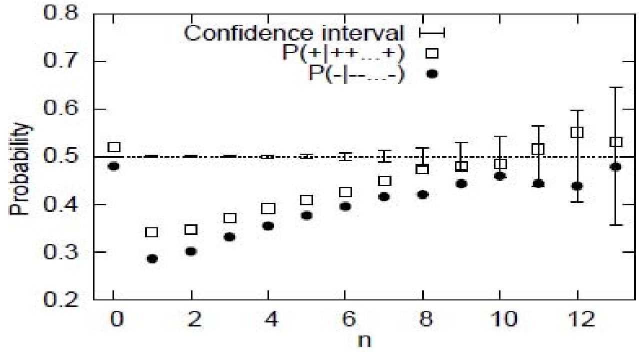


Figure 1-2: Conditional probability of ask price, USDJPY

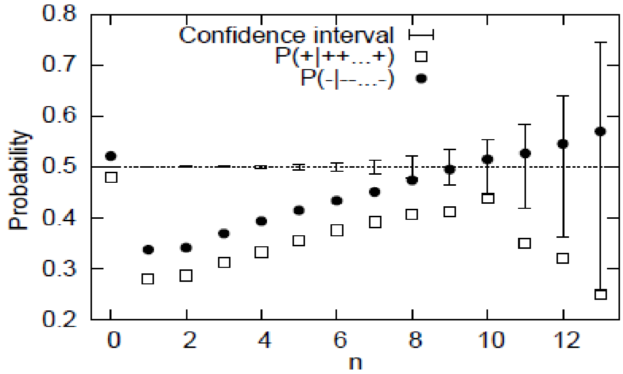


Figure 1-3: Conditional probability of mid-price, USDJPY

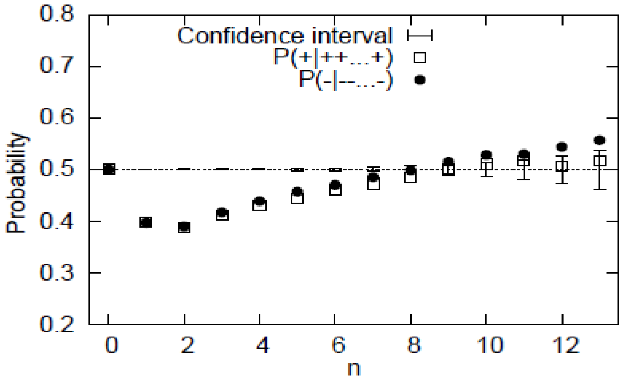


Figure 1-4: Conditional probability of Deal bid price, USDJPY

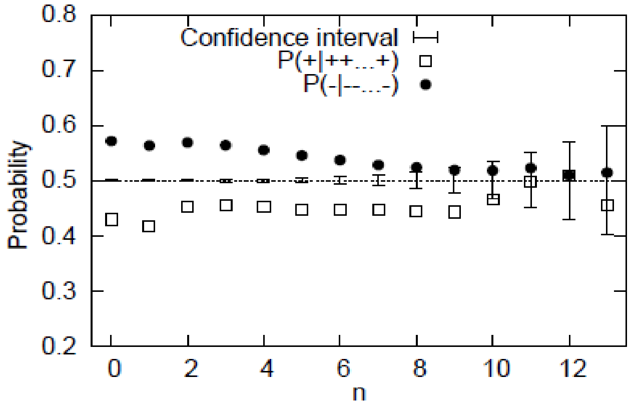


Figure 1-5: Conditional probability of Deal ask price, USDJPY

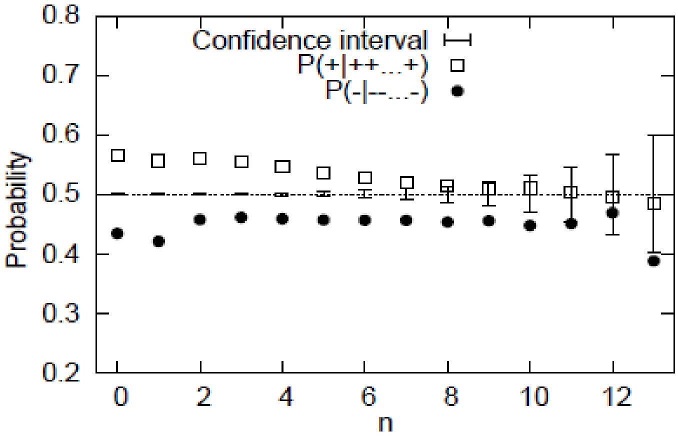


Figure 2-1: Conditional probability of bid price, EURUSD

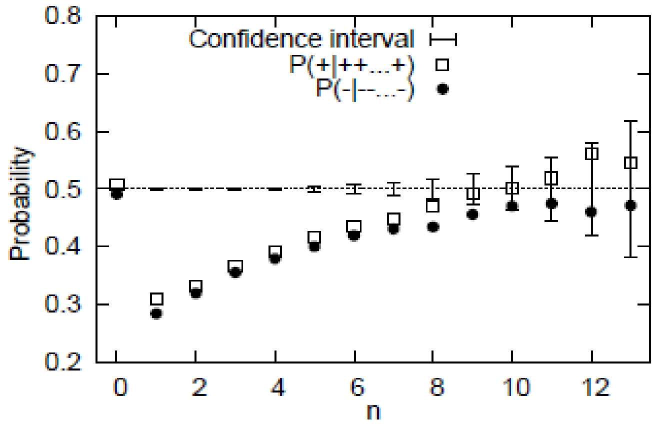


Figure 2-2: Conditional probability of ask price, EURUSD

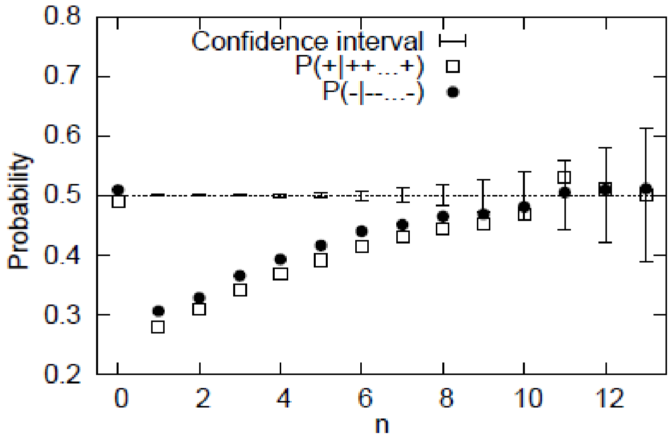


Figure 2-3: Conditional probability of mid-price, EURUSD

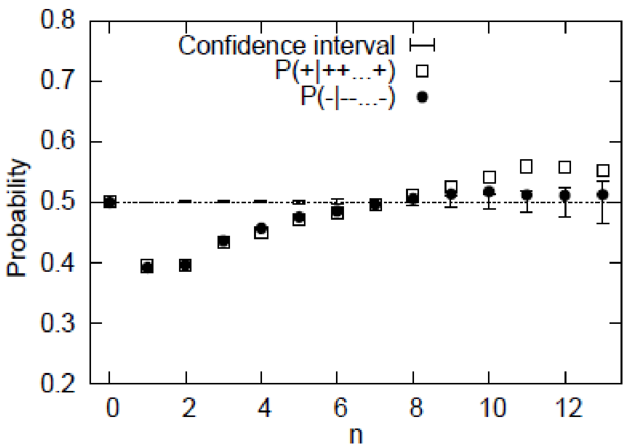


Figure 2-4: Conditional probability of Deal bid price, EURUSD

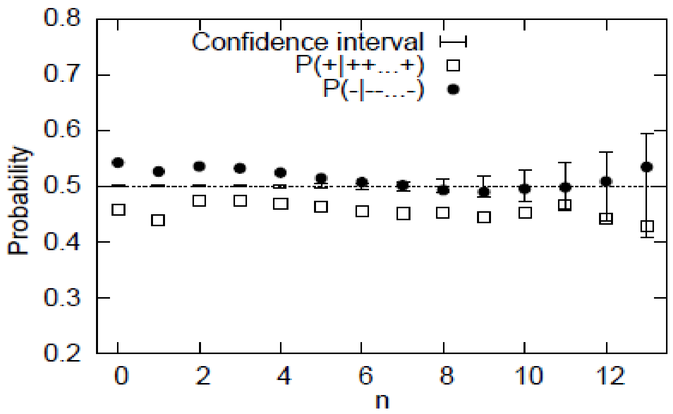


Figure 2-5: Conditional probability of Deal ask price, EURUSD

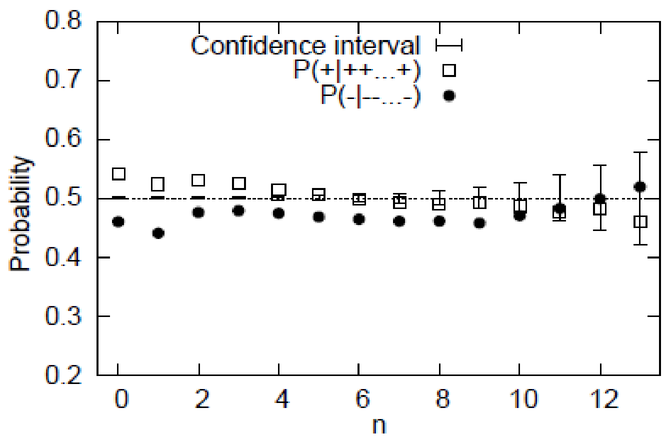


Figure 3-1. USD/JPY Bid

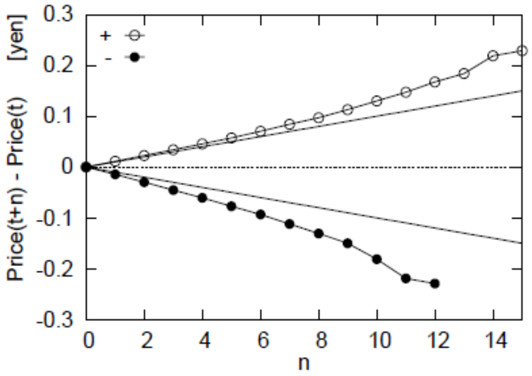


Figure 3-2. USD/JPY Ask

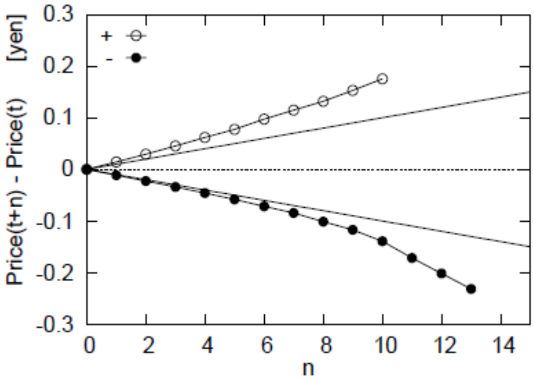


Figure 3-3 Midprice

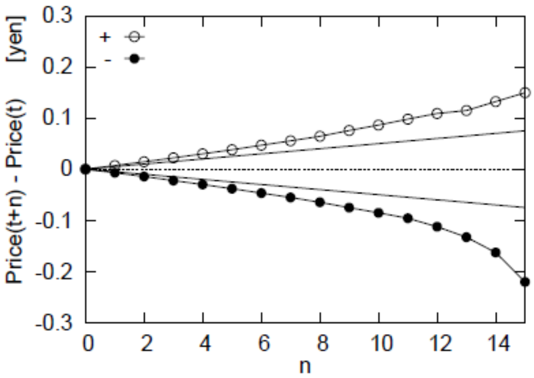


Figure 3-4. USD/JPY Deal bid

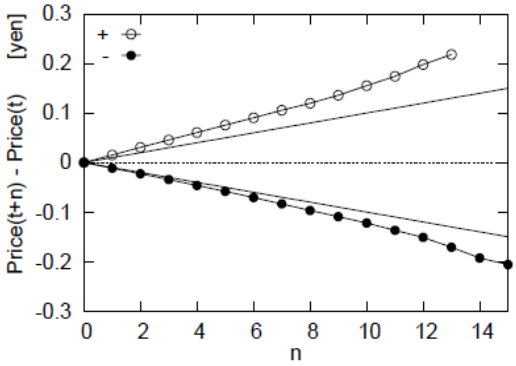


Figure 3-5. USD/JPY Deal Ask

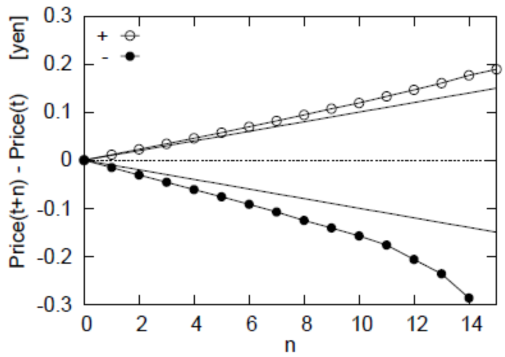


Figure 4-1 Bid, EURUSD

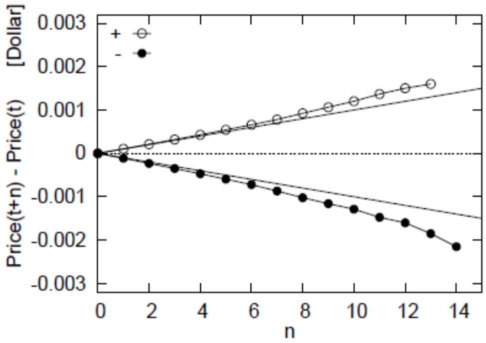


Figure 4-2 Ask, EURUSD

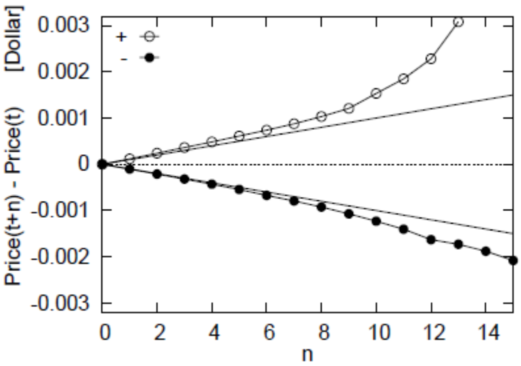


Figure 4-3 Midprice, EURUSD

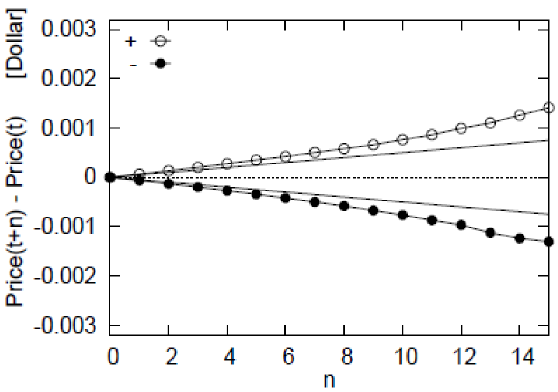


Figure 4-4 Deal bid, EURUSD

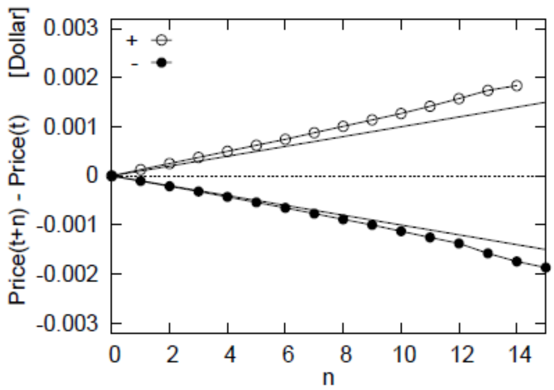
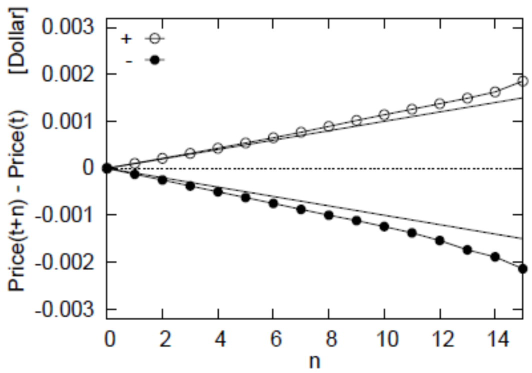
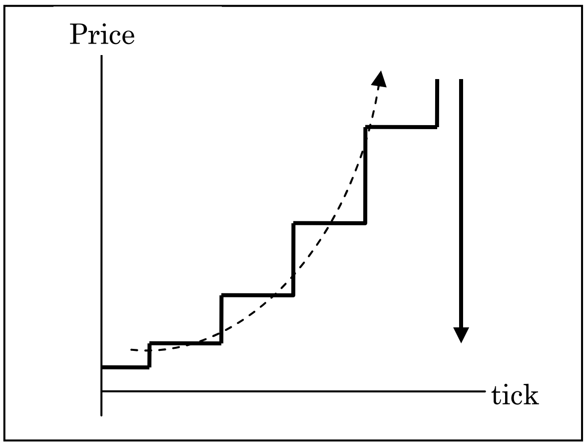


Figure 4-5 Deal ask, EURUSD



Pattern 1:   pattern of a bubble



Pattern 2:   pattern of convergence