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A Mechanism Design for a Solution to the Tragedy of Commons*

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Abstract

We consider the problem of the tragedy of commons in cooperative production economies, and propose a mechanism to resolve the tragedy, taking seriously non-negligible aspects of that problem and the real application of mechanisms. The mechanism permits agents to choose their own labor time freely, and to announce their labor skills freely not only if they choose to understate them, but also if they overstate them. It doubly implements the *proportional solution* [Roemer and Silvestre (1989, 1993)] in Nash and strong equilibria when it is played as a normal game form, while it triply implements the solution in Nash, subgame-perfect, and strong equilibria when it is played as a two-stage extensive game form.

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1 Introduction

It is well-known as the "*tragedy of commons*," that in cooperative production economies, the resource allocations under free access to technology result in "overproduction" and inefficient Nash equilibria. This paper provides a mechanism which can solve this tragedy problem. As a normative solution for the tragedy, we adopt the *proportional solution* [Roemer and Silvestre (1993)] under *joint ownership of the technology*, which assigns Pareto efficient allocations where each agent's output consumption is proportional to his labor contribution. Then, we construct an incentive compatible mechanism which implements the proportional solution.

There are some works on implementation of the proportional solution, such as Suh (1995), Yoshihara (1999, 2000), and Tian (2000), as well as of other social choice correspondences in production economies.[1] However, in most of the literature on implementation in production economies, a non-negligible problem of asymmetric information involved in the production process seems to be treated as a "*black box*." Under any mechanism, each agent is usually required to announce some information, and the outcome function assigns an allocation to each profile of agents' strategies. This implicitly assumes, in the case of production economies with labor input, that the mechanism coordinator is authorized to make agents supply their labor hours to be consistent with the assigned allocation.[2] This happens because the original concern of implementation theory has been in *adverse selection problems*, and such a focus is valid whenever we consider decentralized resource allocations in exchange economies and/or production economies with no labor input. However, in production economies with labor input, such an implicit assumption is not so realisitic.

In this paper, alternatively, we suppose that the coordinator is not authorized to make agents work as he wants; he can monitor each agent's labor hour, but he cannot perfectly monitor each agent's *labor contribution measured in efficiency units*, since he is incapable of observing each agent's *labor skill* or *labor intensity* exercised in the production process. Thus, there may be an incentive for each agent to *overstate* as well as *understate* his own

[1] For example, Hurwicz *et al.* (1995), Hong (1995), Tian (1999) for private ownership production economies with only private goods, Varian (1994) for production economies with externality, and Kaplan and Wettstein (2000) and Corchón and Puy (2002) for cooperative production economies.

[2] Roemer (1989) pointed out this implicit assumption explicitly.

labor skill or labor intensity.[3] Even under such a more realistic model of the tragedy of commons, the incentive compatible mechanism in this paper can implement the solution.

The mechanism we propose here is a type of *sharing mechanism*: each agent can freely supply his labor hour,[4] and he is asked to give some information about his demand for the consumption good and his labor skill. After that, the outcome function only distributes the produced output to agents, according to the given information and the record of their supply of labor hours done. Here, there is no restriction on strategy spaces which prohibits agents from understating or overstating their labor skills. We will show in this study that this mechanism *triply implements* the proportional solution *in Nash, strong Nash, and subgame perfect equilibria.*

In the following discussion, a basic model of economies and sharing mechanisms is defined in Section 2. Section 3 provides a sharing mechanism which implements the proportional solution. Concluding remarks are in Section 4. All the proofs of the theorems here will be relegated to the Appendix.

2 The Basic Model

There are two goods, one of which is an input good (labor time) $x \in \mathbb{R}_+$ to be used to produce the other good $y \in \mathbb{R}_+$.[5] The population in the society is given by the set $N = \{1, \ldots, n\}$, where $2 \leq n < +\infty$. Each agent i's consumption vector is denoted by $z_i = (x_i, y_i)$, where x_i denotes his labor time, and y_i denotes his assigned amount of output consumption. It is assumed that all agents face a common upper bound of labor time $\bar{x}$, where $0 < \bar{x} < +\infty$, so that they have the same consumption set $[0, \bar{x}] \times \mathbb{R}_+$. Each

[3] Tian (2000) constructed a mechanism which implements the proportional solution even if the coordinator does not know the agents' endowment vectors of commodities under the assumption that agents cannot overstate their endowments. As Tian (2000) himself mentioned, such an assumption may be justified when endowments consist only of material goods, since the coordinator can require agents to "place the claimed endowments on the table" (Hurwicz *et al.* (1995)). In our setting where endowments are labor skills, however, such a requirement is no longer forceful, since the coordinator may not inspect the amount of labor skills in advance of production.

[4] Thus, our mechanism is *labor sovereign* (Kranich (1994); Yoshihara (2000a)) which is not satisfied in the previous mechanisms (Suh (1995), Yoshihara (1999, 2000), Tian (2000)).

[5] The symbol $\mathbb{R}_+$ denotes the set of non-negative real numbers.

agent i's preference is defined on $[0,\bar{x}] \times \mathbb{R}_+$ and represented by a utility function $u_i : [0,\bar{x}] \times \mathbb{R}_+ \rightarrow \mathbb{R}$ which is continuous and quasi-concave on $[0,\bar{x}] \times \mathbb{R}_+$, and strictly monotonic (decreasing in labor time and increasing in the share of output) on $[0,\overline{x}) \times \mathbb{R}_{++}$.[6] We denote by $\mathcal{U}$ the class of such utility functions. Each agent i is also characterized by a *labor skill* which is represented by a positive real number, $s_i^t \in \mathbb{R}_{++}$. The superscript t on s_i^t indicates "true," so that s_i^t denotes *agent i's true labor skill.* The universal set of labor skills for all agents is denoted by $\mathcal{S} = \mathbb{R}_{++}$.[7] The labor skill $s_i^t \in \mathcal{S}$ implies i's *labor endowment* per unit of labor time, which is measured in efficiency units. It can also be interpreted as i's *labor intensity* which would be exercised in the production process: that is, i's labor input per unit of labor time measured in efficiency units. Thus, if his *supply of labor time* is $x_i \in [0,\bar{x}]$ and his labor intensity is $s_i^t \in \mathcal{S}$, then it is $s_i^t x_i \in \mathbb{R}_+$ which implies his *labor contribution* to the production process measured in efficiency units. The production technology is described by a production function $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which is assumed to be continuous, strictly increasing, concave, and $f(0) = 0$. For simplicity, we fix a production function f for all economies. Thus, the economy is characterized by a pair of profiles $\mathbf{e} \equiv (\mathbf{u}, \mathbf{s}^t)$ with $\mathbf{u} = (u_1, \ldots, u_n) \in \mathcal{U}^n$ and $\mathbf{s}^t = (s_1^t, \ldots, s_n^t) \in \mathcal{S}^n$. Denote the class of such economies by $\mathcal{E} \equiv \mathcal{U}^n \times \mathcal{S}^n$.

Given $\mathbf{s}^t = (s_1^t, \ldots, s_n^t) \in \mathcal{S}^n$, an allocation $\mathbf{z} = (x_i, y_i)_{i \in N} \in ([0,\bar{x}] \times \mathbb{R}_+)^n$ is *feasible for* $\mathbf{s}^t$ if $\sum y_i \leq f\left(\sum s_i^t x_i\right)$. We denote by $Z(\mathbf{s}^t)$ the set of feasible allocations for $\mathbf{s}^t \in \mathcal{S}^n$. An allocation $\mathbf{z} = (z_1, \ldots, z_n) \in ([0,\bar{x}] \times \mathbb{R}_+)^n$ is *Pareto efficient for* $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ if $\mathbf{z} \in Z(\mathbf{s}^t)$ and there does not exist $\mathbf{z}' = (z'_1, \ldots, z'_n) \in Z(\mathbf{s}^t)$ such that for all $i \in N$, $u_i(z'_i) \geq u_i(z_i)$, and for some $i \in N$, $u_i(z'_i) > u_i(z_i)$. The *proportional solution* [Roemer and Silvestre (1993)] is a correspondence $PR : \mathcal{E} \twoheadrightarrow ([0,\bar{x}] \times \mathbb{R}_+)^n$ such that for each economy $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, any $\mathbf{z} = (x_i, y_i)_{i \in N} \in PR(\mathbf{e})$ is a Pareto efficient allocation for $\mathbf{e}$, where for all $i \in N$, $y_i = \frac{s_i^t x_i}{\sum s_j^t x_j} f\left(\sum s_j^t x_j\right)$.

A *normal-form game form* is a pair $\Gamma = (M, h)$, where $M = M_1 \times \cdots \times M_n$, M_i being the *strategy space of agent* $i \in N$, and $h : M \rightarrow ([0,\bar{x}] \times \mathbb{R}_+)^n$ being the *outcome function* which associates each $\mathbf{m} \in M$ with a unique element $h(\mathbf{m}) \in ([0,\bar{x}] \times \mathbb{R}_+)^n$. The i-th component of $h(\mathbf{m})$ will be denoted by $h_i(\mathbf{m}) \equiv (h_{i1}(\mathbf{m}), h_{i2}(\mathbf{m}))$, where $h_{i1}(\mathbf{m}) \in [0,\overline{x}]$ and $h_{i2}(\mathbf{m}) \in \mathbb{R}_+$. Given

[6] The symbol $\mathbb{R}_{++}$ denotes the set of positive real numbers.

[7] For any two sets X and Y, $X \subseteq Y$ whenever any $x \in X$ also belongs to Y, and $X \subsetneq Y$ if and only if $X \subseteq Y$ and *not* $(Y \subseteq X)$.

$\mathbf{m} \in M$, let $\mathbf{m}_{-i} = (m_1, \cdots, m_{i-1}, m_{i+1}, \cdots, m_n)$. A normal-form game form $\Gamma = (M, h)$ is *labor sovereign* if for every $i \in N$ and every $x_i \in [0, \bar{x}]$, there exists a strategy $m_i \in M_i$ such that for any $\mathbf{m}_{-i} \in M_{-i}$, $h_{i1}(m_i, \mathbf{m}_{-i}) = x_i$.

2.1 Sharing mechanisms

We are interested in labor sovereign game forms, and focus on *sharing mechanisms* that only distribute output among the agents according to their announcements on their private information $(\mathbf{s}, \mathbf{w}) \in \mathcal{S}^n \times \mathbb{R}_+^n$ and their supplied labor time $\mathbf{x} \in [0, \bar{x}]^n$. A *sharing mechanism* is a function $g : \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n \to \mathbb{R}_+^n$ satisfying the following property: for any $\mathbf{s} \in \mathcal{S}^n$, any $\mathbf{x} \in [0, \bar{x}]^n$, and any $\mathbf{w} \in \mathbb{R}_+^n$, $g(\mathbf{s}, \mathbf{x}, \mathbf{w}) = \mathbf{y}$, where $\mathbf{s} = (s_1, \ldots, s_n)$ denotes the agents' reported skills and $\mathbf{w} = (w_1, \ldots, w_n)$ their desired amounts of output consumption. A sharing mechanism g is *feasible* if for any $\mathbf{s}^t \in \mathcal{S}^n$, any $\mathbf{s} \in \mathcal{S}^n$, any $\mathbf{x} \in [0, \bar{x}]^n$, and any $\mathbf{w} \in \mathbb{R}_+^n$, $(\mathbf{x}, g(\mathbf{s}, \mathbf{x}, \mathbf{w})) \in Z(\mathbf{s}^t)$. Note in feasible sharing mechanisms, even without the true information of labor skills $\mathbf{s}^t$, the total amount of output $f\left(\sum s_k^t x_k\right)$ is observable after the production process, since the coordinator can hold all of the produced output. Note also that any feasible sharing mechanism is a *labor sovereign normal-form game form.*[8] We denote by $\mathcal{G}$ the class of such feasible sharing mechanisms.

Given a feasible sharing mechanism $g \in \mathcal{G}$, a *feasible sharing game* is defined for each economy $\mathbf{e} \in \mathcal{E}$ as a non-cooperative game $(N, (\mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+)^n, g, \mathbf{e})$. Fixing the set of players N and their strategy sets $(\mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+)^n$, we simply denote a feasible sharing game $(N, (\mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+)^n, g, \mathbf{e})$ by $(g, \mathbf{e})$.

Given a strategy profile $(\mathbf{s}, \mathbf{x}, \mathbf{w}) \in \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n$, let $\left(\mathbf{s}_{s'_i}, \mathbf{x}_{x'_i}, \mathbf{w}_{w'_i}\right) \in \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n$ be another strategy profile which is obtained by replacing the i-th component (s_i, x_i, w_i) of $(\mathbf{s}, \mathbf{x}, \mathbf{w})$ with (s'_i, x'_i, w'_i). A strategy profile $(\mathbf{s}^*, \mathbf{x}^*, \mathbf{w}^*) \in \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n$ is a (*pure-strategy*) *Nash equilibrium of the feasible sharing game* $(g, \mathbf{e})$ if, for any $i \in N$ and any $(s_i, x_i, w_i) \in \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$,

$$u_i\left(x_i^*, g_i\left(\mathbf{s}^*, \mathbf{x}^*, \mathbf{w}^*\right)\right) \geq u_i\left(x_i, g_i\left(\mathbf{s}^*_{s_i}, \mathbf{x}^*_{x_i}, \mathbf{w}^*_{w_i}\right)\right).$$

Denote by $NE(g, \mathbf{e})$ the set of Nash equilibria of $(g, \mathbf{e})$. An allocation $\mathbf{z} = (x_i, y_i)_{i \in N} \in ([0, \bar{x}] \times \mathbb{R}_+)^n$ is a *Nash equilibrium allocation of the feasible*

[8] More correctly, any feasible sharing mechanism g is also expressed as a game form $\Gamma_g = \left(\mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n, h\right)$ such that for any $(\mathbf{s}, \mathbf{x}, \mathbf{w}) \in \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n$, $h(\mathbf{s}, \mathbf{x}, \mathbf{w}) = (\mathbf{x}, g(\mathbf{s}, \mathbf{x}, \mathbf{w}))$. Clearly, Γ_g is a labor sovereign normal-form game form, which is uniquely corresponding to g.

sharing game $(g, \mathbf{e})$ if there exists $(\mathbf{s}, \mathbf{w}) \in \mathcal{S}^n \times \mathbb{R}_+^n$ such that $(\mathbf{s}, \mathbf{x}, \mathbf{w}) \in NE(g, \mathbf{e})$ and $\mathbf{y} = g(\mathbf{s}, \mathbf{x}, \mathbf{w})$ where $\mathbf{x} = (x_i)_{i \in N}$ and $\mathbf{y} = (y_i)_{i \in N}$. Denote by $NA(g, \mathbf{e})$ the set of Nash equilibrium allocations of $(g, \mathbf{e})$. A feasible sharing mechanism $g \in \mathcal{G}$ is said to *implement* the proportional solution on $\mathcal{E}$ *in Nash equilibria*, if for all $\mathbf{e} \in \mathcal{E}$, $NA(g, \mathbf{e}) = PR(\mathbf{e})$.

A strategy profile $(\mathbf{s}^*, \mathbf{x}^*, \mathbf{w}^*) \in \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}_+^n$ is a *(pure-strategy) strong (Nash) equilibrium of the feasible sharing game* $(g, \mathbf{e})$, if for any $T \subseteq N$ and any $(s_i, x_i, w_i)_{i \in T} \in \mathcal{S}^{\#T} \times [0, \bar{x}]^{\#T} \times \mathbb{R}_+^{\#T}$, there exists $j \in T$ such that

$$u_j\left(x_j^*, g_j(\mathbf{s}^*, \mathbf{x}^*, \mathbf{w}^*)\right) \geq u_j\left(x_j, g_j\left((s_i, x_i, w_i)_{i \in T}, (s_k^*, x_k^*, w_k^*)_{k \in N \setminus T}\right)\right).$$

Denote by $SNE(g, \mathbf{e})$ the set of strong equilibria of $(g, \mathbf{e})$. An allocation $\mathbf{z} = (x_i, y_i)_{i \in N} \in ([0, \bar{x}] \times \mathbb{R}_+)^n$ is a *strong equilibrium allocation of the feasible sharing game* $(g, \mathbf{e})$ if there exists $(\mathbf{s}, \mathbf{w}) \in \mathcal{S}^n \times \mathbb{R}_+^n$ such that $(\mathbf{s}, \mathbf{x}, \mathbf{w}) \in SNE(g, \mathbf{e})$ and $\mathbf{y} = g(\mathbf{s}, \mathbf{x}, \mathbf{w})$. Denote by $SNA(g, \mathbf{e})$ the set of strong equilibrium allocations of $(g, \mathbf{e})$. A feasible sharing mechanism $g \in \mathcal{G}$ is said to *implement* the proportional solution on $\mathcal{E}$ *in strong equilibria*, if for all $\mathbf{e} \in \mathcal{E}$, $SNA(g, \mathbf{e}) = PR(\mathbf{e})$. Moreover, a feasible sharing mechanism $g \in \mathcal{G}$ is said to *doubly implement* the proportional solution on $\mathcal{E}$ *in Nash and strong equilibria*, if for all $\mathbf{e} \in \mathcal{E}$, $NA(g, \mathbf{e}) = SNA(g, \mathbf{e}) = PR(\mathbf{e})$.

2.2 Timing Problem in Sharing Mechanisms

Before discussing our own sharing mechanism for implementing the proportional solution, we should mention the timing problem of strategy-decision in real applications of sharing mechanisms, which is particularly relevent to the case of production economies. Note that the strategic action for (s, w) is only to announce a pair of two real numbers, while the strategic action for x is to engage in production activity by supplying that amount of labor time. Thus, there may be a time difference between the point in time when $(\mathbf{s}, \mathbf{w})$ is announced and the period when $\mathbf{x}$ is exercised. *It implies that there may be at least two polar cases of time sequence of decision making*: the agents may annouce $(\mathbf{s}, \mathbf{w})$ before they supply their labor hours, or they may annouce $(\mathbf{s}, \mathbf{w})$ after each agent supplies his labor time $\mathbf{x}$. In the former case, each agent i may decide his supply of labor time with the knowledge of the messages $(\mathbf{s}, \mathbf{w})$, while in the latter case, he may decide his message (s_i, w_i) with the knowledge of agents' actions $\mathbf{x}$ in production process.

Consequently, we may derive at least the following two types of two-stage game forms from the original normal form sharing mechanism. Given a feasible sharing mechanism $g \in \mathcal{G}$, the *(1) type g-implicit two-stage mechanism* Γ_g^1 is a two-stage extensive game form in which the first stage consists of selecting $(\mathbf{s}, \mathbf{w})$ from $\mathcal{S}^n \times \mathbb{R}_+^n$, the second stage consists of selecting $\mathbf{x}$ from $[0, \bar{x}]^n$, and the final stage assigns an outcome which is the same value as $g(\mathbf{s}, \mathbf{x}, \mathbf{w})$ for every choice $(\mathbf{s}, \mathbf{w}) \in \mathcal{S}^n \times \mathbb{R}_+^n$ in the first stage and every choice $\mathbf{x} \in [0, \bar{x}]^n$ in the second stage. Given $g \in \mathcal{G}$, the *(2) type g-implicit two-stage mechanism* Γ_g^2 is a two-stage extensive game form in which the first stage consists of selecting $\mathbf{x}$ from $[0, \bar{x}]^n$, the second stage consists of selecting $(\mathbf{s}, \mathbf{w})$ from $\mathcal{S}^n \times \mathbb{R}_+^n$, and the final stage assigns an outcome which is the same value as $g(\mathbf{s}, \mathbf{x}, \mathbf{w})$ for every choice $\mathbf{x} \in [0, \bar{x}]^n$ in the first stage and every choice $(\mathbf{s}, \mathbf{w}) \in \mathcal{S}^n \times \mathbb{R}_+^n$ in the second stage.

Given the feasible (1) type g-implicit two-stage game $\left(\Gamma_g^1, \mathbf{e}\right)$ and each strategy profile $(\mathbf{s}, \mathbf{w}) \in \mathcal{S}^n \times \mathbb{R}_+^n$ in the first stage of the game $\left(\Gamma_g^1, \mathbf{e}\right)$, let us denote its corresponding second stage subgame by $\left(\Gamma_g^1(\mathbf{s}, \mathbf{w}), \mathbf{e}\right)$. Let $\mathbf{x}^{\mathbf{e}} : \mathcal{S}^n \times \mathbb{R}_+^n \to [0, \bar{x}]^n$ be a *Nash equilibrium mapping* such that for each $(\mathbf{s}, \mathbf{w}) \in \mathcal{S}^n \times \mathbb{R}_+^n$, $\mathbf{x}^{\mathbf{e}}(\mathbf{s}, \mathbf{w})$ is a (pure-strategy) Nash equilibrium of the subgame $\left(\Gamma_g^1(\mathbf{s}, \mathbf{w}), \mathbf{e}\right)$. Denote the set of such Nash equilibrium mappings in second stage subgames of the game $\left(\Gamma_g^1, \mathbf{e}\right)$ by $\mathbf{X}^{\mathbf{e}}$. A strategy profile $(\mathbf{s}^*, \mathbf{w}^*, \mathbf{x}^{\mathbf{e}*}) \in \mathcal{S}^n \times \mathbb{R}_+^n \times \mathbf{X}^{\mathbf{e}}$ is a (*pure-strategy*) *subgame-perfect (Nash) equilibrium of the feasible (1) type g-implicit two-stage game* $\left(\Gamma_g^1, \mathbf{e}\right)$, if for any $i \in N$ and any $(s_i, w_i) \in \mathcal{S} \times \mathbb{R}_+$,

$$\begin{aligned} & u_i\left(x_i^{\mathbf{e}*}(\mathbf{s}^*, \mathbf{w}^*), g_i\left(\mathbf{s}^*, \mathbf{x}^{\mathbf{e}*}(\mathbf{s}^*, \mathbf{w}^*), \mathbf{w}^*\right)\right) \\ \geq \;& u_i\left(x_i^{\mathbf{e}*}(\mathbf{s}^*_{s_i}, \mathbf{w}^*_{w_i}), g_i\left(\mathbf{s}^*_{s_i}, \mathbf{x}^{\mathbf{e}*}(\mathbf{s}^*_{s_i}, \mathbf{w}^*_{w_i}), \mathbf{w}^*_{w_i}\right)\right), \end{aligned}$$

where $x_i^{\mathbf{e}*}(\mathbf{s}^*, \mathbf{w}^*)$ is the i-th component of the Nash equilibrium strategy profile $\mathbf{x}^{\mathbf{e}*}(\mathbf{s}^*, \mathbf{w}^*)$ in the second stage subgame induced from the strategy choice $(\mathbf{s}^*, \mathbf{w}^*)$ in the first stage.

Given the feasible (2) type g-implicit two-stage game $\left(\Gamma_g^2, \mathbf{e}\right)$ and each strategy profile $\mathbf{x} \in [0, \bar{x}]^n$ in the first stage of the game $\left(\Gamma_g^2, \mathbf{e}\right)$, let us denote its corresponding second stage subgame by $\left(\Gamma_g^2(\mathbf{x}), \mathbf{e}\right)$. Let $\boldsymbol{\omega}^{\mathbf{e}} : [0, \bar{x}]^n \to \mathcal{S}^n \times \mathbb{R}_+^n$ be a *Nash equilibrium mapping* such that for each $\mathbf{x} \in [0, \bar{x}]^n$, $\boldsymbol{\omega}^{\mathbf{e}}(\mathbf{x})$ is a (pure-strategy) Nash equilibrium of the subgame $\left(\Gamma_g^2(\mathbf{x}), \mathbf{e}\right)$. Denote the set of such Nash equilibrium mappings in second stage subgames of the game $\left(\Gamma_g^2, \mathbf{e}\right)$ by $\boldsymbol{\Omega}^{\mathbf{e}}$. A strategy profile $(\mathbf{x}^*, \boldsymbol{\omega}^{\mathbf{e}*}) \in [0, \bar{x}]^n \times \boldsymbol{\Omega}^{\mathbf{e}}$ is a (*pure-strategy*) *subgame-perfect (Nash) equilibrium of the feasible (2) type g-implicit two-stage game* $\left(\Gamma_g^2, \mathbf{e}\right)$, if for any $i \in N$ and any $x_i \in [0, \bar{x}]$,

$$u_i\left(x_i^*, g_i\left(\mathbf{x}^*, \boldsymbol{\omega}^{\mathbf{e}*}\left(\mathbf{x}^*\right)\right)\right) \geq u_i\left(x_i, g_i\left(\mathbf{x}_{x_i}^*, \boldsymbol{\omega}^{\mathbf{e}*}\left(\mathbf{x}_{x_i}^*\right)\right)\right).$$

Denote by $SPE\left(\Gamma_g^1, \mathbf{e}\right)$ (*resp.* $SPE\left(\Gamma_g^2, \mathbf{e}\right)$) the set of subgame-perfect equilibria of $\left(\Gamma_g^1, \mathbf{e}\right)$ (*resp.* $\left(\Gamma_g^2, \mathbf{e}\right)$). An allocation $\mathbf{z} = (x_i, y_i)_{i \in N} \in ([0, \bar{x}] \times \mathbb{R}_+)^n$ is a *subgame-perfect equilibrium allocation of the game* $\left(\Gamma_g^1, \mathbf{e}\right)$ (*resp.* $\left(\Gamma_g^2, \mathbf{e}\right)$) if there exists $(\mathbf{s}, \mathbf{w}, \mathbf{x}^{\mathbf{e}}) \in SPE\left(\Gamma_g^1, \mathbf{e}\right)$ (*resp.* $(\mathbf{x}, \boldsymbol{\omega}^{\mathbf{e}}) \in SPE\left(\Gamma_g^2, \mathbf{e}\right)$) such that $\mathbf{x}^{\mathbf{e}}(\mathbf{s}, \mathbf{w}) = \mathbf{x}$ and $\mathbf{y} = g\left(\mathbf{s}, \mathbf{x}^{\mathbf{e}}(\mathbf{s}, \mathbf{w}), \mathbf{w}\right)$ (*resp.* $\mathbf{y} = g\left(\mathbf{x}, \boldsymbol{\omega}^{\mathbf{e}}(\mathbf{x})\right)$). Denote by $SPA\left(\Gamma_g^1, \mathbf{e}\right)$ (*resp.* $SPA\left(\Gamma_g^2, \mathbf{e}\right)$) the set of subgame-perfect equilibrium allocations of $\left(\Gamma_g^1, \mathbf{e}\right)$ (*resp.* $\left(\Gamma_g^2, \mathbf{e}\right)$). A feasible (1)-type (*resp.* (2)-type) g-implicit two-stage mechanism Γ_g^1 (*resp.* Γ_g^2) is said to *implement* the proportional solution on $\mathcal{E}$ *in subgame-perfect equilibria*, if for all $\mathbf{e} \in \mathcal{E}$, $SPA\left(\Gamma_g^1, \mathbf{e}\right) = PR(\mathbf{e})$ (*resp.* $SPA\left(\Gamma_g^2, \mathbf{e}\right) = PR(\mathbf{e})$). Moreover, a feasible (1)-type (*resp.* (2)-type) g-implicit two-stage mechanism Γ_g^1 (*resp.* Γ_g^2) is said to *doubly implement* the proportional solution on $\mathcal{E}$ *in Nash and subgame-perfect equilibria*, if for all $\mathbf{e} \in \mathcal{E}$, $NA(g, \mathbf{e}) = SPA\left(\Gamma_g^1, \mathbf{e}\right) = PR(\mathbf{e})$ (*resp.* $NA(g, \mathbf{e}) = SPA\left(\Gamma_g^2, \mathbf{e}\right) = PR(\mathbf{e})$). Finally, a feasible (1)-type (*resp.* (2)-type) g-implicit two-stage mechanism Γ_g^1 (*resp.* Γ_g^2) is said to *triply implement* the proportional solution on $\mathcal{E}$ *in Nash, subgame-perfect, and strong equilibria*, if for all $\mathbf{e} \in \mathcal{E}$, $NA(g, \mathbf{e}) = SPA\left(\Gamma_g^1, \mathbf{e}\right) = SNA(g, \mathbf{e}) = PR(\mathbf{e})$ (*resp.* $NA(g, \mathbf{e}) = SPA\left(\Gamma_g^2, \mathbf{e}\right) = SNA(g, \mathbf{e}) = PR(\mathbf{e})$).

3 Implementation of the proportional solution

In the following, we add two additional assumptions.

Assumption 1 (boundary condition):

$$\forall i \in N, \forall z_i \in [0, \bar{x}) \times \mathbb{R}_{++}, \forall z_i' \in [0, \bar{x}] \times \{0\}, u_i(z_i) > u_i(z_i').$$

Assumption 2: *The production function f is continuously differentiable.*

We denote by $f'(x)$ the *derivative of f at x.*

3.1 Nash and strong implementability

In this subsection, we will set aside the timing problem of sharing mechanisms and propose a sharing mechanism as a normal form game form, which doubly implements the proportional solution in Nash and strong equilibria. To propose our mechanism, let us introduce four feasible sharing mechanisms defined as follows:

- g^{PR} is such that for each strategy profile $(\mathbf{s}, \mathbf{x}, \mathbf{w}) \in \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}^n_+$ and real amount of produced output $f\left(\sum s^t_j x_j\right)$,
  $g^{PR}_i(\mathbf{s}, \mathbf{x}, \mathbf{w}) = \frac{s_i x_i}{\sum s_j x_j} f\left(\sum s^t_j x_j\right)$ for all $i \in N$.

- $g^{(\hat{\mathbf{z}},\widehat{\mathbf{s}})}$ with $\widehat{\mathbf{s}} \in \mathcal{S}^n$ and $\hat{\mathbf{z}} = (\hat{\mathbf{x}}, \hat{\mathbf{y}}) \in [0, \bar{x}]^n \times \mathbb{R}^n_+$ is such that for each strategy profile $(\mathbf{s}, \mathbf{x}, \mathbf{w}) \in \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}^n_+$ and real amount of produced output $f\left(\sum s^t_j x_j\right)$,
  $$g^{(\hat{\mathbf{z}},\widehat{\mathbf{s}})}_i(\mathbf{s}, \mathbf{x}, \mathbf{w}) = \begin{cases} \min\left\{\max\left\{0, \hat{y}_i + \widehat{s}_i \cdot f'\left(\sum \widehat{s}_j \hat{x}_j\right) \cdot (x_i - \hat{x}_i)\right\}, f\left(\sum s^t_j x_j\right)\right\} \\ \text{if } \hat{\mathbf{z}} \in Z(\widehat{\mathbf{s}}) \text{ and } x_j = \hat{x}_j,\ s_j = \widehat{s}_j\ (\forall j \neq i), \\ 0 \text{ otherwise,} \end{cases}$$
  for all $i \in N$.

- g^m is such that for each strategy profile $(\mathbf{s}, \mathbf{x}, \mathbf{w}) \in \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}^n_+$ and real amount of produced output $f\left(\sum s^t_j x_j\right)$, and for all $i \in N$,
  $$g^m_i(\mathbf{s}, \mathbf{x}, \mathbf{w}) = \begin{cases} f\left(\sum s^t_j x_j\right) \text{ if } x_i = \mu(\mathbf{x}_{-i}) \text{ and } w_i > \max\{w_j\}_{j \neq i} \\ 0 \text{ otherwise.} \end{cases}$$
  where $\mu(\mathbf{x}_{-i}) \equiv \max_{x_j < \bar{x}} \{x_j\}_{j \neq i} + \frac{\bar{x} - \max_{x_j < \bar{x}}\{x_j\}_{j \neq i}}{2}$.

- g^d is such that for each strategy profile $(\mathbf{s}, \mathbf{x}, \mathbf{w}) \in \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}^n_+$ and real amount of produced output $f\left(\sum s^t_j x_j\right)$, and for all $i \in N$,
  $$g^d_i(\mathbf{s}, \mathbf{x}, \mathbf{w}) = \begin{cases} f\left(\sum s^t_j x_j\right) \text{ if } N^m(\mathbf{s}) = \{i\} \subseteq N^0(\mathbf{x}), \\ 0 \text{ otherwise,} \end{cases}$$
  where $N^m(\mathbf{s}) \equiv \left\{i \in N : s_i = \max\{s_j\}_{j \in N}\right\}$ and $N^0(\mathbf{x}) \equiv \{i \in N : x_i = 0\}$.

Note that $g^{(\hat{\mathbf{z}},\widehat{\mathbf{s}})}$ is designed to implement $\hat{\mathbf{z}}$ in Nash equilibrium under some economy with $\widehat{\mathbf{s}}$. If $\hat{\mathbf{z}}$ is Pareto efficient for some economy with $\widehat{\mathbf{s}}$, say $(\widehat{\mathbf{u}},\widehat{\mathbf{s}})$, then $\hat{\mathbf{z}}$ becomes a Nash equilibrium allocation of the game $\left(g^{(\hat{\mathbf{z}},\widehat{\mathbf{s}})}, (\widehat{\mathbf{u}},\widehat{\mathbf{s}})\right)$, since each agent's attainable allocations by his unilateral deviation from $\hat{\mathbf{z}}$ are the points in the "budget set" defined by the supporting price $f'\left(\sum \widehat{s}_j \hat{x}_j\right)$ at

$\hat{\mathbf{z}}$. Secondly, g^m assigns all of the produced output to only one agent who provides the maximal *interior* amount of labor time and reports a maximal amount of demand for the output, where the scheme $\mu(\mathbf{x}_{-i})$ is introduced to have agents find their best response strategies.

Given $(\mathbf{s},\mathbf{x},\mathbf{w}) = (s_i, x_i, w_i)_{i\in N} \in \mathcal{S}^n \times [0,\bar{x}]^n \times \mathbb{R}^n_+$, let $PR(\mathbf{s},\mathbf{x},\mathbf{w})^{-1} \equiv \{\mathbf{u} \in \mathcal{U}^n : (\mathbf{x},\mathbf{w}) \in PR(\mathbf{u},\mathbf{s})\}$. If $PR(\mathbf{s},\mathbf{x},\mathbf{w})^{-1} \neq \varnothing$, then $(\mathbf{x},\mathbf{w})$ should be a *PR-optimal allocation for some economy with* $\mathbf{s}$. Let us call such a $(\mathbf{s},\mathbf{x},\mathbf{w})$ a *PR-consistent strategy profile.* Note that if $g^{PR}(\mathbf{s},\mathbf{x},\mathbf{w}) = \mathbf{w}$ holds and $(\mathbf{x},\mathbf{w})$ is an interior allocation, then $PR(\mathbf{s},\mathbf{x},\mathbf{w})^{-1} \neq \varnothing$ holds. Given $(\mathbf{s},\mathbf{x},\mathbf{w}) \in \mathcal{S}^n \times [0,\bar{x}]^n \times \mathbb{R}^n_+$, let $N(\mathbf{s},\mathbf{x},\mathbf{w}) \equiv \{i \in N : \exists\,(s'_i, x'_i, w'_i) \in \mathcal{S}\times[0,\bar{x}]\times\mathbb{R}_+ \textit{ s.t. } PR\left(\mathbf{s}_{s'_i}, \mathbf{x}_{x'_i}, \mathbf{w}_{w'_i}\right)^{-1} \neq \varnothing\}$. This $N(\mathbf{s},\mathbf{x},\mathbf{w})$ is the set of *potential deviators* under strategy profile $(\mathbf{s},\mathbf{x},\mathbf{w})$, since any $i \in N(\mathbf{s},\mathbf{x},\mathbf{w})$ can constitute a PR-consistent strategy profile with the others' fixed strategies by changing his strategy from (s_i, x_i, w_i). Given $(\mathbf{s},\mathbf{x},\mathbf{w}) \in \mathcal{S}^n \times [0,\bar{x}]^n \times \mathbb{R}^n_+$ and $i \in N(\mathbf{s},\mathbf{x},\mathbf{w})$, let

$$\mathcal{S}^i(\mathbf{s},\mathbf{x},\mathbf{w}) \equiv \left\{s'_i \in \mathcal{S} : \exists\ (x'_i, w'_i) \in [0,\bar{x}] \times \mathbb{R}_+ \text{ s.t. } PR(\mathbf{s}_{s'_i}, \mathbf{x}_{x'_i}, \mathbf{w}_{w'_i})^{-1} \neq \varnothing\right\}.$$

Note that $\mathcal{S}^i(\mathbf{s},\mathbf{x},\mathbf{w})$ is closed and bounded from below, or otherwise, $\mathcal{S}^i(\mathbf{s},\mathbf{x},\mathbf{w}) = \mathcal{S}$. The latter case occurs if and only if f is linear on $[0,b]$ such that $\sum_{k\neq i} s_k x_k < b$.

We introduce a feasible sharing mechanism $g^* \in \mathcal{G}$ which works in each given $\mathbf{s}^t \in \mathcal{S}^n$ as follows:

For any $(\mathbf{s},\mathbf{x},\mathbf{w}) = (s_i, x_i, w_i)_{i\in N} \in \mathcal{S}^n \times [0,\bar{x}]^n \times \mathbb{R}^n_+$,
Rule 1: If $f(\sum s_j x_j) = f\left(\sum s^t_j x_j\right)$, then
1-1: if $PR(\mathbf{s},\mathbf{x},\mathbf{w})^{-1} \neq \varnothing$, then $g^*(\mathbf{s},\mathbf{x},\mathbf{w}) = g^{PR}(\mathbf{s},\mathbf{x},\mathbf{w})$,
1-2: if $PR(\mathbf{s},\mathbf{x},\mathbf{w})^{-1} = \varnothing$, and $N(\mathbf{s},\mathbf{x},\mathbf{w}) \neq \varnothing$, then
1-2-1: if $\#N(\mathbf{s},\mathbf{x},\mathbf{w}) > 1$, then $g^*(\mathbf{s},\mathbf{x},\mathbf{w}) = \mathbf{0}$,
1-2-2: if $N(\mathbf{s},\mathbf{x},\mathbf{w}) = \{j\}$ for some $j \in N$, then $g^*_j(\mathbf{s},\mathbf{x},\mathbf{w}) = g^{(\widehat{\mathbf{z}},\widehat{\mathbf{s}})}_j(\mathbf{s},\mathbf{x},\mathbf{w})$ and $g^*_i(\mathbf{s},\mathbf{x},\mathbf{w}) = 0$ for any $i \neq j$, where $\widehat{\mathbf{z}} = (\mathbf{x}_{\widehat{x}_j}, \mathbf{w}_{\widehat{w}_j})$ and $\widehat{\mathbf{s}} = \mathbf{s}_{\widehat{s}_j}$ such that $\widehat{s}_j = \arg\min_{s'_j \in \mathcal{S}^j(\mathbf{s},\mathbf{x},\mathbf{w})} \mid s'_j - s_j \mid$ & $PR(\mathbf{s}_{\widehat{s}_j}, \mathbf{x}_{\widehat{x}_j}, \mathbf{w}_{\widehat{w}_j})^{-1} \neq \varnothing$,
1-3: in any other case, $g^*(\mathbf{s},\mathbf{x},\mathbf{w}) = g^m(\mathbf{s},\mathbf{x},\mathbf{w})$.

Rule 2: If $f(\sum s_j x_j) \neq f\left(\sum s^t_j x_j\right)$, then $g^*(\mathbf{s},\mathbf{x},\mathbf{w}) = g^d(\mathbf{s},\mathbf{x},\mathbf{w})$.

It is easy to see that g^* satisfies *forthrightness* (Saijo *et al.* (1996)) and *best response property* (Jackson *et al.* (1994)). Moreover, g^* is a mechanism of the *quantity* type, and so satisfies *self-relevancy* (Hurwicz (1960)). It is also easy to check that the mechanism g^* is feasible.

The mechanism g^* is a combination of the four sharing mechanisms defined above: First, g^* computes the expected amount of produced output $f\left(\sum s_j x_j\right)$ from the data $(\mathbf{s}, \mathbf{x}, \mathbf{w})$ and compares this with the real amount of produced output $f\left(\sum s_j^t x_j\right)$. In the case that these two values coincide, if the strategy profile $(\mathbf{s}, \mathbf{x}, \mathbf{w})$ is PR-consistent, then g^* applies g^{PR} in **Rule 1-1**; if $(\mathbf{s}, \mathbf{x}, \mathbf{w})$ is not PR-consistent, and there is a unique potential deviator, then g^* applies $g^{(\widehat{\mathbf{z}}, \widehat{\mathbf{s}})}$ in **Rule 1-2-2** so as to punish him; for any other case, g^* either applies g^m in **Rule 1-3** or assigns nothing to everyone in **Rule 1-2-1**. Finally, if $f\left(\sum s_j x_j\right)$ and $f\left(\sum s_j^t x_j\right)$ are different, then g^* applies g^d in **Rule 2**.

Note that the data $(\widehat{\mathbf{z}}, \widehat{\mathbf{s}})$ of $g^{(\widehat{\mathbf{z}}, \widehat{\mathbf{s}})}$ in **Rule 1-2-2** is obtained by replacing the unique potential deviator's strategy with an appropriate one, where such an operation is possible by the definition of $N(\mathbf{s}, \mathbf{x}, \mathbf{w})$. Note also that such $(\widehat{\mathbf{z}}, \widehat{\mathbf{s}})$ is essentially uniquely determined: first, $\widehat{s}_j$ is uniquely determined, since if $s_j \in \mathcal{S}^j(\mathbf{s}, \mathbf{x}, \mathbf{w})$, then $\widehat{s}_j = s_j$, while if $s_j \notin \mathcal{S}^j(\mathbf{s}, \mathbf{x}, \mathbf{w})$, then $\mathcal{S}^j(\mathbf{s}, \mathbf{x}, \mathbf{w})$ is bounded from below and $\widehat{s}_j = \min \mathcal{S}^j(\mathbf{s}, \mathbf{x}, \mathbf{w})$. Second, once $\widehat{s}_j$ is uniquely determined, then the other agents' strategies together with $\widehat{s}_j$ give us the unique information about j's *potential consumption vector* $(\widehat{x}_j, \widehat{w}_j)$, whenever the production function f is strictly concave, because of the proportionality of the PR-optimal allocation. Even if f is linear, the ratios between input and output of j's potential consumption vectors should be the same value, by which the corresponding supporting price is uniquely determined.

Before we formally show the performance of g^*, let us briefly explain how the mechanism induces true information of labor skills below:the mechanism g^* only distributes total amounts of output $f\left(\sum s_k^t x_k\right)$ among agents according to the agents' strategies $(\mathbf{s}, \mathbf{x}, \mathbf{w})$, where the coordinator cannot know whether $\mathbf{s} = \mathbf{s}^t$ or not. However, first, if $f\left(\sum s_k x_k\right) \neq f\left(\sum s_k^t x_k\right)$, then clearly $\mathbf{s} \neq \mathbf{s}^t$ holds, and there must be at least one agent, say $j \in N$, who has misreported his labor skill, $s_j \neq s_j^t$, and supplied a positive amount of labor time $x_j > 0$. Then, this agent is definitely punished under the application of **Rule 2**. Secondly, consider the case that $f\left(\sum s_k x_k\right) = f\left(\sum s_k^t x_k\right)$ but $\mathbf{s} \neq \mathbf{s}^t$. Then, there are at least two agents $i, j \in N$ such that $s_i \neq s_i^t$, $s_j \neq s_j^t$, $x_i > 0$, and $x_j > 0$; otherwise there exists at least one agent $j \in N$ such that $s_j \neq s_j^t$ and $x_j = 0$. If the latter case is applied, agents such as j will be punished under the application of **Rule 1-3**. In the former case, one of the agents, $j \in N$, who has misrepresented his skill can induce **Rule 2** by changing from $x_j > 0$ to $x'_j = 0$, together with reporting a sufficiently high level of labor skill, so as to improve his payoff, while the other misreporting agents remain punished. Thus, this case may also not correspond to an equilibrium situation, and the following lemma confirms such an insight:

Lemma 1: *Let the feasible sharing mechanism $g^* \in \mathcal{G}$ be given as above. Given an economy $(\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, let a strategy profile $(\mathbf{s}, \mathbf{x}, \mathbf{w}) \in \mathcal{S}^n \times [0, \bar{x}]^n \times \mathbb{R}^n_+$ be a Nash equilibrium of the game $(g^*, \mathbf{u}, \mathbf{s}^t)$ such that $f\left(\sum s_j x_j\right) = f\left(\sum s^t_j x_j\right)$. Then, it follows that $s_i = s^t_i$ for all $i \in N$ with $x_i > 0$.*

Now, we analyze the performance of g^*.

Theorem 1: *Let Assumptions 1 and 2 hold. Let the feasible sharing mechanism $g^* \in \mathcal{G}$ be given as above. Then, g^** doubly implements *the proportional solution on $\mathcal{E}$* in Nash and strong equilibria.

Note that the mechanism g^* does not depend on the number of agents, and it works even in economies of *two agents*.

3.2 Implementation of the proportional solution with the timing problem

Given the mechanism g^*, we can also derive the *(1) type* and the *(2) type two-stage g^*-implicit extensive game forms* $\Gamma^1_{g^*}$ and $\Gamma^2_{g^*}$ respectively from g^*. Because of the timing problem discussed in section 2.2, g^* may be played as $\Gamma^1_{g^*}$ or $\Gamma^2_{g^*}$. In this situation, the coordinator may not know in advance the *information structure of the two-stage game* induced by $\Gamma^1_{g^*}$ or $\Gamma^2_{g^*}$, even if he has control on the number of stages in the mechanism: this information structure among agents may be characterized as *perfect information*, or as *complete but imperfect information on the first stage*.[9] In such a situation, the double implementability by $\Gamma^1_{g^*}$ (*resp.* $\Gamma^2_{g^*}$) in Nash and subgame perfect equilibria would be strongly attractive, since it keeps the desirable performance of the mechanism without relying on the information structure among agents. Fortunately, the following results would warrant this:

[9] If the game is played as one with perfect information (*resp.* complete but imperfect information), the natural equilibrium notion might be the subgame-perfect one (*resp.* the Nash one).

Theorem 2: *Let Assumptions 1 and 2 hold. Let the feasible sharing mechanism* $g^* \in \mathcal{G}$ *be given as above. Then, the* (1)-type g^*-implicit extensive game form doubly implements *the proportional solution on* $\mathcal{E}$ in Nash and subgame-perfect equilibria.

Theorem 3: *Let Assumptions 1 and 2 hold. Let the feasible sharing mechanism* $g^* \in \mathcal{G}$ *be given as above. Then, the* (2)-type g^*-implicit extensive game form doubly implements *the proportional solution on* $\mathcal{E}$ in Nash and subgame-perfect equilibria.

By the three theorems discussed above, we can summarize as follows:

Corollary: *Let Assumptions 1 and 2 hold. Let the feasible sharing mechanism* $g^* \in \mathcal{G}$ *be given as above. Then, both the* (1)-type *and* the (2)-type g^*-implicit extensive game forms *respectively* triply implement *the proportional solution on* $\mathcal{E}$ in Nash, subgame-perfect, and strong equilibria.

This result implies that the mechanism g^* implements the solution even if it permits each agent various kinds of freedom: he may choose his own supply of labor time freely; he is permitted to overstate his labor skill; he can behave unilaterally or coalitionally; and he can behave strong-rationally, as in the subgame-perfect response, or weak-rationally, as in the Nash-like response.

4 Concluding remarks

We have proposed a feasible sharing mechanism which triply implements the proportional solution in Nash, subgame-perfect, and strong equilibria, even when agents can not only understate, but also overstate their labor skills. The performance of our mechanism is summarized in Table 1, which provides a comparison with other relevant mechanisms.

Insert Table 1 around here.

As shown in Table 1, our mechanism has two undesirable features. First, it lacks continuity. Second, the mechanism fails to meet *balancedness* or *non-wastefulness*. One reason is that the mechanism permits agents to both overstate and/or understate their labor skills. So, it is difficult to find the deviator when only aggregate information $\left(f\left(\sum s_j x_j\right) \text{ and } f\left(\sum s_j^t x_j\right)\right)$ is available.

Therefore, the mechanism basically punishes all agents when there must be a deviator. The other reason is that this mechanism is labor sovereign. The labor sovereign mechanism should accept a profile of the agents' choice of labor time as an outcome, even when it may constitute a non-desirable allocation. Thus, if the mechanism needs to punish potential deviators, it is only possible by reducing their shares of output, which leads it to violate balancedness. We conjecture that there may be a trade-off between labor sovereignty and balancedness. However, it is still an open question whether or not there exists a mechanism which satisfies labor sovereignty and balancedness, and implements the proportional solution.

5 Appendix

Proof of Lemma 1. Suppose there exists $j \in N$ with $s_j \neq s_j^t$ and $x_j > 0$. Let $N_L(\mathbf{s})$ be the set of such j. Since $f(\sum s_i x_i) = f(\sum s_i^t x_i)$, $N_L(\mathbf{s})$ is not a singleton. Moreover, any $j \in N_L(\mathbf{s})$ can obtain $y_j' = f\left(\sum_{i \neq j} s_i^t x_i\right) > 0$ with $s_j' > \max\{s_i\}_{i \neq j}$ and $x_j' = 0$ under **Rule 2**. Note that

$$\begin{aligned}
\sum_{j \in N_L(\mathbf{s})} y_j' &= \sum_{j \in N_L(\mathbf{s})} f\left(\sum_{i \neq j} s_i^t x_i\right) = \sum_{j \in N_L(\mathbf{s})} f\left(\sum_{i \in N_L(\mathbf{s}) \setminus \{j\}} s_i^t x_i + \sum_{i \notin N_L(\mathbf{s})} s_i^t x_i\right) \\
&\geq \sum_{j \in N_L(\mathbf{s})} f\left(s_j^t x_j + \sum_{i \notin N_L(\mathbf{s})} s_i^t x_i\right) \text{ (since } N_L(\mathbf{s}) \text{ is not a singleton)} \\
&\geq f\left(\sum_{j \in N_L(\mathbf{s})} \left(s_j^t x_j + \sum_{i \notin N_L(\mathbf{s})} s_i^t x_i\right)\right) \text{ (since } f \text{ is concave and } f(0) \geq 0) \\
&\geq f\left(\sum_{j \in N_L(\mathbf{s})} s_j^t x_j + \sum_{i \notin N_L(\mathbf{s})} s_i^t x_i\right) \geq \sum_{j \in N_L(\mathbf{s})} y_j \equiv \sum_{j \in N_L(\mathbf{s})} g_j^*(\mathbf{s}, \mathbf{x}, \mathbf{w}).
\end{aligned}$$

If $\sum_{j \in N_L(\mathbf{s})} y_j' > \sum_{j \in N_L(\mathbf{s})} y_j$, then there must be $j \in N_L(\mathbf{s})$ who has an incentive to induce **Rule 2** by $s_j' > \max\{s_i\}_{i \neq j}$ and $x_j' = 0$. If $\sum_{j \in N_L(\mathbf{s})} y_j' = \sum_{j \in N_L(\mathbf{s})} y_j$ and there exists one individual $j \in N_L(\mathbf{s})$ with $y_j' > y_j$, then he has an incentive to induce **Rule 2** by $s_j' > \max\{s_i\}_{i \neq j}$ and $x_j' = 0$. Finally, if $\sum_{j \in N_L(\mathbf{s})} y_j' = \sum_{j \in N_L(\mathbf{s})} y_j$ and $y_j' = y_j$ for all $j \in N_L(\mathbf{s})$, then every $j \in N_L(\mathbf{s})$ has an incentive to change x_j to $x'_j = 0$, since $u_j(x'_j, y'_j) = u_j(0, y_j) > u_j(x_j, y_j)$ by the strict monotonicity of utility functions. Thus, in any case, it contradicts the fact that $(\mathbf{s}, \mathbf{x}, \mathbf{w})$ is a Nash equilibrium. ■

Proof of Theorem 1. Let $(\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ be any given.

(1) Show $PR(\mathbf{u}, \mathbf{s}^t) \subseteq NA(g^*, \mathbf{u}, \mathbf{s}^t)$ for all $(\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$.

Let $\mathbf{z} = (\mathbf{x}, \mathbf{y}) \in PR(\mathbf{u}, \mathbf{s}^t)$. Then, if the strategy profile of agents is $(\mathbf{s}^t, \mathbf{x}, \mathbf{y}) = (s_i^t, x_i, y_i)_{i \in N} \in (\mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+)^n$, then $g^*(\mathbf{s}^t, \mathbf{x}, \mathbf{y}) = \mathbf{y}$ by **Rule 1-1**. Since $\mathbf{s}^t \gg \mathbf{0}$ and $\mathbf{z}$ is an efficient proportional allocation, Assumption 1 implies $\mathbf{x} \gg \mathbf{0}$ and $g_i^*(\mathbf{s}^t, \mathbf{x}, \mathbf{y}) > 0$ for all $i \in N$. Suppose that an individual $j \in N$ deviates from $\left(s_j^t, x_j, y_j\right)$ to $\left(s'_j, x'_j, w'_j\right) \in \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$.

Note first that the deviation cannot induce **Rule 1-3**. If the deviation results in **Rule 1-2-1**, then $g_j^*\left(\mathbf{s}^t_{s'_j}, \mathbf{x}_{x'_j}, \mathbf{y}_{w'_j}\right) = 0$. If the deviation induces **Rule 2**, then $x'_j > 0$. Hence, it must be the case that $g_j^*\left(\mathbf{s}^t_{s'_j}, \mathbf{x}_{x'_j}, \mathbf{y}_{w'_j}\right) = 0$ under **Rule 2**.

If the deviation induces **Rule 1-2-2** with $s'_j \neq s_j^t$, then $x'_j = 0$. So,

$$\begin{aligned} g_j^*\left(\mathbf{s}^t_{s'_j}, \mathbf{x}_{x'_j}, \mathbf{y}_{w'_j}\right) &\leq \widehat{w}_j + \widehat{s}_j \cdot f'\left(\sum_{i \neq j} s_i^t x_i + \widehat{s}_j \widehat{x}_j\right) \cdot \left(x'_j - \widehat{x}_j\right) \\ &= \widehat{w}_j - \widehat{s}_j \widehat{x}_j \cdot f'\left(\sum_{i \neq j} s_i^t x_i + \widehat{s}_j \widehat{x}_j\right) \end{aligned}$$

with some $(\widehat{s}_j, \widehat{x}_j, \widehat{w}_j) \in \mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+$ such that $PR\left(\mathbf{s}^t_{\widehat{s}_j}, \mathbf{x}_{\widehat{x}_j}, \mathbf{y}_{\widehat{w}_j}\right)^{-1} \neq \varnothing$. Suppose $f\left(\sum_{i \neq j} s_i^t x_i + \widehat{s}_j \widehat{x}_j\right) \neq f\left(\sum_{i \neq j} s_i^t x_i + s_j^t x_j\right)$. Since $PR(\mathbf{s}^t, \mathbf{x}, \mathbf{y})^{-1} \neq \varnothing$, it implies that both of the points $\left(\sum_{i \neq j} s_i^t x_i + \widehat{s}_j \widehat{x}_j, f\left(\sum_{i \neq j} s_i^t x_i + \widehat{s}_j \widehat{x}_j\right)\right)$ and $\left(\sum_{i \neq j} s_i^t x_i + s_j^t x_j, f\left(\sum_{i \neq j} s_i^t x_i + s_j^t x_j\right)\right)$ must be on the same line that passes through $(0, 0)$. Since f is concave, f must be linear on a closed interval $\left[0, \max\left\{\sum_{i \neq j} s_i^t x_i + \widehat{s}_j \widehat{x}_j, \sum_{i \neq j} s_i^t x_i + s_j^t x_j\right\}\right]$. Hence, $\widehat{w}_j - \widehat{s}_j \widehat{x}_j \cdot f'\left(\sum_{i \neq j} s_i^t x_i + \widehat{s}_j \widehat{x}_j\right) = 0$. Thus, $g_j^*\left(\mathbf{s}^t_{s'_j}, \mathbf{x}_{x'_j}, \mathbf{y}_{w'_j}\right) \leq 0$. Next, suppose that $f\left(\sum_{i \neq j} s_i^t x_i + \widehat{s}_j \widehat{x}_j\right) = f\left(\sum_{i \neq j} s_i^t x_i + s_j^t x_j\right)$. This implies $\widehat{s}_j \widehat{x}_j = s_j^t x_j$ and

$\widehat{w}_j = y_j$, since $PR\left(\mathbf{s}^t, \mathbf{x}, \mathbf{y}\right)^{-1} \neq \varnothing$ and $PR\left(\mathbf{s}^t_{\widehat{s}_j}, \mathbf{x}_{\widehat{x}_j}, \mathbf{y}_{\widehat{w}_j}\right)^{-1} \neq \varnothing$. Thus,

$$\begin{aligned} g^*_j\left(\mathbf{s}^t_{s'_j}, \mathbf{x}_{x'_j}, \mathbf{y}_{w'_j}\right) &= \widehat{w}_j - \widehat{s}_j \widehat{x}_j \cdot f'\left(\sum_{i \neq j} s^t_i x_i + \widehat{s}_j \widehat{x}_j\right) \\ &= y_j - s^t_j x_j \cdot f'\left(\sum s^t_i x_i\right) = y_j + s^t_j \cdot f'\left(\sum s^t_i x_i\right) \cdot \left(x'_j - x_j\right). \end{aligned}$$

Since $\mathbf{z}$ is Pareto efficient, the deviation gives no additional benefit to j.

Consider the case that the deviation induces **Rule 1-2-2** with $s'_j = s^t_j$. If $g^*_j\left(\mathbf{s}^t, \mathbf{x}_{x'_j}, \mathbf{y}_{w'_j}\right) = g^{(\widehat{\mathbf{z}}, \widehat{\mathbf{s}})}_j\left(\mathbf{s}^t, \mathbf{x}_{x'_j}, \mathbf{y}_{w'_j}\right)$ under **Rule 1-2-2** where $\widehat{\mathbf{z}} = \left(\left(\widehat{x}_j, \widehat{w}_j\right), \left(x_i, y_i\right)_{i \neq j}\right)$, then $\widehat{\mathbf{s}} = \mathbf{s}^t$ and the point $\left(\sum_{i \neq j} s^t_i x_i + s^t_j \widehat{x}_j, f\left(\sum_{i \neq j} s^t_i x_i + s^t_j \widehat{x}_j\right)\right)$ must be on the line that starts from $(0,0)$ and passes through $\left(\sum s^t_i x_i, f\left(\sum s^t_i x_i\right)\right)$, because we have $\left(\widehat{w}_j, \mathbf{y}_{-j}\right) = g^{PR}\left(\mathbf{s}^t, \left(\widehat{x}_j, \mathbf{x}_{-j}\right), \left(\widehat{w}_j, \mathbf{y}_{-j}\right)\right)$ as well as $\mathbf{y} = g^{PR}\left(\mathbf{s}^t, \mathbf{x}, \mathbf{y}\right)$. Since f is concave, f must be linear on a closed interval $\left[0, \max\left\{\sum s^t_i x_i, \sum_{i \neq j} s^t_i x_i + s^t_j \widehat{x}_j\right\}\right]$, which implies $g^*_j\left(\mathbf{s}^t, \mathbf{x}_{x'_j}, \mathbf{y}_{w'_j}\right) = g^{(\mathbf{z}, \mathbf{s}^t)}_j\left(\mathbf{s}^t, \mathbf{x}_{x'_j}, \mathbf{y}_{w'_j}\right)$. Since $g^{(\mathbf{z}, \mathbf{s}^t)}_j\left(\mathbf{s}^t, \mathbf{x}_{x'_j}, \mathbf{y}_{w'_j}\right) \leq y_j + s^t_j \cdot f'\left(\sum s^t_i x_i\right) \cdot \left(x'_j - x_j\right)$ and $\mathbf{z}$ is Pareto efficient, the deviation gives no additional benefit to j.

Next, if the deviation induces **Rule 1-1** with $s'_j \neq s^t_j$, then it must be $x'_j = 0$ under **Rule 1-1**, since $\mathbf{x} \gg \mathbf{0}$. Thus, by the definition of g^{PR} in **Rule 1-1**, it follows that $w'_j = 0$. By Assumption 1, however, this implies that $\left(\mathbf{x}_{x'_j}, \mathbf{y}_{w'_j}\right)$ cannot be Pareto efficient for any economy, which is in contradiction to **Rule 1-1**. Thus, $s'_j = s^t_j$ follows when **Rule 1-1** is induced. However, if the deviation induces **Rule 1-1** with $s'_j = s^t_j$, then f must be linear on a closed interval $\left[0, \max\left\{\sum s^t_i x_i, \sum_{i \neq j} s^t_i x_i + s^t_j x'_j\right\}\right]$, and we obtain

$$g^*_j\left(\mathbf{s}^t, \mathbf{x}_{x'_j}, \mathbf{y}_{w'_j}\right) \leq y_j + s^t_j \cdot f'\left(\sum s^t_i x_i\right) \cdot \left(x'_j - x_j\right).$$

The Pareto efficiency of $\mathbf{z}$ implies no additional benefit for j.

(2) Show $NA\left(g^*, \mathbf{u}, \mathbf{s}^t\right) \subseteq PR\left(\mathbf{u}, \mathbf{s}^t\right)$ for all $\left(\mathbf{u}, \mathbf{s}^t\right) \in \mathcal{E}$.

Let $(\mathbf{s}, \mathbf{x}, \mathbf{w}) = \left(s_i, x_i, w_i\right)_{i \in N} \in \left(\mathcal{S} \times [0, \bar{x}] \times \mathbb{R}_+\right)^n$ be a pure-strategy Nash equilibrium of the feasible sharing game $\left(g^*, \mathbf{u}, \mathbf{s}^t\right)$.

Suppose that $(\mathbf{s}, \mathbf{x}, \mathbf{w})$ induces **Rule 2**. If $N^0(\mathbf{x}) = \varnothing$, then $g^*_i(\mathbf{s}, \mathbf{x}, \mathbf{w}) = 0$ for all $i \in N$. Note that when **Rule 2** is induced, there exists at least an individual $j \in N$ such that $\sum_{i \neq j} s_i x_i \neq \sum_{i \neq j} s_i^t x_i$. Thus, there exists an individual $j \in N$ in this case who can enjoy $g_j^* \left(\mathbf{s}_{s'_j}, \mathbf{x}_{x'_j}, \mathbf{w}_{w'_j} \right) > 0$ with $s'_j > \max \{s_i\}_{i \in N}$ and $x'_j = 0$ under **Rule 2**.

If $N^0(\mathbf{x}) \neq \varnothing$, then everyone $j \in N^0(\mathbf{x})$ can monopolize all $f(\sum s_i^t x_i)$ by supplying $x_j = 0$ while reporting his labor skill as s'_j so that $s'_j > s_i$ for all $i \neq j$ under **Rule 2**. Thus, if $\# N^0(\mathbf{x}) \geq 2$, then no profile of agents' strategies can constitute a Nash equilibrium in **Rule 2**.

If $\# N^0(\mathbf{x}) = 1$ and $\# N \backslash N^0(\mathbf{x}) \geq 2$, then there exists at least an individual $j \in N \backslash N^0(\mathbf{x})$ such that $\sum_{i \in N \backslash (N^0(\mathbf{x}) \cup \{j\})} s_i x_i \neq \sum_{i \in N \backslash (N^0(\mathbf{x}) \cup \{j\})} s_i^t x_i$. In fact, if not, then $(n-2) \cdot \left(\sum_{i \in N \backslash N^0(\mathbf{x})} s_i x_i \right) = (n-2) \cdot \left(\sum_{i \in N \backslash N^0(\mathbf{x})} s_i^t x_i \right)$, which contradicts the fact that **Rule 2** is induced. Thus, there exists an individual $j \in N \backslash N^0(\mathbf{x})$ in this case who can enjoy $g_j^* \left(\mathbf{s}_{s'_j}, \mathbf{x}_{x'_j}, \mathbf{w}_{w'_j} \right) > 0$ with $s'_j > \max \{s_i\}_{i \in N}$ and $x'_j = 0$ under **Rule 2**. If $\# N^0(\mathbf{x}) = 1$ with $N^0(\mathbf{x}) = \{i\}$ and $\# N \backslash N^0(\mathbf{x}) = 1$ with $N \backslash N^0(\mathbf{x}) = \{j\}$, then j can enjoy $g_j^* \left(\mathbf{s}_{s'_j}, \mathbf{x}_{x'_j}, \mathbf{w}_{w'_j} \right) > 0$ with $s'_j = s_j^t$, $x'_j = \frac{\overline{x}}{2}$, and $w'_j > \max \left\{ w_i, f\left(s_j^t x'_j \right) \right\}$ under **Rule 1-3**. Thus, if $\# N^0(\mathbf{x}) = 1$, no profile of agents' strategies can constitute a Nash equilibrium in **Rule 2**.

Suppose that $(\mathbf{s}, \mathbf{x}, \mathbf{w})$ induces **Rule 1-3**. Then, $g_j^*(\mathbf{s}, \mathbf{x}, \mathbf{w}) = 0$ for some $j \in N$. If either $x_j = 0$ or $s_j = s_j^t$, then he can enjoy $g_j^* \left(\mathbf{s}_{s'_j}, \mathbf{x}_{x'_j}, \mathbf{w}_{w'_j} \right) > 0$ with $s'_j = s_j^t$, $x'_j = \mu(\mathbf{x}_{-j})$, and $w'_j > \max \left\{ f\left(s_j^t x'_j \right),\ \max \{w_i\}_{i \neq j} \right\}$, under **Rule 1-3**. If $x_j > 0$ and $s_j \neq s_j^t$, then he can enjoy $g_j^* \left(\mathbf{s}_{s'_j}, \mathbf{x}_{x'_j}, \mathbf{w}_{w'_j} \right) > 0$ with $s'_j > \max \{s_i\}_{i \in N}$ and $x'_j = 0$ under **Rule 2**.

Suppose that $(\mathbf{s}, \mathbf{x}, \mathbf{w})$ induces **Rule 1-2-1**. Then, $g_i^*(\mathbf{s}, \mathbf{x}, \mathbf{w}) = 0$ for all $i \in N$. If $x_j = 0$ for some $j \in N$, then $N(\mathbf{s}, \mathbf{x}, \mathbf{w}) = \{j\}$ or $\varnothing$. Hence $\mathbf{x} \gg \mathbf{0}$. Any $j \in N$ with $s_j = s_j^t$ can enjoy $g_j^* \left(\mathbf{s}_{s'_j}, \mathbf{x}_{x'_j}, \mathbf{w}_{w'_j} \right) > 0$ with $s'_j = s_j^t$, $x'_j = \mu(\mathbf{x}_{-j})$, and $w'_j > \max \left\{ f\left(s_j^t x'_j \right),\ \max \{w_i\}_{i \neq j} \right\}$, under **Rule 1-3** or **Rule 1-2-2**. At the same time, any $j \in N$ with $s_j \neq s_j^t$ can enjoy $g_j^* \left(\mathbf{s}_{s'_j}, \mathbf{x}_{x'_j}, \mathbf{w}_{w'_j} \right) > 0$ with $s'_j > \max \{s_i\}_{i \in N}$ and $x'_j = 0$ under **Rule 2**.

Suppose that $(\mathbf{s}, \mathbf{x}, \mathbf{w})$ induces **Rule 1-2-2**. Then, $N \backslash N(\mathbf{s}, \mathbf{x}, \mathbf{w}) \neq \varnothing$, and $x_i > 0$ and $g_i^*(\mathbf{s}, \mathbf{x}, \mathbf{w}) = 0$ for all $i \in N \backslash N(\mathbf{s}, \mathbf{x}, \mathbf{w})$. Any $j \in N \backslash N(\mathbf{s}, \mathbf{x}, \mathbf{w})$ with $s_j = s_j^t$ can enjoy $g_j^* \left(\mathbf{s}_{s'_j}, \mathbf{x}_{x'_j}, \mathbf{w}_{w'_j} \right) > 0$ with $s'_j = s_j^t$,

$x'_j = \mu(\mathbf{x}_{-j})$, and $w'_j > \max\left\{f\left(s^t_j x'_j\right), \max\{w_i\}_{i\neq j}\right\}$, under **Rule 1-3**. At the same time, any $j \in N\backslash N(\mathbf{s},\mathbf{x},\mathbf{w})$ with $s_j \neq s^t_j$ can enjoy $g^*_j\left(\mathbf{s}_{s'_j}, \mathbf{x}_{x'_j}, \mathbf{w}_{w'_j}\right) > 0$ with $s'_j > \max\{s_i\}_{i\in N}$ and $x'_j = 0$ under **Rule 2**.

Thus, $(\mathbf{s},\mathbf{x},\mathbf{w})$ corresponds to **Rule 1-1**. Then, for $\mathbf{z} = \left(\mathbf{x}, g^{PR}(\mathbf{s},\mathbf{x},\mathbf{w})\right)$, we have $g^*(\mathbf{s},\mathbf{x},\mathbf{w}) = g^{PR}(\mathbf{s},\mathbf{x},\mathbf{w})$, which means $g^*_i(\mathbf{s},\mathbf{x},\mathbf{w}) = \frac{s_i x_i}{\sum s_j x_j} f\left(\sum s^t_j x_j\right)$ for each $i \in N$. Note that by Assumption 1, $\mathbf{x} \gg \mathbf{0}$. Moreover, $f\left(\sum s_j x_j\right) = f\left(\sum s^t_j x_j\right)$ holds. Therefore, Lemma 1 implies it must be the case that $\mathbf{s} = \mathbf{s}^t$.

Since $(\mathbf{s},\mathbf{x},\mathbf{w})$ is a Nash equilibrium, it holds that for all $i \in N$ and all $(s'_i, x'_i, w'_i) \in \mathcal{S} \times [0,\bar{x}] \times \mathbb{R}_+$, $u_i(x_i, g^*_i(\mathbf{s},\mathbf{x},\mathbf{w})) \geq u_i\left(x'_i, g^*_i\left(\mathbf{s}_{s'_i}, \mathbf{x}_{x'_i}, \mathbf{w}_{w'_i}\right)\right)$. At the same time, any $i \in N$ can enjoy

$$g^*_i\left(\mathbf{s}_{s'_i}, \mathbf{x}_{x'_i}, \mathbf{w}_{w'_i}\right) \leq y_i + s^t_i \cdot f'\left(\sum s^t_j x_j\right) \cdot (x'_i - x_i)$$

with $s'_i = s^t_i$ and $w'_i = 0$ under **Rule 1-2-2**. Thus, the facts together imply the Pareto efficiency of $\mathbf{z}$, or $\mathbf{z} \in PR(\mathbf{u}, \mathbf{s}^t)$.

(3) Show $SNA(g^*, \mathbf{e}) = NA(g^*, \mathbf{e})$ for all $\mathbf{e} \in \mathcal{E}$.

Let $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$ be any given. By definition, $SNA(g^*, \mathbf{e}) \subseteq NA(g^*, \mathbf{e})$. Suppose $SNA(g^*, \mathbf{e}) \subsetneq NA(g^*, \mathbf{e})$. Then, there exists $(\mathbf{s},\mathbf{x},\mathbf{w}) \in NE(g^*, \mathbf{e})$ such that for some $T \subsetneq N$ and some $(s'_i, x'_i, w'_i)_{i\in T} \in \mathcal{S}^{\#T} \times [0,\bar{x}]^{\#T} \times \mathbb{R}^{\#T}_+$,

$$u_j\left(x_j, g^*_j(\mathbf{s},\mathbf{x},\mathbf{w})\right) < u_j\left(x'_j, g^*_j\left((s'_i, x'_i, w'_i)_{i\in T}, (s_k, x_k, w_k)_{k\in N\backslash T}\right)\right)$$

for all $j \in T$. Since $(\mathbf{s},\mathbf{x},\mathbf{w}) \in NE(g^*, \mathbf{e})$ corresponds to **Rule 1-1** as is shown in the proof of Theorem 1, $(\mathbf{s},\mathbf{x},\mathbf{w})$ is *PR-consistent*, which implies $\mathbf{x} \gg \mathbf{0}$ under Assumption 1. Hence $\mathbf{s} = \mathbf{s}^t$ by Lemma 1. Note also that $T = N$ is eliminated by Pareto efficiency of $NA(g^*, \mathbf{e})$. By construction of g^*, there is at most one agent who can enjoy a positive amount of output under **Rules 1-2-1**, **1-2-2**, **1-3**, and **2**. Since $g^*_i(\mathbf{s}^t,\mathbf{x},\mathbf{w}) > 0$ for all $i \in N$, $\left((s'_i, x'_i, w'_i)_{i\in T}, (s^t_k, x_k, w_k)_{k\in N\backslash T}\right)$ should induce **Rule 1-1** by Assumption 1. Then, f must be linear on a closed interval $\left[0, \max\left\{\sum s^t_i x_i, \sum_{i\in T} s'_i x'_i + \sum_{k\in N\backslash T} s^t_k x_k\right\}\right]$, and we obtain

$$g^*_j\left((s'_i, x'_i, w'_i)_{i\in T}, \left(s^t_k, x_k, w_{ki}\right)_{k\in N\backslash T}\right) \leq w_j + s^t_j \cdot f'\left(\sum s^t_i x_i\right) \cdot \left(x'_j - x_j\right)$$

for some $j \in T$. The Pareto efficiency of $(x_i, g_i^*(\mathbf{s}^t, \mathbf{x}, \mathbf{w}))_{i \in N}$ implies no additional benefit for this j, which implies a desired contradiction. Thus, $NA(g^*, \mathbf{e}) = SNA(g^*, \mathbf{e})$. ■

Proof of Theorem 2. Since $NA(g^*, \mathbf{e}) = PR(\mathbf{e})$ and $SPA\left(\Gamma^1_{g^*}, \mathbf{e}\right) \subseteq NA(g^*, \mathbf{e})$ for all $\mathbf{e} \in \mathcal{E}$, we have only to show $PR(\mathbf{e}) \subseteq SPA\left(\Gamma^1_{g^*}, \mathbf{e}\right)$ for all $\mathbf{e} \in \mathcal{E}$. First, we will show that in every second stage subgame, there is at least one Nash equilibrium strategy. Let us take a strategy mapping $\mathbf{x}^{\mathbf{e}} : \mathcal{S}^n \times \mathbb{R}^n_+ \to [0, \bar{x}]^n$ such that for each second stage subgame $\left(\Gamma^1_{g^*}(\mathbf{s}, \mathbf{w}), \mathbf{e}\right)$:

$$\text{for all } i \in N,\ x_i^{\mathbf{e}}(\mathbf{s}, \mathbf{w}) = \begin{cases} \frac{\bar{x}}{2} & \begin{array}{l} \text{if } w_i > \max\{w_k\}_{k \neq i},\ s_i = s_i^t, \text{ and} \\ \text{for any } j \neq i \text{ and any } x'_j, \\ PR\left(\mathbf{s}, \left(\frac{\bar{x}}{2}, x'_j, \mathbf{0}_{-\{i,j\}}\right), \mathbf{w}\right)^{-1} = \varnothing \end{array} \\ 0 & \text{otherwise} \end{cases}.$$

Then, we can see that $\mathbf{x}^{\mathbf{e}}(\mathbf{s}, \mathbf{w}) \in NE\left(\Gamma^1_{g^*}(\mathbf{s}, \mathbf{w}), \mathbf{e}\right)$. Note that $g^*(\mathbf{s}, \mathbf{x}^{\mathbf{e}}(\mathbf{s}, \mathbf{w}), \mathbf{w})$ corresponds to **Rule 1-3**. To simplify the notation, let us denote $\mathbf{x}^{\mathbf{e}} = \mathbf{x}^{\mathbf{e}}(\mathbf{s}, \mathbf{w})$ in the following discussion.

Note first that no individual can induce **Rule 1-1** by changing his strategy, and that no individual can enjoy output consumption under **Rule 1-2-1**. If $i \in N$ can induce **Rule 1-2-2** by changing his strategy, then the deviation gives him no additional benefit, or $g_i^*\left(\mathbf{s}, \mathbf{x}^{\mathbf{e}}_{x'_i}, \mathbf{w}\right) = 0$, because it must be the case that $i \notin N\left(\mathbf{s}, \mathbf{x}^{\mathbf{e}}_{x'_i}, \mathbf{w}\right)$. Moreover, if $i \in N$ can induce **Rule 2** by changing his strategy, then it must be the case that $x_i^{\mathbf{e}} > 0$, which implies the deviation gives him no additional benefit, or $g_i^*\left(\mathbf{s}, \mathbf{x}^{\mathbf{e}}_{x'_i}, \mathbf{w}\right) = 0$. Finally, if there exists $i \in N$ who can enjoy a positive amount of output under **Rule 1-3**, then $w_i > \max\{w_k\}_{k \neq i}$, $s_i = s_i^t$, and for any $j \neq i$ and any x'_j, $PR\left(\mathbf{s}, \left(\frac{\bar{x}}{2}, x'_j, \mathbf{0}_{-\{i,j\}}\right), \mathbf{w}\right)^{-1} = \varnothing$. His strategy is already $\frac{\bar{x}}{2}$, which is necessary for him to get a positive output under **Rule 1-3**. Thus, $\mathbf{x}^{\mathbf{e}} \in NE\left(\Gamma^1_{g^*}(\mathbf{s}, \mathbf{w}), \mathbf{e}\right)$.

Now, we will show that for each $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, if $\widehat{\mathbf{z}} = (\widehat{\mathbf{x}}, \widehat{\mathbf{y}}) \in PR(\mathbf{u}, \mathbf{s}^t)$, then there exists a subgame perfect equilibrium whose corresponding outcome is $\widehat{\mathbf{z}}$. Consider the following strategy profile of the extensive game $\left(\Gamma^1_{g^*}, \mathbf{e}\right)$:
(1) In the first stage, every individual i reports $(s_i, w_i) = (s_i^t, \widehat{y}_i)$.
(2) In the second stage:

(2-1): if $(\mathbf{s}, \mathbf{w}) = (\mathbf{s}^t, \widehat{\mathbf{y}})$ is the action profile of all individuals in the first stage, then any $i \in N$ supplies $x_i = \widehat{x}_i$;
(2-2): if $(\mathbf{s}, \mathbf{w}) = \left(\left(s'_j, \mathbf{s}^t_{-j}\right), \left(w'_j, \widehat{\mathbf{y}}_{-j}\right)\right)$, where $s'_j = s^t_j$, $w'_j \neq \widehat{y}_j$, and for all $i \neq j$, $w_i \leq w'_j$, is the action profile of all individuals in the first stage, then for this $j \in N$,

$$x_j = \arg \max_{x^*_j \in \chi_j\left(\mathbf{s}^t, \widehat{\mathbf{x}}_{-j}, \widehat{\mathbf{y}}_{w'_j}\right)} u_j\left(x^*_j, g^*_j\left(\mathbf{s}^t, \widehat{\mathbf{x}}_{x^*_j}, \widehat{\mathbf{y}}_{w'_j}\right)\right)$$

and for all $i \neq j$, $x_i = \widehat{x}_i$, where

$$\chi_j\left(\mathbf{s}^t, \widehat{\mathbf{x}}_{-j}, \widehat{\mathbf{y}}_{w'_j}\right) \equiv \left\{x^*_j \mid N\left(\mathbf{s}^t, \widehat{\mathbf{x}}_{x^*_j}, \widehat{\mathbf{y}}_{w'_j}\right) = \{j\} \text{ or } PR\left(\mathbf{s}^t, \widehat{\mathbf{x}}_{x^*_j}, \widehat{\mathbf{y}}_{w'_j}\right)^{-1} \neq \varnothing\right\};$$

(2-3): in any other case, for all $i \in N$, $x_i = x^{\mathbf{e}}_i$.

Note that for the subgame of (2-1), $\mathbf{x} = \widehat{\mathbf{x}} \in NE\left(\Gamma^1_{g^*}(\mathbf{s}, \mathbf{w}), \mathbf{e}\right)$, since any deviator can enjoy a positive amount of output only under **Rule 1-2-2**. Also, $\mathbf{x} \in NE\left(\Gamma^1_{g^*}(\mathbf{s}, \mathbf{w}), \mathbf{e}\right)$ for the subgame of (2-3), as we have already shown. Moreover, we can see that for the subgame of (2-2), $\mathbf{x} \in NE\left(\Gamma^1_{g^*}(\mathbf{s}, \mathbf{w}), \mathbf{e}\right)$. Note that $g^*(\mathbf{s}, \mathbf{x}, \mathbf{w})$ corresponds to **Rule 1-1** or **Rule 1-2-2** if $\left(\Gamma^1_{g^*}(\mathbf{s}, \mathbf{w}), \mathbf{e}\right)$ corresponds to (2-2). Note also that $\chi_j\left(\mathbf{s}^t, \widehat{\mathbf{x}}_{-j}, \widehat{\mathbf{y}}_{w'_j}\right)$ is non-empty, since $x^*_j = 0$ guarantees $N\left(\mathbf{s}^t, \widehat{\mathbf{x}}_{x^*_j}, \widehat{\mathbf{y}}_{w'_j}\right) = \{j\}$.

Suppose that $g^*(\mathbf{s}, \mathbf{x}, \mathbf{w})$ corresponds to **Rule 1-1** when $\left(\Gamma^1_{g^*}(\mathbf{s}, \mathbf{w}), \mathbf{e}\right)$ corresponds to (2-2). Then, f must be linear on $\left[0, \max\left\{\sum s^t_i \widehat{x}_i, \sum_{i \neq j} s^t_i \widehat{x}_i + s^t_j x_j\right\}\right]$, which implies the Pareto efficiency of $\left((x_j, \widehat{\mathbf{x}}_{-i}), \left(w'_j, \widehat{\mathbf{y}}_{-j}\right)\right)$. Thus, $\mathbf{x} \in NE\left(\Gamma^1_{g^*}(\mathbf{s}, \mathbf{w}), \mathbf{e}\right)$.

Suppose that $g^*(\mathbf{s}, \mathbf{x}, \mathbf{w})$ corresponds to **Rule 1-2-2** when $\left(\Gamma^1_{g^*}(\mathbf{s}, \mathbf{w}), \mathbf{e}\right)$ corresponds to (2-2). Note first that any individual cannot induce **Rule 2**. Moreover, any $i \neq j$ cannot induce **Rule 1-1**, and he can induce **Rule 1-3**, but only to enjoy no output consumption since $w_i \leq w'_j$. Finally, j cannot induce **Rule 1-3**. Thus, $\mathbf{x} \in NE\left(\Gamma^1_{g^*}(\mathbf{s}, \mathbf{w}), \mathbf{e}\right)$.

Now, let us see that the above strategy profile (1)-(2) constitutes a subgame perfect equilibrium of the extensive game $\left(\Gamma^1_{g^*}, \mathbf{e}\right)$. By the strategy profile (1)-(2) of the extensive game $\left(\Gamma^1_{g^*}, \mathbf{e}\right)$, $g^*(\mathbf{s}, \mathbf{x}, \mathbf{w}) = g^*(\mathbf{s}^t, \widehat{\mathbf{x}}, \widehat{\mathbf{y}}) = \widehat{\mathbf{y}}$. Suppose that individual j deviates from (s_j, w_j) to $\left(s'_j, w'_j\right)$ in the first stage. Then by (2-2) and (2-3), he only gets

$$g^*_j\left(\mathbf{s}_{s'_j}, \mathbf{x}\left(\mathbf{s}_{s'_j}, \mathbf{w}_{w'_j}\right), \mathbf{w}_{w'_j}\right) \leq \widehat{y}_j + s^t_j \cdot f'\left(\sum s^t_k \widehat{x}_k\right) \cdot (x_j - \widehat{x}_j).$$

These arguments imply that no individual has any incentive to deviate from (s_j, w_j) in the first stage. Thus, $\widehat{\mathbf{z}} = (\widehat{\mathbf{x}},\widehat{\mathbf{y}}) \in SPA\left(\Gamma^1_{g^*}, \mathbf{u}, \mathbf{s}^t\right)$. ■

Proof of Theorem 3. Since $NA(g^*, \mathbf{e}) = PR(\mathbf{e})$ and $SPA\left(\Gamma^2_{g^*}, \mathbf{e}\right) \subseteq NA(g^*, \mathbf{e})$ for all $\mathbf{e} \in \mathcal{E}$, we have only to show $PR(\mathbf{e}) \subseteq SPA\left(\Gamma^2_{g^*}, \mathbf{e}\right)$ for all $\mathbf{e} \in \mathcal{E}$. First, we will show that in every second stage subgame, there is at least one Nash equilibrium strategy.

Let us define a strategy profile $(\mathbf{s}^{\mathbf{e}}, \mathbf{w}^{\mathbf{e}})$ of the given second stage subgame $\left(\Gamma^2_{g^*}(\mathbf{x}), \mathbf{e}\right)$ as follows:

$$\text{for all } i \in N,\ \left(s^{\mathbf{e}}_i, w^{\mathbf{e}}_i\right) = \begin{cases} \left(s^t_i, 0\right) & \text{if } x_i \neq \mu(\mathbf{x}_{-i}) \\ \left(s^t_i, f\left(s^t_i x_i\right) + 1\right) & \text{otherwise} \end{cases}.$$

Then, we can see that $(\mathbf{s}^{\mathbf{e}}, \mathbf{w}^{\mathbf{e}}) \in NE\left(\Gamma^2_{g^*}(\mathbf{x}), \mathbf{e}\right)$. Note that $g^*(\mathbf{s}^{\mathbf{e}}, \mathbf{x}, \mathbf{w}^{\mathbf{e}})$ corresponds to **Rule 1-3**. Note first that no individual can induce **Rule 1-1** by changing his strategy, and that no individual can enjoy output consumption under **Rule 1-2-1**. If any $i \in N$ can induce **Rule 1-2-2** by changing his strategy, then such a deviation gives him no additional benefit, or $g^*_i\left(\mathbf{s}^{\mathbf{e}}_{s'_i}, \mathbf{x}, \mathbf{w}^{\mathbf{e}}_{w'_i}\right) = 0$, because it must be the case that $i \notin N\left(\mathbf{s}^{\mathbf{e}}_{s'_i}, \mathbf{x}, \mathbf{w}^{\mathbf{e}}_{w'_i}\right)$. Moreover, if any $i \in N$ can induce **Rule 2** by changing his strategy, then it must be the case that $x_i > 0$, which implies this deviation gives him no additional benefit, or $g^*_i\left(\mathbf{s}^{\mathbf{e}}_{s'_i}, \mathbf{x}, \mathbf{w}^{\mathbf{e}}_{w'_i}\right) = 0$. Finally, if any $i \in N$ can induce **Rule 1-3** by changing his strategy, then such a deviation gives him no additional benefit, since the individual i who has $x_i = \mu(\mathbf{x}_{-i})$ is already fixed in the first stage game. Thus, $(\mathbf{s}^{\mathbf{e}}, \mathbf{w}^{\mathbf{e}}) \in NE\left(\Gamma^2_{g^*}(\mathbf{x}), \mathbf{e}\right)$.

Now, we will show that for each $\mathbf{e} = (\mathbf{u}, \mathbf{s}^t) \in \mathcal{E}$, if $\widehat{\mathbf{z}} = (\widehat{\mathbf{x}},\widehat{\mathbf{y}}) \in PR(\mathbf{u}, \mathbf{s}^t)$, then there exists a subgame perfect equilibrium whose corresponding outcome is $\widehat{\mathbf{z}}$. Consider the following strategy profile of the extensive game $\left(\Gamma^2_{g^*}, \mathbf{e}\right)$:

(1) In the first stage, every individual i supplies $\widehat{x}_i > 0$.

(2) In the second stage:

(2-1): if $\widehat{\mathbf{x}}$ is the action profile of all individuals in the first stage, then for any $i \in N$, $(s_i, w_i) = (s^t_i, \widehat{y}_i)$;

(2-2): if $\mathbf{x} = \left(x'_j, \widehat{\mathbf{x}}_{-j}\right) \gg \mathbf{0}$, where $x'_j \neq \widehat{x}_j$, is the action profile of all individuals in the first stage, then

$$\text{for this } j \in N,\ (s_j, w_j) = \left(s^t_j, f\left(s^t_j x'_j\right) + 1\right),$$

$$\text{for all } i \neq j,\ (s_i, w_i) = \begin{cases} \left(s^t_i, \widehat{y}_i\right) & \text{if } x_i \neq \mu(\mathbf{x}_{-i}) \\ \left(s^t_i, f\left(\sum s^t_k x_k\right)\right) & \text{otherwise} \end{cases};$$

(2-3): in any other case, for all $i \in N$, $(s_i, w_i) = (s_i^{\mathbf{e}}, w_i^{\mathbf{e}})$.

Note that for the subgame of (2-1), $(\mathbf{s}, \mathbf{w}) = (\mathbf{s}^t, \widehat{\mathbf{y}}) \in NE\left(\Gamma_{g^*}^2(\mathbf{x}), \mathbf{e}\right)$, since any deviator can enjoy a positive amount of output only under **Rule 1-2-2**. Also, $(\mathbf{s}, \mathbf{w}) \in NE\left(\Gamma_{g^*}^2(\mathbf{x}), \mathbf{e}\right)$ for any subgame $\left(\Gamma_{g^*}^2(\mathbf{x}), \mathbf{e}\right)$ of the case (2-3), as we have already shown. Moreover, we can see that for the subgame of (2-2), $(\mathbf{s}, \mathbf{w}) \in NE\left(\Gamma_{g^*}^2(\mathbf{x}), \mathbf{e}\right)$.

(i) Consider the case that for all $i \neq j$, $x_i \neq \mu(\mathbf{x}_{-i})$. Then, $g^*(\mathbf{s}, \mathbf{x}, \mathbf{w})$ corresponds to **Rule 1-2-2**. Note that for any individual, changing his announcement of his labor skill cannot make him better off. Note also any $i \neq j$ can induce **Rule 1-3** by changing from $w_i = \widehat{y}_i$ to $w_i' = f\left(\sum s_k^t x_k\right)$, which cannot make him better off. Finally, j cannot induce **Rule 1-3** by changing from $w_j = f\left(s_j^t x_j\right) + 1$ to any $w_j' \geq 0$. Thus, $(\mathbf{s}, \mathbf{w}) \in NE\left(\Gamma_{g^*}^2(\mathbf{x}), \mathbf{e}\right)$.

(ii) Consider the case that there exists $i \neq j$ with $x_i = \mu(\mathbf{x}_{-i})$. Then $g^*(\mathbf{s}, \mathbf{x}, \mathbf{w})$ corresponds to **Rule 1-3**. Note first that no individual can induce **Rule 1-1** by changing his strategy, and that no individual can enjoy output consumption under **Rule 1-2-1**. If any $i \in N$ can induce **Rule 1-2-2** by changing his strategy, then such a deviation gives him no additional benefit, or $g_i^*\left(\mathbf{s}_{s_i'}, \mathbf{x}, \mathbf{w}_{w_i'}\right) = 0$, because it must be the case that $i \notin N\left(\mathbf{s}_{s_i'}, \mathbf{x}, \mathbf{w}_{w_i'}\right)$. Moreover, if any $i \in N$ can induce **Rule 2** by changing his strategy, then the deviation gives him no additional benefit, or $g_i^*\left(\mathbf{s}_{s_i'}, \mathbf{x}, \mathbf{w}_{w_i'}\right) = 0$, since $\mathbf{x} \gg \mathbf{0}$. Finally, if any $i \in N$ can induce **Rule 1-3** by changing his strategy, then such a deviation gives him no additional benefit, since the individual i who has $x_i = \mu(\mathbf{x}_{-i})$ is already fixed in the first stage game. Thus, $(\mathbf{s}, \mathbf{w}) \in NE\left(\Gamma_{g^*}^2(\mathbf{x}), \mathbf{e}\right)$.

Now, let us see that the above strategy profile (1)-(2) constitutes a subgame perfect equilibrium of the extensive game $\left(\Gamma_{g^*}^2, \mathbf{e}\right)$. By the strategy profile (1)-(2) of the extensive game $\left(\Gamma_{g^*}^2, \mathbf{e}\right)$, $g^*(\mathbf{s}, \mathbf{x}, \mathbf{w}) = g^*(\mathbf{s}^t, \widehat{\mathbf{x}}, \widehat{\mathbf{y}}) = \widehat{\mathbf{y}}$. Suppose that individual j deviates from $\widehat{x}_j$ to $x_j' \neq \widehat{x}_j$ in the first stage. If $x_j' = 0$, then by (2-3), he only gets $g_j^*\left(\mathbf{s}\left(x_j', \widehat{\mathbf{x}}_{-j}\right), \left(x_j', \widehat{\mathbf{x}}_{-j}\right), \mathbf{w}\left(x_j', \widehat{\mathbf{x}}_{-j}\right)\right) = 0$. If $x_j' > 0$, then by (2-2), he only gets

$$g_j^*\left(\mathbf{s}\left(x_j', \widehat{\mathbf{x}}_{-j}\right), \left(x_j', \widehat{\mathbf{x}}_{-j}\right), \mathbf{w}\left(x_j', \widehat{\mathbf{x}}_{-j}\right)\right) \leq \widehat{y}_j + s_j^t \cdot f'\left(\sum s_i^t \widehat{x}_i\right) \cdot \left(x_j' - \widehat{x}_j\right).$$

These arguments imply that no individual has any incentive to deviate from x_j in the first stage. Thus, $\widehat{\mathbf{z}} = (\widehat{\mathbf{x}}, \widehat{\mathbf{y}}) \in SPA\left(\Gamma_{g^*}^2, \mathbf{u}, \mathbf{s}^t\right)$. ■

6 References

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	Suh (1995)	Yoshihara (1999)	Tian (2000)	Yamada-Yoshihara (2001)
Equilibrium notions	$\cdot NA$ $\cdot SNA$ $\cdot UNA$	$\cdot NA$ $\cdot SNA$ $\cdot UNA$	$\cdot NA$ $\cdot SNA$	$\cdot NA$ $\cdot SNA$ $\cdot SPA$
# of goods	2	2	$m \geq 2$	2
# of agents	$n \geq 2$	$n \geq 3$	$n \geq 2$	$n \geq 2$
endowment information	known	known	unknown; overstatements are prohibited	unknown; overstatements and understatements are possible
labor sovereignty	no	no	no	yes
feasibility	yes	yes	yes	yes
self-relevancy	no	yes	no	yes
best response property	no	yes	no	yes
forthrightness	no	yes	yes	yes
balancedness	no	yes	no	no
continuity	no	no	yes	no

Table 1: Performance of mechanisms implementing PR

where NA means "Nash implementability," SNA means "strong Nash implementability," UNA means "undominated Nash implementability," and SPA means "subgame-perfect implementability."