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Fiscal and Monetary Policy Instruments**

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Incomplete Financial Markets, Irreversibility of Investment, and Fiscal and Monetary Policy Instruments ^{*†}

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ABSTRACT. This paper explores fiscal and monetary instruments to improve long-run welfare when financial markets are incomplete. Here the markets are incomplete in the sense that productive investment is irreversible and uncollateralizable, and that there is no insurance against unobserved idiosyncratic shocks. Only government-issued bonds provide self-insurance. This paper demonstrates that, unlike dynamic models with reversible productive capital, an increase in precautionary savings (self-insurance) by holding liquid bonds reduces, rather than increases, irreversible productive investment. Accordingly, subsidies to promote productive but irreversible investment should be financed in such a way that they do not reduce the capability of consumers to insure themselves against idiosyncratic shocks. In this context, lump-sum subsidies financed by consumption taxes are preferred to fixed and/or proportional investment subsidies financed by either large-scale seigniorage revenues or lump-sum taxes. The combination of lump-sum subsidies and consumption taxes are more redistributive and thus more consumption-smoothing than the other sets of instruments available in this model.

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Keywords: fiscal instruments, irreversibility, incomplete markets, liquidity constraints, risk-sharing.

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1. Introduction This paper explores fiscal and monetary instruments to improve long-run welfare when financial markets are incomplete and economic agents' incomes are not perfectly observable. Here the markets are incomplete in the sense that productive investment is irreversible and uncollaterizable,¹ and that there is no insurance against unobservable idiosyncratic shocks. If incomes are not perfectly observable and that unobservable income shocks cause substantial income dispersion among economic agents, then income taxes, especially progressive ones, are not necessarily effective instruments for redistributive policies in this economy. Such incompleteness of financial markets and imperfectness of income observability are often found in market economies. In particular, we focus fiscal and monetary instruments in this economy to improve social welfare by promoting productive, but irreversible, investment.

In this economy, economic agents may not undertake productive, but irreversible investment. Rather, they may hold more liquid, but less profitable, government-issued securities as a measure of precautionary savings. Government-issued securities are not backed directly by productive assets, but are circulated between generations. Thus, a shift of financial funds from productive investment to government-issued securities may reduce consumption opportunities in the long term.

We examine how subsidies should be granted to those who make irreversible investment decisions, and how these subsidies should be financed. This would shed light on how profitable, but irreversible, investment should be promoted to improve long-run welfare.

On the one hand, subsidies can be in the form of lump-sum transfers or they are proportional to the amount of investment. On the other hand, subsidies can be financed by lump-sum taxes, consumption taxes, or seigniorage revenues. Here we do not consider income taxes because of unobservability of income. Hence, we explore how various combinations of fiscal and monetary policies might be used to remedy this undesirable fund allocation, which favors liquid/unbacked assets at the expenses of illiquid/productive assets. Thus, we investigate, analytically and numerically, the welfare effects of these policy instruments, and in particular, the relative desirability of various types of tax instruments.

To explore effects of these policy instruments, we need a framework of portfolio choice between liquid and illiquid assets in the context of incomplete markets. Here we adopt a

¹Capital investment characterized by such features may include human capital, and intangible assets such as intellectual property right.

theoretical framework proposed by Dutta and Kapur (1998) for the following four reasons. First, its overlapping-generations setup incorporates government-issued securities or fiat money as an instrument of intergenerational transfer. Second, productive investment is assumed to be irreversible and, in addition, uncollateralizable. This assumption subjects investors in productive capital to liquidity constraints. Third, the model is set-up in such a way that there is no insurance market. Thus, consumers cannot insure idiosyncratic shocks directly within their own cohorts. Fourth, under these assumptions, consumers can only partially self-insure idiosyncratic risks by carrying either government securities or fiat money. Thus, though simple, this framework incorporates necessary elements of portfolio choice between liquid and illiquid assets in incomplete markets.

An important policy implication of our investigation is that, in examining fiscal and monetary instruments in an economy with irreversible and uncollateralizable productive investment, we should consider not only the promotion of irreversible investment, but also the capability of consumers to self-insure. We find that lump-sum subsidies to investors who make irreversible investment commitments should be financed in a way to enhance or at least not to block self-insurance capabilities of consumers. In particular, broadly based consumption taxes are more desirable than other instruments since they are less destructive of self-insuring. Large-scale inflationary taxes are not desirable, since high rates of inflation make self-insurance through holding money costly and thereby reduce the self-insurance capabilities of consumers. Less redistributive taxes than consumption taxes, such as lump-sum taxes, impose disproportionate burdens on consumers having negative income shocks, since they are likely to be subject to liquidity constraints. In this way, lump-sum taxes reduce the self-insurance capabilities of consumers as a whole. Broadly based consumption taxes are preferable, since these taxes are less burdensome to consumers having negative income shocks (because they are lower spenders) than consumers having positive income shocks (who are higher spenders). This policy implication is also applicable to the case with unobservable preference shocks.² In addition, we find that lump-sum subsidies are preferable to proportional ones because they distort portfolio choice between liquid and illiquid assets to a lesser extent.

Our model has differences and similarities with existing dynamic macroeconomic models

²A preference shock implies an achieved utility level is different from the one before the shock, for the same income level. Consequently, a preference shock can be viewed as an unobservable shock in income required to achieve the same utility level as before.

with incomplete markets in three aspects.

First, it differs substantially from models with reversible physical capital. In Aiyagari (1995), for example, an increase in precautionary savings leads to the accumulation of physical capital. This is because liquid bonds, which are held as a precautionary measure, are backed by physical capital. Reversibility makes this possible. In contrast, in our framework of irreversibility, liquid assets are unbacked and circulated between generations; therefore, increasing precautionary savings by holding liquid assets reduces irreversible capital.³ Given this difference between the two models, the presence of a strong precautionary savings motive may accompany over-accumulation in the former model, and under-investment in the latter. For this reason, taxes on capital income may enhance steady-state welfare by discouraging capital accumulation in the former model, whereas subsidizing irreversible investment may improve long-run welfare in the latter.

Second, our model is similar to models with incomplete markets in that redistributive taxes help maintain the risk-sharing capabilities of consumers. Varian (1980) and Eaton and Rosen (1980) examine the desirability of redistributive taxation as social insurance in the presence of uninsured idiosyncratic shocks in a static framework. Following researches, including those of Kimball and Mankiw (1989) and Castañeda et al. (2002), explore the positive implications for insurance effects of redistributive taxes using dynamic models. More recent literature attempts to analyze not only the beneficial, but also the detrimental effects of redistributive taxes in a dynamic context. For example, Caucutt et al. (2000) and Conesa and Krueger (2002) consider the distortion arising from labor-supply and investment decisions, while Krueger and Perri (2001) analyze the emergence of tighter constraints on private insurance contracts. Our model of irreversible and uncollateralizable investment is different from those mentioned above, but shares the same motivation.

Third, our model indicates that financing through seigniorage is not necessarily preferable to financing through taxation. Dutta and Kapur (1998) demonstrate that the benefit of an increase in productive investment exceeds the cost of inflation in the case of small-scale seigniorage in this framework. However, we are able to show large-scale seigniorage raises the cost of holding money, and thereby substantially reduces the risk-sharing capabilities of consumers, particularly high-income earners.⁴ In a different context, Bewley (1983)

³In this regard, this model is rather realistic because government-issued securities serves as major liquid instruments enabling self-insurance in most market economies.

⁴This is particularly important for developing economies, since they often show a heavy dependence on

explores the limitations of monetary policy within dynamic models in which fiat money is circulated as a precautionary measure against uninsured idiosyncratic shocks.

This paper is organized as follows. In section 2, we present our framework, which is a modified version of the monetary model proposed by Dutta and Kapur (1998), and explore policy measures for improving social welfare. In section 3, we present several numerical examples to assess the qualitative significance of the results obtained in section 2, especially their global implications. In section 4, we offer concluding remarks. Proofs of propositions and other technical details are relegated to Appendices.

2. Theoretical Framework This section presents a simple monetary model with incomplete markets based on Dutta and Kapur (1998), and derives positive and normative implications of various policy combinations. This section establishes theoretical propositions on global properties in most cases, while it sometimes makes theoretical propositions on local cases in which government-financed subsidies to those who make irreversible investment are close to zero. Robustness of these local propositions will be examined by numerical examples in the next section. We then compare welfare implications of various combinations of fiscal and monetary instruments.

2.1. Basic Framework Consider an economy of overlapping generations of investor-consumers that consists of three cohorts: young, middle-aged, and old. The population mass of each cohort is constant over time, and normalized to unity. An infinite sequence of generations allows for the issuance of government securities including fiat money, which are handed over from one generation to another.

A young generation is endowed with y_0 units of goods, while a middle-aged generation suffers an independently and identically distributed (i.i.d.) income shock, which is explained in the next paragraph. There is no endowment or income shock for the old generation.

Let us explain more about an middle-age income shock. In particular, we assume that middle-aged income is y_h with probability $\frac{1}{2}$ and y_l with probability $\frac{1}{2}$, where $y_h > y_l$. Moreover, these i.i.d. income shocks are assumed to be unobservable, and form part of the private information of each consumer of this generation. Because of the unobservability of middle-aged income risks, no standard claim that is contingent on these idiosyncratic shocks is traded in financial markets.

seigniorage revenues, which may lead to destructive welfare consequences.

Here we depart from the original configuration of Dutta and Kapur in assuming that idiosyncratic risks are unobservable income shocks⁵ rather than unobservable preference shocks. This change makes exposition simple and transparent without altering the model substantially. This assumption does not necessarily imply that individual income is not observable at all. A part of individual income may be observable to even outsiders, and such an observable part may be shared completely among ex-ante identical consumers by standard insurance contracts. Hence, the above setup should be interpreted as abstracting an unobservable part of the whole individual income process.

The difference between unobservable income shocks and preference shocks is not so substantial as it looks, when income in our model is interpreted as *consumable* income after deducting expenses to offset unexpected and uninsured negative preference shocks such as health problems. In such a case, income may be observable, but the true expenses to offset negative shocks are not reliably observable. This implies that *consumable* income, income minus these expenses is unlikely to be observable or taxable.⁶

Let us now consider productive opportunities open to investor-consumers. Following Dutta and Kapur, we assume that only the young generation can undertake productive investment. One unit of investment yields $1 + r$ with certainty two periods later, where r is assumed to be positive ($r > 0$). However, this investment is assumed to be neither reversible nor collateralizable when these investors become middle-aged in the next period.⁷ Under this assumption, this illiquid asset provides no self-insurance against middle-aged idiosyncratic shocks. In the following analysis, let I denote the level of investment undertaken by a representative young investor.

In this framework, insurance contracts (standard contingent claims) are not available, and productive investment is not liquid or collateralizable. Thus, only liquid assets such as fiat money or government bonds may serve as precautionary measures or provide self-

⁵Here we follow Saito and Takeda (2004).

⁶Policy implications from this model do not depend on whether unobservable components are preference shocks or income shocks. As demonstrated later, the advantage of consumption taxes is brought by the fact that unobservable components are reflected in the level of consumption. Preference shocks such as health conditions may not be observable, but they have direct effects on consumption levels. That is, negative (positive) preference shocks result in less (more) consumption. Like in the case with unobservable income shocks, imposing taxes proportional to such consumption would yield redistributive effects in transferring resources from consumers with positive preference shocks to those with negative shocks.

⁷As explained in Dutta and Kapur (1998), this assumption is reasonable if actions undertaken by young investors are unobservable.

insurance for unobservable idiosyncratic shocks. We follow Dutta and Kapur to introduce government-issued assets as intergenerational allocation devices, and thereby allow middle-aged consumers to insure themselves against idiosyncratic shocks by holding such assets from the time they are young. In particular, young investors are allowed to hold m units of government-issued assets, and middle-aged consumers can keep unspent government-issued assets until they reach old age.

These government-issued assets can be interpreted as government bonds with zero rates of real interest when there is no depreciation in them and there are no aggregate shocks. At the same time, they can also be interpreted as fiat money. In particular, when we consider the case of depreciation at the rate of π , it is natural to interpret them as fiat money under nominal price changes, the value of which depreciates by the single-period rate of inflation π . Since we consider monetary policy as well as fiscal one, we adopt the fiat-money interpretation, but fiat money is interchangeable with zero-real-interest-rate government bonds if nominal prices are constant over time.

Now we explain decisions of consumption profiles. We assume that the young do not consume, whereas the middle-aged and the old do. There is no bequest motive. To make a current portfolio choice between liquid assets m and illiquid assets I , and a future consumption plan, a young investor maximizes the following lifetime expected utility function incorporating logarithmic preferences:

$$\begin{aligned} U &\equiv \sum_{i=l,h} \Pr(y_i) \{u(c_1(y_i)) + u(c_2(y_i))\} \\ &= \frac{1}{2}(\ln c_1(y_l) + \ln c_2(y_l)) + \frac{1}{2}(\ln c_1(y_h) + \ln c_2(y_h)), \end{aligned} \quad (1)$$

where $c_1(y)$ is the consumption level for the middle-aged contingent on a realization of middle-aged income y , and $c_2(y)$ is that for the old. We denote indirect lifetime expected utility ($\max_{c_1, c_2, m, I} U$) by W .

Throughout this paper, we focus on the steady-state equilibrium and ignore the transition to equilibrium.

To see basic properties of this model in the case of perfect insurance, let us consider a special case in which the expected middle-aged income is unity, and that the initial endowment y_0 satisfies

$$(1+r)y_0 = \frac{1}{2}(y_h + y_l) = 1. \quad (2)$$

The steady-state first-best allocation, in which idiosyncratic shocks are insured perfectly, implies that a young investor achieves $c_1(y_h) = c_2(y_h) = c_1(y_l) = c_2(y_l) = 1$ by allocating all the young generation's endowment to irreversible investment ($I = y_0$). The corresponding welfare W is equal to zero. Conversely, if a young investor allocates a part of his or her initial endowment to liquid assets for precautionary purposes in the steady-state equilibrium, the level of investment is less than $1/(1+r)$ ($I < y_0$). In this case, consumers may experience a reduction in long-run welfare because of having sacrificed productive opportunities. Thus, an increase in liquid assets may be welfare reducing. Because of this simplicity, we examine this simplifying case (2) extensively in numerical examples of section 3.

In this framework, Dutta and Kapur (1998) and Saito and Takeda (2004) both attempt to identify the financial instruments that might be used to improve welfare. Dutta and Kapur introduce several types of financial intermediation to address the problem of insufficient productive investment, while Saito and Takeda examine the possibility of using dynamic insurance contracts with incentive compatibility constraints to improve welfare. Unlike them, we explore fiscal and monetary policies might that enhance welfare in this context. We examine how a government should finance subsidies to those who can commit to irreversible investment (young investors). In particular, we investigate lump-sum taxes, consumption taxes, and inflation taxes respectively. Note that income taxes cannot be used as an instrument, since income is assumed to be unobservable.

2.2. Cases without subsidies: A frame of reference Before undertaking positive and normative evaluation of various combinations of taxes and subsidies, we first investigate as a frame of reference the case in which there are no changes in nominal prices.

A young investor allocates a portfolio between fiat money (m , measured in real terms) and irreversible investment (I), given the budget constraint $I + m = y_0$ with $0 \leq m \leq y_0$. When middle-aged income y is realized, a middle-aged consumer chooses $c_1(y)$ subject to the liquidity constraint $c_1(y) \leq m + y$. The consumption of an old consumer $c_2(y)$ is financed by the return on investment and unspent money balances; that is, $c_2(y) = (1+r)(y_0 - m) + m + y - c_1$.

The utility-maximization problem is solved backwards. Given a realization $y \in \{y_l, y_h\}$ and m , the Lagrangian for $c_1(y)$ is

$$L(c_1, \lambda, m, y) = \ln c_1 + \ln \{(1+r)(y_0 - m) + m + y - c_1\} + \lambda(m + y - c_1),$$

where λ is the multiplier associated with the liquidity constraint.

If liquidity constraints for middle-aged consumers are not binding, we have

$$c_1(m, y) = c_2(m, y) = \frac{1}{2} ((1+r)(y_0 - m) + m + y), \text{ and } \lambda(m, y) = 0.$$

The consumption profile for middle-aged and old consumers is flat because the rate of time preference is zero. In this case, an inequality $m \geq \frac{(1+r)y_0 - y}{2+r}$ should be satisfied.

If liquidity constraints for middle-aged consumers are binding, we get

$$c_1(m, y) = m + y, \quad c_2(m, y) = (1+r)(y_0 - m), \quad \text{and } \lambda(m, y) = \frac{1}{m + y} - \frac{1}{(1+r)(y_0 - m)}.$$

Since c_1 (c_2) is decreasing (increasing) in m , holding more money helps smooth the consumption profile. In this case, we have $m < \frac{(1+r)y_0 - y}{2+r}$.

The above multiplier λ serves as the shadow price of the liquidity constraint; that is, consumers are willing to pay a price of λ to relax the liquidity constraint by one unit. A higher value of λ implies that the liquidity constraint is more binding. With binding constraints, either greater money holding m or lower opportunity costs r lower λ . With no binding liquidity constraints, we have $\lambda = 0$.

Following Saito and Takeda (2004), we focus on a monetary economy, in which money demand is positive and liquidity constraints are binding only for low-income earners. We give conditions for this property to hold shortly in Proposition 1. In this case it is straightforward to demonstrate that

$$c_1(m, y_l) < c_2(m, y_l) < c_1(m, y_h) = c_2(m, y_h).$$

The second inequality reflects the fact that high-income earners have no liquidity constraints. This ordering of consumption levels is used to prove several propositions in this section.

Consider now money demand. Let indirect utility conditional on real money holdings m be $V(m)$:

$$V(m) \equiv \max_{c_1, c_2, I} \left[\frac{1}{2} \{ \ln c_1(m, y_l) + \ln c_2(m, y_l) \} + \frac{1}{2} \{ \ln c_1(m, y_h) + \ln c_2(m, y_h) \} \right]$$

Then, if money demand is positive so that liquidity constraints are binding only for low-

income earners, we have (see Appendix A, Proof of Proposition 1)

$$V(m) = V_2(m) \equiv \frac{1}{2} \ln(m + y_l) + \frac{1}{2} \ln(1 + r)(y_0 - m) + \ln \left[\frac{1}{2} ((1 + r)(y_0 - m) + m + y_h) \right].$$

Consequently, the optimal holdings of m^* if they are positive must satisfy $\frac{\partial V_2}{\partial m} \Big|_{m=m^*} = 0$, or

$$\frac{1}{m^* + y_l} = \frac{2}{y_0 - m^*} + \frac{2r}{(1 + r)(y_0 - m^*) + m^* + y_h}. \quad (3)$$

The left-hand side of equation (3) is the marginal utility of money from adding one unit to middle-aged low-income earners, while the right-hand side is the marginal disutility from giving up one unit of investment. Lower y_l raises the left-hand side of equation (3), while either larger y_h or lower r reduces the right-hand side. Hence, the optimal money is increased by either a larger income volatilities or a lower opportunity cost of money-holding.

Because there are only two markets (consumption goods and fiat money) in the cross-sectional allocation, Walras' Law implies that we can focus on the following goods-market clearing condition:

$$I(m^*) + \frac{1}{2} [(c_1(m^*, y_h) + c_1(m^*, y_l)) + (c_2(m^*, y_h) + c_2(m^*, y_l))] = y_0 + \frac{1}{2}(y_h + y_l) + (1 + r)I(m^*). \quad (4)$$

The right-hand (left-hand) side of equation (4) represents aggregate supply (demand). It is straightforward to see that this market clearing condition is satisfied if m^* is determined by (3), since (4) is implicitly used to derive (3).

The following proposition summarizes the above discussions and give conditions for binding liquidity constraints and positive money holdings.

Proposition 1: Assume $y_l < (1 + r)y_0 < y_h$. First, liquidity constraints are binding only for low-income earners. Second, money demand m^* is positive when $0 < r < \frac{(y_0 - y_l)(y_0 + y_h)}{(3y_l - y_0)y_0}$. Third, if m^* is positive, then it is increasing in y_h , and decreasing in both y_l and r .

Proof. The third statement has already been proved as discussed above. For the first and second statements, see Appendix A.

From the first statement, it is immediate that the assumption (2) we use extensively in section 3 is a sufficient condition for liquidity constraints to be binding only for low-income earners. From the second statement, we know that more volatile income processes are,

more likely consumers with negative income shocks are liquidity constrained and money demand is positive. This is because higher income volatility expands the range of returns on investment r for positive money demand, since either higher y_h or lower y_l raises $\frac{(y_0 - y_l)(y_0 + y_h)}{(3y_l - y_0)y_0}$.

2.3. Fiscal policy for financing lump-sum subsidies to investors This subsection examines the positive implications for two types of fiscal instruments for financing lump-sum subsidies to young investors, namely (1) lump-sum taxes and (2) consumption taxes. As mentioned earlier, income taxes are not available as an instrument under our assumption of unobservability of income. We assume there is no inflation ($\pi = 0$). Then, real money holdings (m) may also be interpreted as government bonds with zero rates of real interest.

Lump-sum taxes The total amount of lump-sum taxes is denoted by τ_0 . It is assumed that consumers are identifiable in terms of cohort so that different tax burdens can be imposed according to age. The allocation of lump-sum taxes between middle-aged and old consumers is parameterized by $0 \leq k \leq 1$; that is, tax burdens are imposed on only old consumers when $k = 0$, on only middle-aged consumers when $k = 1$, and are broadly based when $k = 0.5$.

The budget constraints of households are

$$\begin{aligned} c_1 &\leq m + y - k\tau_0, \text{ and} \\ c_2 &= (1 + r)(y_0 + s - m) + (m + y - c_1 - k\tau_0) - (1 - k)\tau_0 \\ &= (1 + r)(y_0 + s - m) + m + y - c_1 - \tau_0, \end{aligned}$$

where s denotes the size of the lump-sum subsidies to young investors.

The first panel of Table 1 summarizes optimal consumption and the Lagrange multiplier λ , given money demand m , when liquidity constraints are binding only for low-income earners. As suggested in Table 1, given m , when the tax burden shifts from middle-aged ($k = 1$) to old consumers ($k = 0$), liquidity constraints are relaxed and thus λ is reduced. As shown later, this analytical property plays an important role in determining the welfare effects of lump-sum taxation. Optimal money demand m^* satisfies the following condition:

$$\frac{1}{m^* + y_l - k\tau_0} = \frac{1 + r}{(1 + r)(y_0 + s - m^*) - (1 - k)\tau_0} + \frac{2r}{(1 + r)(y_0 + s - m^*) + m^* + y_h - \tau_0}. \quad (5)$$

Given the balanced-government-budget constraint $s = \tau_0$, optimal money demand is positive when there is lump-sum taxation, if money demand is positive when there are no subsidies.⁸ The following proposition demonstrates global properties of this fiscal policy regime with lump-sum subsidies financed by lump-sum taxes.

Proposition 2: Suppose that money demand is positive and that liquidity constraints are binding only for low-income earners. First, money demand m^* is increasing in the size of the lump-sum subsidies s . Second, if $0 \leq k < 1$, then investment I is increasing in s . Third, a decrease in k (a shift of the tax burden from middle-aged consumers to old consumers) leads to a decrease in m^* and an increase in I .

Proof. See Appendix A.

Proposition 1 shows that money demand is positive and that liquidity constraints are binding only for low-income earners under a fairly general assumption when there is no subsidy ($s = 0$). Thus, the four claims of Proposition 2 always hold around $s = 0$. Proposition 2 shows further that the four claims hold even for large $s > 0$, so long as money demand is positive and liquidity constraints are binding only for low-income earners.⁹ This remark applies all propositions of this paper if not otherwise stated.

One may find one interesting feature in this case. When lump-sum taxes are imposed on only middle-aged consumers ($k = 1$), the optimal amount of fiat money (liquidity assets) increases by the same amount as do the lump-sum subsidies, while productive investment is independent of the size of the subsidies. In other words, the imposition of lump-sum taxation on only middle-aged consumers has no effect on consumption plans or lifetime expected utility. However, this Ricardian neutrality is rather a trivial consequence of the fact that liquidity constraints are irrelevant between young and middle-aged consumers by construction since the young by assumption do not consume at all.

Proposition 2 indicates that, when $0 \leq k < 1$, both investment I and money demand are monotonically increasing in the size of the lump-sum subsidies s . Later in subsection 2.5 we will show that, with relaxed liquidity constraints for low-income earners due to

⁸It follows immediately from equations (3) and (5).

⁹Exact conditions for positive money demand and binding liquidity constraints for low income earners vary with tax and subsidy schemes and become complicated so that we do not present them here.

larger money holding, the promotion of productive investment financed by lump-sum taxes improves welfare. Moreover, we will demonstrate there that the greater the burden of lump-sum taxation on old consumers, the higher the welfare.

Consumption taxes In this subsection, we consider a policy combination in which lump-sum subsidies to young investors are financed by broadly based consumption taxes.

The budget constraints of consumers are

$$(1 + \tau_1)c_1 \leq m + y, \text{ and } (1 + \tau_1)c_2 = (1 + r)(y_0 + s - m) + m + y - (1 + \tau_1)c_1,$$

where τ_1 is the rate of consumption tax. The second panel of Table 1 summarizes optimal consumption and the Lagrange multiplier, given money demand m , when liquidity constraints are binding only for low-income earners. Optimal money demand m^* satisfies the following condition:

$$\frac{1}{m^* + y_l} = \frac{1}{y_0 + s - m^*} + \frac{2r}{(1 + r)(y_0 + s - m^*) + m^* + y_h}. \quad (6)$$

Note that if money demand is positive when there are no subsidies, then optimal money demand is positive when there is consumption-tax financed subsidies.¹⁰

Given the balanced-government-budget constraint $s = \frac{1}{2}\tau_1(c_1(y_h, m^*) + c_2(y_h, m^*) + c_1(y_l, m^*) + c_2(y_l, m^*))$ and the goods-market clearing condition, the equilibrium tax rate is determined by

$$1 + \tau_1 = \frac{(1 + r)(y_0 + s - m^*) + m^* + \frac{1}{2}(y_l + y_h)}{(1 + r)(y_0 + s - m^*) + m^* + \frac{1}{2}(y_l + y_h) - s}. \quad (7)$$

The following proposition demonstrates global properties of the regime with lump-sum subsidies financed by consumption taxes.

Proposition 3: Suppose that money demand is positive and that liquidity constraints are binding only for low-income earners. First, both money demand m^* and productive investment I are increasing in the size of the lump-sum subsidies s . Second, if $0 < r < 1$, money demand (productive investment) is greater (smaller) than when lump-sum taxes are imposed on only old consumers,

¹⁰It follows immediately from equations (3) and (6).

and smaller (greater) than when lump-sum taxes are imposed on only middle-aged consumers, for a given size of the subsidies.

Proof. See Appendix A.

In standard dynamic models with incomplete insurance, a stronger demand for precautionary savings instruments (money in this case) indicates that consumers are exposed to uninsured shocks to a greater extent. This implies that consumers suffer a larger reduction in welfare. In this regard, the welfare effects of lump-sum taxation on only old consumers would be expected to dominate those of broadly based consumption taxation. However, as we will demonstrate later in subsection 2.5, this is not the case in our model of reversible and uncollateralizable investment and no insurance, since consumption taxation is preferable in relation to risk sharing between high- and low-income earners through its redistributive nature.

2.4. Monetary policy for financing lump-sum subsidies to investors Instead of financing by taxes, we now consider money financing for lump-sum subsidies to young investors. Specifically, the government redistributes seigniorage revenues from inflation (inflation taxes) to young investors in a lump-sum manner. This is the case that Dutta and Kapur (1998) and Saito and Takeda (2004) discuss extensively in different contexts.

The budget constraints of consumers are

$$c_1 \leq (1 - \pi)m + y, \text{ and } c_2 = (1 + r)(y_0 + s - m) + (1 - \pi)[(1 - \pi)m + y - c_1],$$

where π denotes the rate of inflation.

The third panel of Table 1 reports optimal consumption and the Lagrange multiplier, given real money holdings m , when liquidity constraints are binding for only low-income earners. According to this table, larger money holdings and a lower rate of inflation help smooth the consumption profile of low-income earners. In addition, a higher rate of inflation distorts the intertemporal consumption allocation even for high-income earners who are free of liquidity constraints.

Optimal real money holdings m^* satisfy the following equation:

$$\frac{1 - \pi}{(1 - \pi)m^* + y_l} = \frac{1}{y_0 + s - m^*} + \frac{2(1 + r) - 2(1 - \pi)^2}{(1 + r)(y_0 + s - m^*) + (1 - \pi)^2 m^* + (1 - \pi)y_h}. \quad (8)$$

Equation (8) indicates that money demand m^* increases in the size of the subsidies s , given the rate of inflation $\pi > 0$, whereas m^* decreases with π given s . However, as explained below, an increase in s accompanies a higher rate of inflation (which increases the cost of holding money). Therefore, an increase in s has positive direct effects, but has negative indirect effects on money demand.

It should be noted that unlike cases in which subsidies are financed by taxation, money demand may not necessarily be positive when they are financed by inflation taxes, even if money demand is positive when there are no subsidies. This difference should be kept in mind between tax financing and money financing.

The equilibrium inflation rate π is determined by the goods-market equilibrium condition (4). It is straightforward to see that inflation is zero ($\pi = 0$) when seigniorage revenues and thus subsidies are zero ($s = 0$). To satisfy the government budget constraint, nominal money supply M must increase by $M_{t+1} - M_t = sP_{t+1} = s(1 + \pi)P_t$ (P is a nominal price) in order to maintain the steady-state monetary equilibrium. The following proposition states the local properties of the money-financing policy (that is, when s is close to zero), with respect to the steady-state rate of inflation and productive investment.

Proposition 4: Suppose that money demand is positive and that liquidity constraints are binding only for low-income earners. A marginal increase in lump-sum subsidies from zero raises both the rate of inflation and productive investment.

Proof. See Appendix A.

It should be noted that Proposition 4 only establishes local properties of money-financing in the neighborhood of $s = 0$. These local properties may be interpreted intuitively as follows. An increase in inflation rates triggered by an increase in seigniorage revenues leads to an increase in the opportunity cost of holding money, but also raises productive investment and expands aggregate output at the expense of money demand. Numerical examples in section 3 suggest that these properties may also hold in the case of large subsidies ($s \gg 0$), but we are so far unable to substantiate this claim theoretically.

2.5. Normative implications of lump-sum subsidies We have so far investigated positive aspects of the macroeconomic policy of providing lump-sum subsidies financed by either lump-sum taxes, consumption taxes, or inflation taxes. In this subsection, we turn to

normative implications of these policy combinations by evaluating lifetime expected utility in the steady-state equilibrium.

First, the following proposition states the welfare implications of lump-sum subsidies financed by different types of lump-sum taxes.

Proposition 5: Suppose that money demand is positive and that liquidity constraints are binding only for low-income earners. First, if $0 \leq k < 1$, lifetime expected utility increases with the size of the subsidies s . Second, a decrease in k (which represents a shift in the tax burden from middle-aged to old consumers) improves welfare.

Proof. See Appendix A.

The above proposition clearly states that, unless $k = 1$ (which represents the Ricardian neutrality case), increasing subsidies to young investors improves welfare by increasing both irreversible investment and money demand; an increase in investment directly expands long-run output, while an increase in money demand relaxes the liquidity constraint for middle-aged consumers.

In addition, a shift in the tax burden from middle-aged to old consumers has redistributive effects on welfare. As the first panel of Table 1 shows, a decrease in k relaxes the liquidity constraint for middle-aged earners, and helps smooth their consumption to a great extent. In the sense that middle-aged low-income consumers bear the highest tax burden relative to their incomes, a lump-sum tax on only old consumers is the most redistributive form of lump-sum taxation. This redistributive effect is welfare improving.

The following proposition states the welfare implications of broadly based consumption taxes in comparison with lump-sum taxes.

Proposition 6: Suppose that money demand is positive and that liquidity constraints are binding only for low-income earners. First, if $0 < r < 2$, lifetime expected utility increases with the size of the subsidies. Second, if $0 < r < 2$ and $\frac{1}{4} \left(\sqrt{1 + \frac{1}{r}} - 1 \right) (y_h - y_l) > \left(1 - \frac{1}{2+r} \right) (y_0 + y_l)$, then the welfare effect of broadly based consumption taxes exceeds that lump-sum taxes on only old consumers ($k = 0$) in the neighborhood of $s = 0$.

Proof. See Appendix A.

As in the case of lump-sum taxes, an increase in the size of the subsidies improves welfare by both expanding long-run production opportunities and relaxing liquidity constraints for middle-aged consumers. In addition, with the above-stated sufficient condition, taxing consumption improves welfare to a greater extent than does imposing lump-sum taxes on only old consumers ($k = 0$), which is the most welfare enhancing form of lump-sum taxation, in the neighborhood of $s = 0$. As shown numerically in the next section, the latter property is likely to hold for large subsidies as well.

We make two comments on the above condition $\frac{1}{4} \left(\sqrt{1 + \frac{1}{r}} - 1 \right) (y_h - y_l) > \left(1 - \frac{1}{2+r} \right) (y_0 + y_l)$. First, as indicated in the proof of this proposition, this condition is a sufficient condition.¹¹ Second, it is easy to show that the left-hand side of the condition is increasing in y_h and decreasing in both y_l and r , while the right-hand side is increasing in both y_l and r . That is, with lower returns on investment r and larger volatility of income (v , defined as $\frac{y_h - y_l}{2}$), this condition is likely to be satisfied. To illustrate the latter property, Figure 1 draws a heavily shaded area where consumption taxes dominate lump-sum taxes in the neighborhood of $s = 0$ under the assumption of equation (2).

As discussed above, when consumers face volatile income shocks, consumption taxes are likely to dominate lump-sum taxes on only old consumers. The welfare-improvement advantage of consumption taxation over lump-sum taxation in such a case is intuitively explained as follows. Substituting equation (7) into the consumption plans reported in the second panel of Table 1 (the consumption plan relating to consumption taxation) yields

$$\begin{aligned} c_1(m^*, y_l) &= m^* + y_l - \kappa_1 s, \\ c_2(m^*, y_l) &= (1+r)(y_0 + s - m^*) - \kappa_2 s, \\ c_1(m^*, y_h) = c_2(m^*, y_h) &= 0.5((1+r)(y_0 + s - m^*) + m^* + y_h - \kappa_3 s), \end{aligned}$$

where $\kappa_1 = (m^* + y_l)/\bar{c}$, $\kappa_2 = (1+r)(y_0 + s - m^*)/\bar{c}$, $\kappa_3 = [(1+r)(y_0 + s - m^*) + m^* + y_h]/\bar{c}$, and $\bar{c} \equiv (1+r)(y_0 + s - m^*) + m^* + 0.5(y_l + y_h)$.

In the above equations, on the one hand, κ_1 (κ_2) represents the rate of tax burden borne by middle-aged (old) low-income earners when the government finances one unit of subsidies s . On the other hand, κ_3 represents the rate of overall tax burden (the sum of

¹¹Even if this is not satisfied, it is possible to numerically find the dominance of consumption taxes over lump-sum taxes. For example, a set of parameters in the next section does not satisfy the condition, but the dominance of consumption taxes over lump-sum taxes still hold as a global property.

middle-aged and old) borne by high-income earners. It is easy to show that $\kappa_1 < \kappa_2 < 1$ and $\kappa_1 + \kappa_2 < \kappa_3$. The latter inequality indicates that the overall tax burden on high-income earners exceeds that on low-income earners. This redistributive effect of consumption taxes contributes to the improvement of welfare.

The dominating welfare effect of consumption taxation over lump-sum taxation suggests what policies are desirable for promoting irreversible investment in the context of incomplete markets. As shown in the previous subsection (Proposition 3), imposing lump-sum taxes on only old consumers is preferable to consumption taxation in relation to expanding long-run consumption opportunities through productive investment. However, on redistributive grounds, consumption taxes are preferable to lump-sum taxes. The latter part of Proposition 6, although it is a local property around $s = 0$, suggests that the redistributive effects of consumption taxes dominates consumption-increasing effects of lump-sum taxes.

Now, we turn our attention from tax financing to money financing. The following proposition states the welfare implications of lump-sum subsidies financed by seigniorage revenues.

Proposition 7: Suppose that money demand is positive and that liquidity constraints are binding only for low-income earners. Then, zero inflation is suboptimal. That is, an increase in s from $s = 0$ improves welfare.

Proof. See Appendix A.

An increase in inflation rates triggered by raising seigniorage revenues improves lifetime expected utility by expanding irreversible investment at the expense of holding money,¹² provided subsidies are positive, but close to zero.

However, we should be careful to infer global properties of money financing from this local result. An even higher rate of inflation (which increases the cost of holding money) generates two types of distortions. As suggested in the third panel of Table 1, a higher rate of inflation tightens up liquidity constraint, and thus reduces middle-aged consumption for low-income earners. In addition, money financing distorts the intertemporal allocation even for high-income earners, who are free from liquidity constraints. Thus, the cost of holding money may dominate the benefit from inflation taxes when subsidies are larger. In fact, this conjecture will be confirmed by numerical examples in section 3.

¹²As Dutta and Kapur (1998) point out, this property corresponds to the Tobin effect.

General conditions become extremely complicated under which welfare is greater in money financing than tax financing, even when subsidies are close to zero. In Appendix B, we identify these conditions in the neighborhood of $s = 0$ in special cases assuming a simplifying assumption (2). There we show that welfare is higher in money financing (i) if returns on investment r are close to zero, or (ii) if income volatility (average absolute deviations of individual income) is not too large. These conditions are intuitive, since lower returns on investment reduce the opportunity cost of holding money, and the redistribution aspect of consumption taxes has a more limited role when income volatility is smaller.

Conversely, if consumers face rather volatile shocks on income with high returns on investment r , then there is a room for inflation taxes to be dominated by consumption taxes in the neighborhood of zero subsidies. To illuminate this local property, Figure 2 draws a lightly shaded area where consumption taxes dominate inflation taxes in the neighborhood of zero subsidies under the assumption of equation (2).

2.6. Subsidies proportional to productive investment Before presenting numerical examples, we briefly consider the case in which a government provides young investors with subsidies proportional to the amount of their irreversible investment. Let ρ be a proportional rate of investment subsidies. Then, the budget constraint for a young investor is

$$(1 - \rho)I + m = y_0.$$

Given that the amount of subsidies is now $\frac{\rho}{1-\rho}(y_0 - m)$, we can derive the optimal holding of fiat money (liquid assets) and consumption plans in the same manner as before.

Because of analytical difficulties in the case of proportional subsidies, we resort to numerical examples in the next section. However, the following basic property of proportional subsidies is easy to understand. Proportional subsidies differ from lump-sum subsidies because they directly increase the opportunity cost of holding money and distort the portfolio allocation between irreversible investment and liquid assets. Such a distortion may reduce lifetime expected utility. This additional distortion in fact makes analytic examination difficult.

3. Numerical Examples In the previous section, we have examined positive and normative implications of policy combinations of subsidies to young investors and taxes on middle-aged and/or old consumers in incomplete markets. To be precise, we have explored

lump-sum subsidies to young consumers who undertake irreversible investment, that are financed by either lump-sum taxes, consumption taxes, or seigniorage revenues.

In this section, we evaluate quantitatively these policy combinations using numerical examples. In particular, this section deals with cases in which the size of subsidies is not only in the neighborhood of zero, but also far from zero.

A set of parameters is chosen as follows. In this section, we make a simplifying assumption explained in subsection 2.1, which is $(1 + r)y_0 = 0.5(y_h + y_l) = 1$, implying that the level of welfare of the first best allocation is equal to zero. In addition, numerical examples consider the case in which liquidity constraints are binding for only middle-aged low-income earners, and in which money demand is positive. For this purpose, we choose low returns on irreversible investment ($r = 0.05$) as well as volatile income fluctuations ($y_h = 1.4$ and $y_l = 0.6$). Numerical results are summarized in Tables 1, 2, and 3 and in Figures 3 through 9.

3.1. Small-scale lump-sum subsidies to investors We begin with cases of relatively small-scale lump-sum subsidies to young investors. Figures 3, 4, and 5 plot welfare levels (lifetime expected utility), the levels of investment, and the level of money demand against the size of lump-sum subsidies for each policy combination. Lump-sum subsidies have no effect on lifetime expected utility or investment when there are lump-sum taxes on only middle-aged consumers ($k = 1$). As discussed in subsection 2.3.1, the Ricardian neutrality effect works perfectly to cancel an increase in after-subsidy income by an increase in money holding.

When k is smaller than unity (more weight is put on old consumers), investment, money demand, and welfare are all increasing in the size of lump-sum subsidies at $s = 0$. In other words, lump-sum subsidies to young investors promote not only irreversible investment, but also money demand. The former aspect helps expand long-run production opportunities, while the latter helps relax liquidity constraints. As k is closer to zero (more weight on old consumers), the promotion of productive investment and the redistribution effect jointly enhance welfare more significantly. Consequently, taxing only old consumers ($k = 0$) dominates any other mode of lump-sum taxing. These numerical results confirm Propositions 2 and 5.

In the case of broadly based consumption taxes, welfare is monotonically increasing in the size of lump-sum subsidies. As in the case of lump-sum taxes, lump-sum subsidies to

young investors promote not only irreversible investment, but also money demand. However, an interesting difference from lump-sum taxes is that welfare is higher in the case of broadly based consumption taxes than in the case of lump-sum taxes with $k = 0$, although investment is larger in the latter case. This means that solving only under-investment problems does not necessarily result in welfare improvement. As discussed in detail below, broadly based consumption taxes achieve more redistribution between high- and low-income earners than do lump-sum taxes. Thanks to this redistributive aspect, welfare improves substantially in the case of consumption taxes regardless of the moderate stimulus to productive investment due to subsidies. These numerical results confirm Propositions 3 and 6.

As Figures 3, 4, and 5 show, when lump-sum subsidies increase marginally from zero, seigniorage-revenue financing is the most effective in promoting investment, money demand, and welfare among the five cases. However, numerical examples about seigniorage-revenue financing reveal an important lesson that is not apparent in Proposition 4. Welfare levels deteriorate quickly as the size of the subsidies increases, although the amount of investment monotonically increases with lump-sum subsidies. As Figure 3 shows, the optimal level of seigniorage is low (at about 0.8% of average middle-aged income).

A major reason for the hump-shaped welfare curve in the case of money financing is that the benefit of solving under-investment in irreversible capital by subsidies is quickly dominated by the cost of money holdings. That is, larger seigniorage revenues increase inflation, which raises the cost of holding money. Costly money holding then makes it difficult not only for young investors to hold money for precautionary reasons, but also makes it costly for high-income earners, even though they are free of liquidity constraints, to transfer resources from middle-age to old age. In this regard, the difference in money financing between small and large subsidies is analogous to the difference between broadly based consumption taxes and lump-sum taxes on only old consumers.

Using Tables 2, 3, and 4, we can view the above properties from different angles. Table 2 evaluates the consumption profiles at five values of lump-sum subsidies for the five policy combinations. In the cases of both lump-sum and consumption taxes, high-income earners can achieve perfect consumption smoothing. The closer is k to zero in the context of lump-sum taxation, the more low-income earners can consume relatively smooth paths at higher levels, while they can enjoy relatively smooth consumption at even higher levels when there

is consumption taxation. In the case of money financing, not only low-income earners, but also high-income earners fail to smooth consumption intertemporarily.

Table 3 calculates the effective rate of tax burdens for each policy combination. The tax burden rate is defined as the percentage ratio of tax payments to total income including endowments and saving balances held in terms of fiat money or government bonds.¹³ As this table shows, high-income (low-income) earners have heavier (lighter) tax burdens under consumption taxes than under broadly based lump-sum taxes ($k = 0.5$). In this respect, broadly based consumption taxes are redistributive relative to broadly based lump-sum taxes. In addition, lump-sum taxes on only middle-aged consumers are the least redistributive in the sense that low-income earners bear the highest burden, while broadly based consumption taxes are the most redistributive in that low-income earners bear the lowest burden.

Table 4 reports the Lagrange multiplier associated with the budget constraint of a low-income middle-aged consumer. As discussed in section 2, the higher the multiplier, the more severely the liquidity constraint binds. Given the size of the subsidies, the value of the multiplier is highest in money financing, while it is lowest in consumption taxes. In other words, the liquidity constraint for a low-income earner is the most severe under money financing and the most relaxed under consumption taxes. These results also confirm the relative advantage of consumption taxes in terms of self-insurance capabilities. Together with the results from Tables 2 and 3, we can conclude that consumption taxation is the most favorable policy instrument in the context of this model.

3.2. Proportional investment subsidies We now turn to the case of small-scale subsidies proportional to the amount of irreversible investment. Figure 6 draws welfare (lifetime expected utility) against the size of the subsidies.¹⁴ As shown in this figure, the welfare effect is similar to the case of lump-sum subsidies. Although not reported here, investment and money demand also reveal similar patterns to the case of lump-sum subsidies. One noticeable difference, however, is that the Ricardian neutrality effect is no longer present in the case of lump-sum taxes with $k = 1$, and that welfare is decreasing in the size of the subsidies because of distortionary effects which are described below. In addition, according

¹³In the case of consumption taxes, the tax burden rate is equal to $\tau/(\tau + 1)$ for consumers except for middle-aged high-income consumers who save a part of income.

¹⁴The size of the subsidies is calculated from $\frac{\rho}{1-\rho}(y_0 - m)$ where ρ is the rate of subsidies.

to Figure 7, which compares two types of subsidies in the case of consumption taxes, welfare under proportional subsidies (the solid lines) is lower than the welfare under lump-sum subsidies (the dotted line).

A major reason for these numerical results is that an allocation between irreversible investment and liquid assets (money) is distorted by proportional subsidies. Proportional subsidies promote irreversible investment only at the expense of money demand, thereby making liquidity constraints more binding for middle-aged low-income earners. In particular, lifetime welfare deteriorates substantially when proportional subsidies are financed by either seigniorage revenues or lump-sum taxes on only middle-aged consumers. As discussed above, in these cases, the capability of middle-aged consumers to self-share income risks is substantially reduced not only by proportional subsidies, but also by less redistributive taxes.

3.3. Large-scale subsidies Finally, we analyze the case of large-scale subsidies. As examined earlier, the cost of money-financed subsidies quickly dominates their benefit even for small-scale subsidies, but in the case of small-scale subsidies financed by tax instruments, welfare increases with the size of the subsidies (except in the Ricardian neutrality case). Thus, we focus on large-scale subsidies financed by either lump-sum taxes (in particular, $k = 0$) or broadly based consumption taxes.

We first consider lump-sum subsidies for productive investment. As shown in Figure 8, lifetime welfare is monotonically increasing even for large subsidies in both cases, while financing by consumption taxes is more desirable than financing by lump-sum taxes in this case. In Figure 8, welfare is increasing in the size of the subsidies, even when $s = 0.3$ (which corresponds to 30% of average middle-aged income), while marginal welfare is slightly diminishing.

We make two comments on the two cases of large-scale lump-sum subsidies. First, this monotonic property holds as long as middle-aged low-income earners are subject to liquidity constraints, and money demand is positive. Second, welfare is still far below the first best allocation (zero in our framework).

Turning to proportional subsidies (Figure 9), financing by consumption taxes is again preferable to financing by lump-sum taxes for large-scale subsidies. However, as these hump-shaped curves indicate, welfare begins to deteriorate when subsidies increase substantially. That is, the benefit of promoting irreversible investment are eventually dominated

by the cost of allocation distortion relating to proportional subsidies.

4. Concluding Remarks This paper has explored the extent to which fiscal and monetary instruments help enhance long-run welfare when financial markets are incomplete in the sense that productive investment is irreversible and uncollaterizable, there is no insurance for idiosyncratic shocks to income, and only government-issued bonds provide self-insurance. Unlike in dynamic models with reversible physical capital, an increase in precautionary savings by holding liquid bonds results in a decrease, rather than an increase, in irreversible productive investment. This paper has demonstrated that subsidies to promote productive, but irreversible, investment should be financed in such a way that a policy instrument does not substantially reduce consumers' capabilities of self-insuring idiosyncratic shocks. When income is not perfectly observable so that income taxes are not effective, lump-sum subsidies financed by consumption taxes are preferred to fixed and/or proportional investment subsidies financed by either large-scale seigniorage revenues or lump-sum taxes. The combination of lump-sum subsidies and consumption taxes are more redistributive and thus more consumption-smoothing than the other sets of instruments available in the context of this model.

The advantage of consumption taxes is brought by the fact that unobservable components of income shocks are reflected in the level of consumption. As a result, imposing taxes proportional to consumption levels could generate redistributive effects from consumers with positive shocks to those with negative shocks. Such a policy implication is also applicable to the case with unobservable preference shocks, because preference shocks have direct impacts on individual consumption.

As has been emphasized throughout the paper, the irreversibility of productive investment plays an essential role in both positive and normative implications. However, in this model, productive capital is completely irreversible before maturity occurs, and only government-issued assets serve as liquid assets. An important extension of this model is to introduce partially irreversible privately-issued assets by allowing for a set of illiquid assets with different maturities. In interaction with macroeconomic policies in such a dynamic context, the term structures may emerge not only for interest rates, but also for liquidity premiums or wedges in the returns between liquid and illiquid assets, while the allocation of productive capital between different maturities is determined endogenously.

Appendix A: Proofs of the propositions

Proof of Proposition 1 1. Because of our simple framework, it is straightforward to show that indirect utility conditional on money holding m , $V(m)$, satisfies

$$V(m) = \begin{cases} V_1(m), & \text{if } m < \frac{(1+r)y_0 - y_h}{2+r}, \\ V_2(m), & \text{if } \frac{(1+r)y_0 - y_h}{2+r} \leq m < \frac{(1+r)y_0 - y_l}{2+r}, \\ V_3(m), & \text{if } m \geq \frac{(1+r)y_0 - y_l}{2+r}, \end{cases}$$

where

$$\begin{aligned} V_1(m) &= \frac{1}{2} \ln(m + y_l) + \frac{1}{2} \ln(m + y_h) + \ln(1+r)(y_0 - m), \\ V_2(m) &= \frac{1}{2} \ln(m + y_l) + \frac{1}{2} \ln(1+r)(y_0 - m) + \ln \left[\frac{1}{2} ((1+r)(y_0 - m) + m + y_h) \right], \\ V_3(m) &= \ln \left[\frac{1}{2} ((1+r)(y_0 - m) + m + y_l) \right] + \ln \left[\frac{1}{2} ((1+r)(y_0 - m) + m + y_h) \right]. \end{aligned}$$

Note that if $m < \frac{(1+r)y_0 - y_h}{2+r}$ ($m < \frac{(1+r)y_0 - y_l}{2+r}$), then liquidity constraints are binding for high income (low income) earners.

The condition $\frac{(1+r)y_0 - y_h}{2+r} < 0$ implies that the range of $V_1(m)$ never overlaps $\{m : 0 \leq m \leq y_0\}$, while the condition $\frac{(1+r)y_0 - y_l}{2+r} > 0$ indicates that the range of $V_2(m)$ and $\{m : 0 \leq m \leq y_0\}$ have an intersection. Furthermore, $V_3(m)$ is monotonically decreasing in m . Therefore, the optimal money m^* (the solution of $\max V(m)$ s.t. $0 \leq m \leq y_0$) lies in the range of $V_2(m)$. This implies that liquidity constraints are binding only for income-low consumers.

2. For the positiveness of m^* , it is sufficient to have $\left. \frac{\partial V_2}{\partial m} \right|_{m=0} = \frac{0.5}{y_l} - \frac{0.5}{y_0} - \frac{r}{(1+r)y_0 + y_h} > 0$. With a bit of algebra, we can show that $r < \frac{(y_0 - y_l)(y_0 + y_h)}{(3y_l - y_0)y_0}$ is necessary and sufficient for the latter to hold.

Proof of Proposition 2 The statement concerning money demand is obvious from equation (6) with $\tau_0 = s$. We prove the remaining statements. Substituting $m' = m^* - s$ and $\tau_0 = s$ into equation (5), we obtain

$$\frac{1}{m' + y_l + (1-k)s} = \frac{1}{y_0 - m' - \frac{1-k}{1+r}s} + \frac{2r}{(1+r)(y_0 - m') + m' + y_h}.$$

The left-hand side of the above equation is decreasing in m' , while its right-hand side is increasing in m' when $r > 0$. Either an increase in s or a decrease in k lowers the left-hand side, but raises the right-hand side. In the presence of positive money demand, m' needs to reduce in order to equate both sides. Thus, investment $I = y_0 - m'$ is increasing in s and decreasing in k .

Proof of Proposition 3 The first statement concerning money demand is obvious from equation (6).

To prove the second statement about money demand, we show that money demand under consumption taxation is equal to that under lump-sum taxation with some $k : 0 < k < 1$.

Let m^* and m^0 be the optimal money demand for consumption taxes and lump-sum taxation with $k = 0$, respectively. m^* satisfies equation (6) or

$$\frac{1}{m^* + y_l} = \frac{1}{y_0 + s - m^*} + \frac{2r}{(1+r)(y_0 + s - m^*) + m^* + y_h},$$

while m^0 satisfies equation (5) with $s = \tau_0$ and $k = 0$, or

$$\frac{1}{m^0 + y_l} = \frac{1}{y_0 + s - m^0 - \frac{s}{1+r}} + \frac{2r}{(1+r)(y_0 + s - m^0) + m^0 + y_h - s}.$$

The left-hand (right-hand) sides of these equations are decreasing (increasing) in money demand. Under the assumption that when $m^* = m^0$, the left-hand sides of both equations are equal to each other, while the right-hand of the former equation is smaller than that of the latter. Hence, $m^* > m^0$.

Let m^1 be the optimal money demand in the case of lump-sum taxation with $k = 1$. m^1 satisfies equation (5) with $s = \tau_0$ and $k = 1$, or

$$\frac{1}{m^1 + y_l - s} - \frac{2r}{(1+r)(y_0 + s - m^1) + m^1 + y_h - s} = \frac{1}{y_0 + s - m^1}.$$

Equation (6) is now

$$\frac{1}{m^* + y_l} - \frac{2r}{(1+r)(y_0 + s - m^*) + m^* + y_h} = \frac{1}{y_0 + s - m^*}.$$

The left-hand (right-hand) sides of the above equations are decreasing (increasing) in money demand. Under the assumption that their right-hand sides are equal to each other when $m^* = m^1$, while the left-hand side of the latter equation is smaller than that of the former because $0 < r < 1$ by assumption, and $c_1(m, y_l) < c_1(m, y_h)$ (or $m + y_l - s < 0.5\{(1+r)(y_0 + s - m) + m + y_h - s\}$) for $s (\geq 0)$ and m . Hence, $m^* < m^1$.

Inferred from the first statement of Proposition 2, there exists some $k : 0 < k < 1$ such that optimal money demand in the case of lump-sum taxation is equal to that in the case of consumption taxation. Since $0 < k < 1$, we get the second statement of Proposition 3 on money holdings.

Because of the fact that $I = y_0 - s + m$ and the second statement of Proposition 2, the statement concerning investment holds true.

Proof of Proposition 4 From the total differential of equation (8), we obtain

$$\left. \frac{d\pi}{ds} \right|_{s=0} = \frac{1}{0.5c_1^*(y_h) + m^* - 0.5c_2^*(y_l)} > 0. \quad (9)$$

Because $I = y_0 + s - m$, for proving $\left. \frac{dI}{ds} \right|_{s=0} > 0$ it is sufficient to show $\left. \frac{dm}{ds} \right|_{s=0} = \left. \frac{\partial m}{\partial s} \right|_{s=0} + \left. \frac{\partial m}{\partial \pi} \right|_{s=0} \left. \frac{d\pi}{ds} \right|_{s=0} < 1$. We have already shown $\left. \frac{d\pi}{ds} \right|_{s=0} > 0$, and we can obtain $\left. \frac{\partial m}{\partial \pi} \right|_{s=0} < 0$ from equation (8). We need only to show $\left. \frac{\partial m}{\partial s} \right|_{s=0} < 1$.

We prove $\left. \frac{\partial m}{\partial s} \right|_{s=0} < 1$ by contradiction. Suppose $\left. \frac{\partial m}{\partial s} \right|_{s=0} \geq 1$. Then, we can choose a sufficiently small $\epsilon > 0$ such that $m' \geq m^* + \epsilon$, where m^* and m' are the optimal money demand with $s = 0$ and $s = \epsilon$, respectively. Given $\pi = 0$, m' must satisfy $\frac{1}{m'+y_l} = \frac{1}{y_0+\epsilon-m'} + \frac{2r}{(1+r)(y_0+\epsilon-m')+m'+y_h}$. It follows from this that $\frac{1}{m'+y_l} < \frac{1}{m^*+y_l}$, and

$$\frac{1}{y_0 + \epsilon - m'} + \frac{2r}{(1+r)(y_0 + \epsilon - m') + m' + y_h} > \frac{1}{y_0 - m^*} + \frac{2r}{(1+r)(y_0 - m^*) + m^* + y_h}.$$

Therefore, $\frac{1}{m^*+y_l} > \frac{1}{y_0-m^*} + \frac{2r}{(1+r)(y_0-m^*)+m^*+y_h}$. But this contradicts the fact that m^* must satisfy equation (3).

Proof of Proposition 5 1. Given the optimal money demand m^* , the welfare level W^l is derived by substituting the consumption plans reported in the first panel of Table 1 into equation (1). We show that $\frac{dW^l}{ds} > 0$ and $\frac{dW^l}{dk} < 0$ below. Applying the envelope theorem to W^l , we find

$$\frac{dW^l}{ds} = -\frac{0.5k}{m^* + y_l - ks} + \frac{0.5(r+k)}{(1+r)(y_0 + s - m^*) - (1-k)s} + \frac{r}{(1+r)(y_0 + s - m^*) + m^* + y_h - s}. \quad (10)$$

Substituting equation (5) with $s = \tau_0$ into equation (10) yields

$$\frac{dW^l}{ds} = \frac{0.5(1-k)}{m^* + y_l - ks} + \frac{0.5(k-1)}{(1+r)(y_0 - m^*) + (r+k)s} > 0,$$

because $0 < k < 1$, $c_1(m^*, y_l) > 0$, and $c_2(m^*, y_l) > 0$.

2. Applying the envelope theorem to W^l , we find

$$\frac{dW^l}{dk} = -\frac{0.5s}{m^* + y_l - ks} + \frac{0.5s}{(1+r)(y_0 + s - m^*) - (1-k)s}.$$

Since $c_1(m^*, y_l) < c_2(m^*, y_l)$ or $m^* + y_l - ks < (1+r)(y_l + s - m^*) - (1-k)s$ given s and k , we can establish that $\frac{dW^l}{dk} < 0$.

Proof of Proposition 6 1. Given the optimal money demand m^* , the welfare level W^c is derived by substituting the consumption plans reported in the second panel of Table 1 into equation (1). We prove $\frac{dW^c}{ds} > 0$ below.

Substituting equation (7) into W^c and applying the envelope theorem to W^c , we find

$$\frac{dW^c}{ds} = \frac{0.5}{y_0 + s - m^*} + \frac{1+r}{(1+r)(y_0 + s - m^*) + m^* + y_h} - \frac{2(1+r)}{\bar{c}} + \frac{2r}{\bar{c} - s}, \quad (11)$$

where $\bar{c} = (1+r)(y_0 + s - m^*) + m^* + 0.5(y_l + y_h)$. Substituting the first order condition (6) into (11) and noting $\frac{2r}{\bar{c}-s} > \frac{2r}{\bar{c}}$, we obtain

$$\frac{dW^c}{ds} > \frac{0.5}{m^* + y_l} + \frac{1}{(1+r)(y_0 + s - m^*) + m^* + y_h} - \frac{2}{\bar{c}}. \quad (12)$$

The right-hand side of (12) is decreasing in m^* . In fact, the third term is decreasing in m^* . The sum of the first and the second terms is also decreasing, because the first derivative of both terms is as follows.

$$-\frac{0.5}{[m^* + y_l]^2} + \frac{r}{[(1+r)(y_0 + s - m^*) + m^* + y_h]^2} < -0.5 \left[\frac{1}{c_1(m^*, y_l)^2} - \frac{1}{c_1(m^*, y_h)^2} \right] < 0,$$

where $c_1(m^*, y_l) = m^* + y_l$ and $c_1(m^*, y_h) = 0.5((1+r)(y_0 + s - m^*) + m^* + y_h)$. The first (second) inequality is established because $0 < r < 2$ ($c_1(m^*, y_l) < c_1(m^*, y_h)$).

Thus, it is sufficient to show that the right-hand side of (12) is positive at $m^* = \frac{(1+r)(y_0+s)-y_l}{2+r}$, which is the largest money demand such that liquidity constraints are binding for low-income earners. Because $(1+r)(y_0 + s - m^*) = m^* + y_l$ in this case, we find

$$\frac{dW^c}{ds} > \frac{0.5}{m^* + y_l} + \frac{0.5}{m^* + 0.5(y_l + y_h)} - \frac{2}{m^* + y_l + m^* + 0.5(y_l + y_h)}.$$

Denoting $a \equiv m^* + y_l (> 0)$ and $b \equiv m^* + 0.5(y_l + y_h) (> 0)$, we obtain

$$\frac{dW^c}{ds} > \frac{0.5}{a} + \frac{0.5}{b} - \frac{2}{a+b} > \frac{0.5(b-a)^2}{ab(a+b)^2} > 0.$$

2. We prove that $\left. \frac{dW^c}{ds} \right|_{s=0} - \left. \frac{dW^l}{ds} \right|_{s=0} > 0$ below. If $s = 0$, then both taxation cases deliver the same optimal money demand m^* and the same welfare level. The marginal welfare in the case of

consumption taxation at $s = 0$ is derived from equation (11):

$$\left. \frac{dW^c}{ds} \right|_{s=0} = \frac{0.5}{y_0 - m^*} + \frac{1+r}{(1+r)(y_0 - m^*) + m^* + y_h} - \frac{2}{\tilde{c}}, \quad (13)$$

where $\tilde{c} = (1+r)(y_0 - m^*) + m^* + 0.5(y_l + y_h)$. On the other hand, the marginal welfare in the case of lump-sum taxation with $k = 0$ is derived from equation (10):

$$\left. \frac{dW^l}{ds} \right|_{s=0} = \frac{0.5r}{(1+r)(y_0 - m^*)} + \frac{r}{(1+r)(y_0 - m^*) + m^* + y_h}. \quad (14)$$

Substituting (3) into $\left. \frac{dW^c}{ds} \right|_{s=0} - \left. \frac{dW^l}{ds} \right|_{s=0}$ yields

$$\left. \frac{dW^c}{ds} \right|_{s=0} - \left. \frac{dW^l}{ds} \right|_{s=0} = \frac{1}{1+r} \left[\frac{0.5}{m^* + y_l} + \frac{1}{(1+r)(y_0 - m^*) + m^* + y_h} \right] - \frac{2}{\tilde{c}}. \quad (15)$$

Since the term in the square brackets of (15) is equal to the first and the second terms of the right-hand side of (12), the right-hand side of (15) is decreasing in m^* .

Thus, as in the first part of this proposition, it is sufficient to show that the right-hand side of (15) is positive at $m^* = \frac{(1+r)y_0 - y_l}{2+r}$. Denoting $a \equiv m^* + y_l = (1+r)(y_0 - m^*) (> 0)$ and $b \equiv m^* + 0.5(y_l + y_h) (> 0)$, we obtain

$$\left. \frac{dW^c}{ds} \right|_{s=0} - \left. \frac{dW^l}{ds} \right|_{s=0} > \frac{0.5}{1+r} \left[\frac{1}{a} + \frac{1}{b} \right] - \frac{2}{a+b} = \frac{0.5(b-a)^2 - 2rab}{(1+r)ab(a+b)}.$$

A sufficient condition for $\left. \frac{dW^c}{ds} \right|_{s=0} - \left. \frac{dW^l}{ds} \right|_{s=0} > 0$ is $(b-a)^2 > 4rab$. Since $a = \frac{1+r}{2+r}(y_0 + y_l)$ and $b = a + \frac{1}{2}(y_h - y_l)$, this condition is rewritten as $\frac{1}{4} \left(\sqrt{1 + \frac{1}{r}} - 1 \right) (y_h - y_l) > \left(1 - \frac{1}{2+r} \right) (y_0 + y_l)$.

Proof of Proposition 7 Given the optimal money demand m^* , the welfare level W^c is derived by substituting the consumption plans reported in the third panel of Table 1 into equation (1). We prove $\left. \frac{dW^p}{ds} \right|_{s=0} > 0$ below. Applying the envelope theorem to W^p at $s = 0$ ($\pi = 0$), we find

$$\begin{aligned} \left. \frac{dW^p}{ds} \right|_{s=0} &= \frac{0.5}{y_0 - m^*} + \frac{1+r}{(1+r)(y_0 - m^*) + m + y_h} \\ &+ \left[\frac{(1+r)(y_0 - m^*) - m^*}{(1+r)(y_0 - m^*) + m^* + y_h} - \frac{0.5m^*}{m^* + y_l} - 0.5 \right] \left. \frac{d\pi}{ds} \right|_{s=0}. \end{aligned} \quad (16)$$

Substituting equations (3) and (9) into the right-hand side of equation (16), we obtain the following condition for $\left. \frac{dW^p}{ds} \right|_{s=0} > 0$:

$$0.25 (c_1(m^*, y_h) - c_2(m^*, y_l)) \left[\frac{1}{c_1(m^*, y_l)} - \frac{1}{c_1(m^*, y_h)} \right] > 0,$$

where $c_1(m^*, y_h) = 0.5((1+r)(y_0 - m^*) + m^* + y_h)$, $c_1(m^*, y_l) = y_l + m^*$, and $c_2(m^*, y_l) = (1+r)(y_0 - m^*)$. The above inequality is established because $c_1(m^*, y_h) > c_2(m^*, y_l) > c_1(m^*, y_l) > 0$ when only low-income earners are subject to liquidity constraints.

Appendix B: Welfare comparison between inflation taxes and consumption taxes

In this appendix, we discuss conditions under which the level of welfare is higher in the case of money financing than in the case of consumption taxation in the neighborhood of $s = 0$. To make analysis simple and intuitive further, we make an additional simplifying assumption (2) in addition to those of Propositions 1 through 7.

Suppose that $(1+r)y_0 = 0.5(y_l + y_h) = 1$, or equation (2) holds. Then, welfare is higher for money financing than consumption taxation in the neighborhood of $s = 0$ either (i) if the return on investment r is close to zero, or (ii) if income volatility v , defined below, is not too large. Exact meaning of “close to” and “not too large” in this statement becomes clear in the following discussion.

For this purpose, we examine the sign of $\left. \frac{dW^p}{ds} \right|_{s=0} - \left. \frac{dW^c}{ds} \right|_{s=0}$ below. With $(1+r)y_0 = 0.5(y_l + y_h) = 1$ and $v \equiv 1 - y_l = y_h - 1$ together with equations (13) and (16), the condition for $\left. \frac{dW^p}{ds} \right|_{s=0} - \left. \frac{dW^c}{ds} \right|_{s=0} > 0$ is rewritten as $\phi(m, r, v) > 0$, where

$$\phi(m, r, v) = \frac{v + (6+r)m}{2(2-rm)} + \frac{1 - (2+r)m}{2+v-rm} - \frac{0.5m}{m+1-v} - 0.5.$$

We investigate the conditions for $\phi > 0$ below.

Before proving the first statement (i), we will show that if r is decreasing and close to zero (so that y_0 is close to unity), the money demand increases to $0.5v$. Indeed, using the simplifying assumption (2), we can write equation (3) as $\psi(m, r) = 0$, where

$$\psi(m, r) \equiv \frac{1}{m+1-v} - \frac{1}{\frac{1}{1+r} - m} - \frac{2}{\frac{2+v}{r} - m}. \quad (17)$$

Since $\psi(m, r)$ is increasing (decreasing) in r (m), an decrease in r raises the optimal money m^* such that $\psi(m^*, r) = 0$. Furthermore, $m^* = \frac{v}{2}$ solves $\psi(m^*, 0) = 0$, and $\psi(m, r)$ is continuous in

r . Thus, $\lim_{r \rightarrow 0} m^* = \frac{v}{2}$ holds.

Then, for any income volatility $0 < v < 1$,

$$\phi(0.5v, 0, v) = \frac{(1-v)v^2}{(2+v)(2-v)} > 0.$$

Since ϕ is continuous in r , $\lim_{r \rightarrow 0} \phi > 0$. Thus, if r is close to zero, then $\phi > 0$.

When r is positive, the optimal money demand is less than $0.5v$, which implies that ϕ may be negative when there is a large v . For example, when $r = 0.05$ and $v = 0.99$, $\phi < 0$ is numerically confirmed.

We next explore the range of v satisfying $\phi > 0$. To prove the second statement (ii), we first show $\frac{\partial \phi}{\partial r} > 0$. Given a $v > 0$ and m ,

$$\frac{\partial \phi}{\partial r} = \frac{2+v+6m}{2(2-rm)^2}m - \frac{1+v+2m}{(2+v-rm)^2}m > \frac{m(m-0.5v)}{(2+v-rm)^2} > 0,$$

because $0 < m < 0.5v$. The optimal money is determined implicitly by v and r from equation (3). As a sufficient condition, we then determine the range of v that satisfies $\phi > 0$ for any m at $r = 0$. When $r = 0$,

$$\phi(m, 0, v) = \frac{v^2}{4(2+v)} + 0.5m \left\{ \xi - \frac{1}{m+2-v} \right\},$$

where $\xi = \frac{3v+2}{2+v}$. Since $\frac{\partial \phi}{\partial m} = 0.5\xi - \frac{0.5(1-v)}{(m+1-v)^2}$, ϕ takes the minimum value with $\tilde{m} = \sqrt{y_l/\xi} - y_l$, and its value is calculated as $\phi(\tilde{m}, 0, v) = \frac{v^2}{4(2+v)} - 0.5 \left(1 - \sqrt{\xi(1-v)} \right)^2$. The condition for $\phi(\tilde{m}, 0, v) > 0$ is equivalent to $\zeta(v) > 0$, where

$$\zeta(v) = v - \sqrt{2(2+v)} + \sqrt{2(3v+2)(1-v)}.$$

It is easy to show that $\zeta(0) = 0$, $\zeta'(0) > 0$, and $\zeta(1) = -1$. These equations imply that $\zeta > 0$ when v is not too large.

For example, $\zeta(0.6) > 0$ and $\zeta(0.7) < 0$ are confirmed numerically when r is close to zero. It should be noted that the above condition is sufficient. For example, when $v = 0.7$ and $r = 0.3$, inflation taxation still dominates consumption taxation. Our numerical examples presented in section 3 consider the case in which v is relatively large, but r is small so that a sufficiently large positive money demand emerges. In such a case, money financing is the most favorable possible policy instrument in the neighborhood of $s = 0$.

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Table 1: Analytical forms of consumption profiles and Lagrange multipliers in the case of lump-sum subsidies

	lump-sum tax
$c_1(m, y_l)$	$m + y_l - k\tau_0$
$c_2(m, y_l)$	$(1 + r)(y_0 + s - m) - (1 - k)\tau_0$
$\lambda(m, y_l)$	$\frac{1}{m + y_l - k\tau_0} - \frac{1}{(1+r)(y_0 + s - m) - (1-k)\tau_0}$
$c_1(m, y_h)$	$0.5((1 + r)(y_0 + s - m) + m + y - \tau_0)$
$c_2(m, y_h)$	$c_1(m, y_h)$
$\lambda(m, y_h)$	0
	consumption tax
$c_1(m, y_l)$	$\frac{m + y_l}{1 + \tau_1}$
$c_2(m, y_l)$	$\frac{(1 + r)(y_0 + s - m)}{1 + \tau_1}$
$\lambda(m, y_l)$	$(1 + \tau_1) \left(\frac{1}{m + y_l} - \frac{1}{(1+r)(y_0 + s - m)} \right)$
$c_1(m, y_h)$	$\frac{0.5}{1 + \tau_1} ((1 + r)(y_0 + s - m) + m + y_h)$
$c_2(m, y_h)$	$c_1(m, y_h)$
$\lambda(m, y_h)$	0
	inflation tax
$c_1(m, y_l)$	$(1 - \pi)m + y_l$
$c_2(m, y_l)$	$(1 + r)(y_0 + s - m)$
$\lambda(m, y_l)$	$\frac{1}{(1 - \pi)m + y_l} - \frac{1 - \pi}{(1 + r)(y_0 + s - m)}$
$c_1(m, y_h)$	$0.5 \left(\frac{1 + r}{1 - \pi} (y_0 + s - m) + (1 - \pi)m + y_h \right)$
$c_2(m, y_h)$	$(1 - \pi)c_1(m, y_h)$
$\lambda(m, y_h)$	0

Table 2: Consumption profiles in the case of lump-sum subsidies

lump-sum tax ($k = 0$)	$s = 0.000$	$s = 0.005$	$s = 0.010$	$s = 0.015$
$c_1(y_l)$	0.7636	0.7637	0.7638	0.7639
$c_2(y_l)$	0.8282	0.8283	0.8285	0.8286
$c_1(y_h) = c_2(y_h)$	1.1959	1.1960	1.1962	1.1963
lump-sum tax ($k = 0.5$)	$s = 0.000$	$s = 0.005$	$s = 0.010$	$s = 0.015$
$c_1(y_l)$	0.7636	0.7637	0.7637	0.7638
$c_2(y_l)$	0.8282	0.8283	0.8283	0.8284
$c_1(y_h) = c_2(y_h)$	1.1959	1.1960	1.1960	1.1961
lump-sum tax ($k = 1$)	$s = 0.000$	$s = 0.005$	$s = 0.010$	$s = 0.015$
$c_1(y_l)$	0.7636	0.7636	0.7636	0.7636
$c_2(y_l)$	0.8282	0.8282	0.8282	0.8282
$c_1(y_h) = c_2(y_h)$	1.1959	1.1959	1.1959	1.1959
consumption tax	$s = 0.000$	$s = 0.005$	$s = 0.010$	$s = 0.015$
$c_1(y_l)$	0.7636	0.7641	0.7647	0.7652
$c_2(y_l)$	0.8282	0.8288	0.8294	0.8300
$c_1(y_h) = c_2(y_h)$	1.1959	1.1955	1.1950	1.1946
seigniorage revenues	$s = 0.000$	$s = 0.005$	$s = 0.010$	$s = 0.015$
$c_1(y_l)$	0.7636	0.7509	0.7355	0.7145
$c_2(y_l)$	0.8282	0.8443	0.8634	0.8883
$c_1(y_h)$	1.1959	1.2042	1.2143	1.2281
$c_2(y_h)$	1.1959	1.1857	1.1739	1.1585

Table 3: Effective rates of tax burdens in the case of tax-financing and lump-sum subsidies (unit: %)

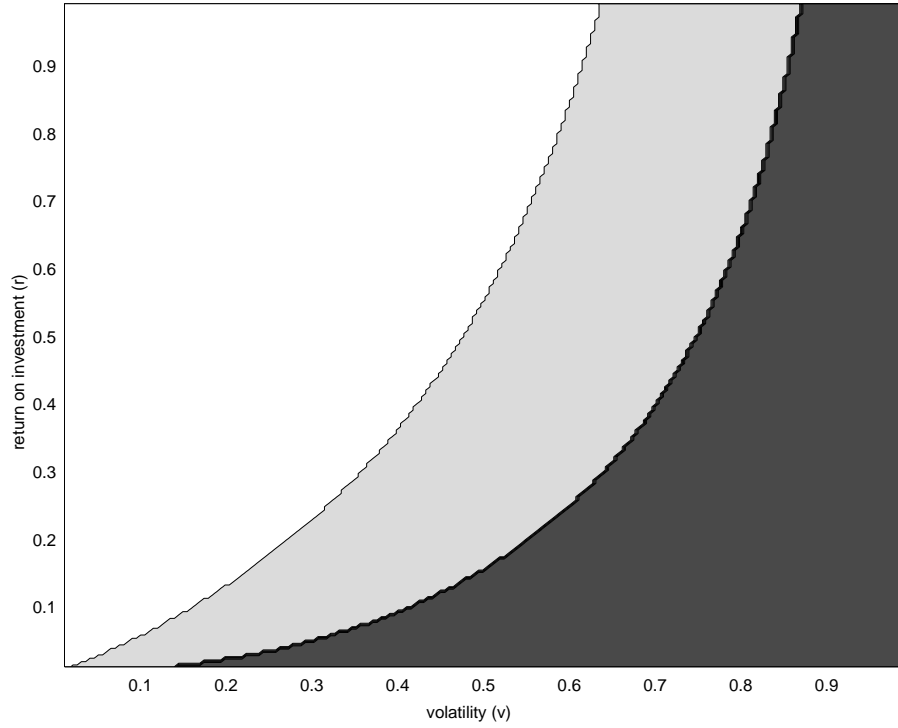
lump-sum tax ($k = 0$)	$s = 0.000$	$s = 0.005$	$s = 0.010$	$s = 0.015$
middle-aged low-income	0.00	0.00	0.00	0.00
old low-income	0.00	0.60	1.19	1.78
middle-aged high-income	0.00	0.00	0.00	0.00
old high-income	0.00	0.42	0.83	1.24
lump-sum tax ($k = 0.5$)	$s = 0.000$	$s = 0.005$	$s = 0.010$	$s = 0.015$
middle-aged low-income	0.00	0.33	0.65	0.97
old low-income	0.00	0.30	0.60	0.90
middle-aged high-income	0.00	0.16	0.32	0.48
old high-income	0.00	0.21	0.42	0.62
lump-sum tax ($k = 1$)	$s = 0.000$	$s = 0.005$	$s = 0.010$	$s = 0.015$
middle-aged low-income	0.00	0.65	1.29	1.93
old low-income	0.00	0.00	0.00	0.00
middle-aged high-income	0.00	0.32	0.64	0.95
old high-income	0.00	0.00	0.00	0.00
consumption tax	$s = 0.000$	$s = 0.005$	$s = 0.010$	$s = 0.015$
middle-aged low-income	0.00	0.25	0.50	0.75
old low-income	0.00	0.25	0.50	0.75
middle-aged high-income	0.00	0.19	0.38	0.57
old high-income	0.00	0.25	0.50	0.75

(1) The effective rate of tax burdens is defined as the percentage ratio of tax payment relative to total income including endowment as well as saving balances held in terms of fiat money or government bonds.

Table 4: Lagrange multipliers in the case of lump-sum subsidies

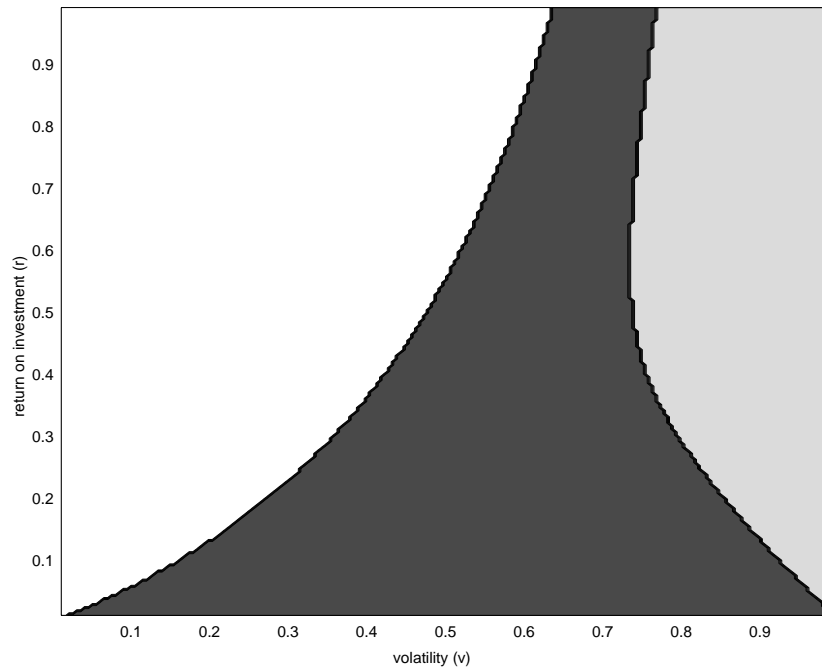
financing instrument	$s = 0.000$	$s = 0.005$	$s = 0.010$	$s = 0.015$
seigniorage revenues	0.10218	0.16553	0.23995	0.33776
lump-sum tax ($k = 0$)	0.10218	0.10217	0.10215	0.10214
lump-sum tax ($k = 0.5$)	0.10218	0.10217	0.10217	0.10216
lump-sum tax ($k = 1$)	0.10218	0.10218	0.10218	0.10218
consumption tax	0.10218	0.10215	0.10212	0.10209

Figure 1: Welfare comparison between the consumption taxation and the lump-sum taxation in the neighborhood of zero subsidies



- (1) A non-shaded area represents the case with zero money demand. A heavily shaded area depicts the case where the consumption taxation dominates the lump-sum taxation on only old consumers in the neighborhood of zero subsidies, while a lightly shaded area draws the reverse case.
- (2) An income volatility v is defined as $\frac{1}{2}(y_h - y_l)$.
- (3) It is assumed that $(1+r)y_0 = \frac{1}{2}(y_h - y_l) = 1$.

Figure 2: Welfare comparison between the inflation taxation and the consumption taxation in the neighborhood of zero subsidies



(1) A non-shaded area represents the case with zero money demand. A lightly shaded area depicts the case where the consumption taxation dominates the inflation taxation in the neighborhood of zero subsidies, while a heavily shaded area draws the reverse case.

(2) An income volatility v is defined as $\frac{1}{2}(y_h - y_l)$.

(3) It is assumed that $(1 + r)y_0 = \frac{1}{2}(y_h - y_l) = 1$.

Figure 3: Welfare comparison in the case of lump-sum subsidies

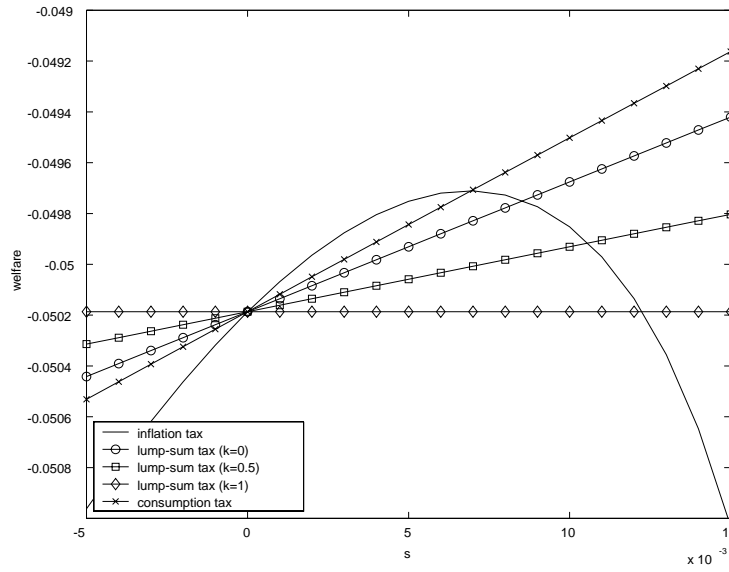


Figure 4: Physical investment comparison in the case of lump-sum subsidies

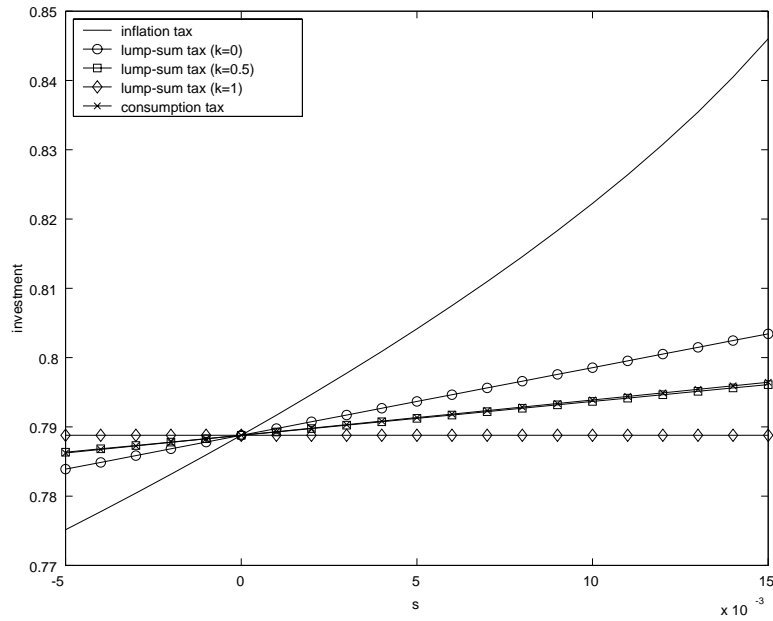


Figure 5: Money demand comparison in the case of lump-sum subsidies

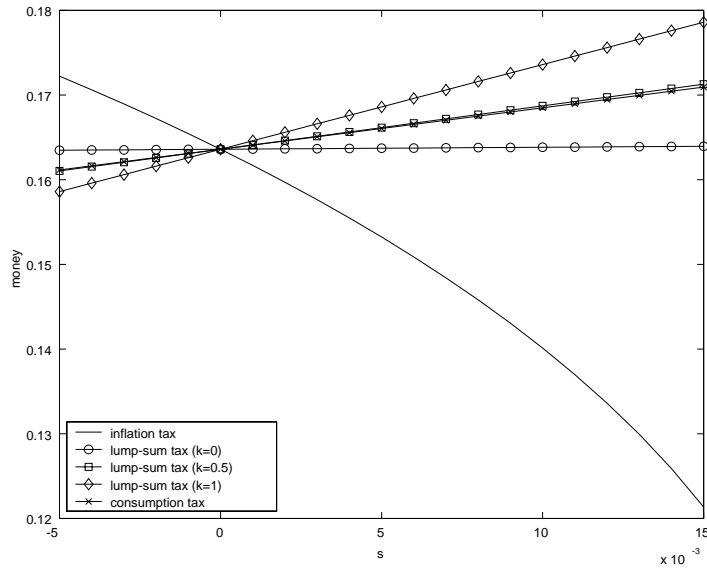


Figure 6: Welfare comparison in the case of subsidies proportional to investment

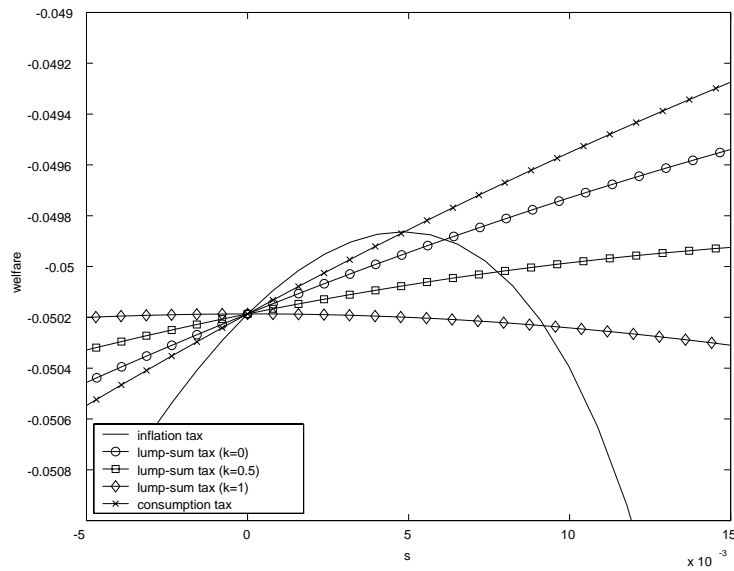


Figure 7: Welfare comparison in the case of consumption taxes

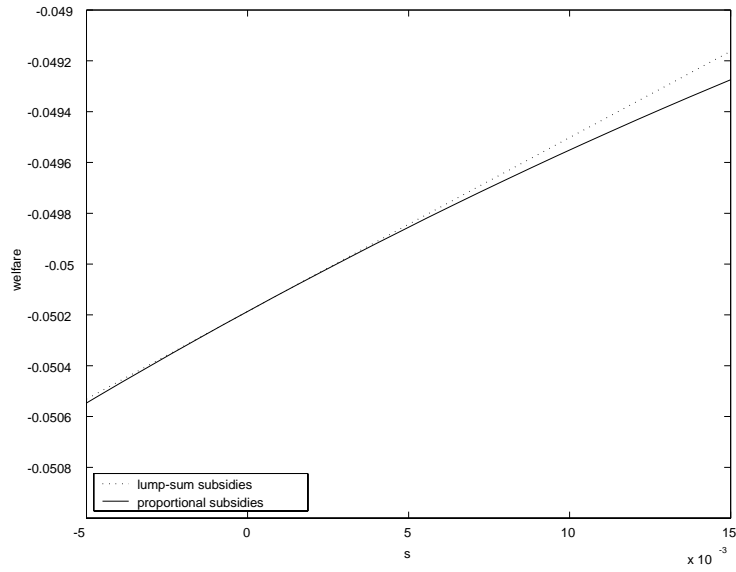


Figure 8: Welfare comparison between consumption taxes and lump-sum taxes ($k = 0$) in the case of *large-scale* lump-sum subsidies

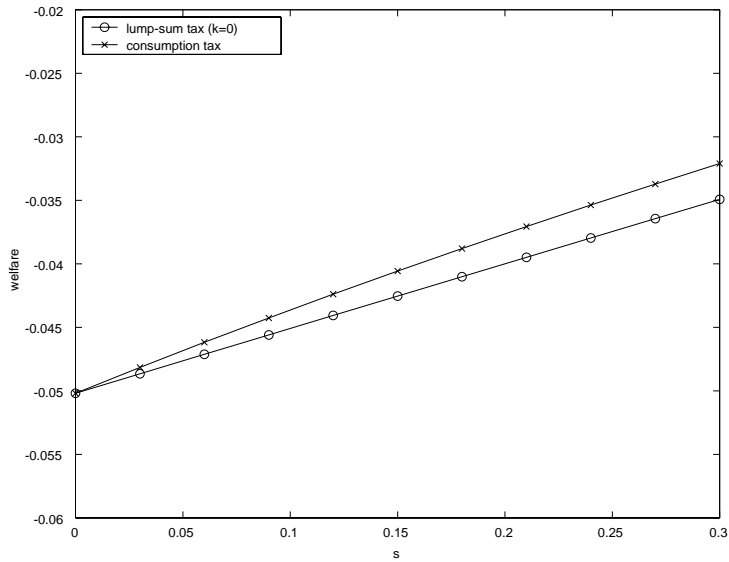


Figure 9: Welfare comparison between consumption taxes and lump-sum taxes ($k = 0$) in the case of *large-scale* subsidies proportional subsidies

