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Social Norms and Rationality of Choice*

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Abstract. Ever since Sen (1993) criticized the notion of internal consistency of choice, there exists a widespread perception that the standard rationalizability approach to the theory of choice has difficulties coping with the existence of external social norms. This paper introduces a concept of *norm-conditional rationalizability* and shows that external social norms can be accommodated so as to be compatible with norm-conditional rationalizability by means of suitably modified revealed preference axioms in the theory of rational choice on general domains à la Richter (1966; 1971) and Hansson (1968).

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1 Introduction

In his Presidential Address to the Econometric Society, Sen (1993) argued against *à priori* imposition of requirements of internal consistency of choice such as the weak and the strong axioms of revealed preference, Arrow's (1959) axiom of choice consistency, and Sen's (1971) condition α, and investigated the implications of eschewing these internal choice consistency requirements.

The gist of his criticism can be neatly summarized in terms of his own example that goes as follows. Let C be the *choice function* that specifies, for any admissible non-empty set S of feasible alternatives, a non-empty subset $C(S)$ of S, which is to be called the *choice set* of S. Then Sen (1993, p.500) poses the following question: "[C]an a set of choices really be seen as consistent *or* inconsistent on purely internal grounds *without* bringing in something *external* to choice, such as the underlying objectives or values that are pursued or acknowledged by choice?" To bring his point into clear relief, Sen invites us to examine the following two choices:

$$C(\{x, y\}) = \{x\} \text{ and } C(\{x, y, z\}) = \{y\}.$$

As Sen rightly points out, this pair of choices violates most of the standard choice consistency conditions including the weak and the strong axioms of revealed preference, Arrow's axiom of choice consistency, and Sen's condition α. It is arguable and indeed Sen (1993, p.501) argues that this seeming inconsistency can be easily resolved if only we know more about the person's choice situation: "Suppose the person faces a choice at a dinner table between having the last remaining apple in the fruit basket (y) and having nothing instead (x), forgoing the nice-looking apple. She decides to behave decently and picks nothing (x), rather than the one apple (y). If, instead, the basket had contained two apples, and she had encountered the choice between having nothing (x), having one nice apple (y) and having another nice one (z), she could reasonably enough choose one (y), without violating any rule of good behavior. The presence of another apple (z) makes one of the two apples decently choosable, but this combination of choices would violate the standard consistency conditions . . . even though there is nothing particularly 'inconsistent' in this pair of choices . . . ."

On the face of it, Sen's argument to this effect may seem to go squarely against the theory of rationalizability à la Arrow (1959), Richter (1966; 1971), Hansson (1968), Sen (1971), Suzumura (1976a) and many others, where the weak axiom of revealed preference is a necessary condition for rationalizability.[1] The purpose of this paper is to develop a new

[1] Recollect that the standard theory of rationalizability has an important point of bifurcation depending

concept of *norm-conditional rationalizability* and build a bridge between rationalizability theory and Sen's criticism. In essence, what emerges from our theory is the peaceful co-existence of a norm-conditional rationalizability theory and Sen's elaborated criticism against internal consistency of choice.

More precisely, we introduce a model of choice where external norms are taken into consideration by specifying all pairs consisting of a feasible set and an element of this set with the interpretation that this element is prohibited from being chosen from this set by the relevant system of social norms. Norm-conditional rationalizability then requires the existence of a preference relation such that, for each feasible set in the domain of the choice function, the chosen elements are at least as good as all elements in the set *except* for those that are prohibited by the social norm. This approach is very general because no restrictions are imposed on how the system of social norms comes about—any specification of a set of pairs as described above is possible. The traditional model of rational choice is included as a special case—the case that obtains if the set of prohibited pairs is empty. It is important to emphasize that, unlike earlier approaches that attempt to incorporate external social norms into models of choice, our framework does not rely on implicit assumptions such as, for example, everyone in a society having the same preferences and a decision-maker should refrain from choosing the unique best element according to such a *common* preference relation; see, for instance, Baigent and Gaertner (1996). We will return to this issue in the concluding section of this paper.

Apart from this introduction, the paper consists of four sections. Section 2 is devoted to the preliminary analysis of preference relations and their extensions to (complete) orderings. Section 3 introduces the concept of external norms and relates them to the concept of a choice function. Section 4 defines the crucial concept of norm-conditional rationalizability and shows how the standard theory of rationalizability can be modified

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on the specification of choice domains. The classical theory of revealed preference due originally to Samuelson (1938; 1947; 1948; 1950) and Houthakker (1950) was concerned with the choice functions on the domains of competitive budgets, whereas the expansion of the choice functional theory beyond the narrow confinement of competitive consumers due to Arrow (1959) and Sen (1971) had a constraint of its own, and presupposed that the domains were confined to the finite sets of alternatives. See, also, Aizerman and Aleskerov (1995), and Schwartz (1976) for further work along this line. It was Richter (1966; 1971), Hansson (1968) and Suzumura (1976a; 1977; 1983) who explored the general rationalizability theory without these domain constraints, thereby making the theory universally applicable to whatever choice contexts we may want to specify. Recent years have witnessed further development of the general theory of rationalizability in the tradition of Richter and Hansson without any external norm. See Bossert, Sprumont and Suzumura (2005; 2006) and Kim and Richter (1986), among others.

in such a way that the core essence of rationalizability theory can be kept intact in the presence of external norms. Section 5 concludes with remarks on some related literature.

2 Preference Relations

Let X be a universal non-empty set of alternatives and let $R \subseteq X \times X$ be a (binary) relation on X. The asymmetric factor $P(R)$ of R is given by $(x, y) \in P(R)$ if and only if $(x, y) \in R$ and $(y, x) \notin R$ for all $x, y \in X$, and the symmetric factor $I(R)$ of R is defined by $(x, y) \in I(R)$ if and only if $(x, y) \in R$ and $(y, x) \in R$ for all $x, y \in X$.

The *transitive closure* $tc(R)$ of a relation R is defined by letting, for all $x, y \in X$,

$$\begin{aligned}(x, y) \in tc(R) \ \Leftrightarrow \ & \exists K \in \mathbb{N}, \exists x^0, \ldots, x^K \in X: \\ & x = x^0 \ \& \ (x^{k-1}, x^k) \in R \ \forall k \in \{1, \ldots, K\} \ \& \ x^K = y.\end{aligned}$$

For any binary relation R, $tc(R)$ is the smallest transitive superset of R.

A relation $R \subseteq X \times X$ is *reflexive* if, for all $x \in X$,

$$(x, x) \in R$$

and R is *complete* if, for all $x, y \in X$ such that $x \neq y$,

$$(x, y) \in R \vee (y, x) \in R.$$

R is *transitive* if, for all $x, y, z \in X$,

$$[(x, y) \in R \ \& \ (y, z) \in R] \ \Rightarrow \ (x, z) \in R.$$

It is clear that R is transitive if and only if $R = tc(R)$. A *quasi-ordering* is a reflexive and transitive relation and an *ordering* is a complete quasi-ordering.

R is *consistent* if, for all $x, y \in X$,

$$(x, y) \in tc(R) \ \Rightarrow \ (y, x) \notin P(R).$$

This notion of consistency is due to Suzumura (1976b) and it is equivalent to the requirement that any cycle must be such that all relations involved in this cycle are instances of indifference—strict preference cannot occur. To facilitate the understanding of this concept, we may define the *consistent closure* $cc(R)$ of R as the smallest consistent superset of R. This is the concept coined by Bossert, Sprumont and Suzumura (2005), which may be written explicitly as follows. For all $x, y \in X$,

$$(x, y) \in cc(R) \ \Leftrightarrow \ (x, y) \in R \vee [(x, y) \in tc(R) \ \& \ (y, x) \in R].$$

Clearly, for any binary relation R, we have $R \subseteq cc(R) \subseteq tc(R)$ and R is consistent if and only if $R = cc(R)$. It is easy to verify that consistency implies (but is not implied by) the well-known *acyclicity* axiom which rules out the existence of strict preference cycles (cycles composed entirely of instances of strict preference). Consistency and *quasi-transitivity,* which requires that $P(R)$ is transitive, are independent. Transitivity implies consistency but the reverse implication is not true in general. However, if R is reflexive and complete, consistency and transitivity are equivalent.

A relation R^* is an *extension* of R if and only if $R \subseteq R^*$ and $P(R) \subseteq P(R^*)$. If an extension R^* of R is an ordering, we refer to R^* as an *ordering extension* of R. One of the most fundamental results on extensions of binary relations is due to Szpilrajn (1930) who showed that any transitive and asymmetric relation has a transitive, asymmetric and complete extension. The result remains true if asymmetry is replaced with reflexivity, that is, *any quasi-ordering has an ordering extension.* Arrow (1951, p.64) stated this generalization of Szpilrajn's theorem without a proof and Hansson (1968) provided a proof on the basis of Szpilrajn's original theorem. While the property of being a quasi-ordering is sufficient for the existence of an ordering extension of a relation, this is not necessary. As shown by Suzumura (1976b), consistency is *necessary and sufficient* for the existence of an ordering extension; see Suzumura (1976b, pp.389–390).

3 Norms and Choices

A choice situation is described by a feasible set S of alternatives, where S is a non-empty subset of X. Social norms such as those discussed in the introduction can be expressed by identifying feasible sets and alternatives that are not to be chosen from these feasible sets. For example, suppose there is a feasible set $S = \{x, y\}$, where x stands for selecting nothing and y stands for selecting (a single) apple. Now consider the feasible set $T = \{x, y, z\}$ where there are two (identical) apples y and z available. The social norm not to take the last apple can easily and intuitively be expressed by requiring that the choice of y from S is excluded, whereas the choice of y (or z) from T is perfectly acceptable. In general, norms of that nature can be expressed by identifying all pairs (S, w), where $w \in S$, such that w is not supposed to be chosen from the feasible set S. To that end, we use a set $\mathcal{N}$, to be interpreted as the set of all pairs (S, w) of a feasible set S and an element w of S such that the choice of w from S is prevented by the social norm under consideration.

More formally, suppose $\mathcal{X}$ is the power set of X excluding the empty set. A choice function is a mapping $C\colon \Sigma \to \mathcal{X}$ such that $C(S) \subseteq S \setminus \{z \in S \mid (S,z) \in \mathcal{N}\}$ for all $S \in \Sigma$, where $\Sigma \subseteq \mathcal{X}$ with $\Sigma \neq \emptyset$ is the domain of C. Let $C(\Sigma)$ denote the image of Σ under C, that is, $C(\Sigma) = \cup_{S \in \Sigma} C(S)$. As is customary, we assume that $C(S)$ is non-empty for all sets S in the domain of C. Thus, using Richter's (1971) term, the choice function C is assumed to be *decisive*. To ensure that this requirement does not conflict with the restrictions imposed by the norm $\mathcal{N}$, we require $\mathcal{N}$ to be such that, for all $S \in \Sigma$, there exists $x \in S$ satisfying $(S,x) \notin \mathcal{N}$. The set of all possible norms satisfying this restriction is denoted by $\mathbf{N}$.

This model of norm-conditional choice may appear somewhat restrictive at first sight because it specifies pairs of a feasible set and *single objects* not to be chosen from that set. One might want to consider the following seeming generalization of this approach: instead of only including pairs of the form (S,x) with $x \in S$ when defining a system of norms, one could include pairs such as (S,S') with $S' \subseteq S$, thus postulating that the subset S' should not be chosen from S. Contrary to first appearance, this does not really provide a more general model of norm-conditional rationalizability because, in order to formulate our notion of norm-conditional rationality, we require that a chosen element $x \in C(S)$ has to be at least as good as all feasible elements except those that are already excluded by the social norm according to a norm-conditional rationalization—that is, x has to be at least as good as all $y \in S$ except for those $y \in S$ such that $(S,y) \in \mathcal{N}$. Allowing for pairs (S,S') does not provide a more general notion of norm-conditional rationalizability because the subset of S, the elements of which have to be dominated by a chosen object, can be obtained in any arbitrary way from the subsets S' such that S' cannot be selected from S according to the social norm. For simplicity of exposition, we work with the simpler version of our model introduced above but note that this formulation does not involve any loss of generality when it comes to the definition of norm-conditional rationality employed in this paper.

Returning to Sen's example involving the norm "do not choose the last available apple," we can, for instance, define the universal set $X = \{x,y,z\}$, the domain $\Sigma = \{S,T\} \subsetneq \mathcal{X}$ with $S = \{x,y\}$ and $T = \{x,y,z\}$, and the social norm described by the set $\mathcal{N} = \{(S,y)\}$. Thus, the social norm requires that $y \notin C(S)$ but no restrictions are imposed on the choice $C(T)$ from the set T—that is, this social norm represents the requirement that the last available apple should not be chosen.

4 Norm-Conditional Rationalizability

The notion of rationality explored in this paper is conditional on a system of social norms $\mathcal{N} \in \mathbf{N}$ as introduced in the previous section. In contrast with the classical model of rational choice, an element x that is chosen by a choice function C from a feasible set $S \in \Sigma$ need not be considered at least as good as *all* elements of S by a rationalizing relation, but merely at least as good as all elements $y \in S$ such that $(S, y) \notin \mathcal{N}$. That is, if the choice of y from S is already prohibited by the norm, there is no need that x dominates such an element y according to the rationalization. Needless to say, the chosen element x itself must be admissible in the presence of the prevailing system of social norms.

To make this concept of norm-conditional rationalizability precise, let a system of social norms $\mathcal{N} \in \mathbf{N}$ and a feasible set $S \in \Sigma$ be given. An $\mathcal{N}$-*admissible set* for $(\mathcal{N}, S)$, $A^{\mathcal{N}}(S) \subseteq S$, is defined by letting, for all $x \in S$,

$$x \in A^{\mathcal{N}}(S) \;\Leftrightarrow\; (S, x) \notin \mathcal{N}.$$

Note that, by assumption, $A^{\mathcal{N}}(S) \neq \emptyset$ for all $\mathcal{N} \in \mathbf{N}$ and for all $S \in \Sigma$.

We say that a choice function C on Σ is $\mathcal{N}$-*rationalizable* if and only if there exists a binary relation $R^{\mathcal{N}} \subseteq X \times X$ such that, for all $S \in \Sigma$ and for all $x \in S$,

$$x \in C(S) \;\Leftrightarrow\; x \in A^{\mathcal{N}}(S) \;\&\; [\forall y \in A^{\mathcal{N}}(S) : (x, y) \in R^{\mathcal{N}}].$$

In this case, we say that $R^{\mathcal{N}}\mathcal{N}$-*rationalizes* C, or $R^{\mathcal{N}}$ is an $\mathcal{N}$-*rationalization* of C.

To facilitate our analysis of $\mathcal{N}$-rationalizability, a generalization of the notion of the *direct revealed preference relation* $R_C \subseteq X \times X$ of a choice function C is of use. For all $x, y \in X$,

$$(x, y) \in R_C \;\Leftrightarrow\; \exists S \in \Sigma : x \in C(S) \;\&\; y \in A^{\mathcal{N}}(S).$$

The *(indirect) revealed preference relation* of C is the transitive closure $tc(R_C)$ of the direct revealed preference relation R_C.

We consider three basic versions of norm-conditional rationalizability. The first is $\mathcal{N}$-rationalizability by itself, where an $\mathcal{N}$-rationalization $R^{\mathcal{N}}$ does not have to possess any additional property (such as reflexivity, completeness, consistency or transitivity). This notion of rationalizability is equivalent to $\mathcal{N}$-rationalizability by a reflexive relation (this is also true for the standard definition of rationalizability without social norms; see Richter (1971)). The second is $\mathcal{N}$-rationalizability by a consistent relation (again, reflexivity can be added and an equivalent condition is obtained; see Bossert, Sprumont and Suzumura

(2005)). Finally, we consider $\mathcal{N}$-rationalizability by a transitive relation which, again as in the classical case, turns out to be equivalent to $\mathcal{N}$-rationalizability by an ordering; see Richter (1966; 1971).

We first provide three preliminary results. The following lemma states that the direct revealed preference relation R_C must be respected by any $\mathcal{N}$-rationalization $R^{\mathcal{N}}$. This observation parallels that of Samuelson (1948; 1950) in the traditional framework; see also Richter (1971).

Lemma 1 *Let $\mathcal{N} \in \mathbf{N}$ be a system of social norms and let C be a choice function. If $R^{\mathcal{N}}$ is an $\mathcal{N}$-rationalization of C, then $R_C \subseteq R^{\mathcal{N}}$.*

Proof. Suppose that $R^{\mathcal{N}}$ is an $\mathcal{N}$-rationalization of C and $x, y \in X$ are such that $(x, y) \in R_C$. By definition of R_C, there exists $S \in \Sigma$ such that $x \in C(S)$ and $y \in A^{\mathcal{N}}(S)$. Because $R^{\mathcal{N}}$ is an $\mathcal{N}$-rationalization of C, we obtain $(x, y) \in R^{\mathcal{N}}$. Thus, $R_C \subseteq R^{\mathcal{N}}$ must be true. ■

Analogously, any consistent $\mathcal{N}$-rationalization $R^{\mathcal{N}}$ must respect not only the direct revealed preference relation R_C but also its consistent closure $cc(R_C)$.

Lemma 2 *Let $\mathcal{N} \in \mathbf{N}$ be a system of social norms and let C be a choice function. If $R^{\mathcal{N}}$ is a consistent $\mathcal{N}$-rationalization of C, then $cc(R_C) \subseteq R^{\mathcal{N}}$.*

Proof. Suppose that $R^{\mathcal{N}}$ is a consistent $\mathcal{N}$-rationalization of C and $x, y \in X$ are such that $(x, y) \in cc(R_C)$. By definition of the consistent closure of a binary relation, $(x, y) \in R_C$ or $[(x, y) \in tc(R_C)$ and $(y, x) \in R_C]$ must hold. If $(x, y) \in R_C$, $(x, y) \in R^{\mathcal{N}}$ follows from **Lemma 1**. If $[(x, y) \in tc(R_C)$ and $(y, x) \in R_C]$, there exist $K \in \mathbb{N}$ and $x^0, \ldots, x^K \in X$ such that $x = x^0$, $(x^{k-1}, x^k) \in R_C$ for all $k \in \{1, \ldots, K\}$ and $x^K = y$. By **Lemma 1**, $(x^{k-1}, x^k) \in R^{\mathcal{N}}$ for all $k \in \{1, \ldots, K\}$ and, thus, $(x, y) \in tc(R^{\mathcal{N}})$. Furthermore, $(y, x) \in R_C$ implies $(y, x) \in R^{\mathcal{N}}$ by **Lemma 1** again. If $(x, y) \notin R^{\mathcal{N}}$, it follows that $(y, x) \in P(R^{\mathcal{N}})$ in view of $(y, x) \in R^{\mathcal{N}}$. Because $(x, y) \in tc(R^{\mathcal{N}})$, this contradicts the consistency of $R^{\mathcal{N}}$. Therefore, $(x, y) \in R^{\mathcal{N}}$. Thus, $cc(R_C) \subseteq R^{\mathcal{N}}$ must be true. ■

Finally, if transitivity is required as a property of an $\mathcal{N}$-rationalization $R^{\mathcal{N}}$, this relation must respect the transitive closure $tc(R_C)$ of R_C.

Lemma 3 *Let $\mathcal{N} \in \mathbf{N}$ be a system of socal norms and let C be a choice function. If $R^{\mathcal{N}}$ is a transitive $\mathcal{N}$-rationalization of C, then $tc(R_C) \subseteq R^{\mathcal{N}}$.*

Proof. Suppose that $R^{\mathcal{N}}$ is a transitive $\mathcal{N}$-rationalization of C and $x, y \in X$ are such that $(x, y) \in tc(R_C)$. By definition of the transitive closure of a binary relation R_C, there exist $K \in \mathbb{N}$ and $x^0, \ldots, x^K \in X$ such that $x = x^0$, $(x^{k-1}, x^k) \in R_C$ for all $k \in \{1, \ldots, K\}$ and $x^K = y$. By **Lemma 1**, we obtain $x = x^0$, $(x^{k-1}, x^k) \in R^{\mathcal{N}}$ for all $k \in \{1, \ldots, K\}$ and $x^K = y$. Repeated application of the transitivity of $R^{\mathcal{N}}$ implies $(x, y) \in R^{\mathcal{N}}$. Thus $tc(R_C) \subseteq R^{\mathcal{N}}$ must hold. ■

We are now ready to identify a necessary and sufficient condition for each one of these notions of $\mathcal{N}$-rationalizability of a choice function. To obtain a necessary and sufficient condition for simple $\mathcal{N}$-rationalizability (that is, $\mathcal{N}$-rationalizability by a binary relation $R^{\mathcal{N}}$ that does not have to possess any further property), we follow Richter (1971) by generalizing the relevant axiom in his approach in order to accommodate an externally imposed system of norms $\mathcal{N}$. This leads us to the following axiom.

$\mathcal{N}$-conditional direct-revelation coherence: For all $S \in \Sigma$ and for all $x \in A^{\mathcal{N}}(S)$,

$$[\forall y \in A^{\mathcal{N}}(S) : (x, y) \in R_C] \;\Rightarrow\; x \in C(S).$$

Our first result establishes that this property is indeed necessary and sufficient for $\mathcal{N}$-rationalizability.

Theorem 1 *Let $\mathcal{N} \in \mathbf{N}$ be a system of social norms and let C be a choice function. C is $\mathcal{N}$-rationalizable if and only if C satisfies $\mathcal{N}$-conditional direct-revelation coherence.*

Proof. "Only if." Suppose $R^{\mathcal{N}}$ is an $\mathcal{N}$-rationalization of C. Let $S \in \Sigma$ and $x \in A^{\mathcal{N}}(S)$ be such that $(x, y) \in R_C$ for all $y \in A^{\mathcal{N}}(S)$. By **Lemma 1**, $(x, y) \in R^{\mathcal{N}}$ for all $y \in A^{\mathcal{N}}(S)$, which implies $x \in C(S)$ because $R^{\mathcal{N}}$ is an $\mathcal{N}$-rationalization of C.

"If." Suppose C satisfies $\mathcal{N}$-conditional direct-revelation coherence. We complete the proof by showing that $R^{\mathcal{N}} = R_C$ is an $\mathcal{N}$-rationalization of C. Let $S \in \Sigma$ and $x \in A^{\mathcal{N}}(S)$.

Suppose first that $x \in C(S)$. By definition, it follows immediately that $(x, y) \in R_C = R^{\mathcal{N}}$ for all $y \in A^{\mathcal{N}}(S)$.

Conversely, suppose that $(x, y) \in R_C = R^{\mathcal{N}}$ for all $y \in A^{\mathcal{N}}(S)$. It follows that $\mathcal{N}$-conditional direct-revelation coherence immediately implies $x \in C(S)$. Thus, C is $\mathcal{N}$-rationalizable by $R^{\mathcal{N}} = R_C$. ■

As is the case for the traditional model of rational choice on general domains, it is straightforward to verify that $\mathcal{N}$-rationalizability by a reflexive relation is equivalent to

$\mathcal{N}$-rationalizability without any further properties of an $\mathcal{N}$-rationalization; this can be verified analogously to Richter (1971). However, adding completeness as a requirement leads to a stronger notion of $\mathcal{N}$-rationalizability; see again Richer (1971).

Next, we examine $\mathcal{N}$-rationalizability by a consistent relation, which is equivalent to $\mathcal{N}$-rationalizability by a reflexive and consistent relation. As in the traditional case, adding completeness, however, leads to a stronger property, namely, one that is equivalent to $\mathcal{N}$-rationalizability by an ordering; see Bossert, Sprumont and Suzumura (2005) for an analogous observation in the traditional model.

The requisite necessary and sufficient condition is obtained from $\mathcal{N}$-conditional direct-revelation coherence by replacing R_C with its consistent closure $cc(R_C)$.

$\mathcal{N}$-conditional consistent-closure coherence: For all $S \in \Sigma$ and for all $x \in A^{\mathcal{N}}(S)$,

$$[\forall y \in A^{\mathcal{N}}(S) : (x, y) \in cc(R_C)] \;\Rightarrow\; x \in C(S).$$

We obtain

Theorem 2 *Let $\mathcal{N} \in \mathbf{N}$ be a system of social norms and let C be a choice function. C is $\mathcal{N}$-rationalizable by a consistent relation if and only if C satisfies $\mathcal{N}$-conditional consistent-closure coherence.*

Proof. The proof is analogous to that of **Theorem 1**. All that needs to be done is replace R_C with $cc(R_C)$ and invoke **Lemma 2** instead of **Lemma 1**. ■

Our final result establishes a necessary and sufficient condition for $\mathcal{N}$-rationalizability by a transitive relation which is equivalent to $\mathcal{N}$-rationalizability by an ordering. We leave it to the reader to verify that the proof strategy employed by Richter (1966; 1971) in the traditional case generalizes in a straightforward manner to the norm-dependent model when establishing that transitive $\mathcal{N}$-rationalizability is equivalent to $\mathcal{N}$-rationalizability by an ordering.

The requisite necessary and sufficient condition is obtained from $\mathcal{N}$-conditional direct-revelation coherence by replacing R_C with its transitive closure $tc(R_C)$.

$\mathcal{N}$-conditional transitive-closure coherence: For all $S \in \Sigma$ and for all $x \in A^{\mathcal{N}}(S)$,

$$[\forall y \in A^{\mathcal{N}}(S) : (x, y) \in tc(R_C)] \;\Rightarrow\; x \in C(S).$$

We obtain

Theorem 3 *Let $\mathcal{N} \in \mathbf{N}$ be a system of social norms and let C be a choice function. C is $\mathcal{N}$-rationalizable by a transitive relation if and only if C satisfies $\mathcal{N}$-conditional transitive-closure coherence.*

Proof. Again, the proof is analogous to that of **Theorem 1**. All that needs to be done is replace R_C with $tc(R_C)$ and invoke **Lemma 3** instead of **Lemma 1**. ■

5 Conclusion

Instead of summarizing the main contents of this short paper, let us conclude with two remarks on the literature with some relevance to the present paper.

(1) Shortly after the publication of Sen's criticism against internal consistency of choice, Baigent and Gaertner (1996) presented an axiomatic characterization of what can be called the *never-choose-the-uniquely-largest* choice function. This choice function was motivated by an alternative interpretation of Sen's example cited in the Introduction, which is due to Sen himself. Although the characterized choice function is not without interest, the characterizing axioms are too complex to be easily intuitively interpretable. Besides, there is no discussion in Baigent and Gaertner on the compatibility between external social norms and the general theory of rationalizability.

(2) It was Sen (1997) who made an important step towards the norm-conditional theory of rationalizability through the concept of *self-imposed choice constraints,* excluding the choice of some alternatives from permissible conducts. According to Sen's (1997, p.769) scenario, "the person may first restrict the choice options ... by taking a 'permissible' subset $K(S)$, reflecting *self-imposed* constraints, and then seek the maximal elements $M(K(S), R)$ in $K(S)$."[2] Despite an apparent family resemblance between Sen's concept of self-imposed choice constraints and our concept of norm-conditionality, Sen did not go as far as to bridge the idea of norm-induced constraints and the theory of rationalizability as we did in this paper.

It is hoped that the present paper provides the missing link in the existing literature and shows that external social norms can be internalized by means of a suitably modified revealed preference theory.

[2]For any $S \subseteq X$ and $R \subseteq X \times X$, $M(S, R)$ is the set of R-maximal points in S, that is to say, $M(S, R) = \{x^* \in S \mid \forall x \in S\colon (x, x^*) \notin P(R)\}$.

References

Aizerman, M. and F. Aleskerov (1995), *Theory of Choice*, North-Holland, Amsterdam.

Arrow, K.J. (1951; second ed. 1963), *Social Choice and Individual Values*, John Wiley & Sons, New York.

Arrow, K.J. (1959), "Rational choice functions and orderings," *Economica* **26**, 121–127.

Baigent, N. and W. Gaertner (1996), "Never choose the uniquely largest: A characterization," *Economic Theory* **8**, 239–249.

Bossert, W., Y. Sprumont and K. Suzumura (2005), "Consistent rationalizability," *Economica* **72**, 185–200.

Bossert, W., Y. Sprumont and K. Suzumura (2006), "Rationalizability of choice functions on general domains without full transitivity," *Social Choice and Welfare* **27**, 435–458.

Hansson, B. (1968), "Choice structures and preference relations," *Synthese* **18**, 443–458.

Houthakker, H. (1950), "Revealed preference and the utility function," *Economica* **17**, 159–174.

Kim, T. and M.K. Richter (1986), "Nontransitive-nontotal consumer theory," *Journal of Economic Theory* **38**, 324–363.

Richter, M.K. (1966), "Revealed preference theory," *Econometrica* **34**, 635–645.

Richter, M.K. (1971), "Rational choice," in J. Chipman, L. Hurwicz, M. Richter, and H. Sonnenschein (eds.), *Preferences, Utility, and Demand*, Harcourt Brace Jovanovich, New York, pp. 29–58.

Samuelson, P.A. (1938), "A note on the pure theory of consumer's behaviour," *Economica* **5**, 61–71.

Samuelson, P.A. (1947), *Foundations of Economic Analysis*, Harvard University Press, Cambridge, Mass.

SAMUELSON, P.A. (1948), "Consumption theory in terms of revealed preference," *Economica* **15**, 243–253.

SAMUELSON, P.A. (1950), "The problem of integrability in utility theory," *Economica* **17**, 355–385.

SCHWARTZ, T. (1976), "Choice functions, 'rationality' conditions, and variations of the weak axiom of revealed preference," *Journal of Economic Theory* **13**, 414–427.

SEN, A.K. (1971), "Choice functions and revealed preference," *Review of Economic Studies* **38**, 307–317.

SEN, A.K. (1993), "Internal consistency of choice," *Econometrica* **61**, 495–521.

SEN, A.K. (1997), "Maximization and the act of choice," *Econometrica* **65**, 745–779.

SUZUMURA, K. (1976a), "Rational choice and revealed preference," *Review of Economic Studies* **43**, 149–158.

SUZUMURA, K. (1976b), "Remarks on the theory of collective choice," *Economica* **43**, 381–390.

SUZUMURA, K. (1977), "Houthakker's axiom in the theory of rational choice," *Journal of Economic Theory* **14**, 284–290.

SUZUMURA, K. (1983), *Rational Choice, Collective Decisions and Social Welfare,* Cambridge University Press, New York.

SZPILRAJN, E. (1930), "Sur l'extension de l'ordre partiel," *Fundamenta Mathematicae* **16**, 386–389.